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Stay Safe from
COVID - 19
and Learn at home
Live interaction on
Presentation of Data Using Spreadsheets
3:30pm - 4:30pm
12 April 2020
Speaker
Mr. Chanchal Dass
Former Engineer & World Oil Awardee (as a Thinker)
Register in the link
https://bit.ly/2xaMzry
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DASS SRL
DASS Scientific Research Labs Pvt. Ltd.

विद्यया ऽमृतमश्नुते

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

एन सी ई आर टी
NCERT

विज्ञान एवं गणित शिक्षा विभाग

Department of Education in Science and Mathematics

केन्द्रीय शैक्षिक प्रौद्योगिकी संस्थान

Central Institute of Educational Technology

Digital India
Power To Empower

Launch of

3D Simplified Mathematics

In Indian Schools, Colleges and Universities

12th April 2020

DASS SRL
DASS Scientific Research Labs Pvt. Ltd.

ACKNOWLEDGEMENTS

Ramesh Pokhriyal 'Nishank'
HON'BLE MINISTER OF HRD

First Question:

What is 3D Simplified Mathematics?

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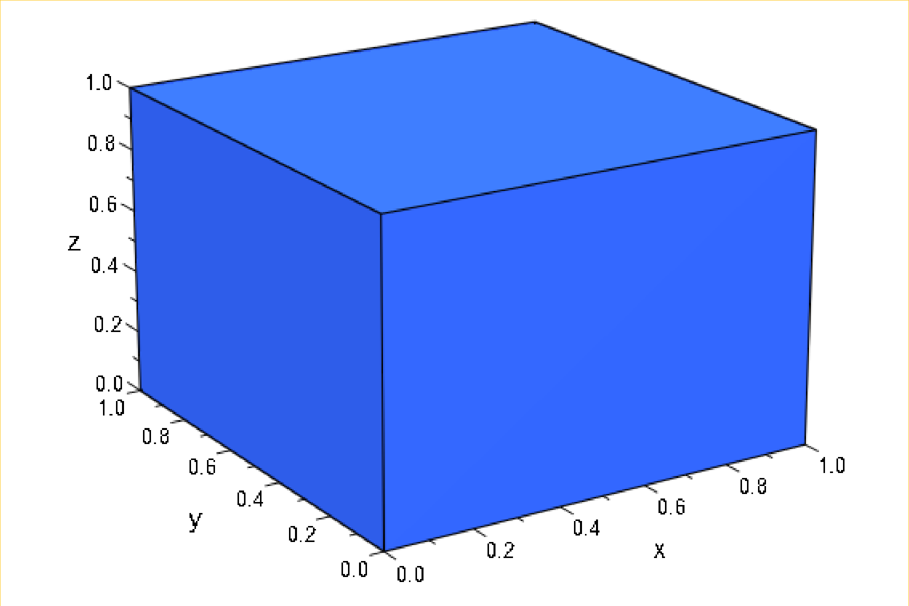

We live in 3D world but we don't have easy tool to capture and animate 3D world Dynamism.

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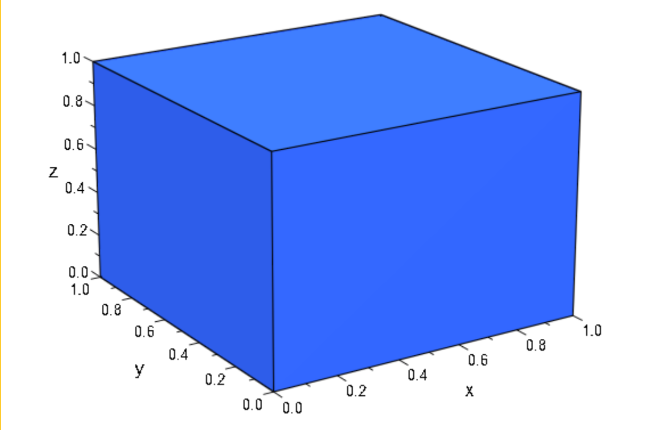

***Objective*:**

To Establish
"Vishwa Ganit Shala" *to*
Promote Simplified
Mathematical Concepts
Through
"Ganit Charcha"
Campaign

Looking The Problem from National Perspective

- Providing quality education and increasing literacy rate remained a challenge for long throughout the globe.
- It is more challenging for India due to its vastness, poverty and diversity.
- Due to the advancement of technology, the situation is aggravating day by day.
- With the advancement of technologies, the knowledge base is increasing at a much faster rate than usual

•

विद्यया ऽ मृतमश्नुते

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- The real meaning of quality education and the meaning of improving the literacy rate is getting wider and wider.
- In addition to improving literacy rate, now new challenge has added up for improving numerical and digital literacy rate.
- For solving this multifaceted problem, we require innovative solutions where objective should be to provide quality education and improve mass numerical and digital literacy rate within a short span of time.
- This could be achieved through the “3D Simplified Mathematics” initiated by Dass Scientific Research Labs Pvt Ltd.

::::Summary::::

Providing High Quality Education to Masses in a Short Span of Time

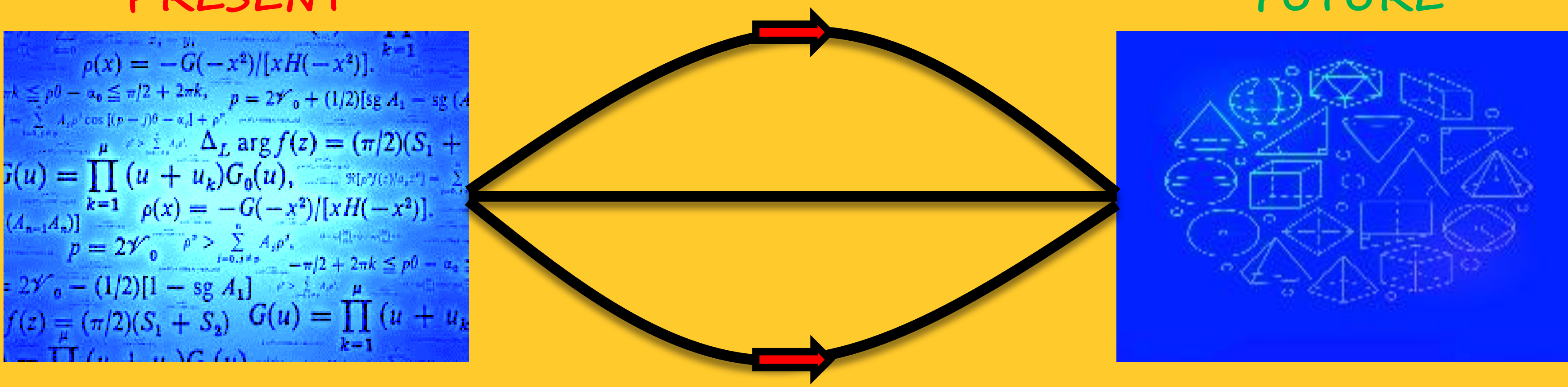
Our Goal

DASS SRL
DASS Scientific Research Labs Pvt. Ltd.

3D Simplified Mathematics

PRESENT

FUTURE



Maths means Symbols, Equations, Formulas

Changing Perception about mathematics

Maths means Points, Lines, Surfaces, Solids

Pain Point

4

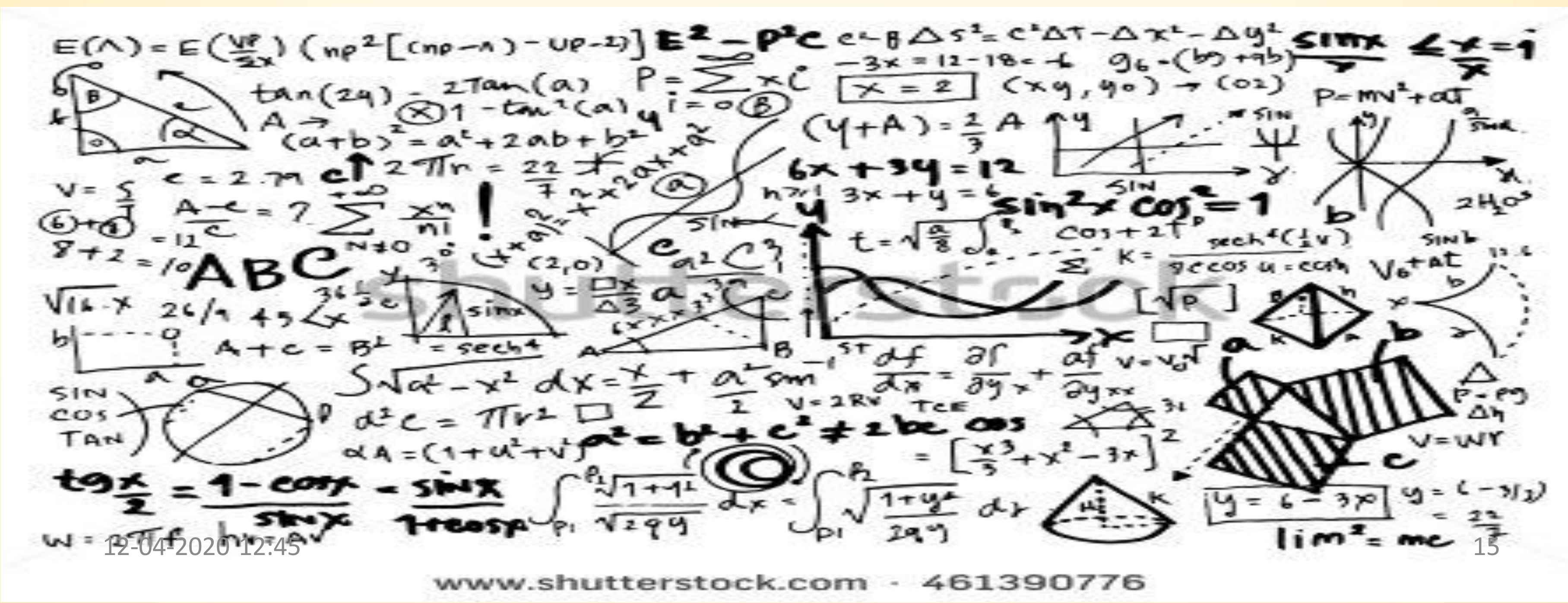
The Pain:

- Difficulty in learning mathematics is a global problem. Millions of student face this problem every year.

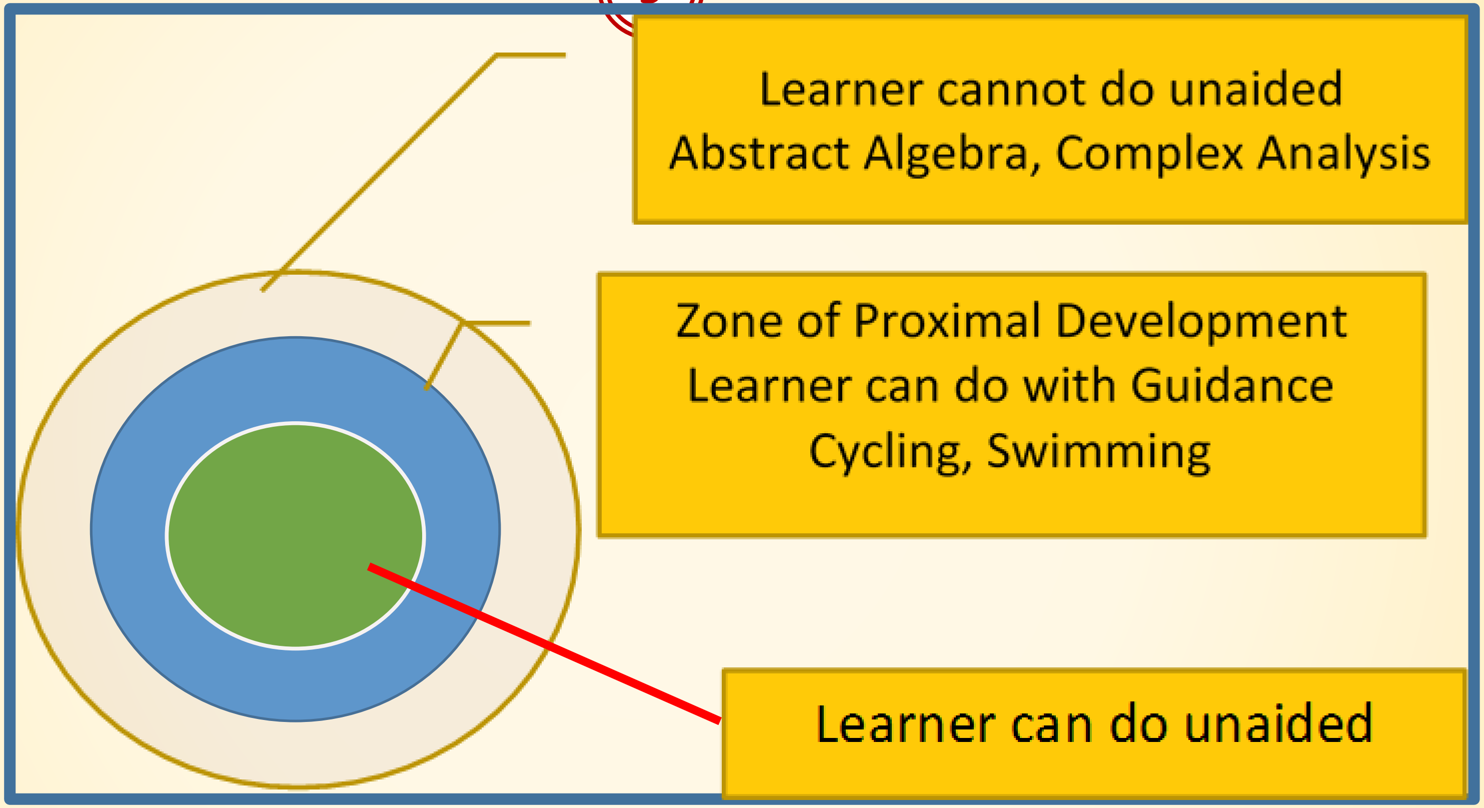
The Solution: Geometry Based Mathematics Teaching

- We have developed an easy way of converting any mathematical concepts into geometrical shapes without any programming knowledge.
- This makes mathematics learning easy, enjoyable and the learning time reduces dramatically.

Simplified Mathematical Concepts



Product Delivery Mode-Zone of Proximal Development



Our Product Delivery Mode is Based on Philosophy of ZPD

Delivery Mode

We have two type of delivery model for our product.

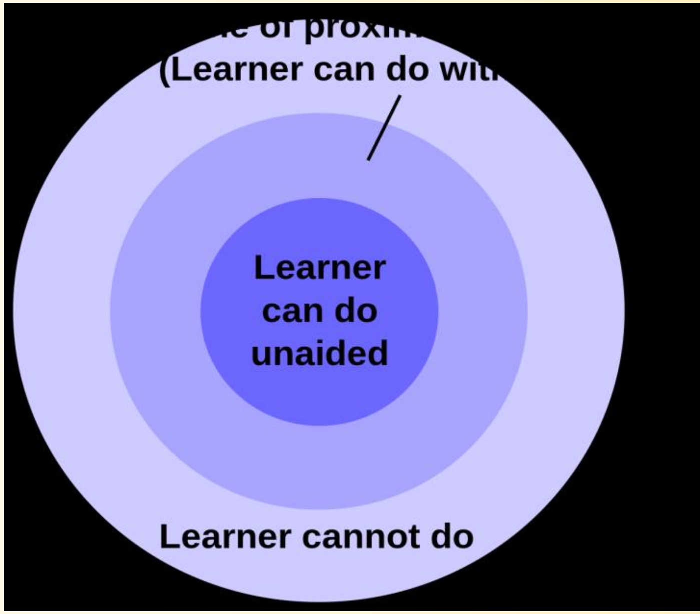
1. Physical Mode: Short duration workshops

One of the delivery mode is through conducting short duration mathematics workshops where main focus is to help participants to use technology for solving their mathematical problems. In this mode, we provide few innovative mathematical tool through which participants can convert any mathematical concepts into geometrical shapes without any programming knowledge. This helps in clear understanding of mathematical concepts.

Delivery Mode-ZPD

2. ONLINE MENTORING MODE:

- This is a membership based service. The registered members get online support in clearing their doubts about any mathematical concepts.



For registration:

http://bit.ly/MathWrokshopRegistration

What We Offer

- In three days time, we discuss few basic concepts in Mathematics, and
- Provide few basic tools to solve mathematical problems. These techniques are used to convert all mathematical concepts into geometrical shapes.

- With the help of these knowledge, one can address all the mathematical problems in their professional career.
- We also provide Online Support through our “Online Mathematics Mentoring Program”. For this service one has to be member of this program.
- www.dasssrl.com

Why new math teaching technique is required

- I reproduce few lines from the preface of the book "Linear Algebra and its application", by Prof. Gilbert Strang:

  I personally believe that many more people need linear algebra than calculus. Issac Newton might not agree! But he is not teaching mathematics in 21st Century (and may be he was not a great teacher, but we will give him benefit of doubt).

Why new math teaching technique required

Certainly the laws of physics are well expressed by differential equations. Newton needed calculus – quite right.

But the scope of science and engineering and management (and life) is now so much wider, and linear algebra has moved into a central place.

My Observation: World has not only become wider but changing at a faster rate.

Linear Algebra vs Calculus

- We face problems in learning calculus of functions of one or two variables. Think of a situation, if we have 10 or more variables.

- First step is to linearize, convert curves into tangent lines, convert surfaces into planes.

Why new math teaching technique required

- According to Prof John Vince, author of the book “Geometric Algebra for Computer Graphics”, wrote in the preface of the book”

In December 2006, I posted my manuscript on “Vector analysis for computer graphics”, to Springer and looked forward to a short rest before embarking upon another book. But whilst surfing the internet, and probably before my manuscript had reached its destination, I discovered a strange topic called “Geometric Algebra (GA)”. Advocates of Geometric Algebra (GA) were claiming that a revolution was coming and that the cross product was dead. I couldn’t believe my eyes.

I had just written a book about vectors extolling the power and benefits of the cross product and now moves were afoot to have it banished.

Why new math teaching technique required

- **Majority of us still not aware about FRACTALS**

- **As per John Archibald Wheeler:**

  NOBODY WILL BE CONSIDERED SCIENTIFICALLY LITERATE TOMORROW WHO IS NOT FAMILIAR WITH FRACTALS.

From Scoonews

- The issue has been raised by **Dr Sanjay Parva** of Scoonews in his article, titled, "Being a tech savvy teacher", in the Straight Talk Column as mentioned below:

  ++++++++++++++++++++++++++++++++++++

  "It is not easy to become a teacher. It is difficult. You need a great deal of patience to become one. And if you are tech savvy teacher, you need twice as much. Technology is making rapid inroads into everything that we do now and that we will do in near future. In order to be a teacher of tomorrow, teachers today need to gain an edge in using technology. They need to be tech savvy.

From Scoonews

- Though there is no rocket science involved, however, the first step towards being a tech savvy teacher is to accept that the technology is going to assume greater proportion in the way students would need to be taught in future. It is a question of attaining certain basic skills. The most basic skill needed is to accept that the time has come for the teachers to learn certain new means of imparting education, which have come up because the classroom dynamics has changed.

From Scoonews

- Children are widely exposed to technology outside the classroom. In the classroom, sooner or later, they will begin to find themselves out of place if they do not find the same flux of technology there as they find outside. A typical student's attention span, reveals psychologists last a maximum of 15 minutes and the same depends on several factors like emotion, motivation, time of the day and enjoyment.

From Scoonews

- Most of the students have to spent at least 40 minutes or more in a classroom, looking at the same blackboard, scribbling at the same notebook, ogling at the same book and blankly looking at the same teacher. This is very monotonous something a teacher would never know because a student would never dare to challenge the system or the teacher. This society is brought up like that. Even if a teacher is wrong, he or she is right! Use of technology, that is to say tech enabled tools and methods to teach, has been found to be an effective method to break the monotony, keep the students alert and interested and encourage participation. Encouraged participation leads to expanded thought process and vision".

Use of Technology

- In many situations, we required to solve polynomial equations. There are formulas for solving linear and quadratic equations. But what about the equations of power 3 or more?

- In his book on MATLAB, Rudra Pratap writes, a cubic equation may take pages and trying to solve a polynomial having power more than four by hand, one has to be insane. Trying to solve a 4x4 matrix by hand, you are either borderline insane or you live in a civilization without computers.

Use of Technology

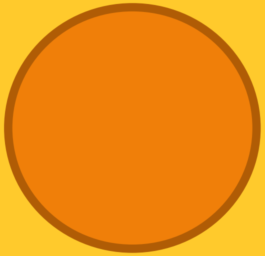
- Tally
- Abacus
- Calculator
- Slide rule
- Log Tables
- MS Math
- MS Excel
- GeoGebra
- MSW LOGO
- MS Word
- Matlab
- XaoS
- Scilab
- Mathematica
- Mapple

Theme: Ganit Charcha

- The idea behind the "Ganit Charcha" is to inculcate a culture of general discussion of mathematics in common parlance.
- India has enormous contribution towards the advancements of Science, Technology, Engineering, Medicine and Mathematics.
- The innovative thought of promoting mathematics through mass communication with "Ganit Charcha/ Math Boliye" Campaign will add another dimension to the advancement of modern civilization.

Examples: Ganit Charcha: topics may be -

1. How will you find the area of the given circle?



2. Why we study Limits?
3. What is eigen vector?
4. What is the use of series in calculus?
5. How to derive the formula cos(A+B).

Proposal for Consideration

- 1. Recognize DASS SRL as Resource Change Agent for Teaching Mathematics and Science
- 2. Associate Schools, Colleges and Universities informally with DASS SRL for supplemental support to the formal mathematics teaching methodologies
- 3. Associate students to join Online Mentoring Program on membership basis
- 4. To conduct 3-day workshop of "3D Simplified Mathematics" for students of secondary standard and above, faculties and interested professions.
- 5. Enter into an MOU to facilitate above steps.

- Communication Matters

Join Ganit Charcha Campaign

Chanchal Dass

Whatsapp-8320172787

Philosophy: Seeing is Believing

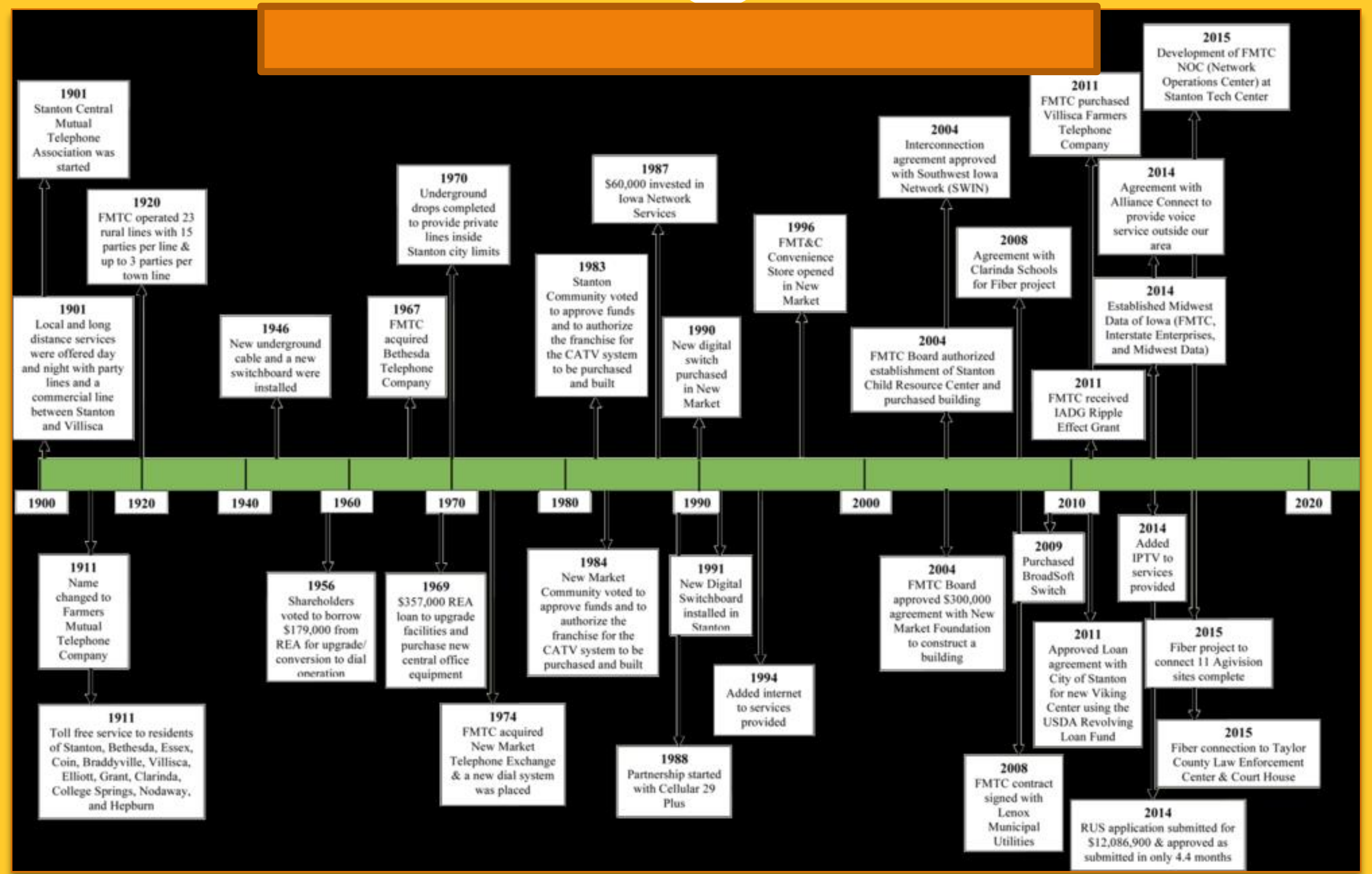
Inclined Plane

Floating Rotator

Philosophy: A picture worth thousands words

I hear - I forget, I see – I remember, I do – I understand

Looking Mathematics From Historical Perspective



Conventional Mathematics Teaching Techniques

There are many mathematics teaching technique available then why a new technique is required?

Conventional Mathematics Teaching Techniques

Inductive Method	Heuristic Method
Deductive Method	Problem Solving Method
Analytic Method	Laboratory Method
Synthetic Method	Active Learning

Introduction of new Mathematics Teaching Techniques

3D Simplified Geometry Based Mathematics Teaching Technique

CHANCHAL DASS, FIE

CHAIRMAN, DASS SCIENTIFIC RESEARCH LABS (P) LTD

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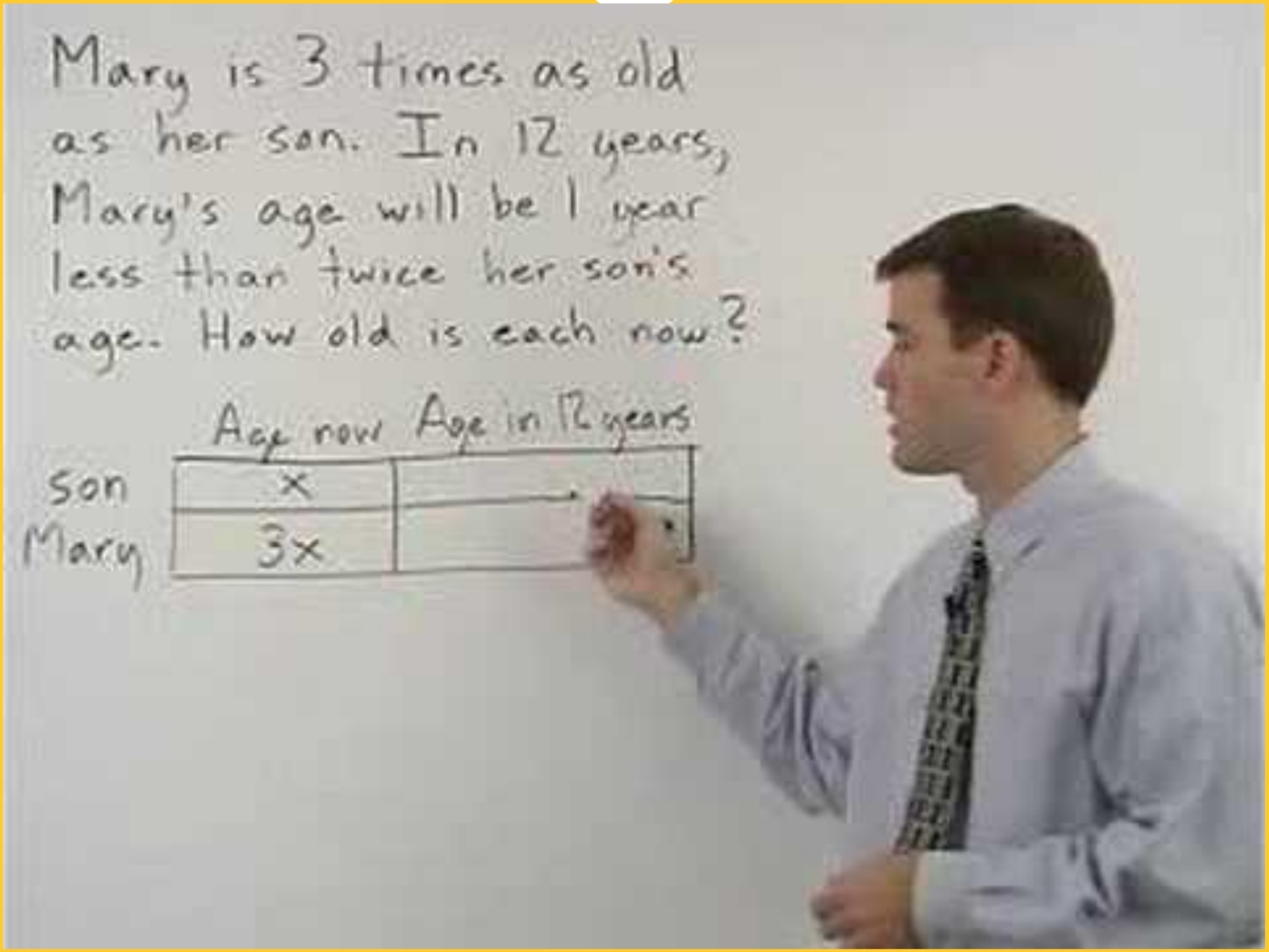
Mathematics from Generation Perspective



Mathematics-Intuition and Technology

Unknown Territory

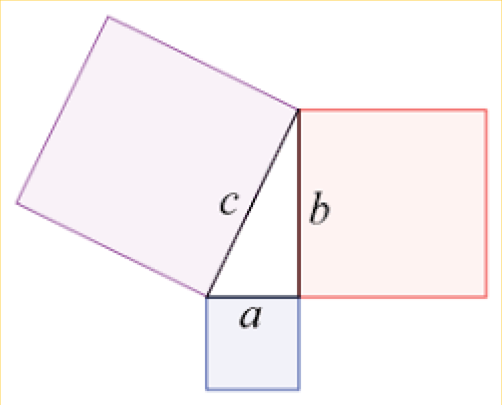
Where to Start

- What is the domain of mathematicians?

Rationale

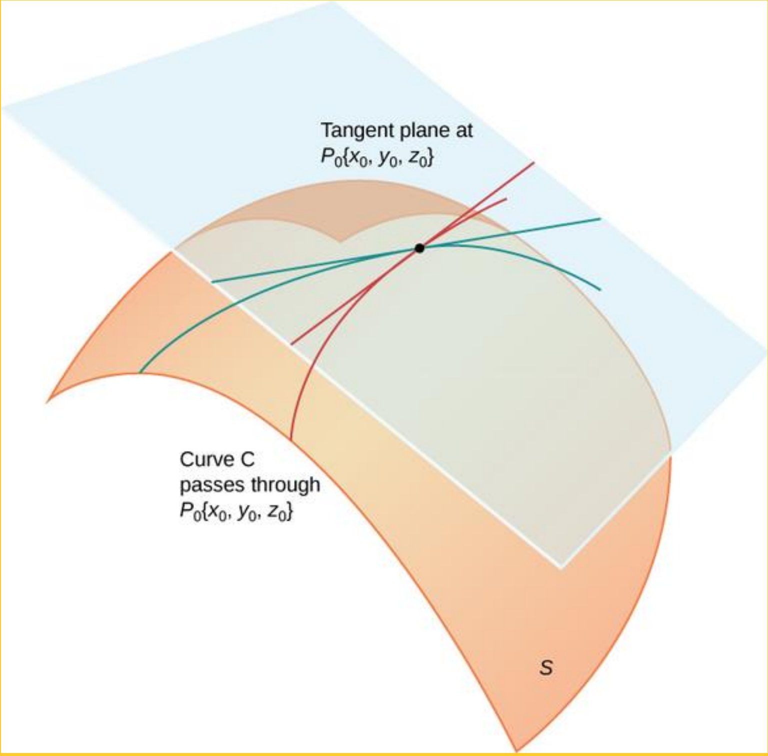
Many of the mathematics concepts are based on geometry.
Few examples:

- Irrational Numbers, Pythagoras theorem



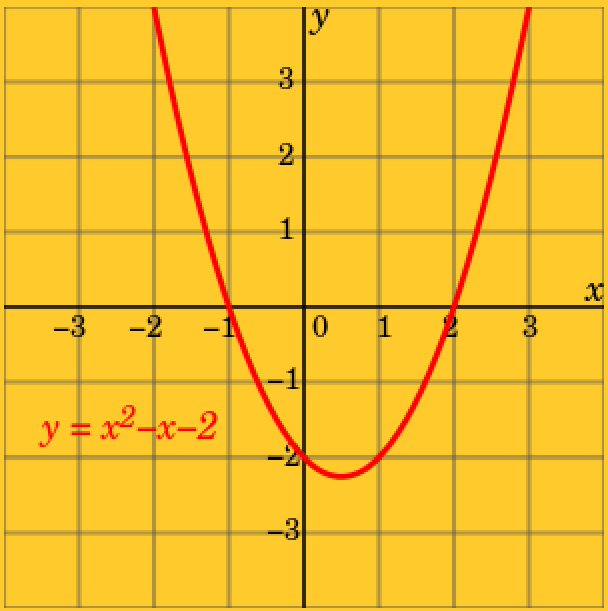
Rationale

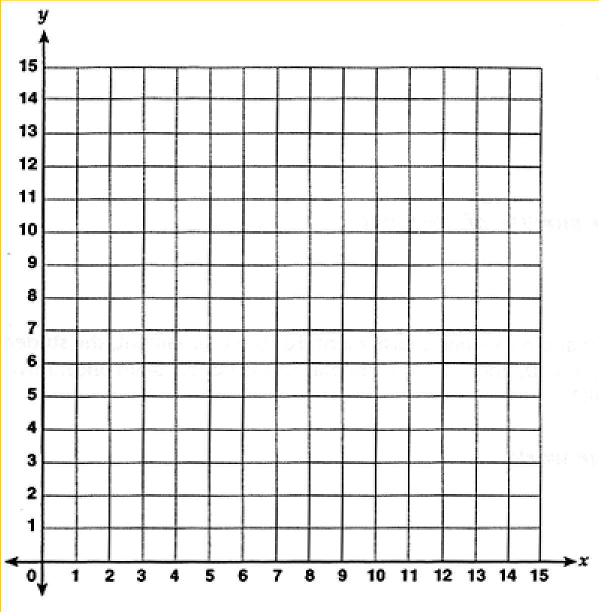
- Calculus, Limit, Tangent Line, Tangent Plane



Rationale

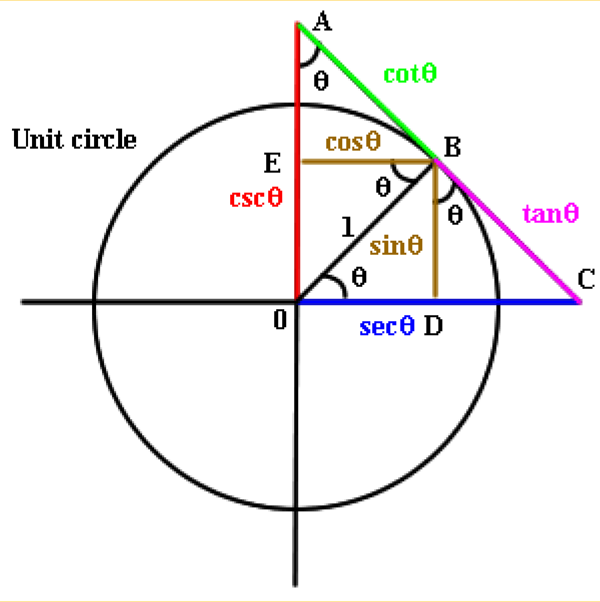
- Imaginary numbers, Complex analysis, Quadratic Equations

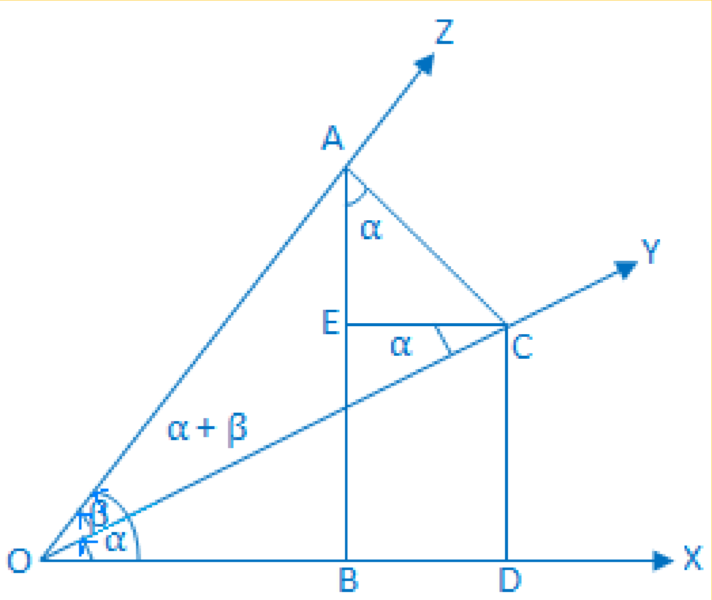




Rationale

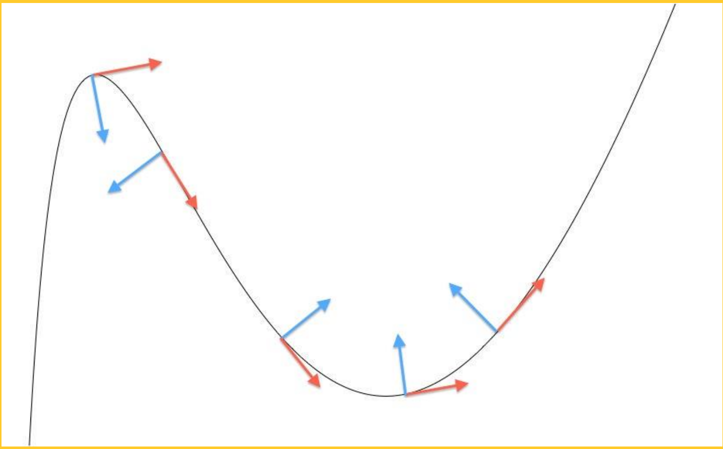
- Trigonometric Identities

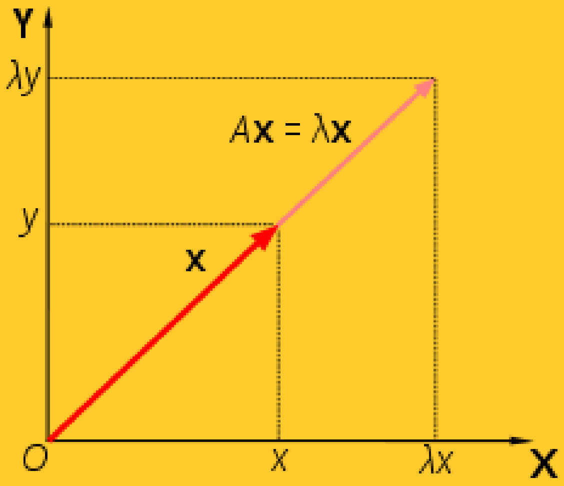




Rationale

- Matrices, Vectors, Eigen Value, Eigen Vectors





Rationale

- Linear Algebra, Matrices, Determinants

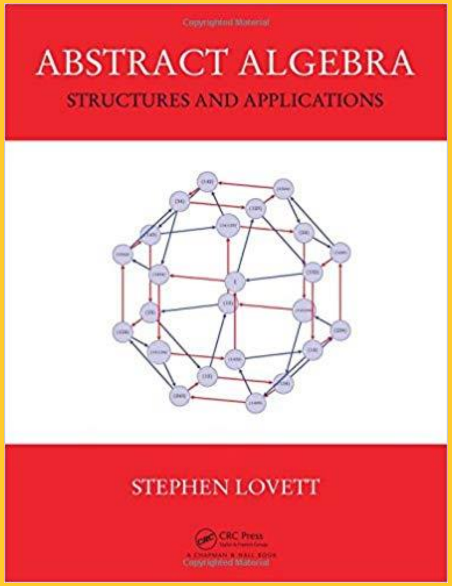
Use Cramer's Rule to write the value of x as a ratio of 2x2 determinants.

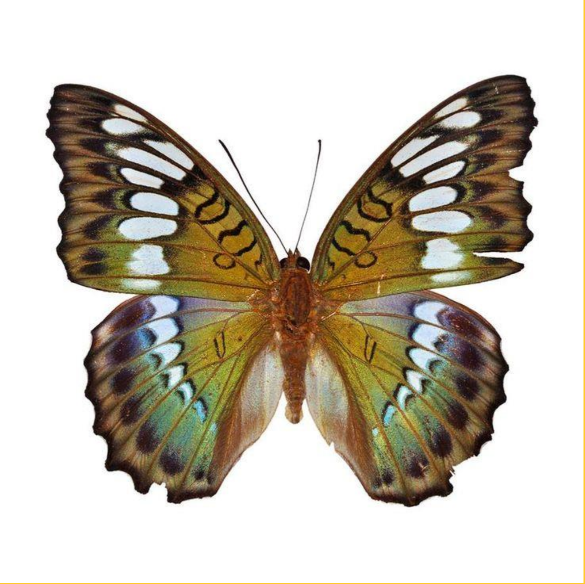
$$x + 3y = -1$$
$$5x + 4y = -7$$

$$x = \frac{\begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix}} \rightarrow x = \frac{\begin{vmatrix} -1 & 3 \\ -7 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix}}$$

Rationale

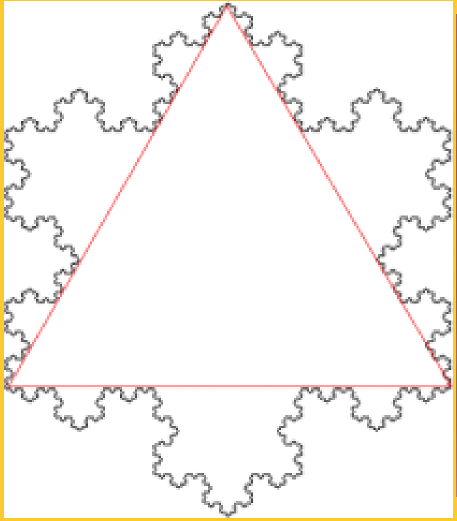
- Abstract Algebra, Group Theory, Symmetry





Rationale

- Nature, Fractal Geometry, Dimension, Lack of differentiability



The Journey

Making Mathematics Popular

Main Objective

Make Advanced Mathematical Concepts Simple and Promote These Simplified Mathematical Concepts Globally

Other Objective

Take participants to the

"Path of Self Discovery and Self Learning"

Self Discovery

- Why wheels are circular?

- Can it be Triangular?
- Can it be Square?
- Can it be Pentagonal?

Self Discovery

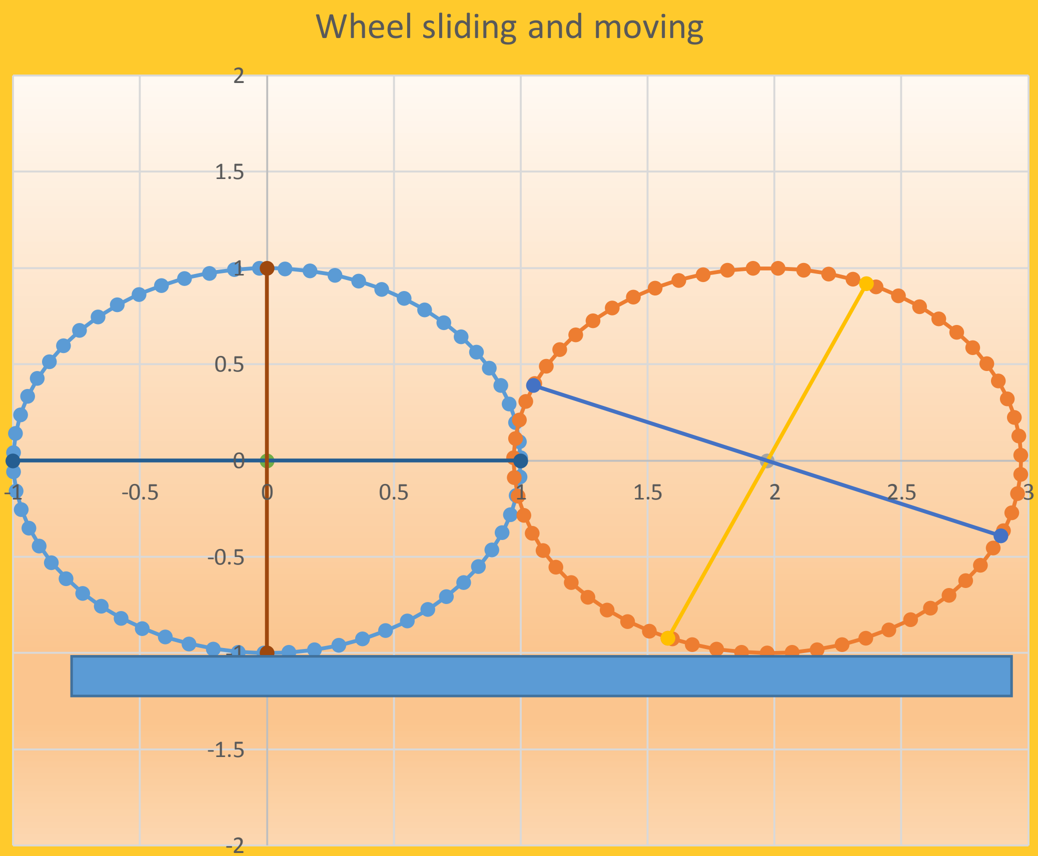
- Why wheels are circular?

- Can it be Triangular?
- Can it be Square?
- Can it be Pentagonal?

- The answer is yes:

Self Discovery

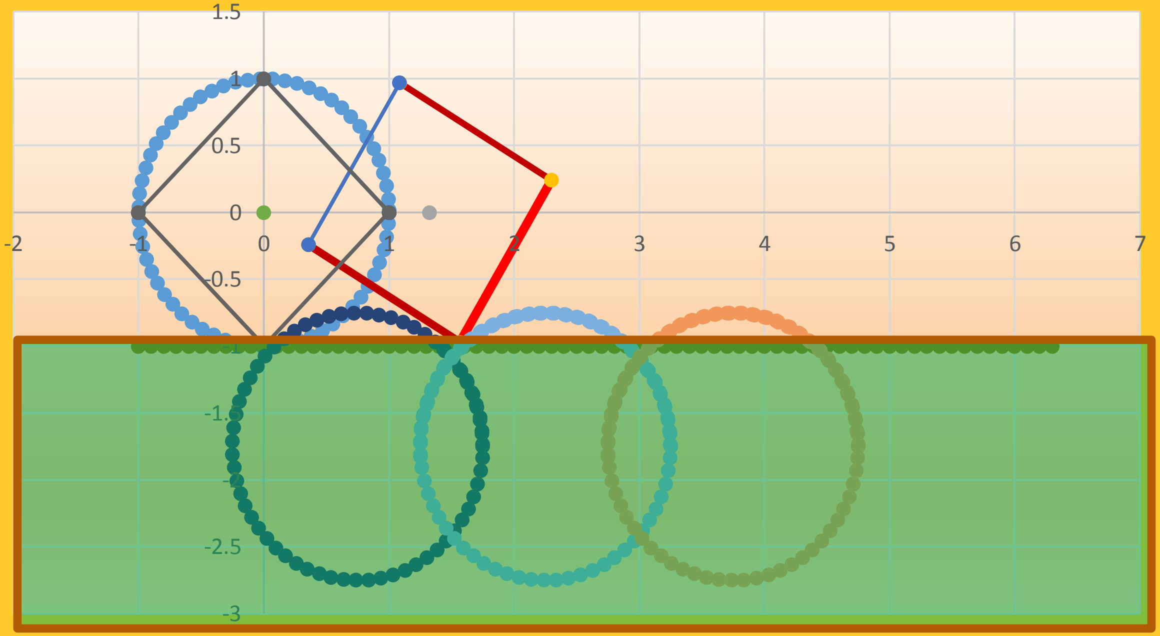
- Why wheels are circular?
- Can it be Square?
- The answer is yes:



Self Discovery

- Why wheels are circular?
- Can it be Square?
- The answer is yes:

Wheel sliding and moving



Dass Scientific Research Labs Private Limited

7th June 2012

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DIPP Recognition No-977

Crafting Preferred Engineers

National Institute of Oceanography

समुचित ज्ञान समन्वय

National Design Business Incubation(NDBI)

NATIONAL DESIGN
BUSINESS INCUBATOR

NATIONAL INSTITUTE OF DESIGN

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। योगः कर्मसु कौशलम् ।

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Indian Institute of Technology

IIT Gandhinagar

Indian Institute of Technology Gandhinagar


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Hoga Bhala"
Established vide Gujarat State Act-21 of 2007
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ঢাকা বিশ্ববিদ্যালয়

5th and 6th June 2018

DASS SRL

DASS Scientific Research Labs Pvt. Ltd.

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adani | Institute of Infrastructure

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SOUTH ASIAN
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Feedback From Previous Participants

Ujjwal Rane,
Alumni IITM 1988

Learned a lot of new & very surprising facts & skill. It will also help me in teaching.

Gopal Menon
Alumni IITB 1975

Excellent Presentation. I have learnt new ideas which can be taught to my students

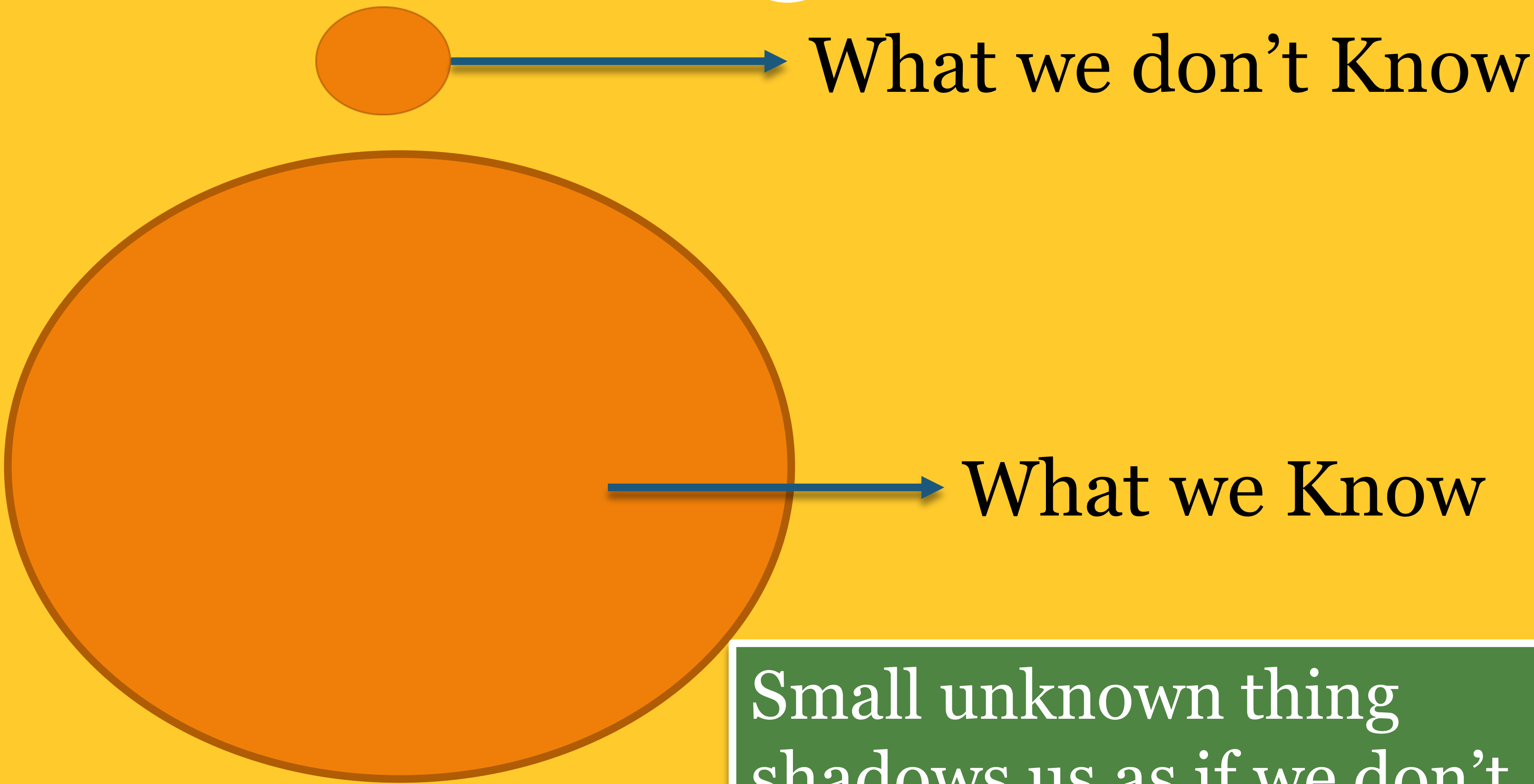
Our Research Findings

"Humans are born Math Literate"

"They don't know that they know Math"

We don't teach math, we make awareness about math

Known VS Unknown



Small unknown thing shadows us as if we don’t know mathematics

IPR

Application Details	
APPLICATION NUMBER	1969/MUM/2013
APPLICATION TYPE	ORDINARY APPLICATION
DATE OF FILING	08/06/2013
APPLICANT NAME	CHANCHAL DASS
TITLE OF INVENTION	SYSTEM AND METHOD FOR MATHEMATICS TEACHING AND DEMONSTRATION
REQUEST FOR EXAMINATION DATE	31/05/2014
PUBLICATION DATE (U/S 11A)	29/08/2014
FIRST EXAMINATION REPORT DATE	28/05/2018

Advisors

Dr Asim Banerjee

Dr Ranadhir Mukhopadhaya

Officially Started on 7th June 2014 at Gujarat Technological University

Inaugurated By VC-GTU

About The Speaker- Chanchal Dass, FIE

Inventor	• MZWC Technology, Contract Bridge Gaming App, Math Teaching and Demonstration Technology
Qualifications	• AMIE (Mechanical Engineering), PGD 'Operations Research', MBA (Finance)
Experience	• 8 Yrs in HFCL, 21 Yrs in ONGC, 7 Yrs in Dass OTPL/ Dass SRL
Skill	• Reservoir Engineering, well testing, Reservoir Simulation, EOR, Sick Well Analysis, Work-over job planning, Chemical Flooding (FPR- Sanand, Jhalora)
Awards & Recognition	• World Oil Award, SPE President Award, SPE Regional Service Award, ONGC Director / Regional Director's Award, IIGP-2011, NASSCOM 10000 Startup Initiative, PALF 2015, CII Innovation Award 2015 and many more
Membership	• Society of Petroleum Engineers, Society of Petroleum Geophysicist, Fellow of Institution of Engineers, Indian Mathematical Society, American Mathematics Society, ACBL
Publications	• SPE IOGCE, SPEIOGCEC, Forums, ATW, NATC, IPTC as presenter, Session Chair, Committee Member and many more
Countries Visited	• US(2), France(2), Holland, Belgium, Germany, Luxembourg, Switzerland, Cairo, Qatar, Dubai(5), Sri Lanka, Abu Dhabi, Sharjah, China(3), Malaysia(4), Thailand(2), Singapore, Georgia, Unitel Kindom, Azarbijan, Bangladesh, Nepal

Brain Storming: Math is not Hard

- Math is not hard if we know how to Handle it.
- Everyone in this world do math in some form or other.

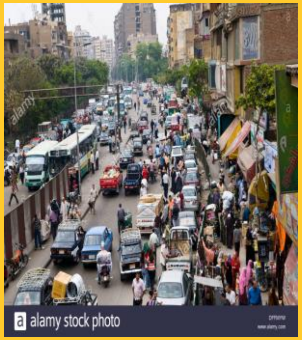
Survey Report ()

- Total Participant –
- Math is hard –
- Math is not hard -

All are Mathematicians !!!

- **Housewives- Great Mathematicians**

•Maintaining Household Expenditure- Financial Management



•Driving Vehicle
Dynamics, Accident, Time, Sp

•Transaction in market place

Math is everywhere
Animal Kingdom

- Tiger >> Dear

- Kingfisher >> Fish

- Honey Bee >> Honeycomb

Math is everywhere
Bird Kingdom

- Nest

- Crow and Cuckoos

- Migratory Birds

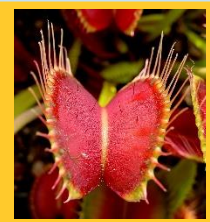
Math is everywhere-Plants

•**Trees, Leaf, Branches, Flowers – Follows definite**
Patterns, reasoning, structure, symmetry, mathematics
(Many guided by Fibonacci Number, Golden Ratios)

Fibonacci was born around 1170.
Michael Maestlin, first to publish a decimal approximation of the golden ratio, in 1597

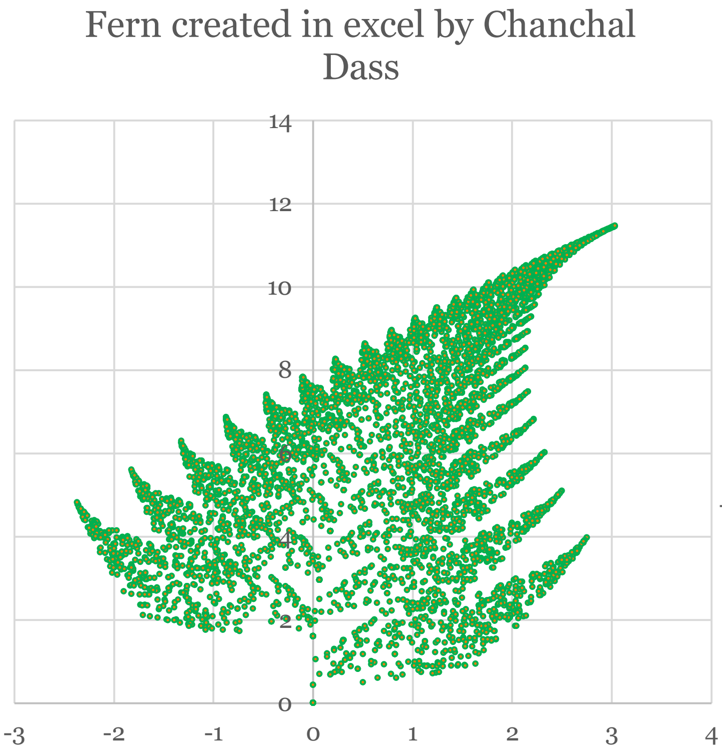
• Sense of Direction (Sunflower)
• Sense of Season
• Sense of Touch
• Sense of Time (Touch-me-not)

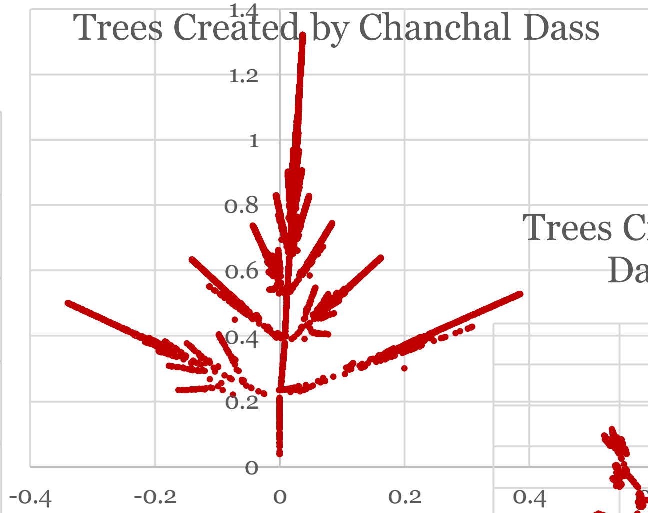


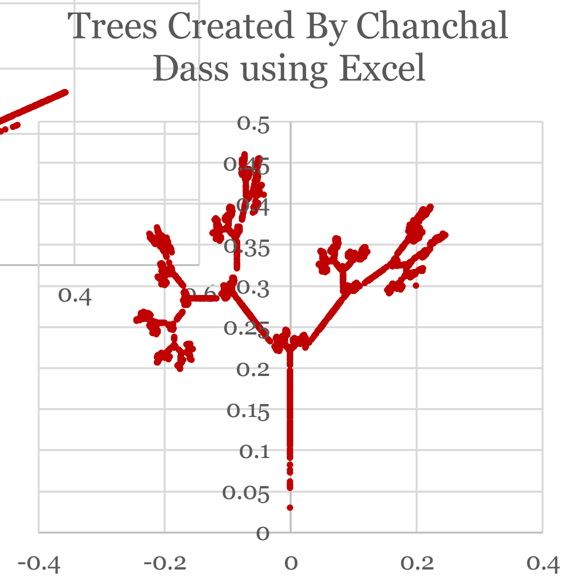



•**Fractals**
(Cauliflower, Fern)


Pattern and Mathematics in Botany





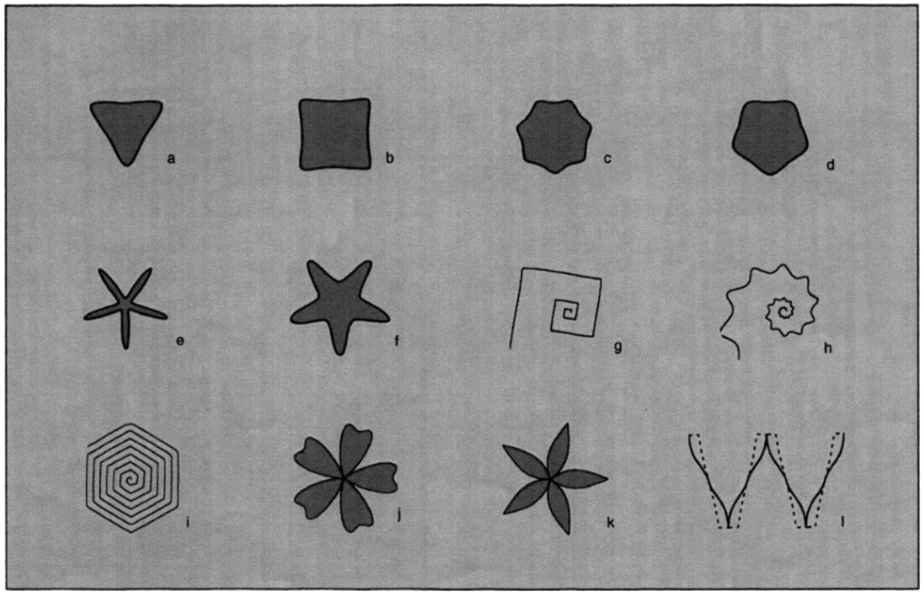


Can you see any specialty or pattern?

They are created from randomly chosen points

Pattern and Mathematics in Botany

- **The Geometrical Beauty of Plants** Hardcover – Import, 16 Apr 2017 – by Johan Gielis (Author)



Can you see any specialty or pattern?

They are all Polygons

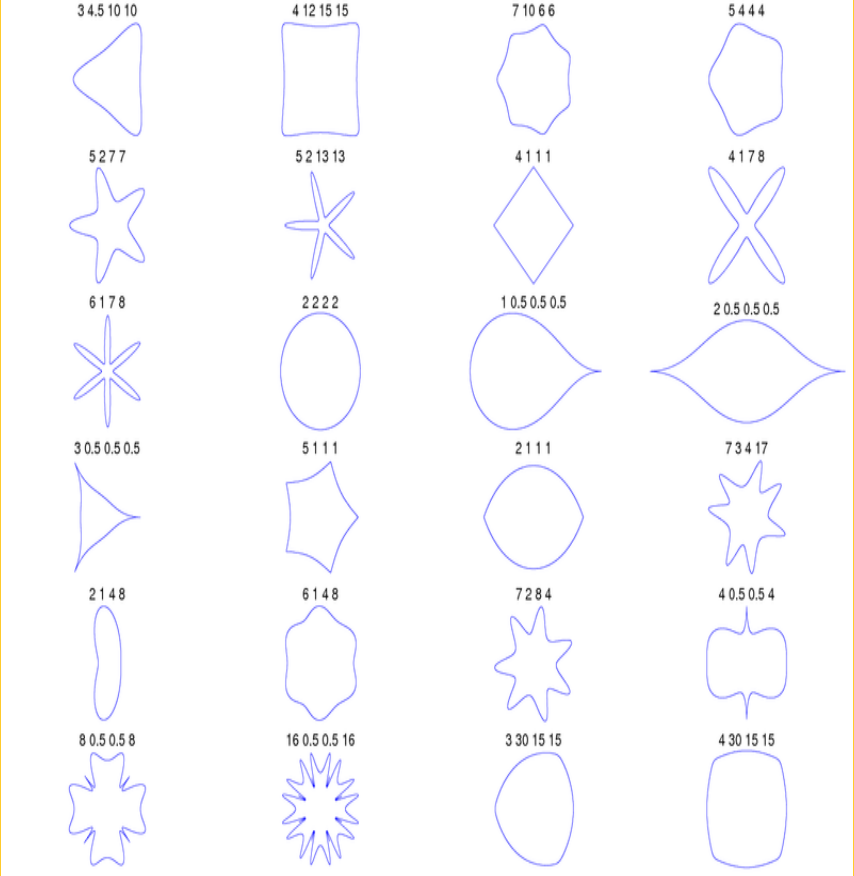
One Formula for Universal Shapes

$$r(\varphi) = \left[\left| \frac{\cos\left(\frac{m\varphi}{4}\right)}{a} \right|^{n2} + \left| \frac{\sin\left(\frac{m\varphi}{4}\right)}{b} \right|^{n3} \right]^{-\frac{1}{n_1}}$$

Super Formula

Natures Shapes

From Wiki: Meaning of Polygon: The word "polygon" derives from the Greek adjective πολύς (*polús*) "much", "many" and γωνία (*gōnía*) "corner" or "angle". It has been suggested that γόνυ (*gónu*) "knee" may be the origin of "gon".[1]



Polygons

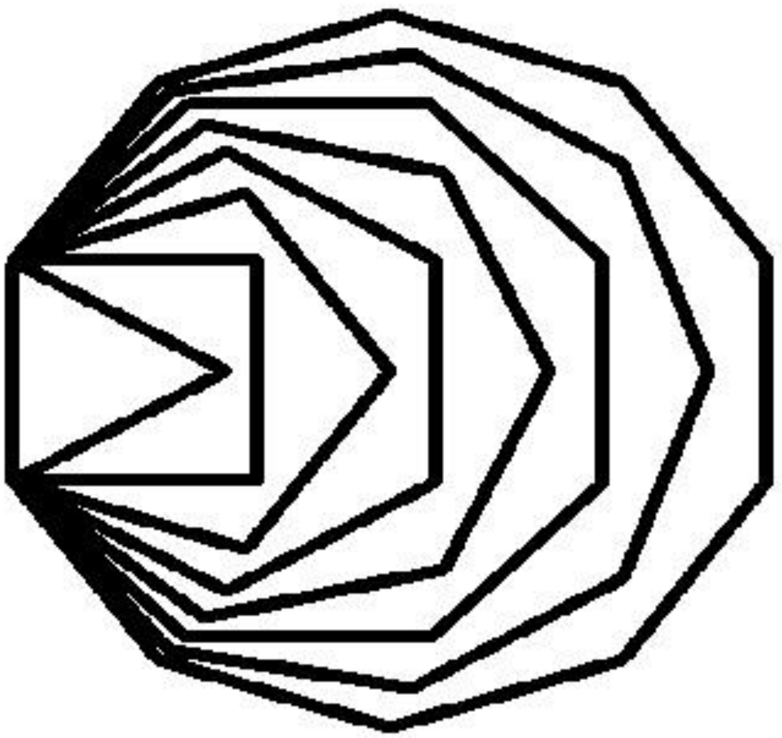
- Zerogon
- Monogon
- Bigon
- Trigon
- Quadrugon
- Pentagon
- Hexagon
- Heptagon
- Octagon

Creating a polygon following a pattern repetitively

Drag and turn

Repeat n time for n-gon

Polygons Created in LOGO



Fibonacci sequence: Golden Ratio

A Sl No	B Fabonacci Sequence	C Golden Ratio=1.618
1	0	
2	1	#DIV/0!
3	1	1
4	2	2
5	3	1.5
6	5	1.666667
7	8	1.6
8	13	1.625
9	21	1.615385
10	34	1.619048
11	55	1.617647

Fibonacci sequence: Golden Ratio

Take any number, add one to it then square root it, repeat the same process recursively, you get golden ratio.

				H
D	E	F	G	x=sqrt(x+1)
Square Root	Add One	D+E	Sqrt(F)=1.618	X^2-X-1=0
100	1	101	10.04988	From Equation
10.04988	1	11.04988	3.324135	1.618034
3.324135	1	4.324135	2.079456	From Series
2.079456	1	3.079456	1.754838	1.618034
1.754838	1	2.754838	1.65977	From Golden Ratio
1.65977	1	2.65977	1.63088	1.618034
1.63088	1	2.63088	1.621999	From Step function
1.621999	1	2.621999	1.619259	1.618182
1.619259	1	2.619259	1.618412	
1.618412	1	2.618412	1.618151	
1.618151	1	2.618151	1.61807	

Continuous Fractions, Series (1+sqrt(5))/2

Fibonacci sequence: Golden Ratio

Continuous Fractions =ones=1+1/(1+1/(1+1/(1+1/(1+1/(1+1/(1+1/(1+1/(1+1)))))))),
From Series, From (1+sqrt(5))/2, From recursion…From ratios

							H
A	B	C	D	E	F	G	x=sqrt(x+1)
Sl No	Fabonacci Sequence	Golden Ratio=1.618	Square Root	Add One	D+E	Sqrt(F)=1.618	X^2-X-1=0
1	0		100	1	101	10.04988	From Equation
2	1	#DIV/0!	10.04988	1	11.04988	3.324135	1.618034
3	1	1	3.324135	1	4.324135	2.079456	From Series
4	2	2	2.079456	1	3.079456	1.754838	1.618034
5	3	1.5	1.754838	1	2.754838	1.65977	From Golden Ratio
6	5	1.666667	1.65977	1	2.65977	1.63088	1.618034
7	8	1.6	1.63088	1	2.63088	1.621999	From Step function
8	13	1.625	1.621999	1	2.621999	1.619259	1.618182
9	21	1.615385	1.619259	1	2.619259	1.618412	
10	34	1.619048	1.618412	1	2.618412	1.618151	
11	55	1.617647	1.618151	1	2.618151	1.61807	

Math is everywhere-Nature



NATURE

•In Nature>> Nothing is Random
(Clouds, Mountains, Rivers, Fire, Coast Line)
>>Many are Fractals

Then Why Math Appears Hard

1. We do math for others

2. We do math without knowing why are we doing the math

Example: If $y=x^2$, what is dy/dx?

3. Learning math in isolation: When asked to draw a line parallel to a given line from a point not on line. It is expected to apply transversal theorem.

Then Why Math Appears Hard

4. What is the solution of a Quadratic equation? Where from it comes?

5. What is the difference between Algebra and Linear Algebra

6. Many feels-I am comfortable without mathematics, then why to load brain

7. Use of mnemonics

Where to start

Evolution of Mathematics

Why math is required

Where to Start

- What is the domain of mathematicians?

Why math is required?
Why do we require mathematics?

- Counting
- Comparing
- Exchanging
- Measuring
- Time Keeping
- Constructing
- Transforming
- Calculating
- Painting
- Colouring
- Singing
- Changing
- Identifying
- Characterizing
- Predicting
- Arranging
- Grouping
- Tiling
- Many more

Evolution of Mathematics

- Natural Numbers
- Whole Number
- Integer
- Rational Number
- Irrational Numbers
- Real Numbers
- Imaginary Numbers
- Complex Numbers
- Logarithmic Numbers
- Prime Numbers
- Quaternion and many more

ADDITION
SUBTRACTION
MULTIPLICATION
DIVISION
EXPONENTIATION

ARITHMETICS
GEOMETRY
ALGEBRA
LINEAR ALGEBRA
TRIGONOMETRY
CALCULUS
STATISTICS
MANY OTHERS

Evolution of Mathematics

Numbers	Need	Binary Operations	Branches
Natural Numbers	Counting	Addition +	Arithmetic
Whole Numbers	Comparing	Subtraction -	Algebra
Fractional Numbes	Measuring	Multiplication *	Geometry
Integer Numbers	Measuring	Multiplication *	Geometry
Rationals	Dividing	Division /	Trigonometry

Evolution of Mathematics

Numbers	Need	Binary Operations	Branches
Irrationals **Hippasus** **Transcendentals**	**Probability**	**Exponentiation ^**	**Calculus**
Real s **Logarithms(Time)**	**Relation** **Function** **Calculus**		**Statistics**
Imaginary	**Exponentiation**		**Linear Algebra**
Complex	**Quantification**		**Many More**
Quaternion	**Qualitative**		
Vectors	**Characterization**		
Matrices, Sets			

Number System-CBSE

Numbers	Class
Natural Numbers	I, II, III, IV
Integers	VI, VII
Decimals/Ratios	VI
Whole Numbers	VI
Rational Numbers	VII, VIII
Irrationals	IX
Real Numbers	X
Sets	XI
Complex Numbers	XI
Matrix/ Determinant	XII(Part-I)
Vectors	XII(Part-II)
Logarithm	Not included

Different Branches Of Mathematics

Sl No	Branches	Class
1	**Geometry, Mensuration**	**I**
2	**Arithmetic**	**I**
3	**Statistics / Data Handling**	**I**
4	**Pattern**	**I**
5	**Algebra**	**VI**
6	**Coordinate Geometry**	**VIII**
7	**Probability**	**VII**
8	**Trigonometry**	**X**
9	**Complex Algebra**	**XI**
10	**Calculus**	**XI**
11	**Linear Algebra**	**XII(Part-I)**
12	**Vector Algebra**	**XII(Part-II)**
13	**Linear Programming**	**XII(Part-II)**

Different Branches/Topics of Mathematics

Branches	Topics
1. Arithmetic	14. Series
2. Geometry	15. Statics
3. Statistics	16. Dynamics
4. Mensuration	17. Modern Algebra
5. Algebra	18. Integral Calculus
6. Coordinate Geometry	19. Differen. Equations
7. Probability	20. Vector Calculus
8. Trigonometry	21. Topology
9. Complex Algebra	22. Fractal
10. Differential Calculus	23. AI
11. Linear Algebra	24. Graph Theory
12. Vector Algebra	25. Fuzzy Logic
13. Linear Programming	26. Vedic Math

NCERT – Class-1 Syllabus-most difficult one

WHAT IS MATH ???

Know more about a system and its behaviour...

WHAT IS MATH ???

What is the definition of Math

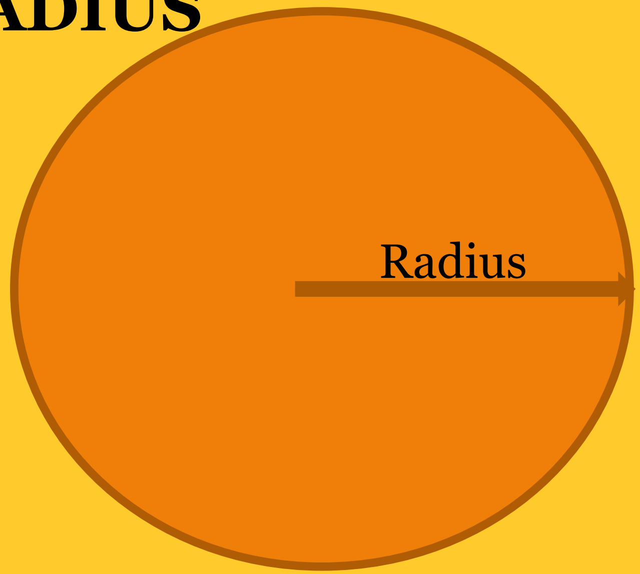
Definition of Mathematics

- Problem:

- How can you find the area of a given circle?

EVERY SYSTEM IS HAVING SOME PARAMETERS…

- **Example: AREA OF CIRCLE IS RELATED TO ITS RADIUS**

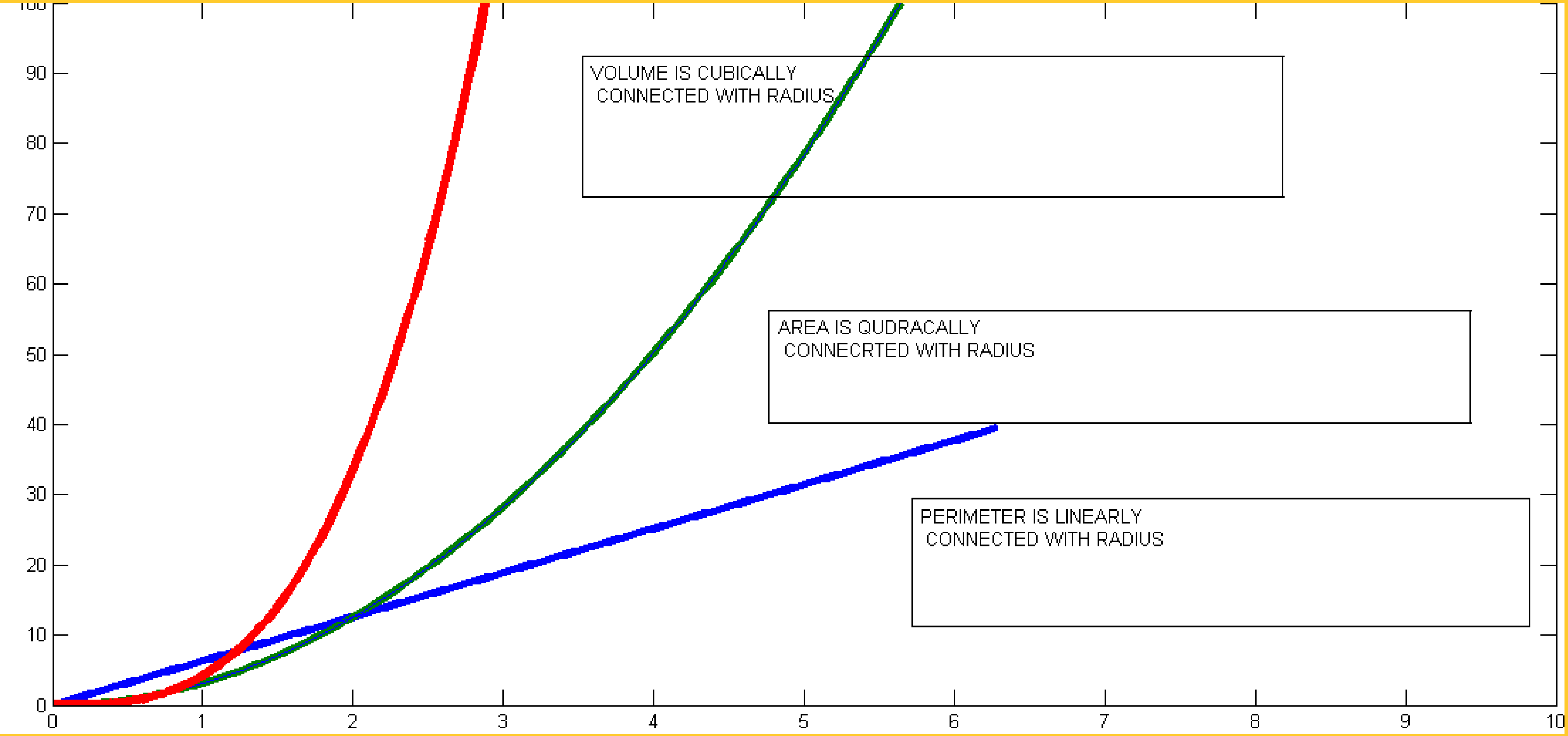


For a circle, Math can be defined as finding the area of the circle when radius is known…

<u>IN A CIRCLE</u>

AREA RELATED TO RADIUS

System Approach



How this information help?

How does these information help

Known vs Unknown

Breaking the problems at micro level

Finding Points

So Mathematics can be defined as process of finding an unknown parameter when a know parameter is given.

Collection of ordered points forms a line

HOW MATH CAN BE MADE EASY?

$KE = \frac{1}{2} mv^2$ $F = ma$ $V = IR$

- World Is Made Of Infinite Number Of Systems. If We Try To Remember All The System Separately Then It Will Become Herculean Task.

Good News: Good News: Good News

- When Systems Are Infinite, A Good News Is That The Line Representing Their Relationship Is Finite. So if we learn about lines, we can solve any problem.

$ax + by = c$ $V = 4/3\Pi R^3$ $C = 2\pi r$

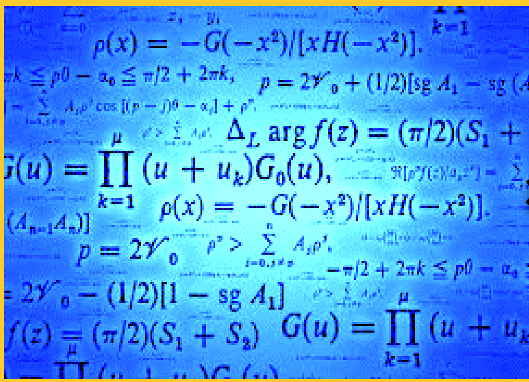
- And the Answer is : Math can be made easy By Proper Understanding of Lines

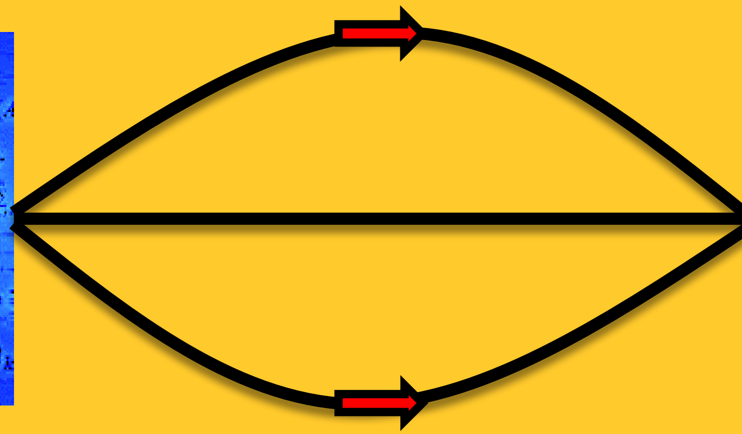
Our Goal

DASS SRL
DASS Scientific Research Labs Pvt. Ltd.

3D Simplified Mathematics

PRESENT





FUTURE

Maths means Symbols, Equations, Formulas

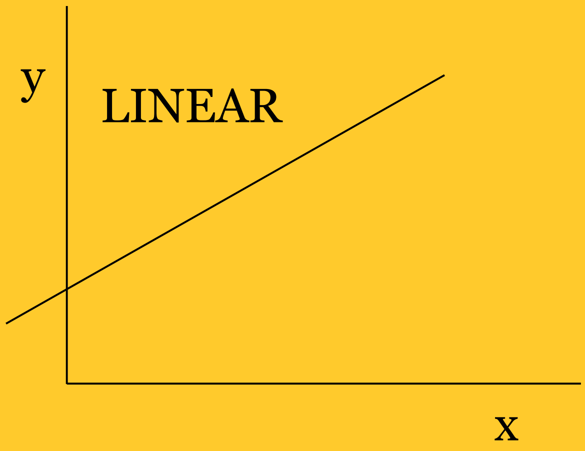
Changing Perception about mathematics

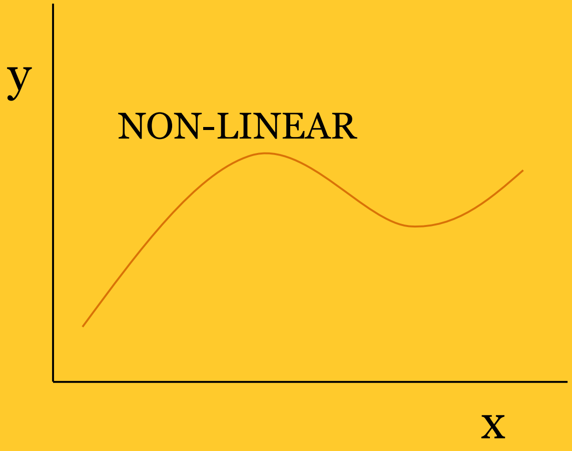
Maths means Points, Lines, Surfaces, Solids

If we can learn all the characteristics of a line, we can solve problems related to any system.

RELATIONSHIP - EITHER LINEAR OR NON LINEAR

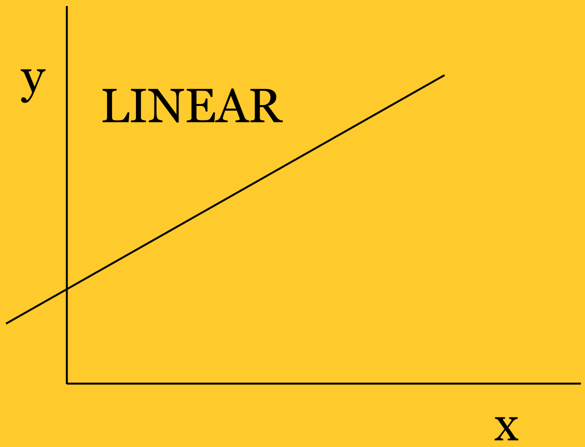
GEOMETRIC RELATIONSHIP BETWEEN VARIABLES
Lines and Curves:

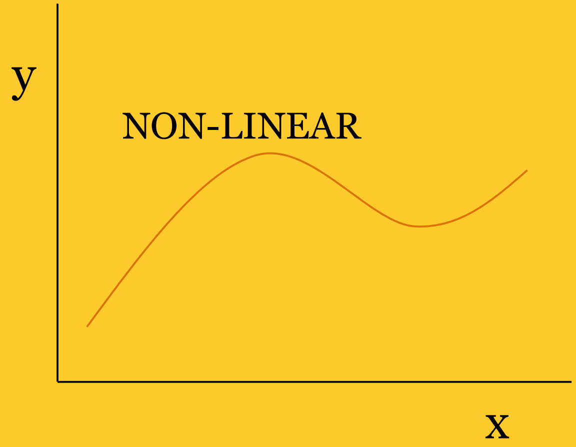




RELATIONSHIP - EITHER LINEAR OR NON LINEAR

If we can efficiently handle lines, we can construct any objects, create graph of a function and solve any type of equations





RELATIONSHIP - EITHER LINEAR OR NON LINEAR

Characteristics of STRAIGHT LINE:

- Equation of a straight line:

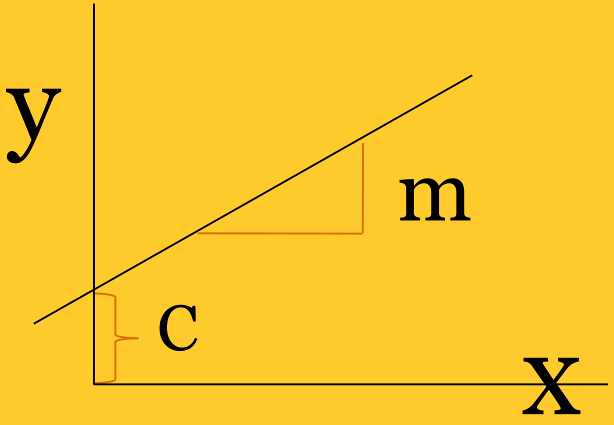
$$y=mx+c$$

x=independent variable

y=dependent variable

m=slope>>constant

c=y-intercept



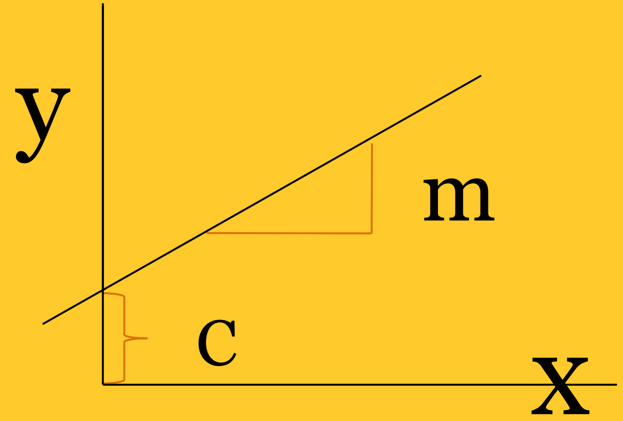
STRAIGHT LINE: GEOMETRIC MEANING:

- <u>DIFFERENCE AMONG</u>:
- Variables, Parameters, Constants

$$y = mx + c$$

y=5*x+9

A=pi( )*r^2



pi->constant

x=independent variable

y=dependent variable

m=slope->constant->parameter

c=intercept->constant->parameter

note: m and c are constant for a particular line

Integrating Geometry and Algebra
Conversion From Points To Equation And Equation To Points

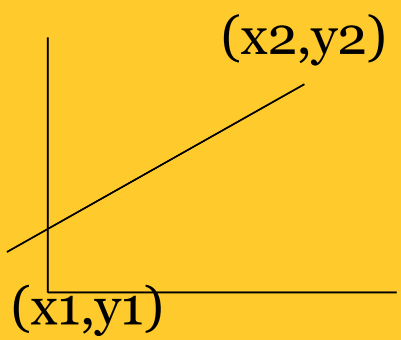
- A point consists of 2 elements(x, y)
- Any straight line is formed from two given points.
- **<u>SLOPE</u>**: If Point A is (5,2) and B is (9,8), then

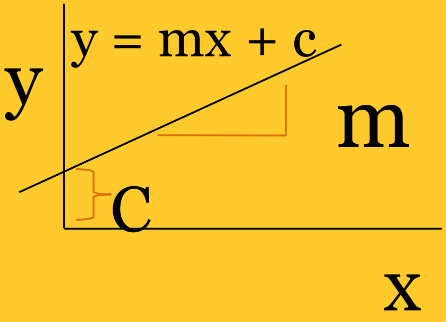
  m=(8-2)/(9-5)=6/4=3/2.

By putting the value of m in the equation of line and value of any one of the given point, c can be calculated.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$c = y - mx$$





STRAIGHT LINES:

- x=independent variable
  y=dependent variable
  m=slope
  c=intercept
  note: m and c are constant for a particular line

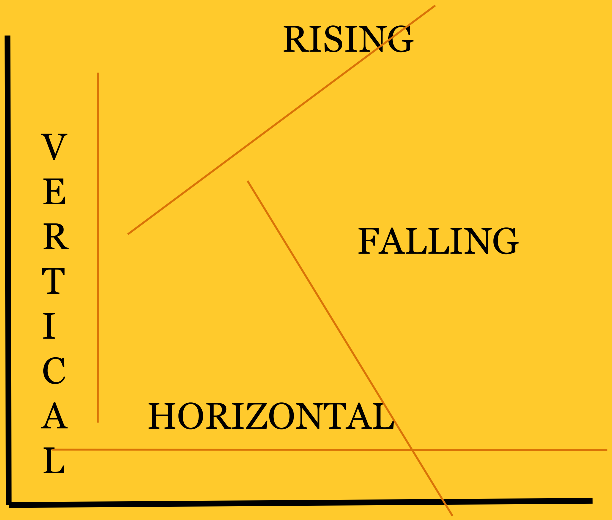
How many types of Straight Line?

Different Types of STRAIGHT LINES:

- Depending Upon Slope, the line is of two type:
  - Rising: Slope(m)=Positive Value
  - Falling: Slope(m)=Negative Value

Horizontal Line: Slope=0
Vertical Line: Slope=Infinity

$$y = mx + c$$



Slope and Angle of Inclination Relationship

Relationship:

- Note: slope varies from zero to infinity when line moves from angle 0 to 90 degree.

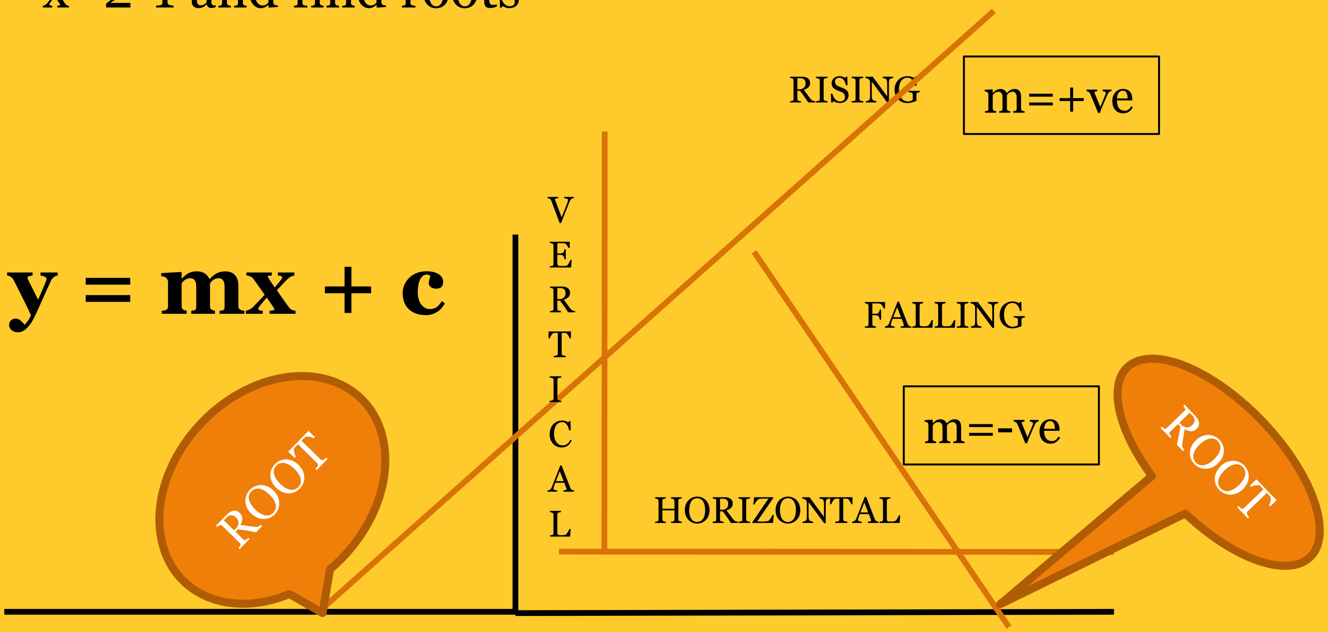
- Slope=m= tan(Ø)= dy/dx

Roots

- What is the root of an equation and how it look like geometrically

Root of An Equation:

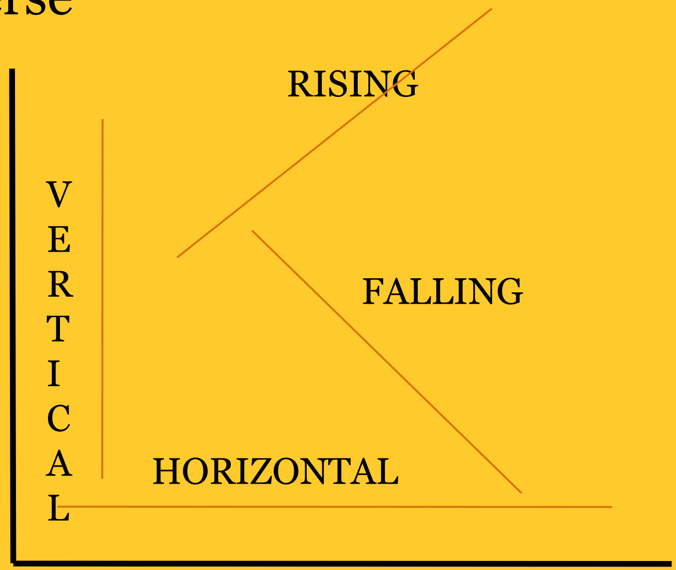
- Finding solution of an equation means finding the roots of the equation. Root of an equation means a point on x-axis where the value of y is zero.
- Draw y=x^2-1 and find roots



Characteristics of STRAIGHT LINE:

1. x=independent variable
2. y=dependent variable
3. m=slope
4. c=Intercept
5. x =Root, the value at y=0
6. Monotonicity (Rising or Falling)/ Inverse
7. One-to-one
8. Strictly increasing/decreasing
9. Onto

$$\mathbf{y = mx + c}$$

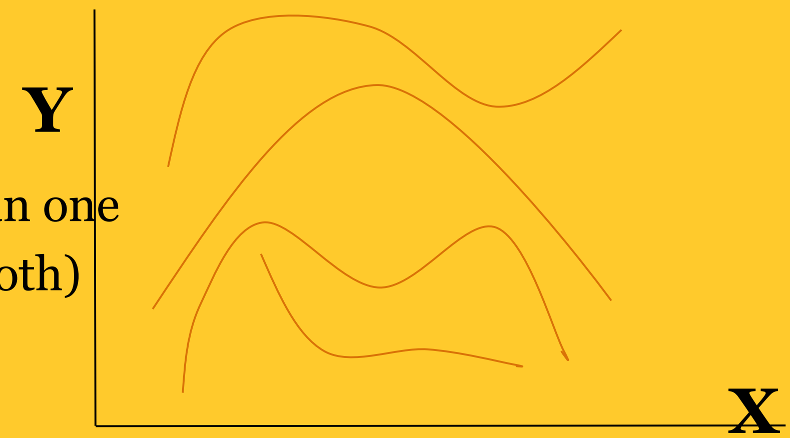


Non-Linear Systems Represented by Curved Lines

- $Y=ax^2+bx+c$
- $Y=ax^3+bx^2+cx+d$
- $Y=ax^4+bx^3+cx^2+dx+e$

Characteristics of Curved Lines

1. x=independent variable
2. y=dependent variable
3. m=slope>>**CHANGES>parameter**
4. c=Intercept>>constant>parameter
5. r=roots, value of x at y=0, can be more than one
6. Monotonocity (Rising And/or Falling or Both)
7. Bends
8. Turning points
9. Point of inflexion
10. Radius of curvature
11. Asymptotes- Asymptote is some boundary beyond which a curve will not pass.
12. Maxima / Minima
13. Concave / Convex (cup / cap)
14. One to one correspondence



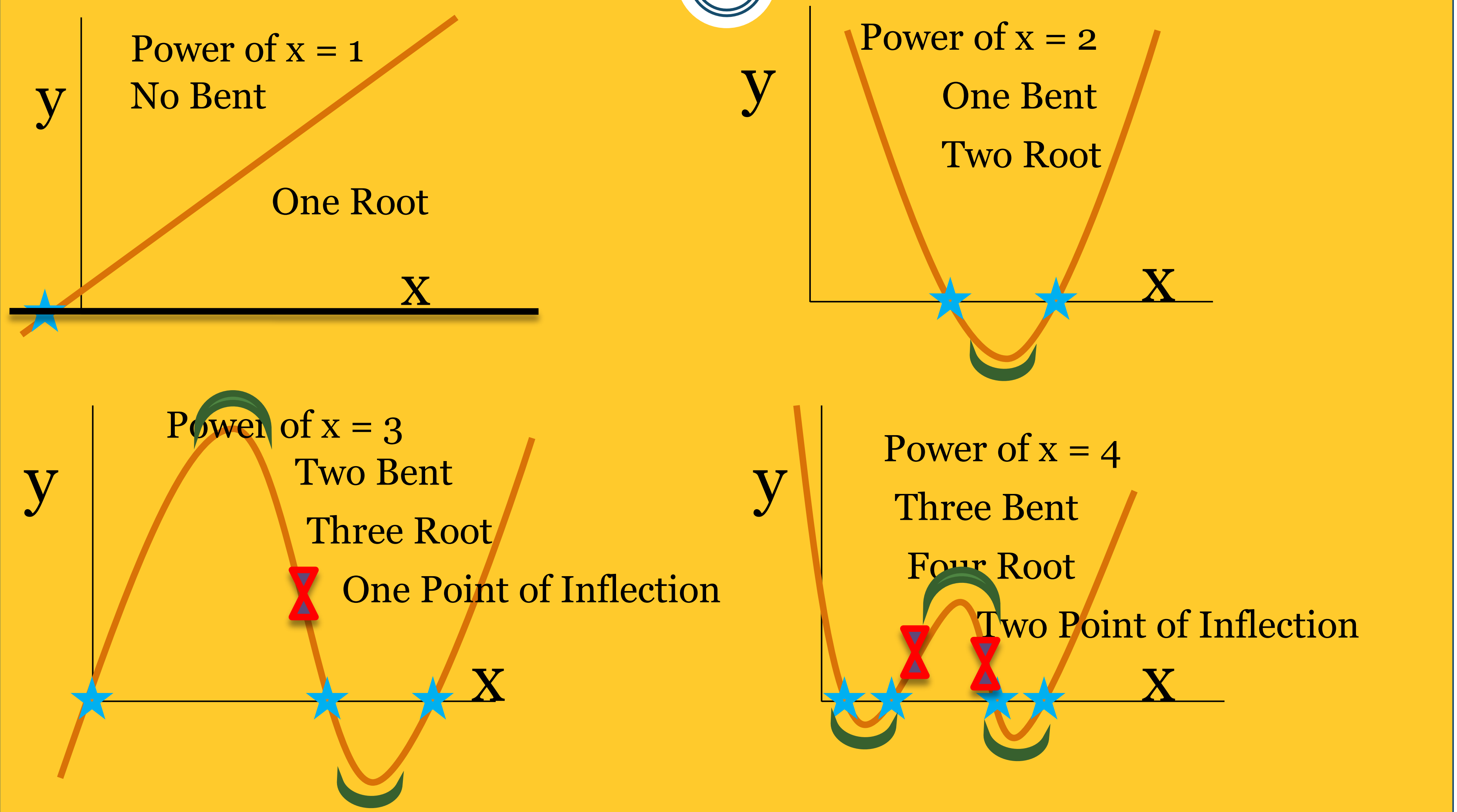
Shape of Non-linear Functions

- Point-1: No. of roots equal to highest power of independent variable (x).
- Point-2: No. of bends is one less than power of independent variable (x).
- Point-3: No. of point of inflexion is one less than the no of bends.

Curved lines:

Power of x	Equation	Roots	Bends	Point Of Inflexion
1	$Y=mx+c$	1	0	0
2	$Y=ax^2+bx+c$	2	1	0
3	$Y=ax^3+bx^2+cx+d$	3	2	1
4	$Y=ax^4+bx^3+cx^2+dx+e$	4	3	2

CURVED LINES



Characteristics of Curved Lines

1. x=independent variable
2. y=dependent variable
3. m=slope>>CHANGES>parameter
4. c=Intercept>>constant>parameter
5. r=roots, value at y=0, can be more than one
6. Monotonocity (Rising And/or Falling or Both)
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11. Asymptotes
12. Maxima / Minima
13. Concave / Convex (cup / cap)
14. One to one correspondence
15. Monotonic / Inverse
16. Explicit/ Implicit
17. Algebraic/ Transcendental
18. Point of Rectification
19. Saddle Point
20. Osculation
21. Curvature

How does these information help

Known vs Unknown

Breaking the problems at micro level
Two Elements Represents a

Points

Describing Math Through Points

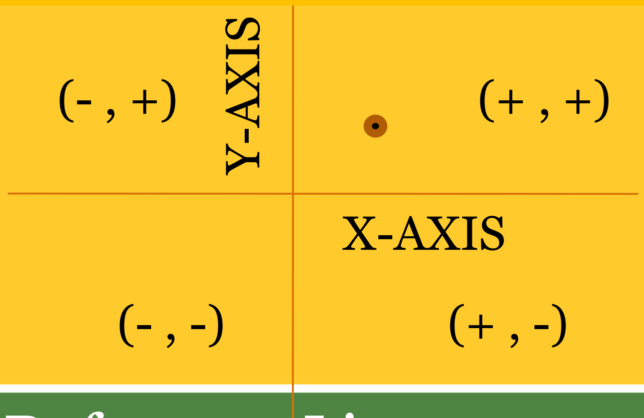
Define a Point?

What is the position of the point?

Points have dimension=0

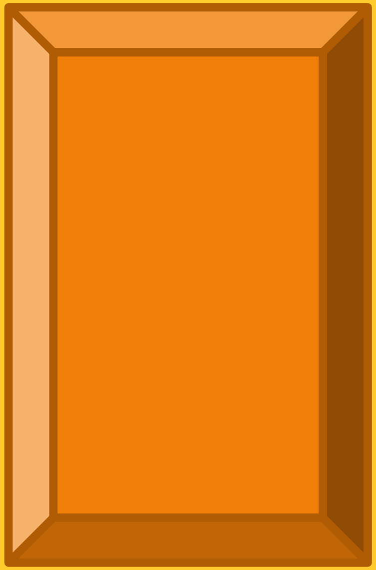
What is Point????

Note-Point described without reference is meaningless.



- Reference Lines
- Domain and Range
- Scale (Nano/Micro/Macro/Mega)
- Dimension

Formation of objects
Dimensions

Drag and Create

1. 0 Dimension - Point
2. 1 Dimension – Line
3. 2 Dimension – Square
4. 3 Dimension – Cube
5. 4 Dimension – Hyper Cube
6. 5 Dimension – Hyper Hyper Cube
7. 0.2 , 0.75 Dimension - Fractals

Drag and Create: Dimensions

Object	Vertex	Edges	Faces	Solids	Hyper Solid	Dimension
Point	1	0	0	0	0	0
Line	2	1	0	0	0	1
Square	4	4	1	0	0	2
Cube	8	12	6	1	0	3
Hyper Cube	16	32	24	8	1	4
Hyper Hyper Cube	32	80	80	40	10	5
Relationship in 3d Objects: Vertex + Face=Edge+2						

Platonic Solids

1. TETRAHEDRON
2. HEXAHEDRON
3. OCTAHREDRON
4. ICOSAHEDRON
5. DODECAHEDRON

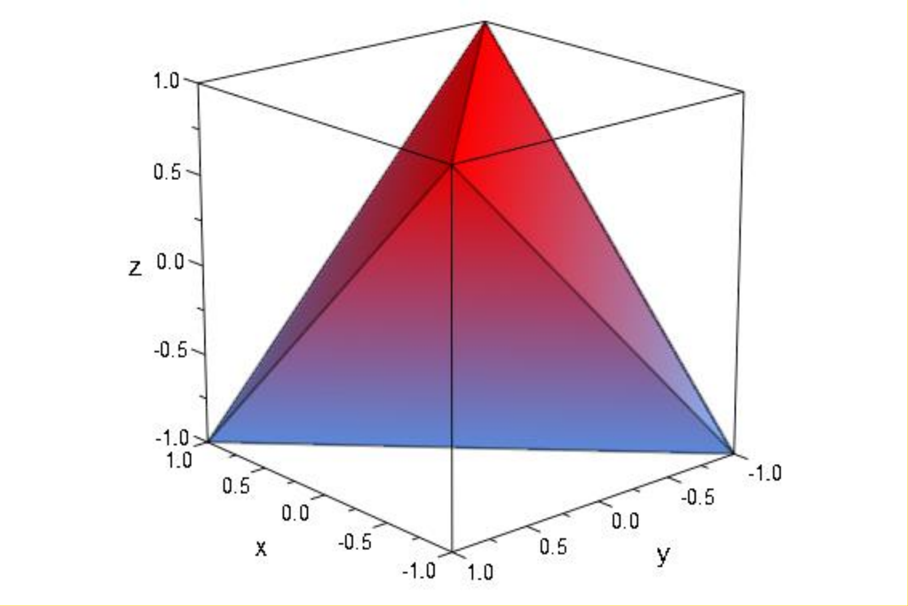
Characteristics:

In three-dimensional space, a **Platonic solid** is a regular, convex polyhedron. It is constructed by congruent regular polygonal faces with the same number of faces meeting at each vertex. Five solids meet those criteria:

Platonic Solids


```
tetra:=plot::Tetrahedron(Center=[0,0,0], Radius=1):
    plot(tetra)
```

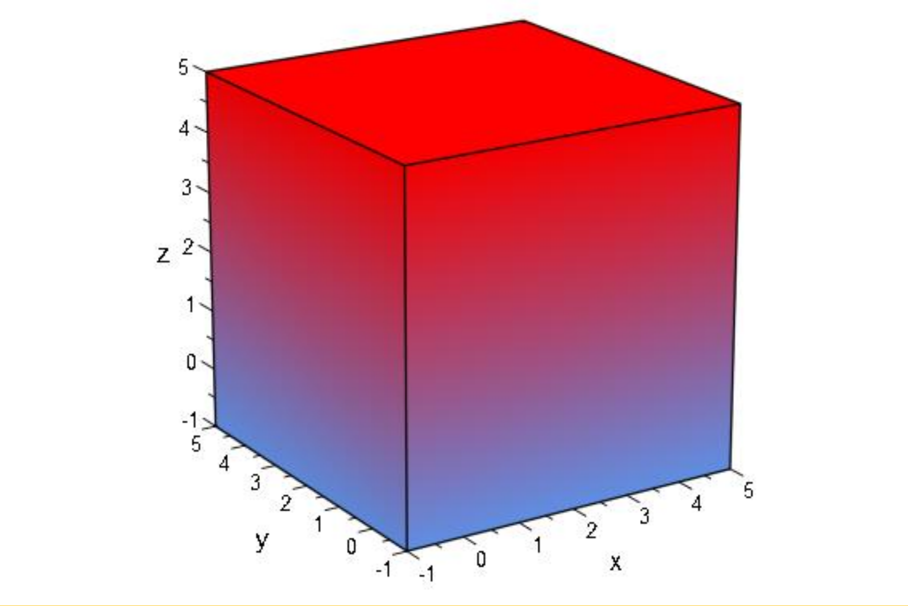
Faces - 4



Platonic Solids


```
hexahedron:=plot::Hexahedron(Center=[2,2,2],Radius=3):
    plot(hexahedron)
```

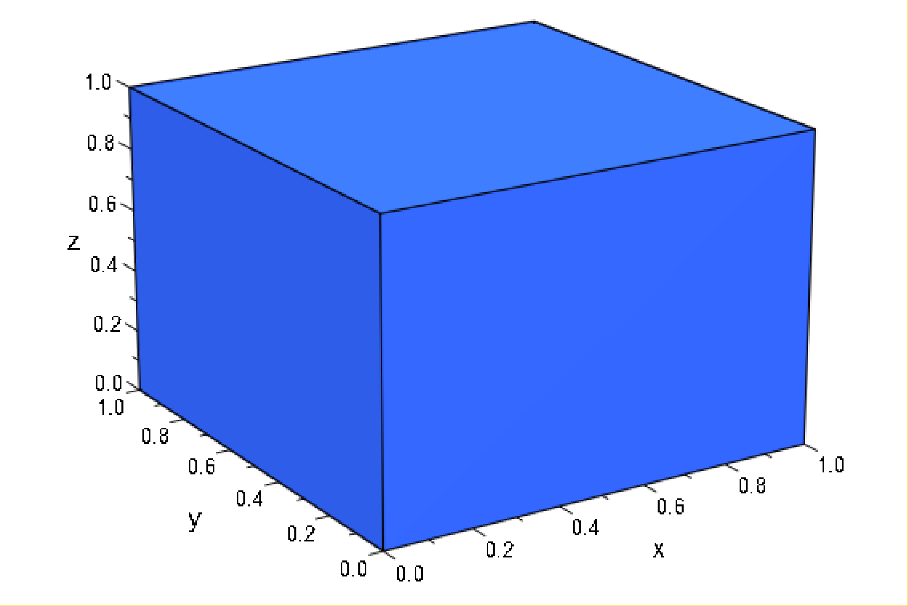
Faces : 6



Platonic Solids

Cube:

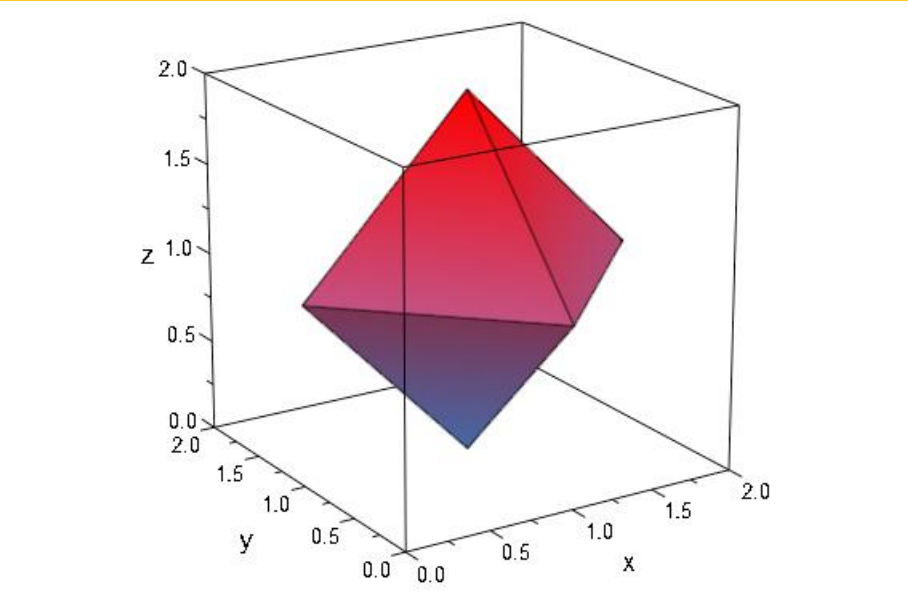
cube:=plot::Box([0,0,0],[1,1,1]):
plot(cube)



Platonic Solids


```
octa:=plot::Octahedron(Center=[1,1,1],Radius=1):
    plot(octa)
```

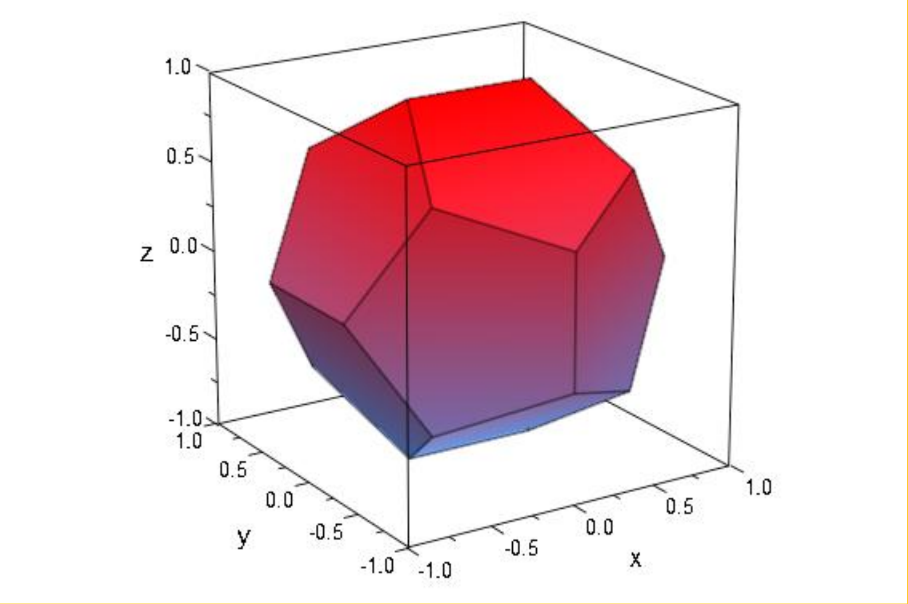
Faces : 8



Platonic Solids


```
dodeca:=plot::Dodecahedron(Center=[0,0,0],Radius=1):
    plot(dodeca)
```

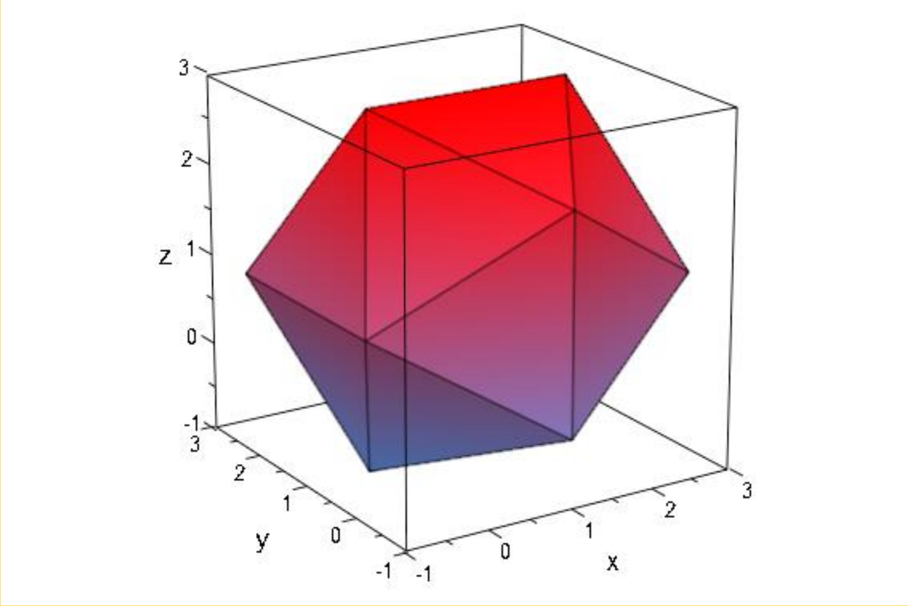
Faces: 12



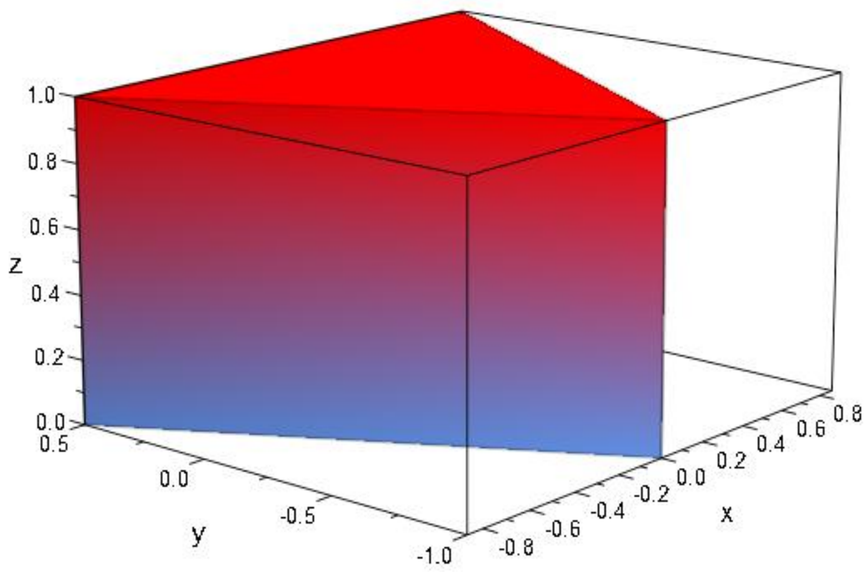
Platonic Solids

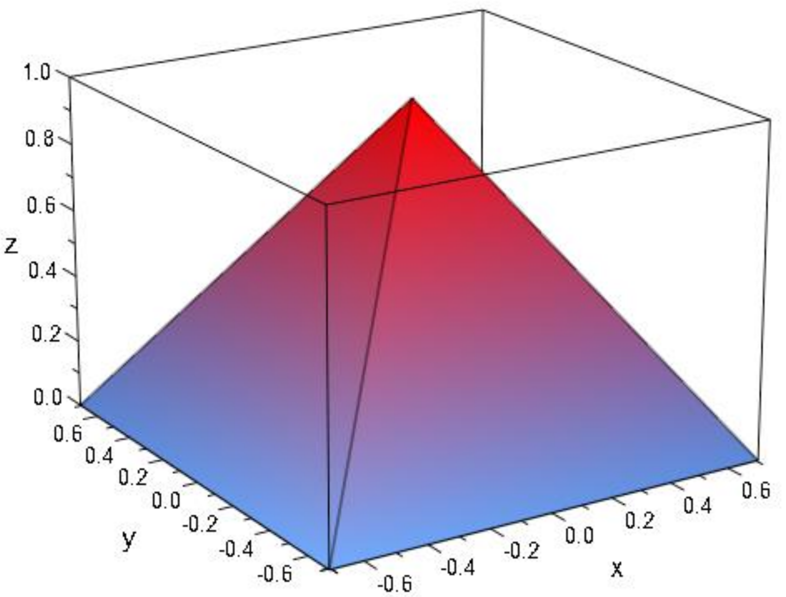
icosa:=plot::Icosahedron(Center=[1,1,1],Radius=2):
plot(icosa)

Faces : 20



Solids-Prism, Pyramid





Representation of Points

Points have a definite position in a Specific Coordinate System

- **Points**
  - Represented by coordinates
    - (x, y)
      where x and y are the coordinates of a point
  - Expressed as Vector/Matrix
    - Matrix of (1x2) dimension: a=[x, y] which form elements of matrix

Draw a Points, Vectors, Lines, Triangles

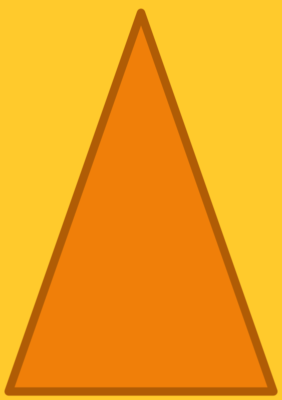
Excel Commands:

(7, 9)

(3,5)

(0,3)

(0,0)



STRAIGHT LINES:

- x=independent variable
  y=dependent variable
  m=slope
  c=intercept
  note: m and c are constant for a particular line

How many types of Straight Line?

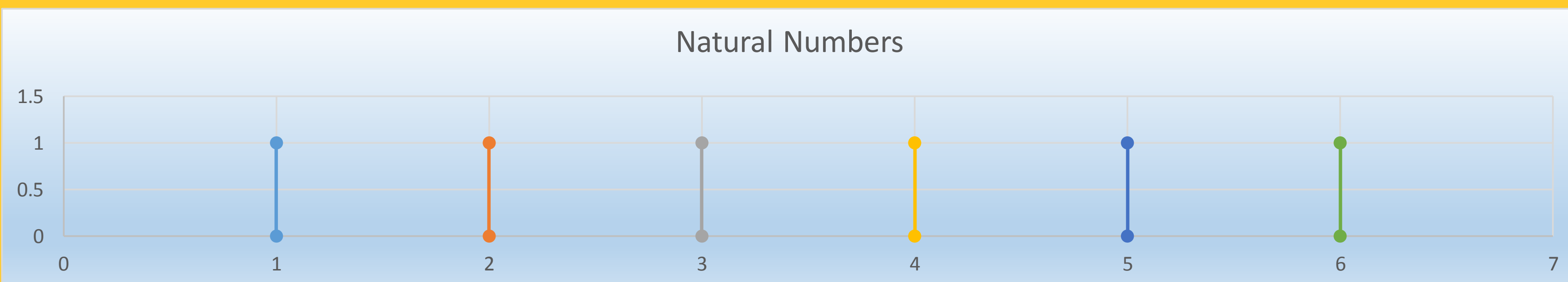
Moving from static to dynamic world

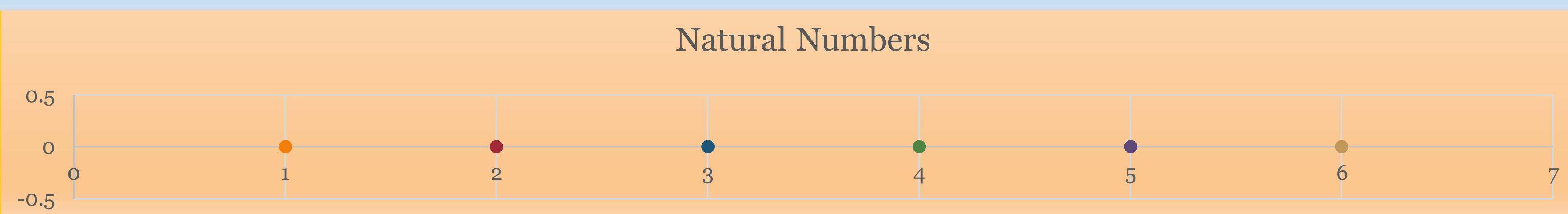
- With points we can construct any geometrical object.
- Every thing in this world are dynamic. We should now find the tools to capture dynamic world.

One, Two and Three Dimension Mathematics

- 1 One dimensional Mathematics

- R=5, A=25, Volume=125





Numbers

- Numbers started with discrete points in positive direction in a line as natural numbers
- Then numbers extended to negative direction in same line as whole number and integers.
- Then discrete points started filling with Fractional Numbers
- Then came rational numbers which changed the discrete nature of numbers to continuous numbers and we got number line.
- Then came irrationals which are the numbers that does not have any place in number line .

Binary Operations

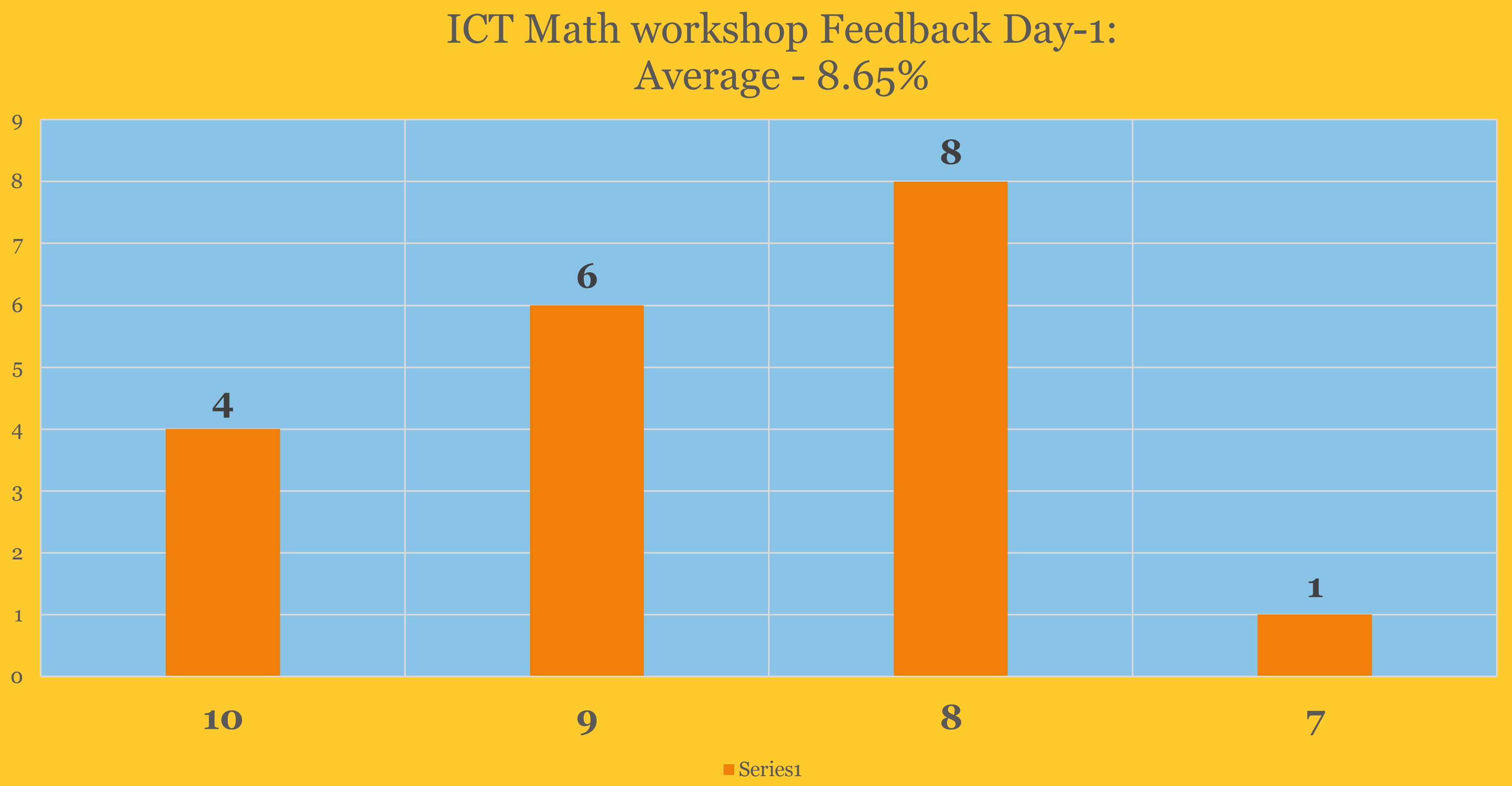
- Binary operations started with additions
- Then multiplication was discovered which is repeated addition.
- Then subtraction started which is addition in negative direction.
- Then division came which is repeated subtraction
- Then came exponentiation which is repeated multiplication.

One dimensional Mathematics

- All most 70% of our social requirement for leading a comfortable life is met with the one dimensional mathematics which come naturally.

- So nobody feels uncomfortable without knowing higher mathematics.

Day-1: ICT Math Workshop Feedback:



Remarks and Ratings from Previous Workshop

- Gopal Menon
- 1) B Tech, IIT, Bombay (1975),
- 2) B Sc (Maths) (2010)
- Retired, Mathematics Tutor at NGO

- Rating-10/10
- Remark-Excellent Presentation, I have learnt new ideas which can be taught to my students.

Remarks and Ratings from Previous Workshop

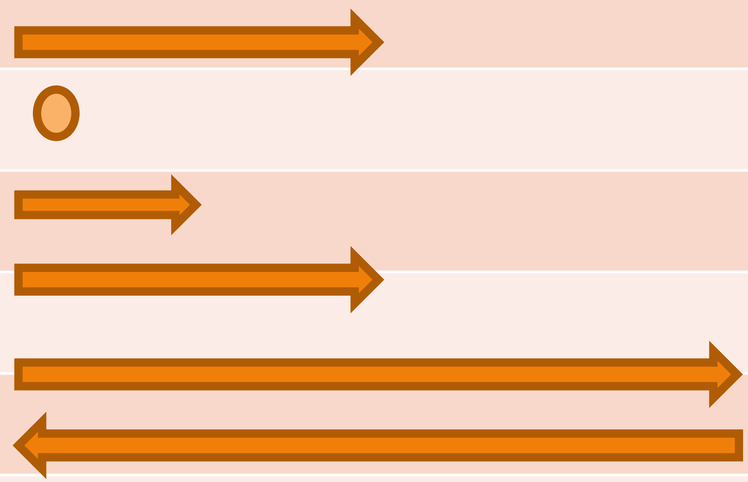
- Ujjwal Rane
- M.Tech. (Machine Dynamics) IIT(1988), Madras, M.S. (Computer Aided Geometric Design) - Arizona State University
- Engineering consultant, Partner in a Publishing business
- Rating-9/10
- Remark: Learned a lot of new & very surprising facts and Skill

Scalar Multiplication

Scalar Multiplication of 2 numbers(a,t): at= a x t

For a number 'a', depending upon the value of 't', the value of 'a*t' remains same, decreases, increases or changes sign.

a	t	a x t	Geometrical Representation
5			
5	0	5x0=0	
5	0.5	5x.5=2.5	
5	1	5x1=5	
5	2	5x2=10	
5	-2	5x-2=-10	Plot these results in excel



Moving from static to dynamic world

- Representation of a point or a vector by its coordinates as [x, y], [x, y, z], can be viewed as well organized ordered numbers.
- This type of organized numbers is termed as MATRIX. It can have any number of rows/ columns.

$$\begin{bmatrix} 5 \end{bmatrix}$$

(1x1) matrix

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(2row x 1column) matrix

$$\begin{bmatrix} 2 & 3 \end{bmatrix}$$

(1x2) matrix

Column

$$\begin{bmatrix} 3 & 2 & 5 \\ 7 & 1 & 0 \\ 9 & 4 & 6 \end{bmatrix}$$

Row

(3x3) matrix

Moving from static to dynamic world

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(1x2) matrix

$$\begin{bmatrix} 3 & 2 & 5 \\ 7 & 1 & 0 \\ 9 & 4 & 6 \end{bmatrix}$$

Column

Row

(3x3) matrix

Moving in the Space: Matrix Multiplication

- Input (x, y)
- Output(x*, y*)
- Easy way to move a point is matrix Multiplication

Matrix Multiplication

Matrix Multiplication:

- Rule: Number of columns of 1st matrix should be equal to number of rows of 2nd matrix.

$$[m \times a] * [a \times n] = [m \times n]$$

- A 1x2 matrix forms when a 1x2 matrix is multiplied by a 2x2 matrix

$$\begin{bmatrix} x & y \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ax + cy & bx + yd \end{bmatrix}$$

- [x,y] is a row matrix represents a point.
- The (2x2) matrix is a transformation matrix.
- We are interested to know the effects of changes in individual elements of a transformation matrix on the resultant point.

Capturing world dynamism through Matrix Multiplication

- Matrix multiplication of a 1xn matrix by a nxn matrix results in a 1xn matrix
- Importance of Matrix Multiplications – It helps in transformations
- Through these transformations, we can capture the dynamism around the world.

Dynamic World
Type of Transformations

- Scaling
- Reflection
- Shearing
- Rotation
- Translation
- Projection

Transformation and Matrices

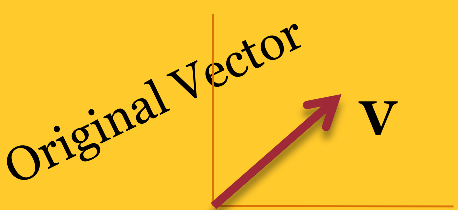
- Let a position vector (v) =[5,3]
- The Transformation Matrix (t)= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- Then Resultant Vector (vt) = [Xt Yt]

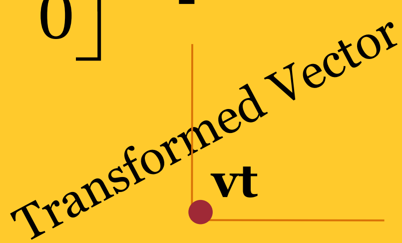
CASE 1:

a	b	c	d
0	0	0	0

$$t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$v*t = vt = \begin{bmatrix} 5 & 3 \end{bmatrix} * \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5*0+3*0 & 5*0+3*0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$





Result: Multiplication of a vector by a zero matrix produces a Zero Vector.

SCALING

Observation from Cases:

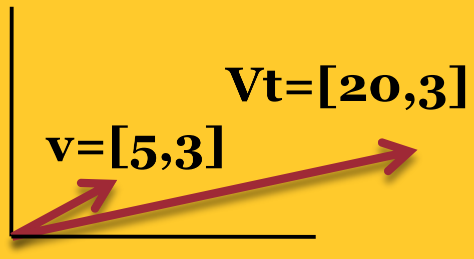
$$[x \quad y] * \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} = [ax \quad dy]$$

- A transformation matrix whose primary diagonal elements are non-zero, results scaling in both axis.
- If a=d, then scaling are equal.
- When a=d>1, b=0, c=0 then pure enlargement occurs.
- If b=0, c=0 , 0<a<1, 0<d<1, then a compression of coordinates of vectors occurs.

- Similarly, as in the above cases,

a	b	c	d
1	0	5	1

$$t = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$



Produces a shear proportional to X coordinate.

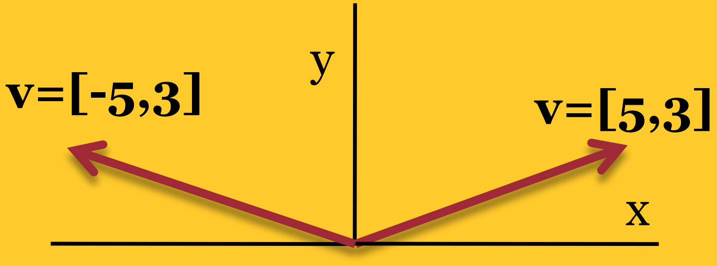
$$\begin{bmatrix} 5 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 5*1+3*5 & 5*0+3*1 \end{bmatrix} = \begin{bmatrix} 20 & 3 \end{bmatrix}$$

- **<u>NOTE 2:</u>**
  OFF DIAGONAL TERMS produces Shear.

REFLECTION

- If ‘a’ and/or ‘d’ are negative, then reflection through a plane or axis occurs.

$$[5 \quad 3] * \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [-5 \quad 3]$$



1. **If a=-1, reflection through Y axis occurs.**
2. **If d=-1, then reflection through X axis occurs.**
3. **If a=-1, d=-1, then reflection through origin occurs.**

Note=Reflection Occurs In 3d

Reflection

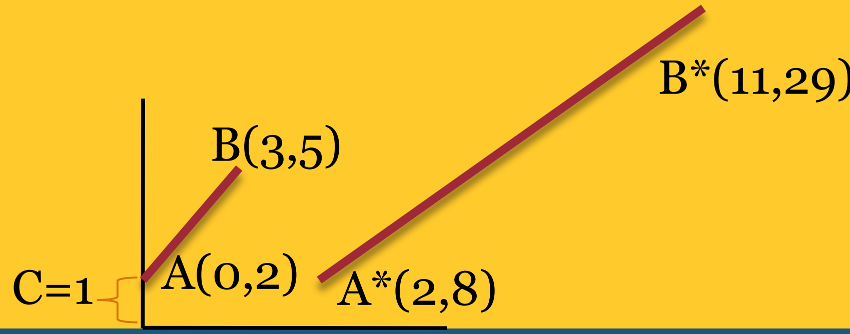
- The 2D rotation in the xy plane occurs entirely in the two dimensional plane about an axis normal to the xy plane, a reflection is a 180° rotation out into 3d space and back into 2d space about an axis in the xy plane.
- A reflection about y=0, i.e., x axis is obtained by transformation matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Reflection about x=0, i.e, y axis is obtained by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
- Reflection about y=x is obtained by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- Reflection about y=-x is obtained by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Transformation of Straight Lines

- A straight line can be defined by two position vectors which specify the coordinates of its end points.
- Let A=[0 2] and B=[3,5] are two position vectors joining the endpoints of a line AB. Now, let t= $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ be the transformation matrix.

$$\begin{bmatrix} 0 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0*2+2*1 & 0*3+2*4 \\ 3*2+5*1 & 3*3+5*4 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 11 & 29 \end{bmatrix}$$

A*B* are the endpoints of the Transformed Line



NOTE 4: Transformation of a line changes the length and Orientation

Transformation of Parallel Lines

Note-5: Slope of parallel line (m) is same

- A 2x2 transformation matrix transforms a pair of parallel lines into another pair of parallel lines.
- Let AB// EF (slope=m)
- If AB is transformed by a transformation matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then,

$$\begin{bmatrix} xt_1 & yt_1 \\ xt_2 & yt_2 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix}$$

Contd..

$$m^* = \left[\frac{(bx_2 + dy_2) - (bx_1 + dy_1)}{(ax_2 + cy_2) - (ax_1 + cy_1)}\right] = \left[\frac{b(x_2 - x_1) + d(y_2 - y_1)}{a(x_2 - x_1) + c(y_2 - y_1)}\right]$$

$$m^* = \left[\frac{b + dm}{a + cm}\right]$$

$$t = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

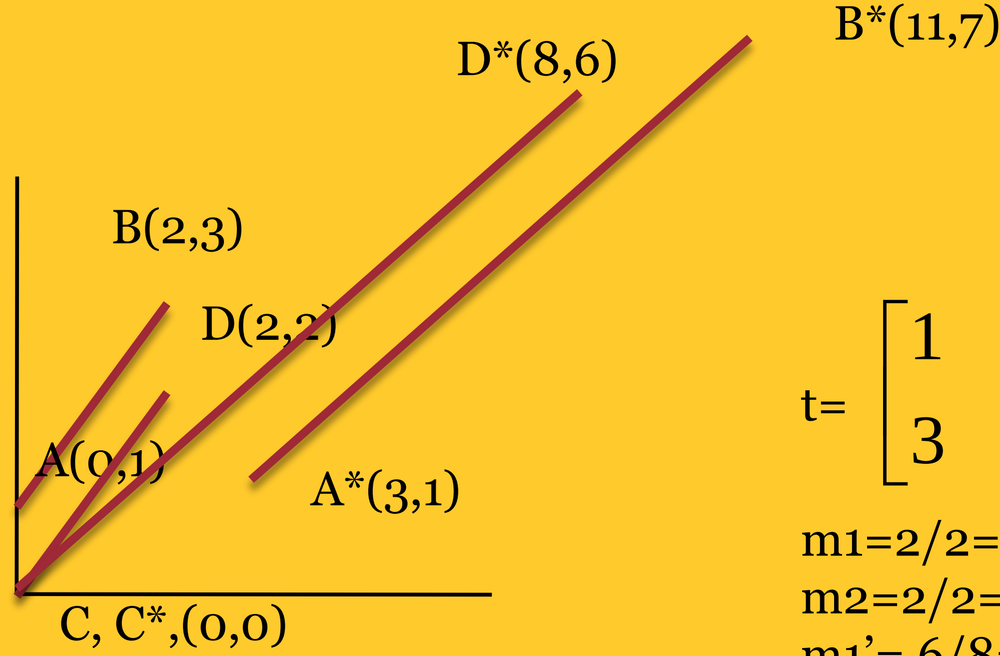
- m* is independent of coordinates
- m* is same for both A*B* and E*F*.
- <u>Note 6:</u>

Parallel Lines Transforms into parallel lines when operated by a general 2x2 transformation matrix (Affine Transformation).

Parallel Lines Remain Parallel After Transformation

0 1
2 3

- 0 0
  2 2



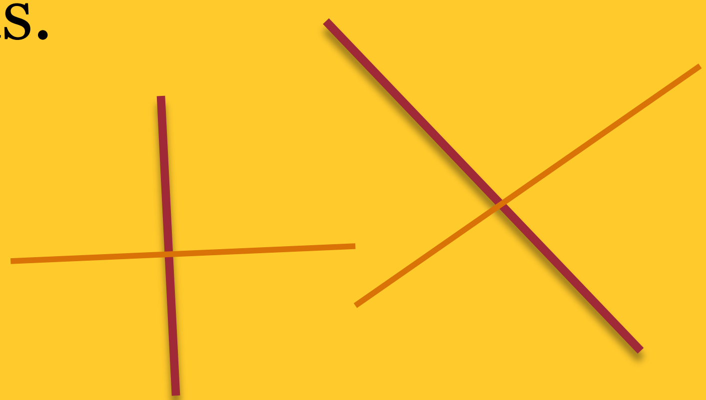
$$t=\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$$

m1=2/2=1
m2=2/2=1
m1'= 6/8=0.75
m2'= 6/8=0.75

Transformation of Intersecting Lines

- Two intersecting lines ⟹ have a common point which means that a solution to the pair of equations representing the lines exists.
- Let the two equations be-

$$y = m_1 x + c_1$$
$$y = m_2 x + c_2$$
$$or$$
$$-m_1 x + y = c_1$$
$$-m_2 x + y = c_2$$
$$or$$
$$[x \quad y]\begin{bmatrix} -m_1 & -m_2 \\ 1 & 1 \end{bmatrix} = [c_1 \quad c_2]$$
$$M^{-1} = \begin{bmatrix} \dfrac{1}{m_2 - m_1} & \dfrac{m_2}{m_2 - m_1} \\ \dfrac{-1}{m_2 - m_1} & \dfrac{-m_1}{m_2 - m_1} \end{bmatrix}$$



Matrix calc

Angle May not be preserved

2x-y=1
x+y=5

X	Y	2	1	1	5
		-1	1		
1	5	1/3	- 1/3	2	3
		1/3	2/3		
2	3	2	1	1	5
		-1	1		

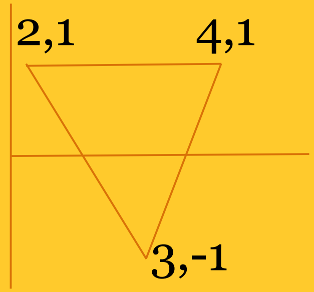
Transformation of Plane
Rotation

- Let ABC be the triangle formed by A(3, -1), B(4, 1) and C(2, 1) and the triangle is rotated 90˚ clockwise. Then,

$$t= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Hence the Transformed triangle is

$$A^*B^*C^* = \begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -1 & 4 \\ -1 & 2 \end{bmatrix}$$



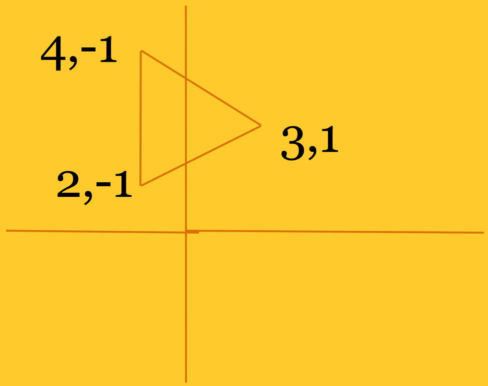
- The general rotation about the origin is governed by, t=

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

- The rotation is positive counter clock wise

Rotation in Excel

Rotation Matrix = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

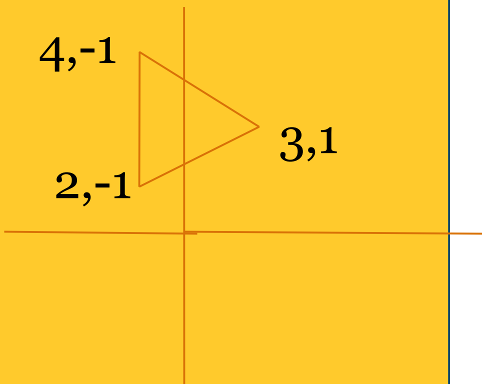


- Draw a rotation matrix in Excel and insert slider to show the dynamic condition of rotation

Rotation

- The determinant of rotation transformation matrix is $(\cos\theta \times \cos\theta)-(\sin\theta \times -\sin\theta)= \cos^2\theta+ \sin^2\theta=1$
- The transpose of rotation transformation matrix

$$t^T= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Since $t^* t^t= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Which indicates that $t^t=t^{-1}$

- **NOTE 8:**

  The determinant of a pure rotational matrix is +1. Such matrices are called orthogonal matrix (Find it in Excel).
- **Inverse Matrix= Transpose Matrix**

Reflection

- The transformation matrices having determinant=-1 produces reflection.
- **<u>NOTE 9:</u>**

  Two successive reflection about two lines passing through origin, results pure rotation.

- **<u>NOTE 10:</u>**

  The reflection matrices are orthogonal meaning that its transpose is its inverse.

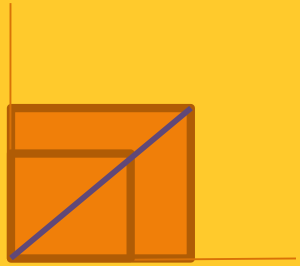
$$t^T = t^i$$

Transformation of the Square

- Transformation matrix operates in every point in the plane.
- Under 2x2 transformation, origin remains invariant. This transformation may be interpreted as stretching of original object into a new shape.
- Let ABCD is a unit rectangle with ABCD= $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$
- The 2x2 transformation matrix is t= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- A*B*C*D*= $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ a & b \\ a+c & b+d \\ c & d \end{bmatrix}$

0	0
2	3
6	5
4	2
0	0

2 3
4 2



Transformation of the Square

- Results: A*=origin is not affected, A=A*
- Coordinates of B* is changed to the first row of matrix.
- Coordinates of D* is changed to second row of transformation matrix.
- Coordinates of C* is a+c and b+d
- The determinant of the transformation matrix determines the scaling factors.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ a & b \\ a+c & b+d \\ c & d \end{bmatrix}$$

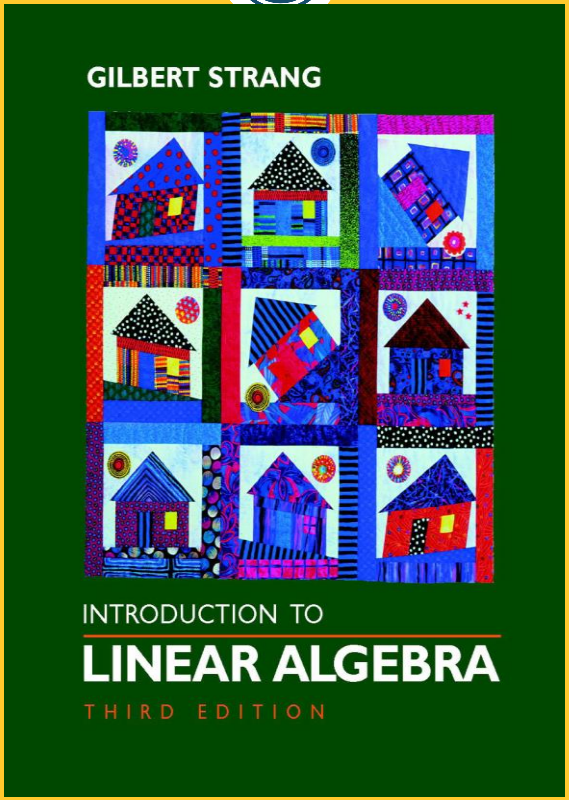
2 3
4 2

Introduction of Homogeneous Coordinates

- By Matrix multiplication all points Transformed except origin and translation can not be achieved.

- Homogeneous Coordinates help for complete transformation .. Rotation, Scaling, Shear, Reflection, translations…
- Homogeneous Coordinates helps in shifting of origin also.

Linear Algebra

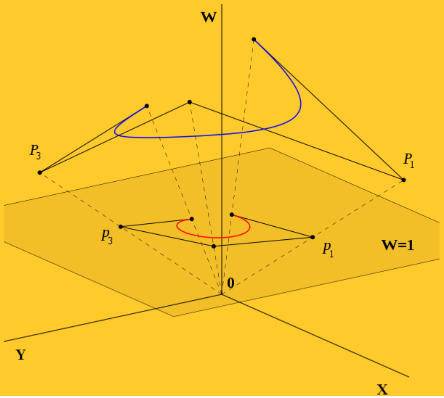


Gilbert Stang also not used Homogeneous Coordinate System

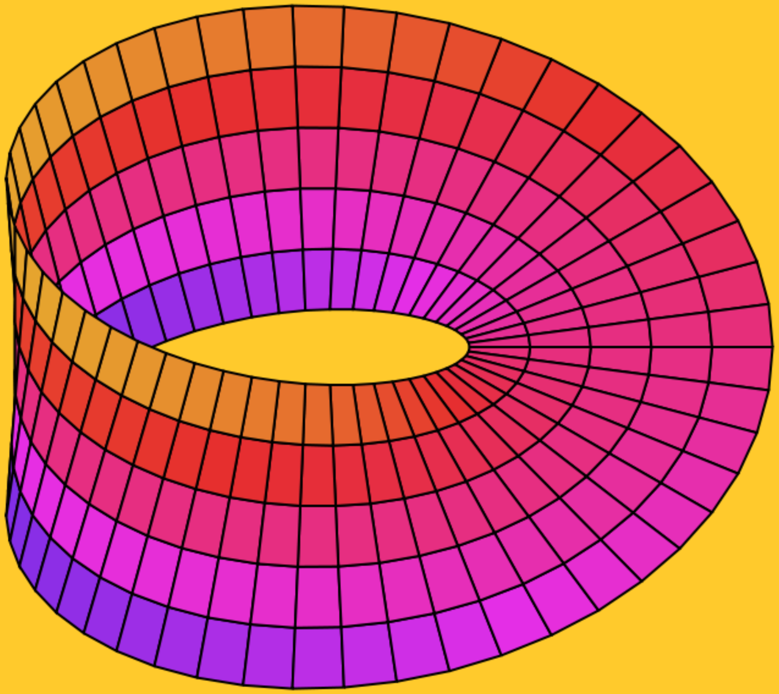
Homogeneous Coordinates

- The number of coordinates required is, in general, +1 than the dimension of the projective space being considered.
- For example, 3 homogeneous coordinates required to specify a point on a projective plane. 4 homogeneous coordinates are required to specify a point on a projective space and so on.
- The origin in homogeneous coordinate system in 2D is (0,0,1) and not (0,0) or (0,0,0).
- If 2D Cartesian coordinate is (x, y), corresponding homogeneous coordinate is (x,y,1).

German mathematicians
August Ferdinand Möbius 1827



From WIKI



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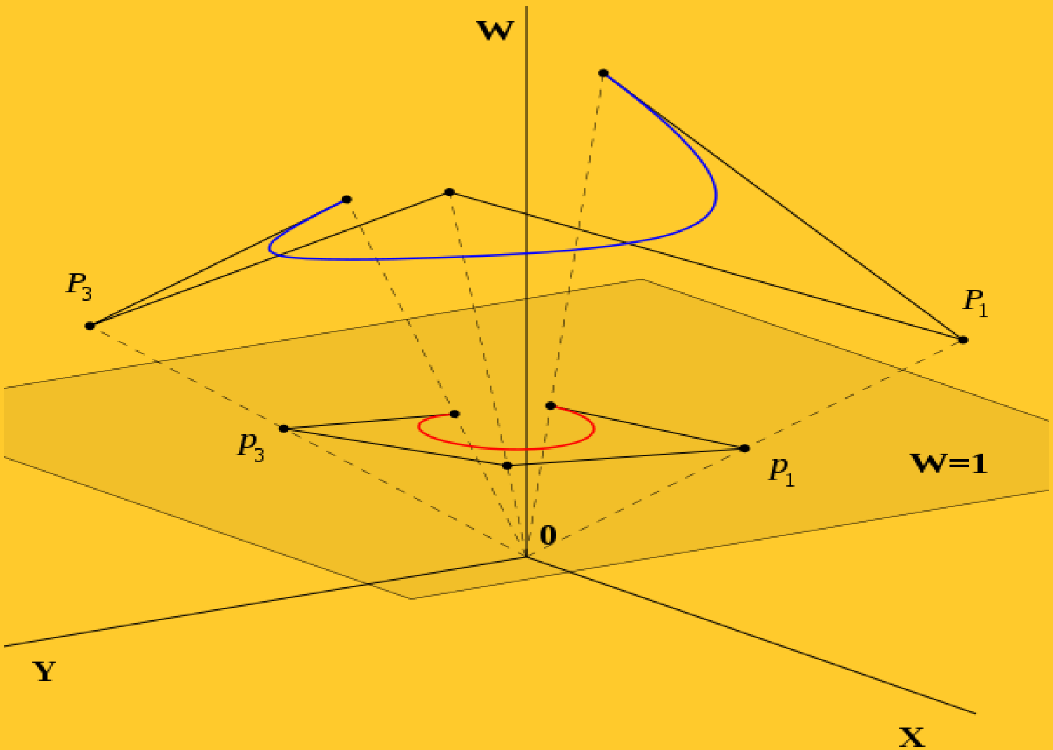
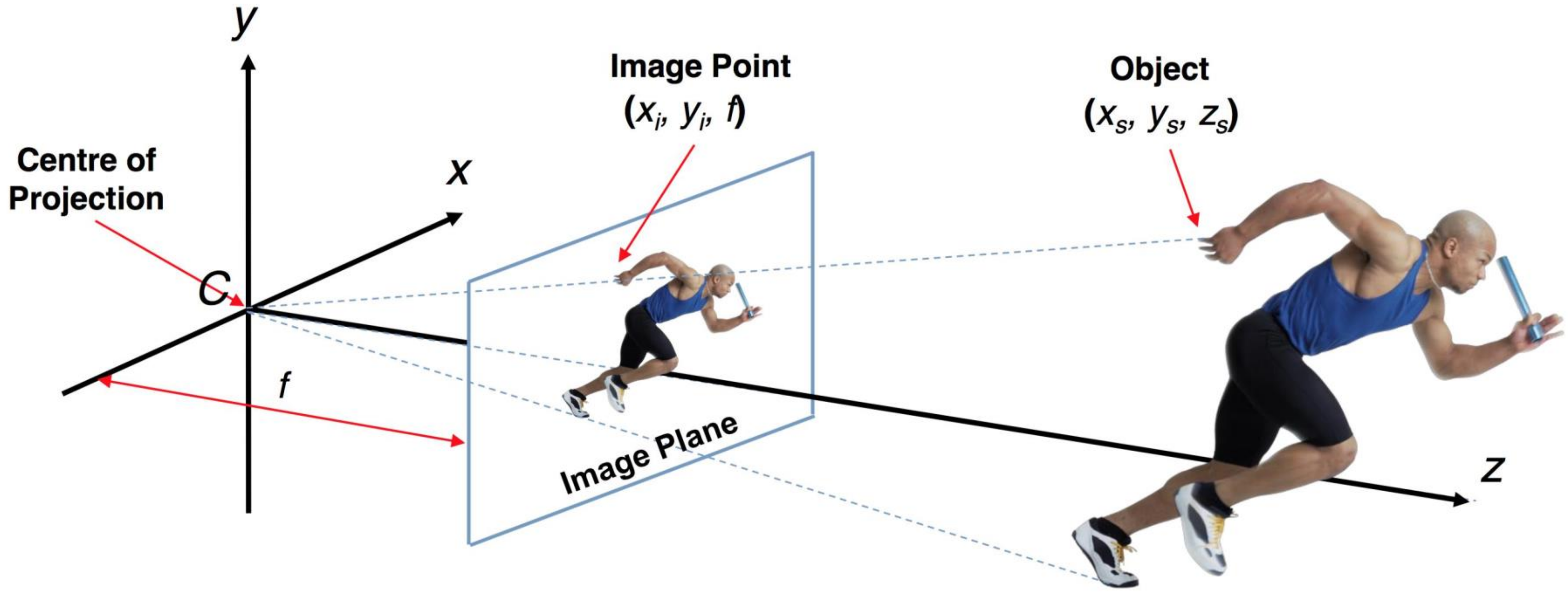


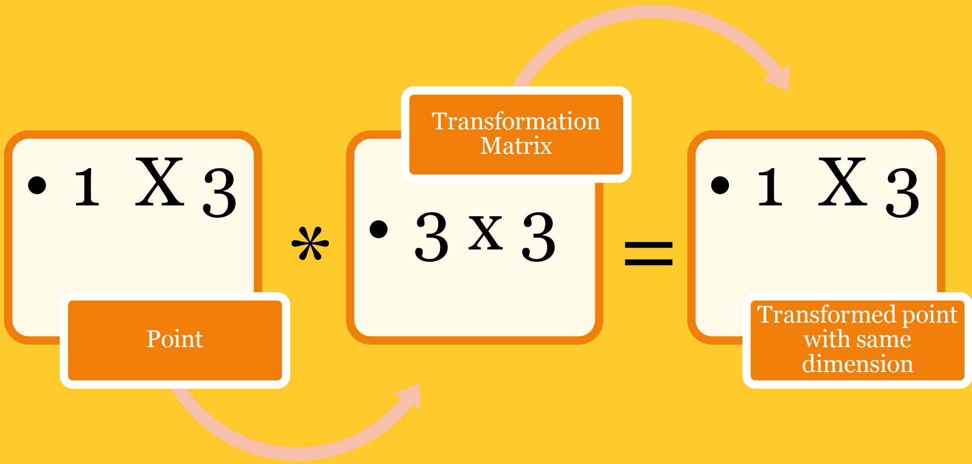
German mathematicians
August Ferdinand Möbius 1827

From Internet



$$x_i = f\frac{x_s}{z_s}, \quad y_i = f\frac{y_s}{z_s}$$

Homogeneous Coordinates



- So for a point (x,y) in 2D system is represented by a point (x,y,1) in homogeneous coordinate system.
- As the point is (1x3) matrix, the transformation matrix , t, is given by t= $\begin{bmatrix} a & b & p \\ c & d & q \\ l & m & s \end{bmatrix}$

Translation

- [5, 3]+[5, 4]=[10, 7]
- Let v=[5,3,1] and t= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$ =[10, 7,1]

  Then vt=[10,7,1]

- [0, 0]+[5,4]=[5,4]
- For origin, v=[0,0,1], t= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$ =[5, 4, 1]

  Vt=[5,4,1] indicating that origin has shifted.

Note-Try it in excel.

- **NOTE 16:**

  Homogeneous coordinates help in translation and shifting of all points.

Technique of projection..

- v=[x y 1] t= $\begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix}$

  vt= [x, y, px+qy+1]

- Here h=px+qy+1. We can divide the coordinates of original vector by h=px+qy+1 to bring the points back to the homogeneous plane whose h value is 1.

Projection in Homogeneous Coordinates

$$\begin{bmatrix} 1 & 3 & 1 \\ 4 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 4 & 1 & 6 \end{bmatrix} \Rightarrow h = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

- Now if we change the transformed coordinates to h=1 plane, then we get $\begin{bmatrix} 1/5 & 3/5 & 1 \\ 4/6 & 1/6 & 1 \end{bmatrix}$
- This action of converting h=1 can be termed as PROJECTION.
- Try in Excel for a triangle

Projection
A Geometric interpretation of homogeneous coordinates

- The general 3x3 transformation matrix for 2D homogeneous coordinates can be subdivided into four parts-

- t=
$$\left[\begin{array}{cc|c} a & b & p \\ c & d & q \\ \hline m & n & s \end{array}\right]$$

- The a, b, c, d elements produce scaling, rotation, reflection and shearing,
- m, n produces translation,
- p, q of the third column produces projection.
- s produces scaling

How this knowledge help in learning Mathematics

- Now we can create any object – in 2-dimension as well as 3 – dimensions.
- We can transform these objects
- We can create graphs of one variable, two variable functions.
- We can demonstrate Geometry, Trigonometry, Algebra, Coordinate Geometry, Vectors, Complex Functions easily.
- We can demonstrate all mathematical concepts related to any branch of mathematics and remove the abstractness of mathematics.

Coordinate Systems

- Quadrants: The axes of a two-dimensional Cartesian system divide the plane into four infinite regions, called quadrants, each bounded by two half-axes.

2nd Quadrants,
--+

1st Quadrants,
++

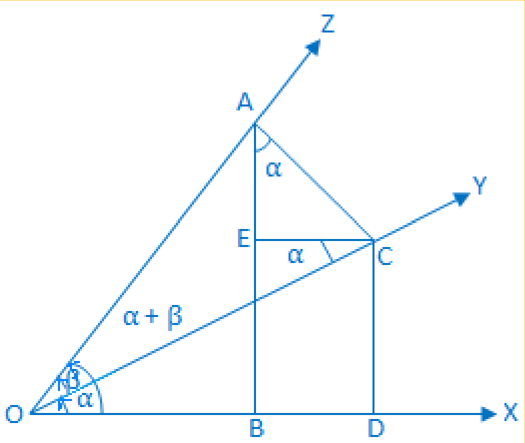
3rd Quadrants, -
--

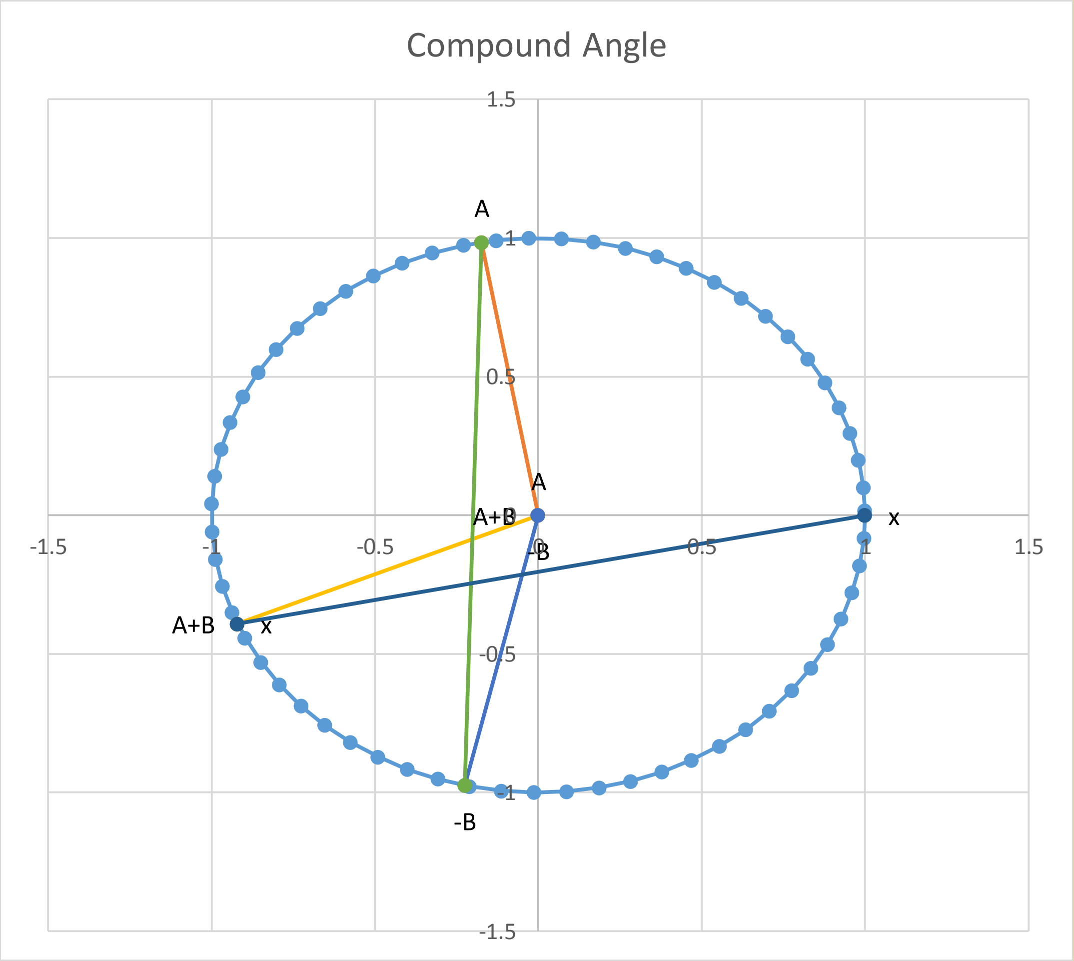
4th Quadrants,
+-

Compound Angle identities

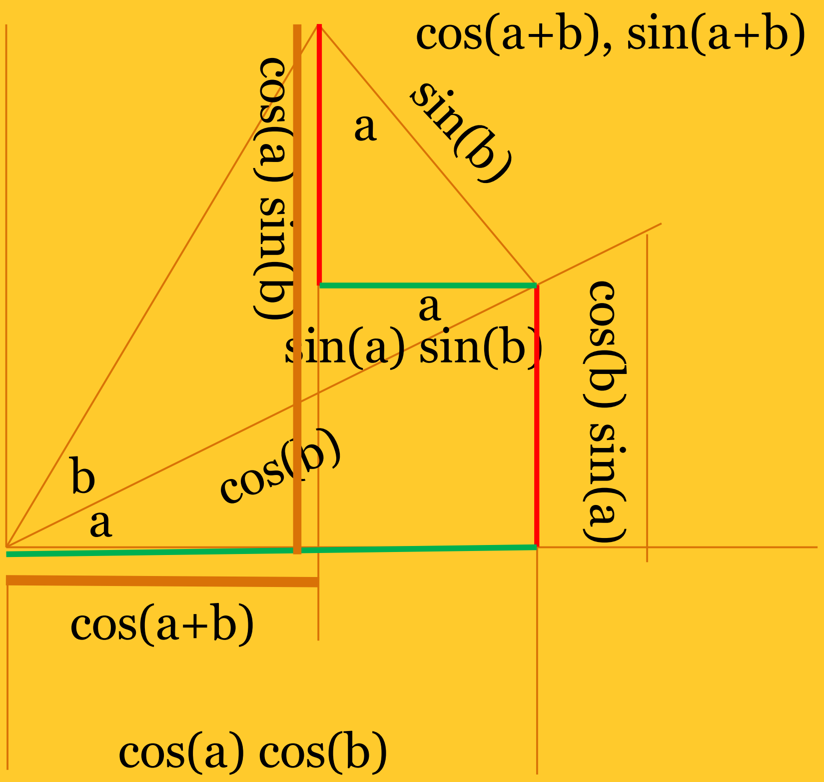
We equate lengths

A-B,XA+B





Compound Angle identities



cos(a+b)=cos(a)cos(b)-sin(a)sin(b)
When a=b,
cos(a+a)=cos(a)cos(a)-sin(a)sin(a)
cos(2a)=cos^2(a)-sin^2(a)

sin(a+b)=sin(a)cos(b)+cos(a)sin(b),

When a=b,
sin(a+a)=sin(a)cos(a)+cos(a)sin(a)
sin(2a)=2 sin(a)cos(a)

Remember the geometry, not the formula

Animating a function and it's Derivative

- Excel

*y=x^3+3*x^2+2*x+5*

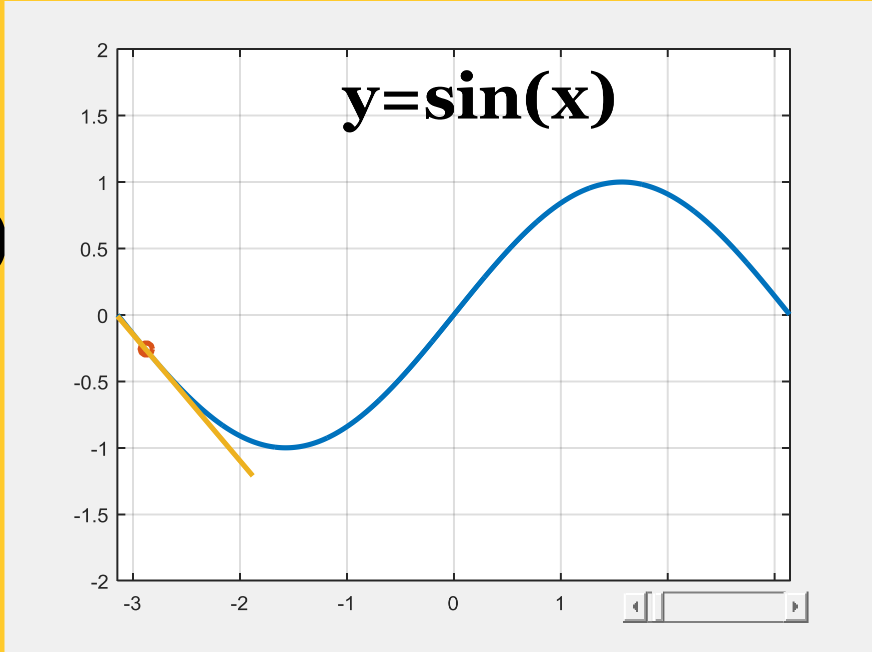
Find dydx at x=3

*y=3*x^2+6*x+2*

Find the area below the curve for x=0 to 5

Construction Methods and Animations

- Function of Single Variable, y=sin(x)
- t=-pi:.01:pi;
- x=sin(t)
-
- plot(t,x,'LineWidth',2.5)
- axis([-pi pi -2 2])
- grid

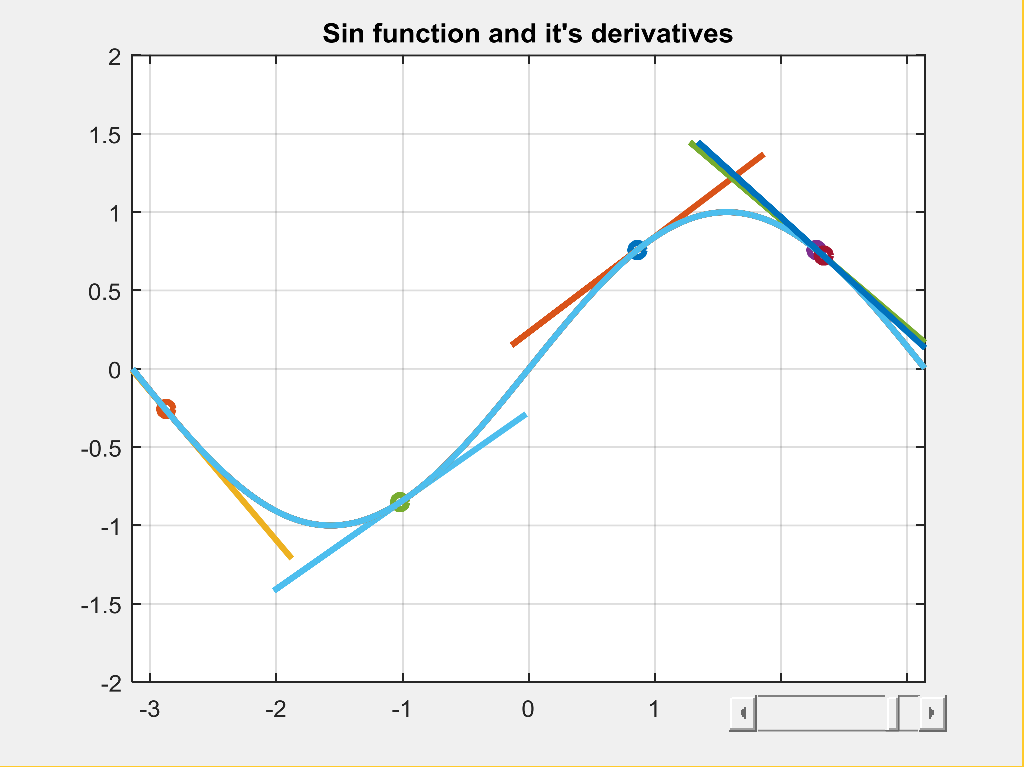


Putting a Slider in Script file

- Animating derivative of a Function of Single Variable
- % 1. Create a figure and axes
- f = figure()
- ax=axis()
- % 2. Create slider
- sld = uicontrol('Style', 'slider','Min',-pi,'Max',pi,'Value',-pi+.2,...
- 'Position', [400 20 120 20],'Callback', @sldcall);
-
- % 3. Create a call back
- function sldcall(source,event)
- val = get(source,'Value')

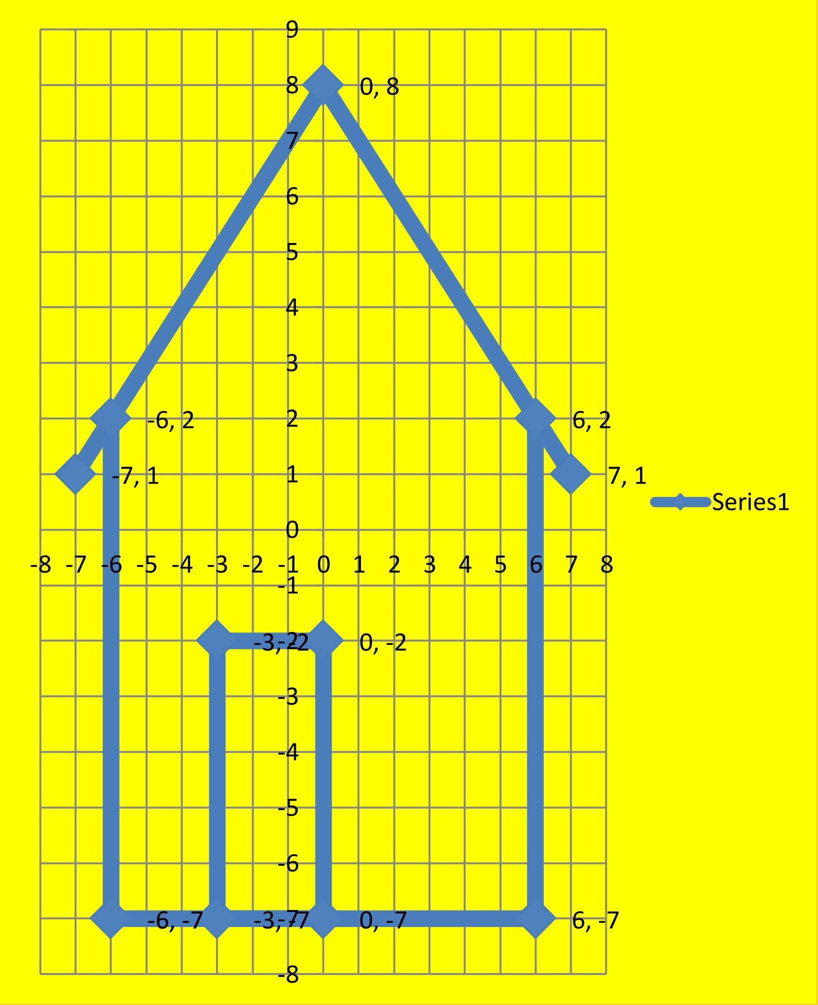
Animating Derivative of Sin function


```
plot(t,x,'LineWidth',2.5)
   axis([-pi pi -2 2])
   grid
   hold on
   x1=val
   y1=sin(x1)
  plot(x1,y1,'o','LineWidth',2.5)
  x2=x1+.1
  y2=sin(x2)
  m=(y2-y1)/(x2-x1)
  c=y1-m*x1

  x3=x1-1
  x4=x1+1
  y3=x3*m+c
  y4=x4*m+c
  plot([x1,x3,x4],[y1,y3,y4],'LineWidth',2.5)
```

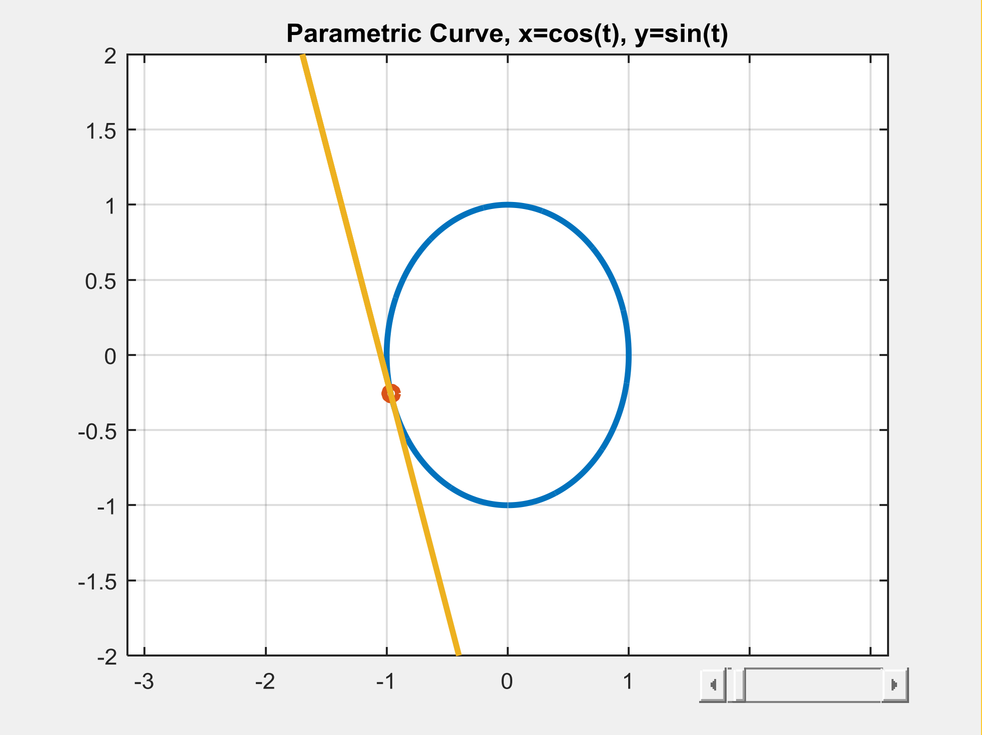


Creating a House

Vertex	x	y
1	-6	-7
2	-6	2
3	-7	1
4	0	8
5	7	1
6	6	2
7	6	-7
8	-3	-7
9	-3	-2
10	0	-2
11	0	-7
12	-6	-7



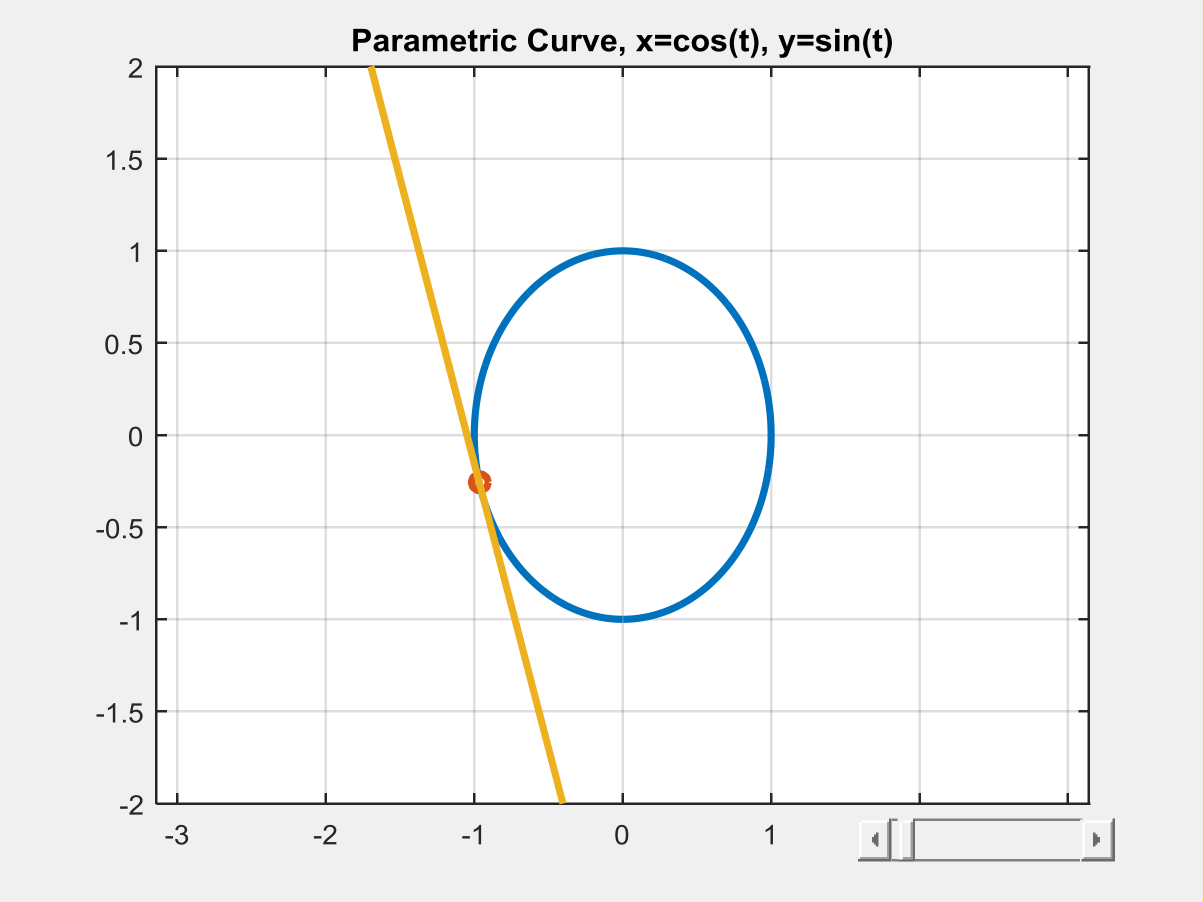
Parametric Curve with Derivative



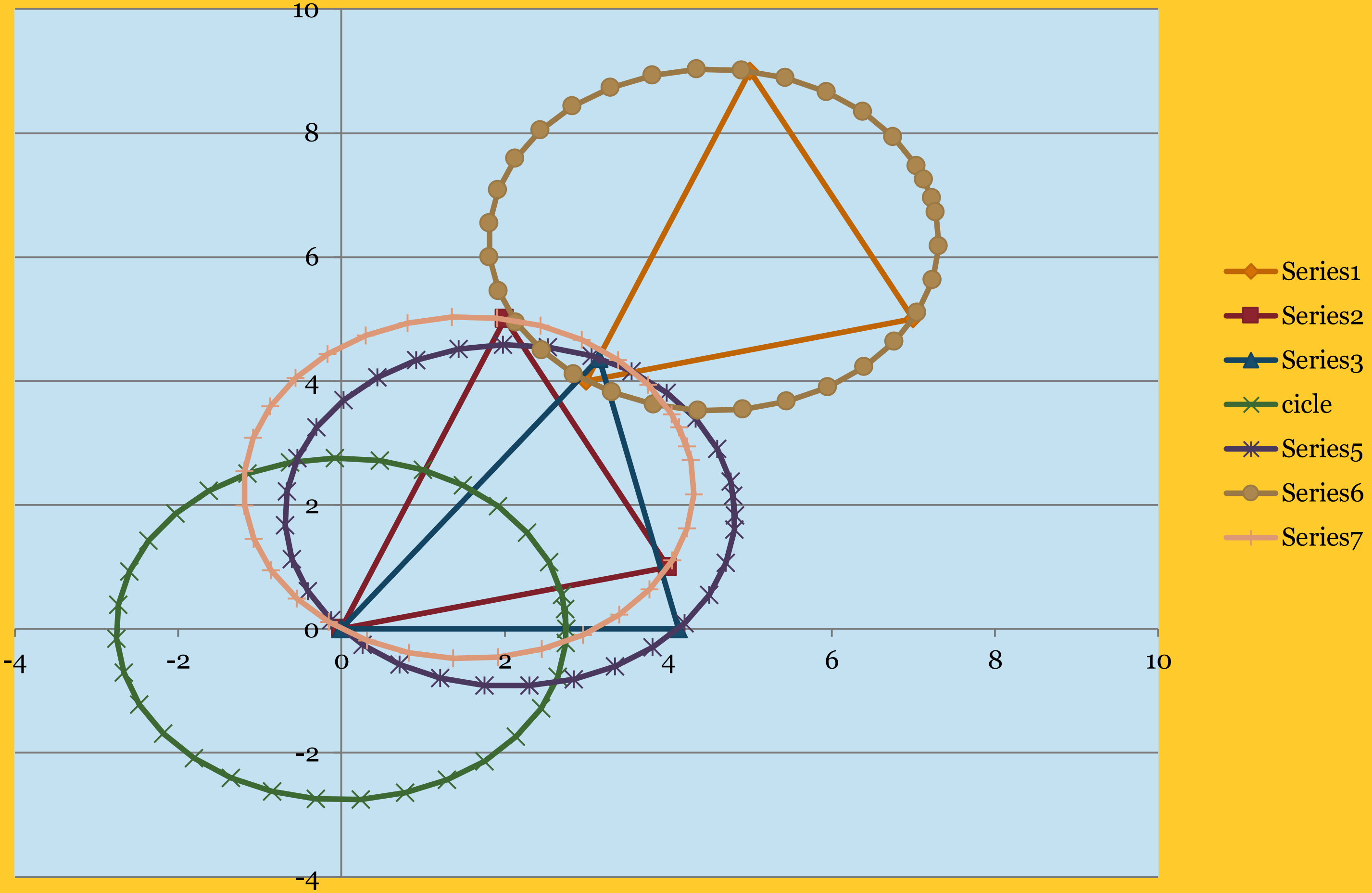

```
% 4. Write the code
   t=-pi:.01:pi;
   x=sin(t)
   y=cos(t)
      plot(x,y,'LineWidth',2.5)
     axis([-pi pi -2 2])
     grid
     hold on
     t=val
     x1=cos(t)
     y1=sin(t)
   plot(x1,y1,'o','LineWidth',2.5)
plot([x1,x3,x4],[y1,y3,y4],'LineWidth',2.5)
```

Parametric Curve with Derivative

x2=cos(t+.1)
y2=sin(t+.1)
m=(y2-y1)/(x2-x1)
c=y1-m*x1
x3=x1-1
x4=x1+1
y3=x3*m+c
y4=x4*m+c



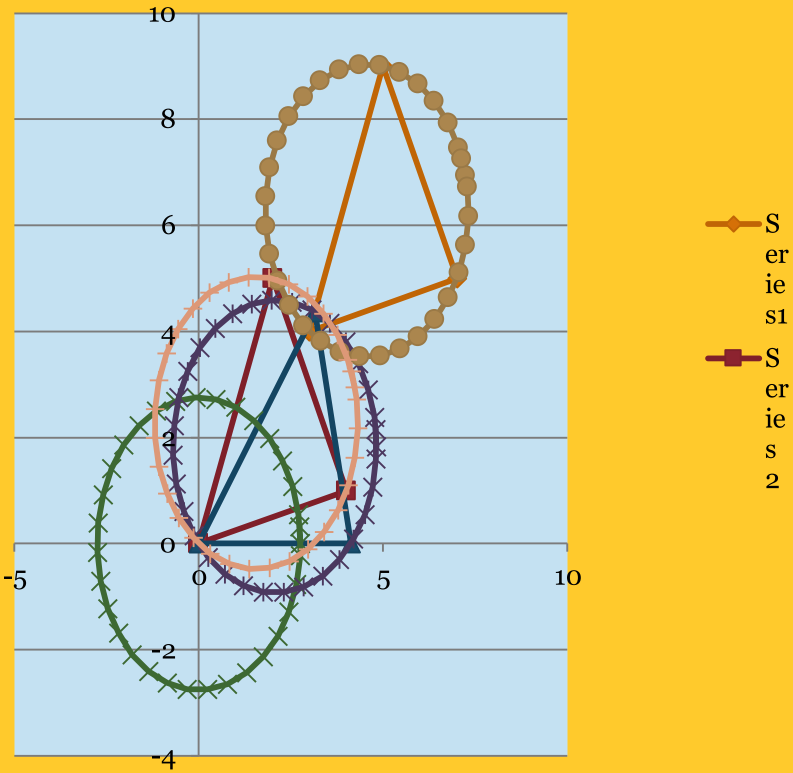
Creating a Circle from Three Point



Creating a Circle from Three Point

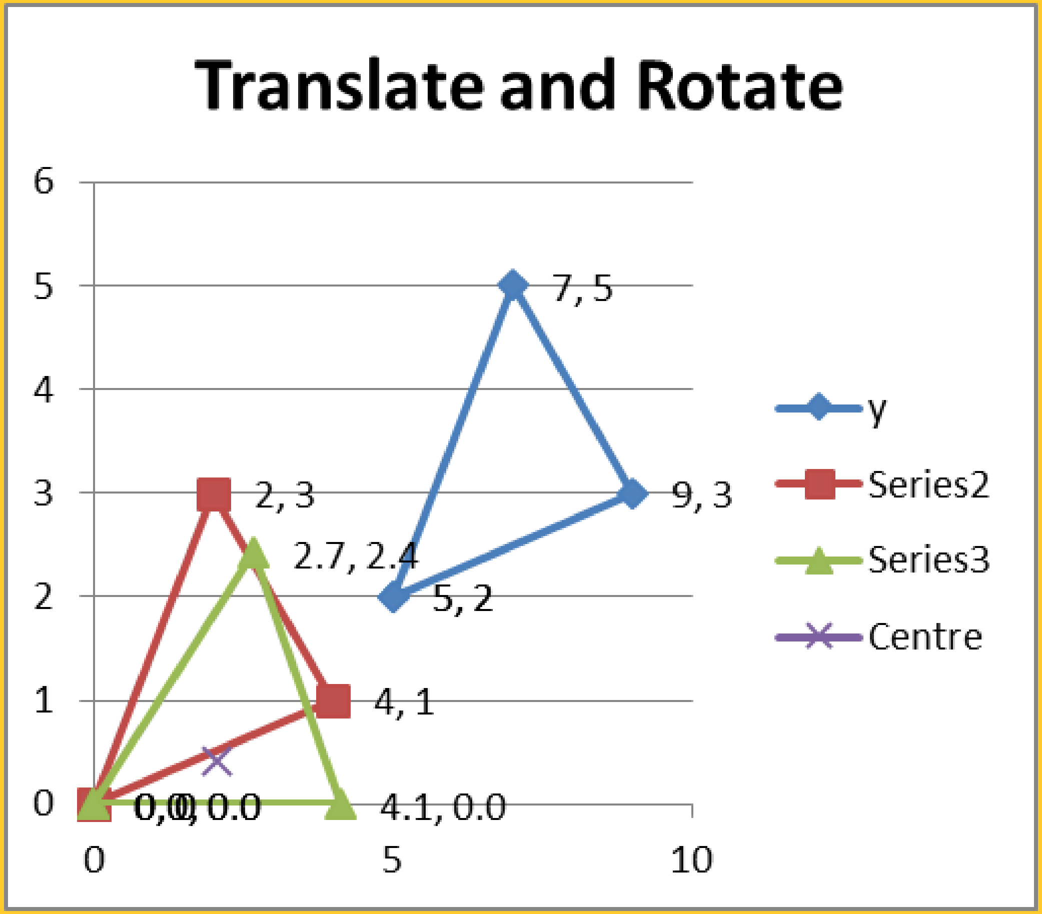
- Step-1: Find the center (h, k) of the circle from three given points.
- If we directly go for solving this, it will be very complex task.
- We try it graphically.

1. Translate all point so that one point coincide with origin
2. Rotate so that other point coincide with x-axis.
3. Calculate h=x2/2,
4. Calculate k=x3/2y3(x3-x2)+y3/2
5. Calculate r =sqrt(h^2+k^2)
6. Draw circle



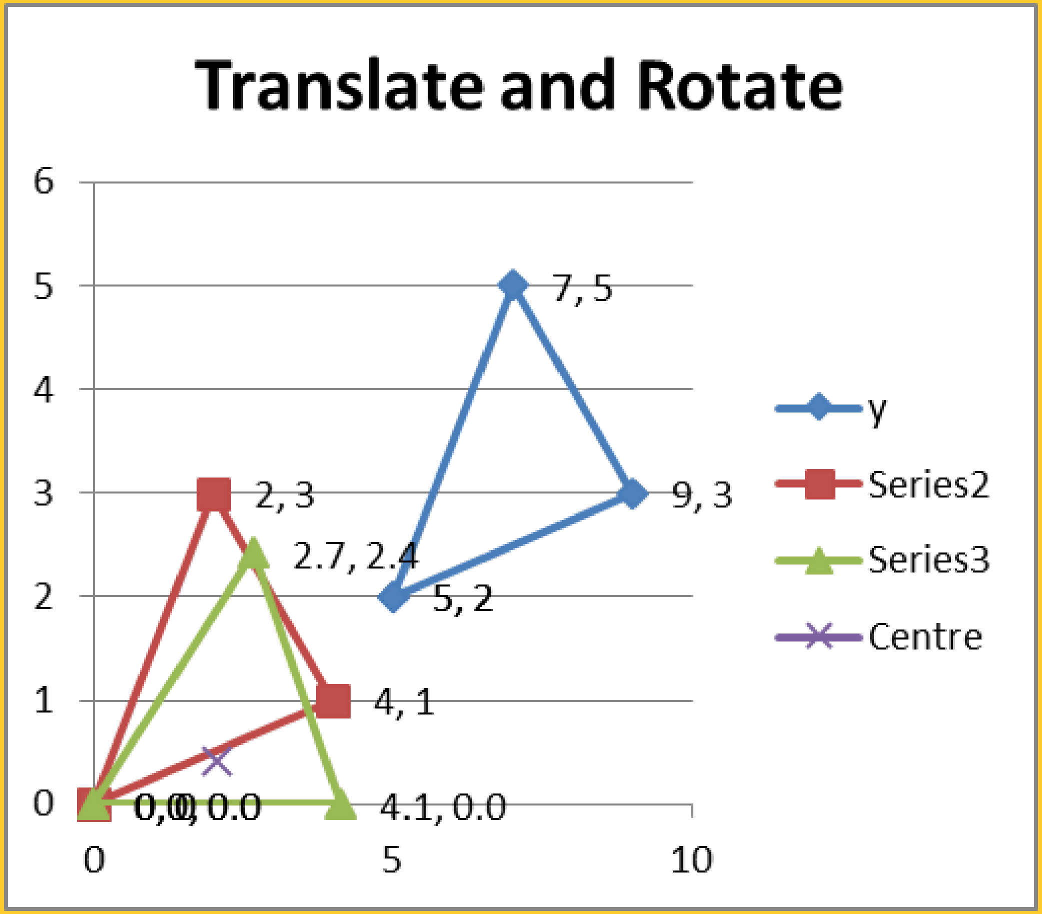
Creating a Circle from Three Point


```
%1. Draw The Points
p1=[5,2]
p2=[9,3]
p3=[7,5]
plot([p1(1)],[ p1(2)],'o')
hold on
plot([p2(1)],[ p2(2)],'o')
plot([p3(1)],[ p3(2)],'o')
grid

%2 Create the Triangle
x=[p1(1), p2(1), p3(1),p1(1)]

y=[p1(2), p2(2) p3(2),p1(2)]
o=[1 1 1 1]
plot(x,y)
axis([-2 10 -2 10 ])
```



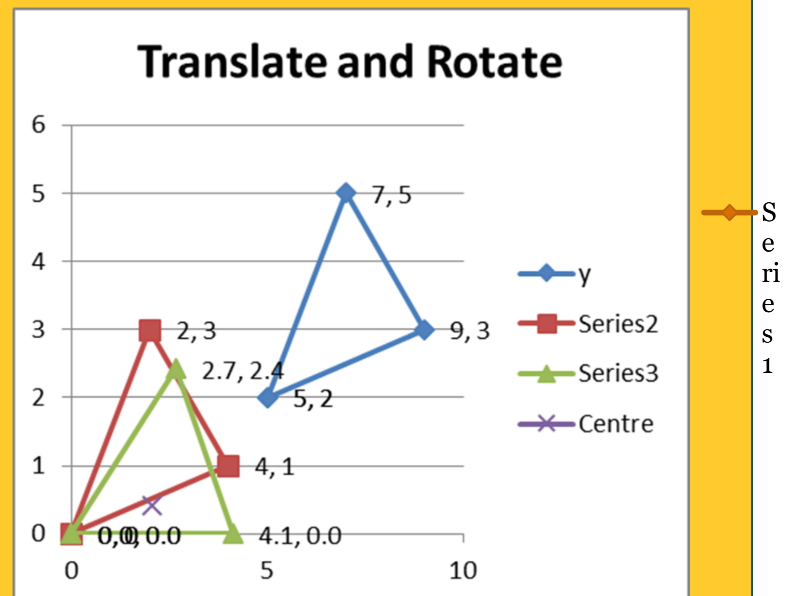
Creating a Circle from Three Point


```
%1. Draw The Points
p1=[5,2]
p2=[9,3]
p3=[7,5]
plot([p1(1)],[ p1(2)],'o')
hold on
plot([p2(1)],[ p2(2)],'o')
plot([p3(1)],[ p3(2)],'o')
grid

%2 Create the Triangle
x=[p1(1), p2(1), p3(1),p1(1)]

y=[p1(2), p2(2) p3(2),p1(2)]
o=[1 1 1 1]
plot(x,y)
axis([-2 10 -2 10 ])
```



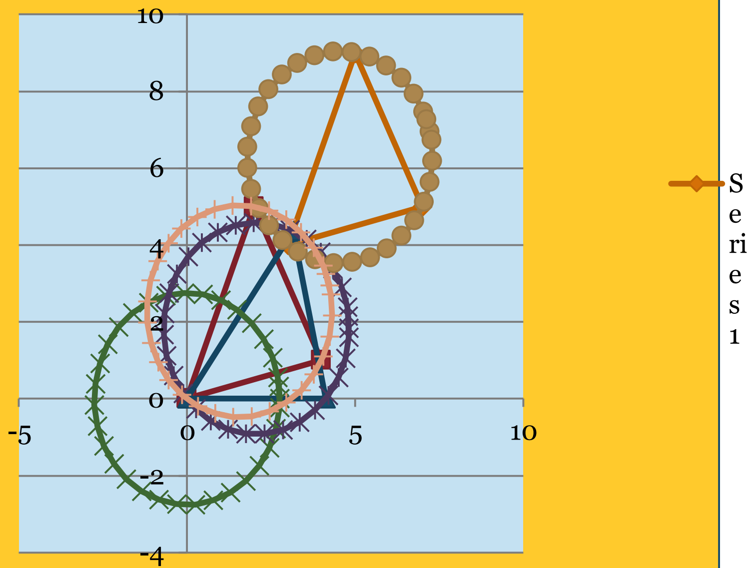
Creating a Circle from Three Point


```
%3Translate to Origin
m=[x;y;o]'
tt=[1 0 0; 0 1 0; -5 -2 1]
itt=inv(tt)
mt=m*tt
   x=mt( : , 1 )'
   y=mt( : , 2)'
plot(x,y)
tant=(mt(2,2)-mt(1,2))/(mt(2,1)-mt(1,1))
t=atan(tant)
```



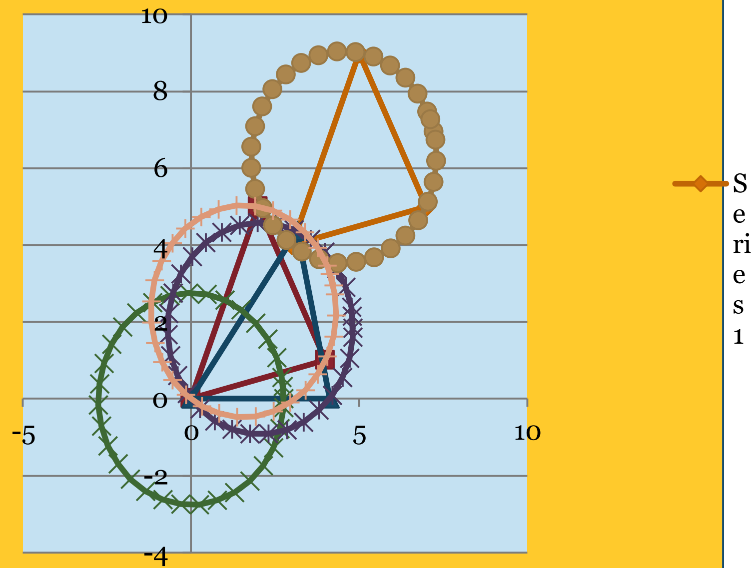
Creating a Circle from Three Point

%4Rotation
tr=[cos(t) -sin(t) 0;sin(t) cos(t) 0;0 0 1]
mr=mt*tr
itr=inv(tr)
x=mr(:, 1)'
y=mr(: , 2)'
plot(x,y)



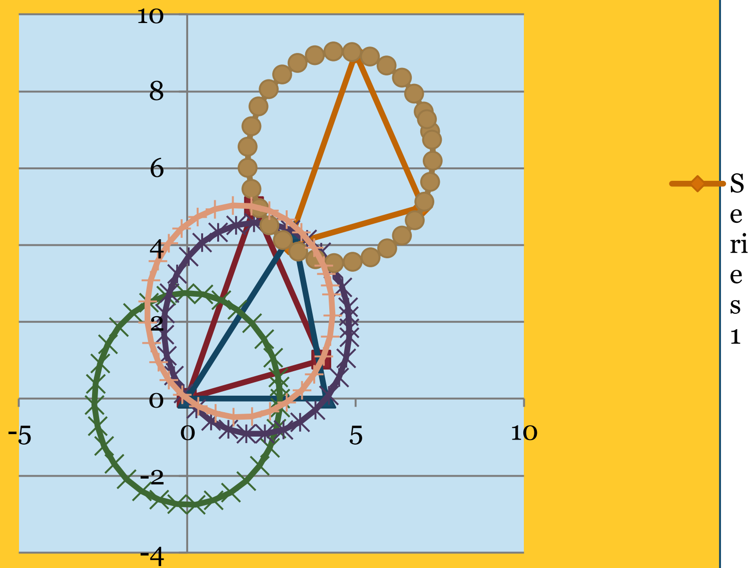
Creating a Circle from Three Point


```
%5Calculate the centre, h and
k and draw the circle
h=mr(2,1)/2
k=(mr(3,1)/(2*mr(3,2)))*(mr(
3,1)-mr(2,1))+mr(3,2)/2
r=sqrt(h^2+k^2)
t=0:.1:2*pi
x=r*cos(t)
y=r*sin(t)
h=y*0+1
plot(x,y)
```



Creating a Circle from Three Point

%6Placing the circle at h and k
m=[x;y;h1]'
m=m*[1 0 0; 0 1 0; 2.0616 .4123 1]
%7Placing the circle to original position
newm=m*itr*itt
x=newm(: ,1)
y=newm(: ,2)
plot(x,y)
p=[1 0 0 ; 0 1 0 ; h k 1]
newp=p*itr*itt
plot(newp(3,1), newp(3,2),'o')



Creating a Circle from Three Point

m =

5	2	1
9	3	1
7	5	1
5	2	1

tt =

1	0	0
0	1	0
-5	-2	1

mt =

0	0	1
4	1	1
2	3	1
0	0	1

tant = 0.2500; t = 0.2450

tr =

0.9701	-0.2425	0
0.2425	0.9701	0
0	0	1.0000

mr =

0	0	1.0000
4.1231	0	1.0000
2.6679	2.4254	1.0000
0	0	1.0000

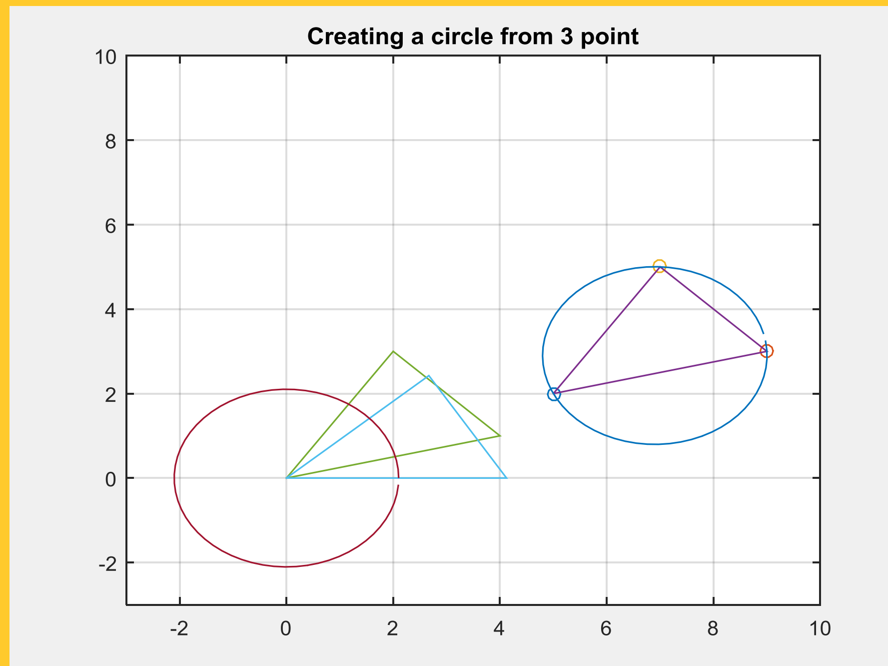
Creating a Circle from Three Point

mr =

0	0	1.0000
4.1231	0	1.0000
2.6679	2.4254	1.0000
0	0	1.0000

h = 2.0616

k = 0.4123



Creating a Circle from Three Point

mr =

0	0	1.0000
4.1231	0	1.0000
2.6679	2.4254	1.0000
0	0	1.0000

h = 2.0616

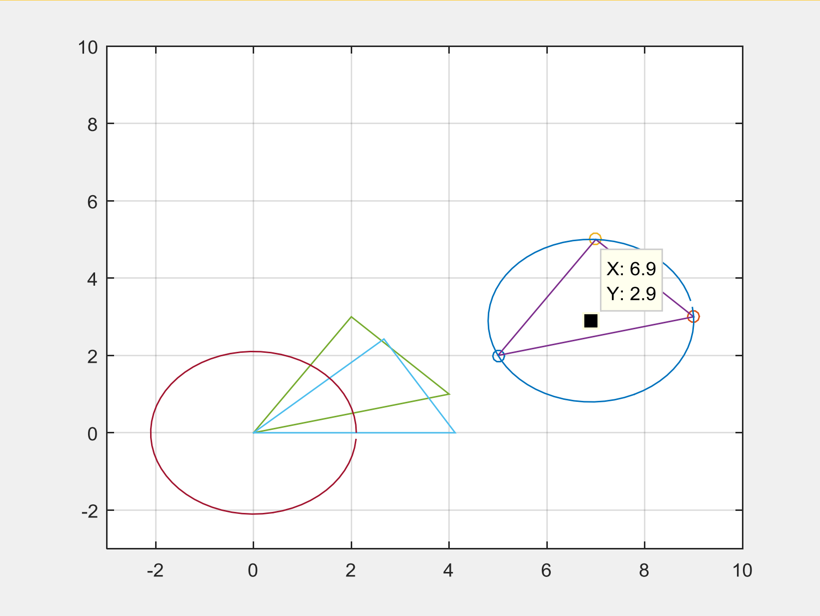
k = 0.4123

p =

1.0000	0	0
0	1.0000	0
2.0616	0.4123	1.0000

newp =

0.9701	0.2425	0
-0.2425	0.9701	0
6.9000	2.9000	1.0000

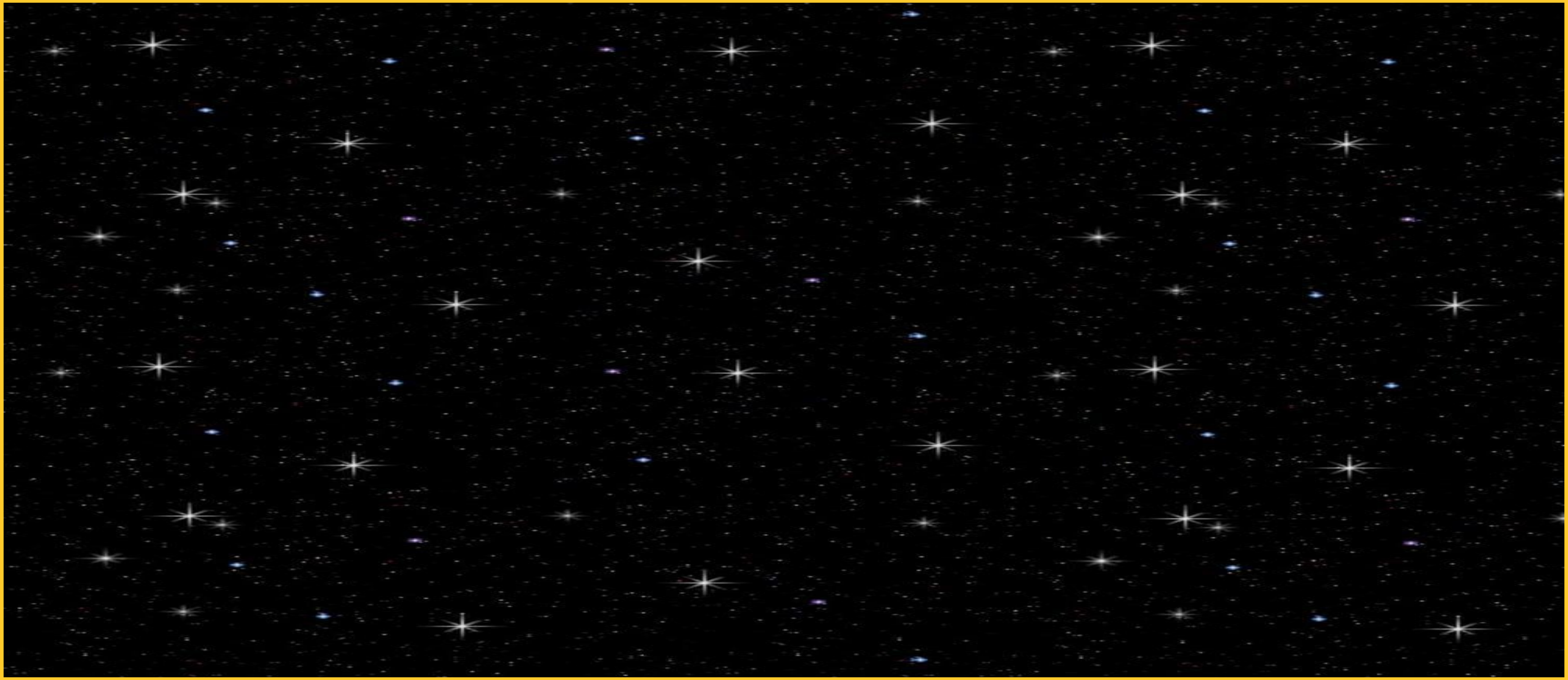


Exploring 3D World

- In this session we will explore three dimensional world around us

Physical World vs Visual World

If I ask , what is this photograph?
You will answer Night sky, Star. I can not differ.
God has given us an incredible gift – our eyes. It can be tiny but it is so powerful that we can see these stars at an infinite distance.



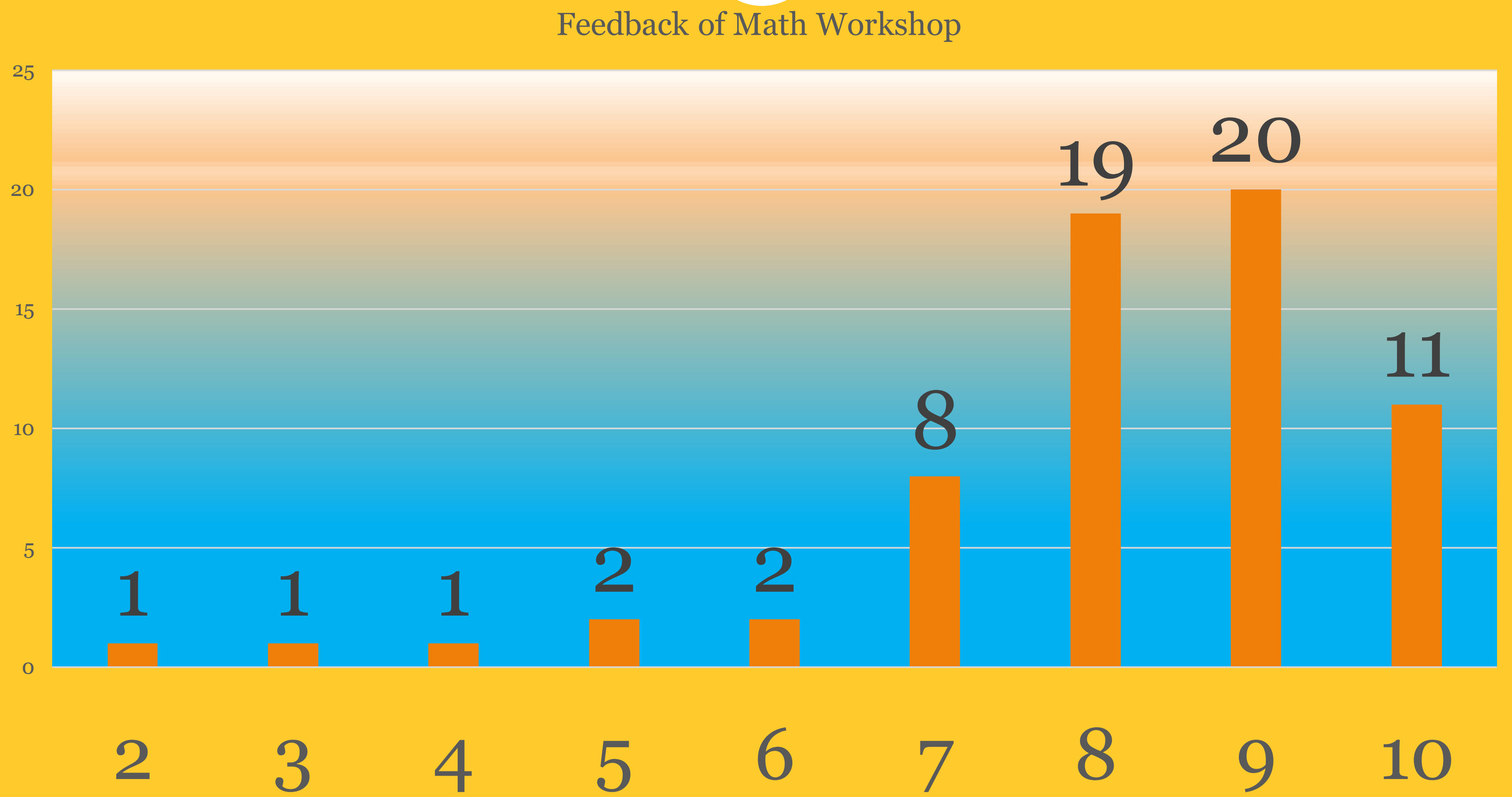
Physical World vs Visual World

- Similarly if I ask you what is this? You will answer, it is railway track. I will say, no it is not a rail track. You will argue. But I will stick in my word.

- And the problem lies here.
- What we see is different from the real world.

Capturing 3 dimensional world

Feedback Day-1 & 2



Three Dimensional Transformations

- Transformation matrix for Three Dimensional transformation in Cartesian coordinate is
- Point= $\alpha = [x, y, z]$

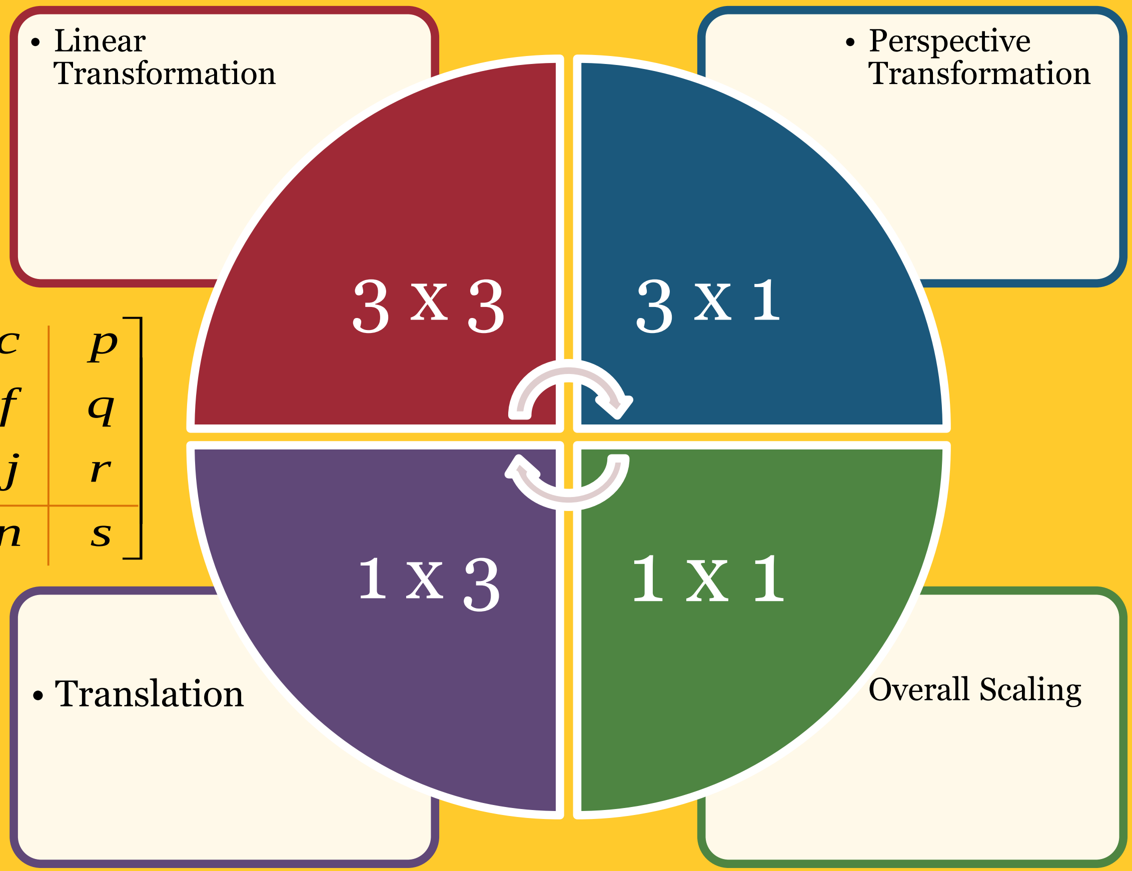
t= $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

- The equivalent homogeneous coordinates in 3d is
- Point $= \alpha = [x, y, z, h]$

[T]= $\begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ l & m & n & s \end{bmatrix}$

Three Dimensional Transformation

$$[T]=\left[\begin{array}{ccc|c} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ \hline l & m & n & s \end{array}\right]$$



Three Dimensional Scaling (Local)

- ts= $\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ produces local scaling about x, y, z coordinate axis.

- ts= $\begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Reduces x coordinate by ½ and y coordinate by 1/3 and z unchanged.

Three Dimensional Scaling (Overall)

- ts= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$
- When s<1, h reduced to less than 1, converting h=1 produces enlargement.
- When s>1, h>1, when h is made equal to 1, produces compression.
- Main diagonal produces scaling
- The overall scaling can also be achieved by means of uniform local scaling factor 1/s.

Three Dimensional Shearing

- The off diagonal terms of 3x3 upper left sub matrix of the generalized 4x4 transformation matrix produces shearing.
- $t_{sh} = \begin{bmatrix} 1 & b & e & 0 \\ c & 1 & f & 0 \\ g & i & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is a shear matrix.
- The origin remains unaffected.

3 Dimensional Rotation

- There is no formula for 3d rotation
- We play a trick to achieve 3d rotation
- 2D rotation formula for rotation will be used

Three Dimensional Rotation

- Before considering three dimensional rotation about an arbitrary axis, let us examine rotation about x axis. For rotation about x-axis, the x coordinates of the position vectors do not change and the transformation matrix is given by

$$t=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Contd..

- The rotation about Z-axis is:

$$t = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The rotation about Y-axis is:

$$t = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The requirement of pure rotation is determinant=+1

Three Dimensional Reflection

- The reflection occurs through a plane. During the reflection through XY plane, the Z coordinate value reversed in sign.
- $tz = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for XY Plane
- $tx = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for YZ Plane
- $ty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for XZ Plane

Three Dimensional Translation

$$[Tr] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix}$$

Multiple Transformations

- Successive transformations can be combined or concatenated into a single 4x4 transformation that yields the same results. Since matrix methods are non-commutative, the order of multiplication is important ([A] [B] ≠ [B] [A]).

Visualization

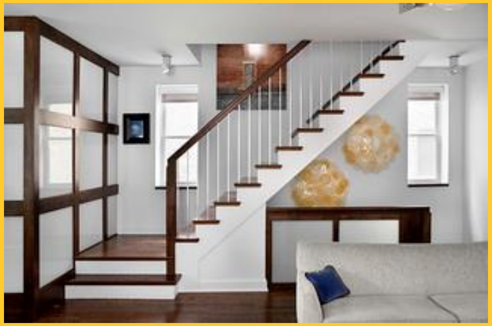
We live in a three dimensional world. The world is made of different types of object. These objects can be of different dimensions.

Some may be plane figures others may be solid shape. Same objects looks different when seen from different angles and distances. Hence understanding of visualization process or visualization technique is very important.

Visualization

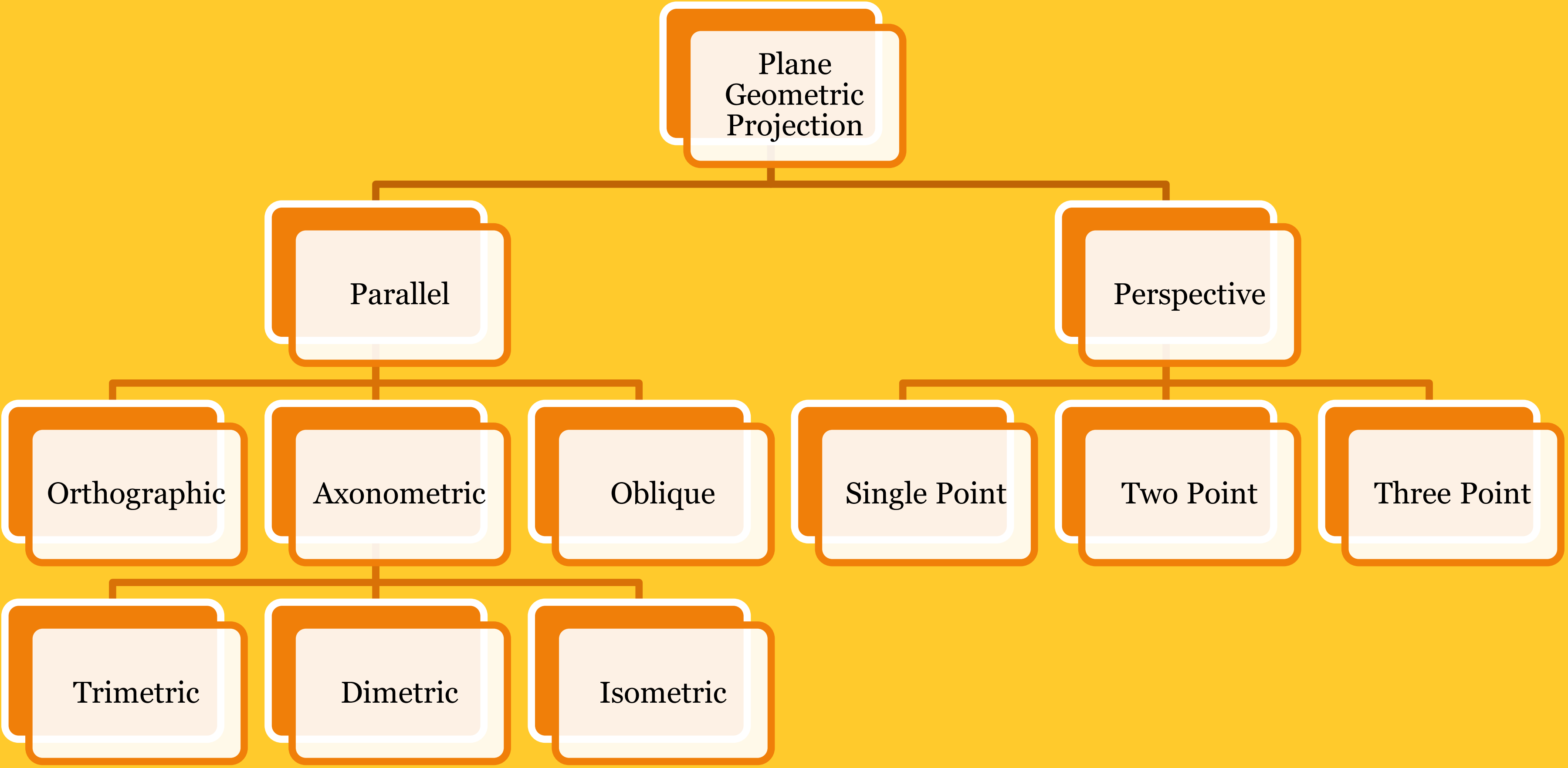
SL	CLASS	CHAPTER	TITLE
1	III	1	Where to look from (Visualization and Pattern)
2	VII	15	Visualizing solid shapes

Visualization

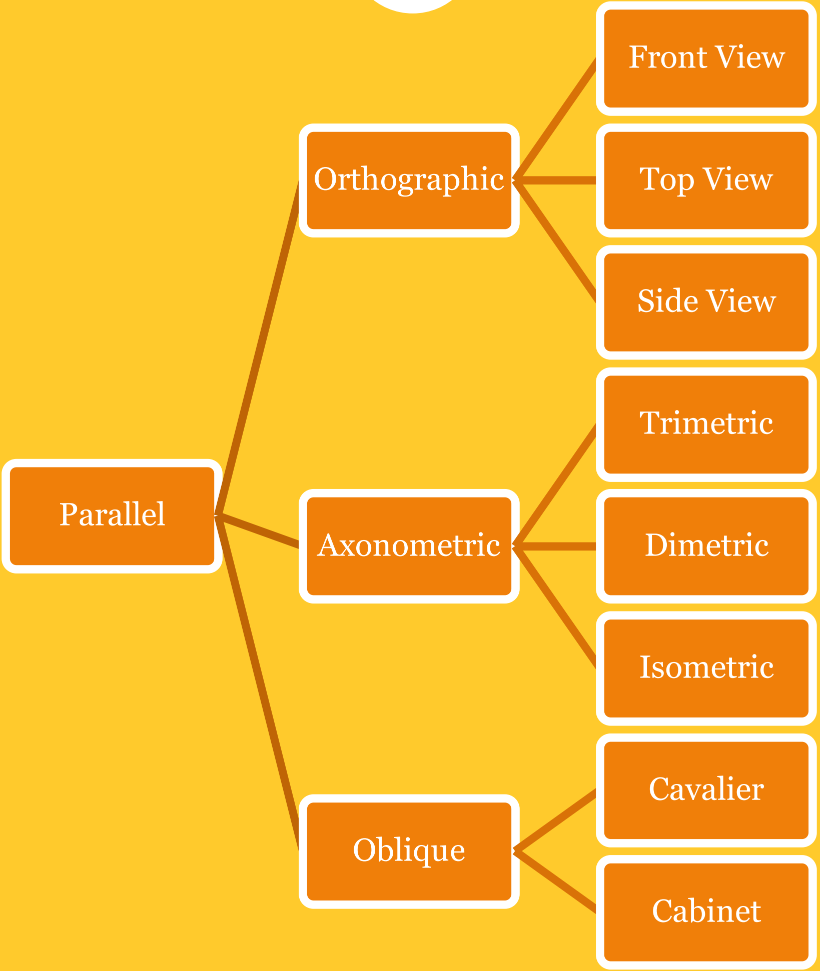


Plane Geometric Projection

Type of Projections



Type of Parallel Projections



Orthographic Projections

- It is the projection where one of the coordinate plane is zero.
- The matrix for projection into x=0, y=0, z=0 are given below:
- $$P_X = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Parallel Projection-Axonometric

- In orthographic projection we can view only one face. In axonometric projection at least three adjacent faces are shown.
- This is achieved by rotation, translation and then projection in one plane, generally z=0 plane.
- The center of projection is at infinity. As the faces are not parallel to the plane of projection, the projection does not show true shape.
- However, parallel lines equally fore shortened.
- The fore shortening factor is the ratio of the projected length to the true length.
- There are three types of Axonometric projection:

  (1) Trimetric (2) Dimetric and (3) Isometric

Axonometric Projections

- An axonometric projection is used to show three adjacent faces of an object.

- After a rotation with few degree, translations to a desired distance, the result is then projected from a centre of projection at infinity onto one of the coordinate planes, usually z=0 plane or xy plane.

Parallel Projection-Axonometric-Trimetric

- Axonometric projection is formed by first rotating the object in y direction, then by x-direction followed by projection in z=0 plane.
- In Trimetric Projection, the foreshortening factor in x , y and z direction is different.
- The Projection matrix is:

$$\text{Proty}=\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Protx}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\text{Pz}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{T=Proty*Protx*Pz},$$

Parallel Projection-Axonometric-Trimetric

- In Trimetric Projection, the foreshortening factor in x , y and z direction is calculated by applying the concatenated transformation matrix to the unit vectors along the principal axes.
- The Projection matrix is:

$$Proty=\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Protx=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Pz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T=Proty*Protx*Pz, U=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Parallel Projection-Axonometric-Trimetric

- In Trimetric Projection, the foreshortening factor in x , y and z direction is calculated by applying the concatenated transformation matrix to the unit vectors along the principal axes.
- Let, the angle of rotation in y is p, and the angle of rotation in x is t.
- The Projection matrix is:

$$Proty=\begin{bmatrix} cp & 0 & sp & 0 \\ 0 & 1 & 0 & 0 \\ -sp & 0 & cp & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Protx=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & ct & st & 0 \\ 0 & -st & ct & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Pz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T=Proty*Protx*Pz,\ T=\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, U=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Parallel Projection-Axonometric-Trimetric

- The calculation of foreshortening factor:

$$\text{Proty}=\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Protx}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Pz}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\text{T=Proty*Protx*Pz, U}=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \text{T}=\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

U*=U*T

$$\text{Fx}=\sqrt{xx^2+xy^2}, \text{Fy}=\sqrt{xy^2+yy^2}, \text{Fz}=\sqrt{xz^2+yz^2}$$

$$\text{Fx}=\sqrt{cp^2+spst^2}, \text{Fy}=\sqrt{0^2+ct^2}, \text{Fz}=\sqrt{sp^2+-cpst^2}$$

Parallel Projection-Axonometric-Dimetric

- In diametric projection, two the three foreshortening factor is equal.
- The third one is arbitrary.

- calculation of foreshortening factor:

Proty= $\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Protx= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Pz= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

T=Proty*Protx*Pz, U= $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T= $\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Original length of the unit vector = 1.

Fx= $\sqrt{xx^2 + xy^2}$, Fy= $\sqrt{xy^2 + yy^2}$, Fz= $\sqrt{xz^2 + yz^2}$

Fx= $\sqrt{cp^2 + spst^2}$, Fy= $\sqrt{0^2 + ct^2}$, Fz= $\sqrt{sp^2 + -cpst^2}$

Parallel Projection-Axonometric-Dimetric

- In diametric projection, two the three foreshortening factor is equal.
- The third one is arbitrary.

- calculation of foreshortening factor:

T=Proty*Protx*Pz, U=$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T=$\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

U*=U*T

- Original length of the unit vector = 1.

Fx=$\sqrt{xx^2 + xy^2}$, Fy=$\sqrt{xy^2 + yy^2}$, Fz=$\sqrt{xz^2 + yz^2}$

Fx=$\sqrt{cp^2 + spst^2}$, Fy=$\sqrt{0^2 + ct^2}$, Fz=$\sqrt{sp^2 + -cpst^2}$

For two foreshortening factors to be equal, p=asin(fz/$\sqrt{2 - fz^2}$)

t=asin(+-fz/$\sqrt{2}$)

Parallel Projection-Axonometric-Dimetric

- Calculation of foreshortening factor:

U*=U*T

- Original length of the unit vector = 1.

Fx=$\sqrt{xx^2 + xy^2}$, Fy=$\sqrt{xy^2 + yy^2}$, Fz=$\sqrt{xz^2 + yz^2}$

Fx=$\sqrt{cp^2 + spst^2}$, Fy=$\sqrt{0^2 + ct^2}$, Fz=$\sqrt{sp^2 + -cpst^2}$

For two foreshortening factors to be equal,

p=asin(+-fz/$\sqrt{2 - fz^2}$)

t=asin(+-fz/$\sqrt{2}$)

- It is mentioned that FF is between 0 to 1.
- For each fz between 0 to 1, there are four possible diametric projections depending on angles p and t.

Parallel Projection-Axonometric-Isometric

- In isometric projection, all three foreshortening factors are equal.
- calculation of foreshortening factor:

T=Proty*Protx*Pz, U= $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T= $\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Original length of the unit vector = 1.

Fx= $\sqrt{xx^2 + xy^2}$, Fy= $\sqrt{xy^2 + yy^2}$, Fz= $\sqrt{xz^2 + yz^2}$

Fx= $\sqrt{cp^2 + spst^2}$, Fy= $\sqrt{0^2 + ct^2}$, Fz= $\sqrt{sp^2 + -cpst^2}$

- For isometric view, Fx=Fy=Fz. This can be achieved when FF=0.8165
- There are four isometric view with p=+_45 degree and t=+-35.26 degree.

Parallel Projection-Oblique Projection

- The center of projection is at infinity.
- The projectors intersect the projection plane at an oblique angle.
- Faces parallel to the plane of projection shows true shape and size.
- Faces that are not parallel to the plane of projection are distorted.

- The Projection matrix is:

$$P\_oblique=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Parallel Projection-Oblique Projection

- In two-dimension the projector P'O can be obtained from PO, where PO is the unit vector by translating P to the point P' at (-a –b 1)

$$P\_oblique=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

- Where a=f*cos(t) and b=f*sin(t) and f is the projected length of the z-axis unit vector, i.e., the foreshortening factor and t is the angle between projected z-axis and x-axis.

Parallel Projection-Oblique Projection

- P_oblique= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,
- Where a=f*cos(t) and b=f*sin(t)
- Let b=angle of projector and the plane of projection. Then b=acot(f)
- When b=90 degree, f=0. When f=1, b=acot(1)=45 degree. This condition is called Cavalier Projection.
- When foreshortening factor is ½, the angle t=acot(1/2)=63.435 degree.

Perspective Transformations

- When any of the first three elements of the fourth column of the homogeneous coordinate is non-zero, a perspective transformation results.
- It is a transformation in one 3d space to another 3d space.
- In perspective transformation parallel lines converge, object size is reduced with increasing distance from the centre of projection and non- uniform foreshortening of the lines in the object as a function of orientation and distance of the object from the centre of projection occurs.
- Center of projection at a finite distance from the Projection Plane.

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Perspective Transformations

- Perspective Transformation Matrix

$$P=[x,y,z,1],\ Tp=\begin{bmatrix}1 & 0 & 0 & p\\ 0 & 1 & 0 & q\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix},\ Tz=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

P*=P*Tp*Tz=[x y 0 px+qy+rz+1]

- The point is to be brought back to the h=1 plane.
- This is done by dividing all elements of

[x y z px+qy+rz+1] by px+qy+rz+1.

Perspective Transformations-Single Point

- Perspective Transformation Matrix

$$P=[x,y,z,1],\ Tp=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix},\ Tz=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$P^*=P*Tp*Tz=[x\ y\ 0\ rz+1]$

- The point is to be brought back to the h=1 plane.
- This is done by dividing all elements of

[x y z rz+1] by rz+1.

Perspective Transformations-Single Point

- The projection of P(x, y, z) in the projection plane is given by P'(x*, y*, z*)
- The relation can be established by,
  - $x^*/zc = x/(zc-z)$, $x^* = x/(1-z/zc)$, Let, $r = -1/zc$, gives, $x^* = x/(1+rz)$
  - $y^*/zc = y/(zc-z)$, $y^* = y/(1-z/zc)$, let $r = -1/zc$, gives, $y^* = y/(1+rz)$
  - The result is same.
  - The perspective projection matrix is T=Tp*Tz.
  - $T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Perspective Transformations-Single Point

- The perspective projection matrix is T=Tp*Tz.
- T= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- This matrix produces perspective projection on to z=0 plane from a center of projection, zc=-1/r on the z-axis.
- Perspective projection occurs in two steps-first is the perspective transformation and then projection.
- The perspective transformation image intersects the z-axis at z=+1/r.
- This intersection point represents the intersection point of the line parallel to z-axis and z-axis at infinity into the finite point at z=+1/r on the z-axis. This point is called vanishing point.
- Vanishing point lies on the opposite side of the plane of projection at an equal distance of 1/r.
- All lines parallel to z axis passes through [0 0 1/r 1] point, the vanishing point.

Perspective Transformations-Single Point

- The perspective projection in x-axis is T=Tp*Tz.
- $T=\begin{bmatrix}1 & 0 & 0 & p\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

and xc=[-1/p 0 0 1], vanishing point = [1/p 0 0 1]

- The perspective projection in y-axis is T=Tp*Tz.
- $T=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & q\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$
- and yc=[0 -1/q 0 1], vanishing point = [0 1/p 0 1]

Perspective Transformations-Two Point

- Single point perspective projection, does not provide adequate perception of the three dimensional shape of the object. Two point perspective projection helps in this direction.

- The perspective projection matrix is T=Tp*Tz.
- Tp= $\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Tz= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, **T=Tp*Tz**
- **This matrix produces perspective projection on to z=0 plane from two center of projection, xc=-1/p on the x-axis and yc=-1/q on y axis..**
- Vanishing point lies on the opposite side of the plane of projection at an equal distance of 1/p and 1/q in x and y axis.
- All lines parallel to x and y axis passes through [1/p 0 0 1] and [0 1/q 0 1] point in respective axes, the vanishing points.

Perspective Transformations-Three Point

- For adequate perception of the three dimensional shape of the object, three point perspective projection is used.
- The perspective projection matrix is T=Tp*Tz.
- $Tp=\begin{bmatrix}1 & 0 & 0 & p\\ 0 & 1 & 0 & q\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix}, Tz=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$, **T=Tp*Tz**
- **This matrix produces perspective projection on to z=0 plane from three center of projection, xc=-1/p on the x-axis and yc=-1/q on y axis and zc=-1/r in z-axis.**
- Vanishing point lies on the opposite side of the plane of projection at an equal distance of 1/p, 1/q and 1/r in x, y and z axis.
- All lines parallel to x, y and z axis passes through [1/p 0 0 1], [0 1/q 0 1] and [0 0 1/r 1] point in respective axes, the vanishing points.

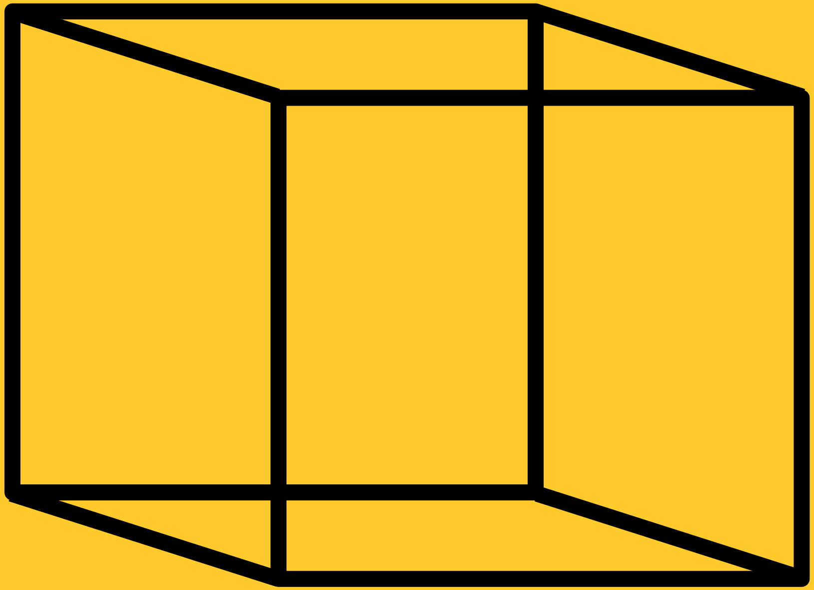
Size of Transformation Matrix

- 2D Plane – The transformation matrix for a 2d object is a 3x3 matrix
- 3D Space – The transformation matrix of a 3d object is a 4x4 matrix
- The number of points representing the object can be infinite but the transformation matrix is only 3x3 or 4x4 matrix.

Capturing 3 dimensional world

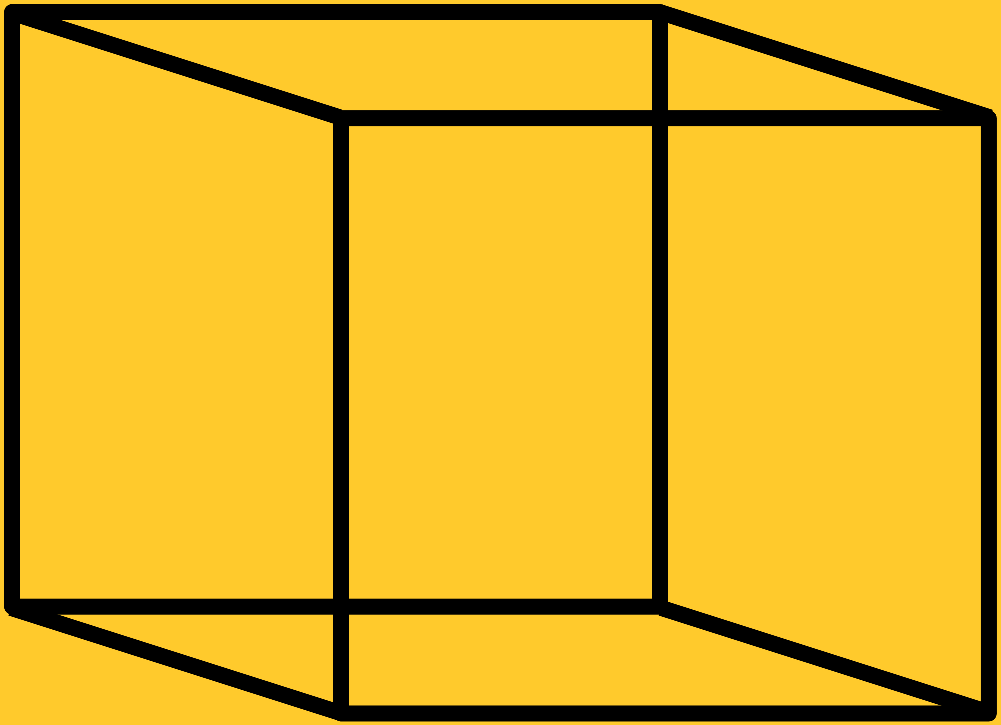
Physical World vs Visual World

Similarly if I ask you what is this? You will answer cube. I will say, no it is not cube. You will argue. But I will stick in my word.



And the problem lies here.

A 3d cube in 2d



With the projection tool available to us, now we can create 3d objects in 2d Plane

Creating a cuboid in 3d

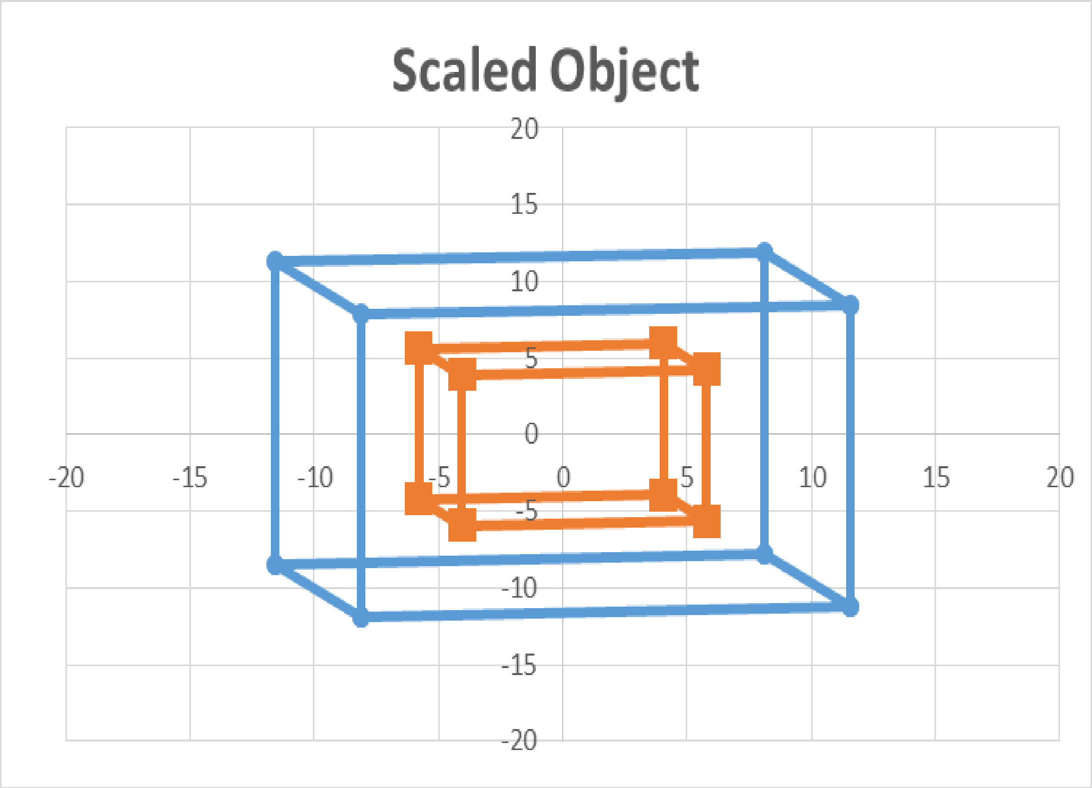
x	y	z	h
0	0	1	1
2	0	1	1
2	3	1	1
0	3	1	1
0	0	1	1
0	0	0	1
2	0	0	1
2	3	0	1
0	3	0	1
0	3	1	1
2	3	1	1
2	3	0	1
2	0	0	1
2	0	1	1
0	0	1	1
0	0	0	1
0	3	0	1

py	py	py	py
=Cos(x)	0	=Sin(x)	0
0	1	0	0
=-sin(x)	0	=Cos(x)	0
0	0	0	1

1	0	0	0
0	=Cos(y)	=sin(y)	0
0	=-sin(y)	=cos(y)	0
0	0	0	1

pz	pz	pz	pz
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	1

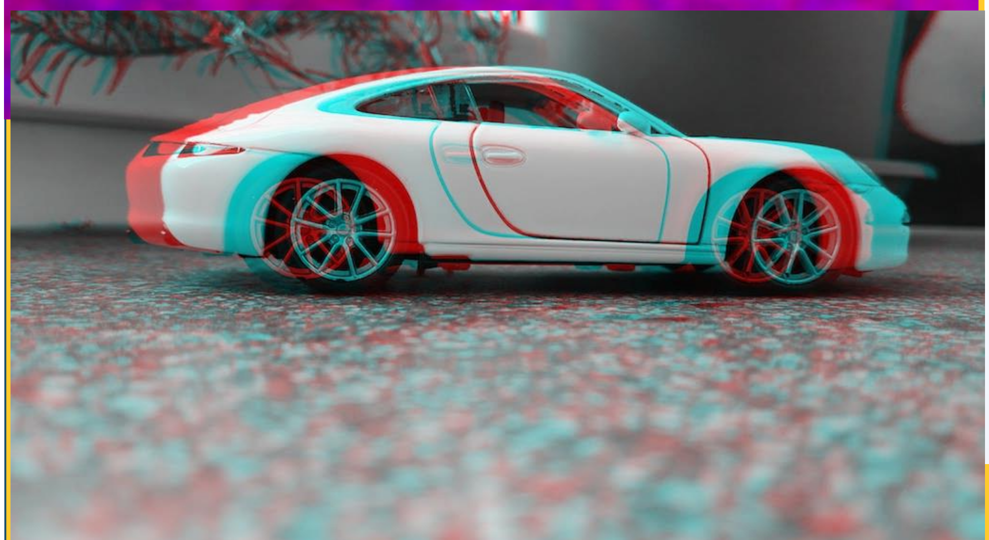
Transforming 3 dimensional objects



Scale Down Matrix

0.5	0	0	0
0	0.5	0	0
0	0	0.5	0
0	0	0	1

Depth Perception



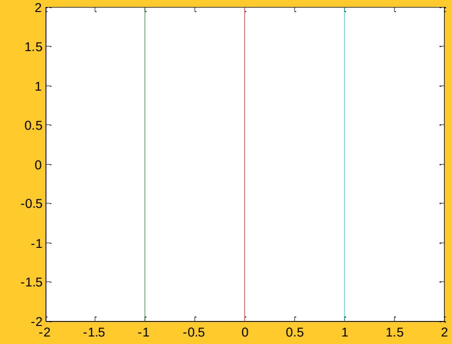
Functions of 2 variable: Plotting xy Grid

- x y grid is the domain of a two variable function z=f(x,y)

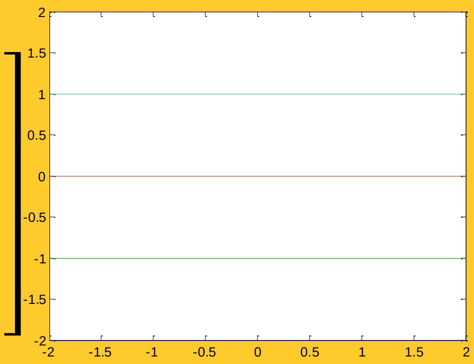
- It is very important to learn how to make a grid
- Matlab Commands:
- X=-2:1:2
- Y=X'
- [x,y]=meshgrid(X,Y)
- plot(x,y)
- figure
- plot(y,x)
- figure
- plot(x,y,y,x)

xy Grid Data

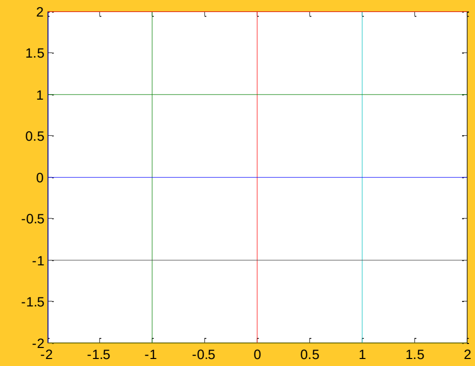
- X = [-2 -1 0 1 2]
- Y = [-2 -1 0 1 2]'
- $x = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}$
- y =
- $\begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix}$



y-grid



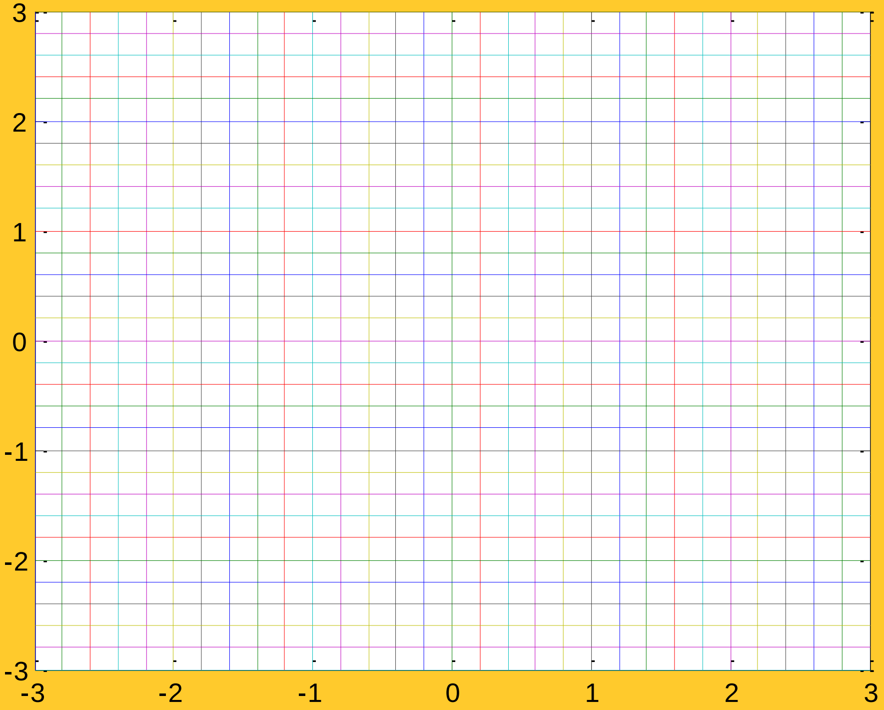
x-grid



x-y grid

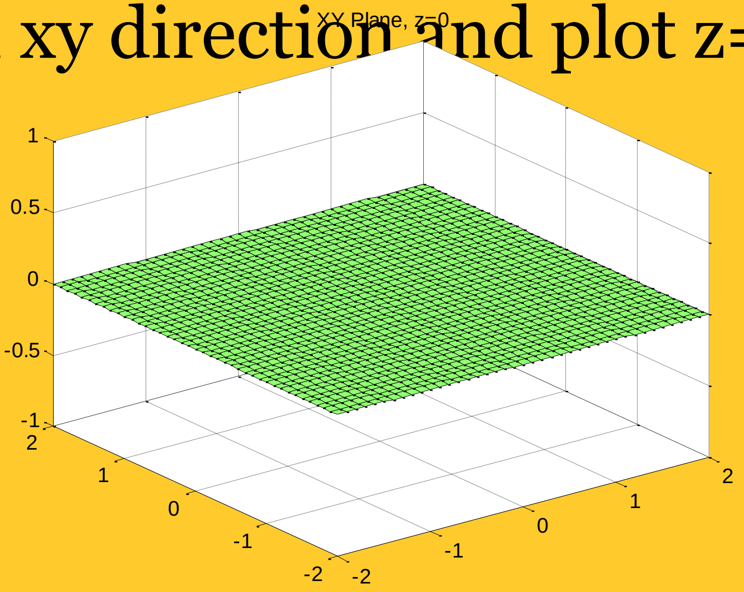
Mesh Grid

- xx=-3:.2:3
- yy=xx'
- [x,y]=meshgrid(xx,yy)
- plot(x,y,y,x)



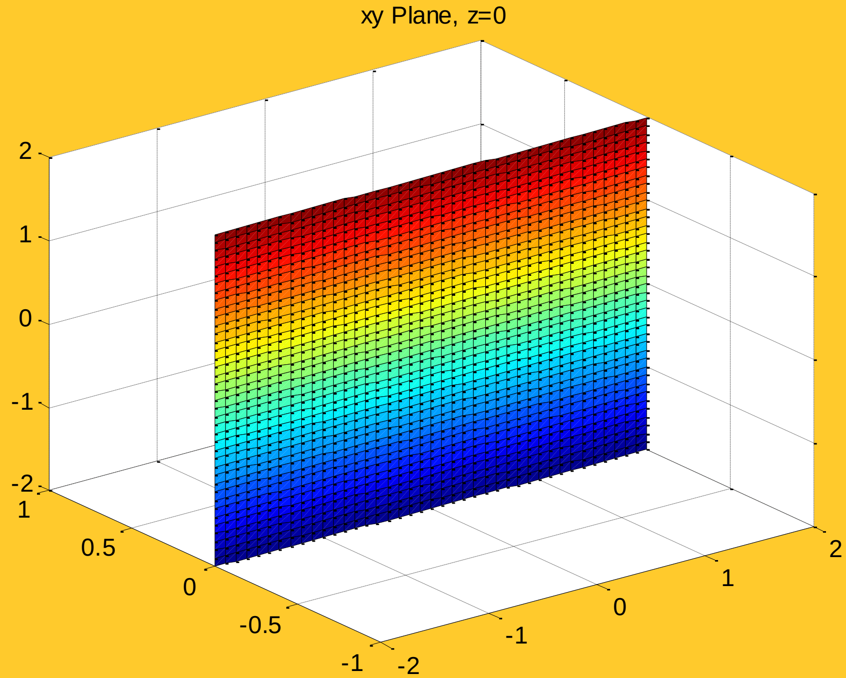
Creating XY Plane, Z=0

- A xy plane is that whose Z value is 0
- We have already to know how to create a grid. Now we will create a grid in xy direction and plot z=0 value in this grid.
- X=-2:.1:2;
- Y=X';
- [x,y]=meshgrid(X,Y);
- z=x.*y*0-0;
- surf(x,y,z)



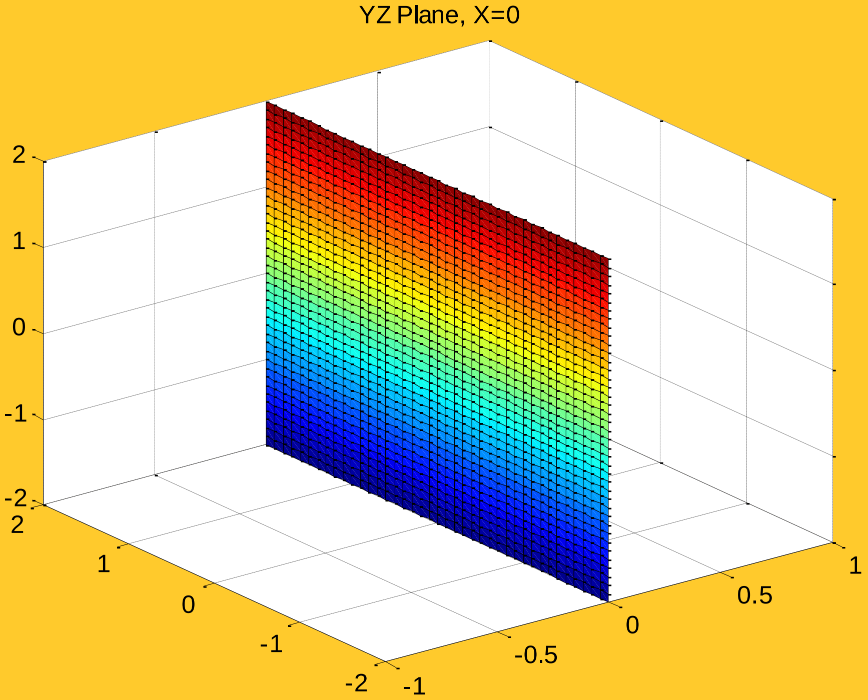
Creating Y=0 plane or xz plane

- A xz plane is that whose y value is 0
- We have already to know how to create a grid. Now we will create a grid in xz direction and plot y=0 value in this grid.
- %XZ PLANE
- X=-2:.1:2;
- Z=X';
- [x,z]=meshgrid(X,Z);
- y=x.*z.*0+0;
- figure
- surf(x,y,z)



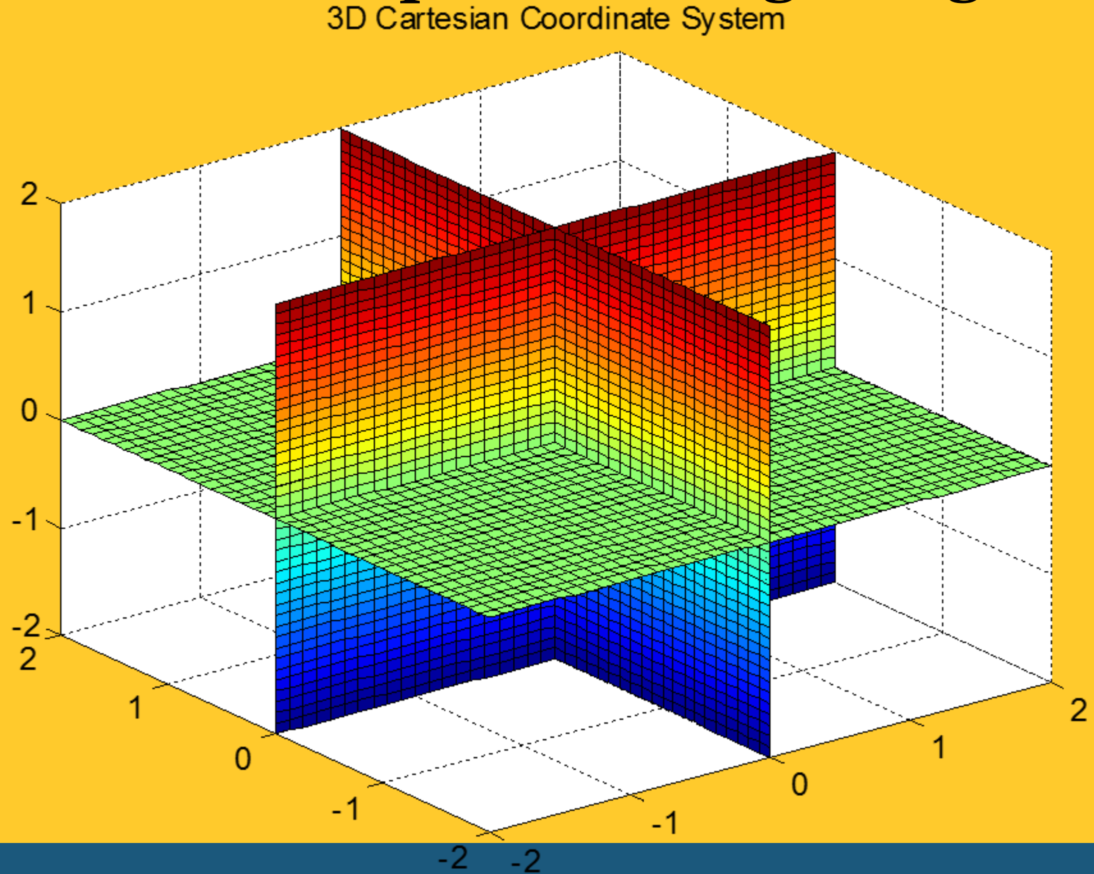
Creating YZ Plane, x=0

- A yz plane is that whose y value is 0
- We have already to know how to create a grid. Now we will create a grid in xz direction and plot y=0 value in this grid.
- %yz PLANE
- Y=-2:.1:2;
- Z=Y';
- [y,z]=meshgrid(Y,Z);
- x=y.*z.*0+0;
- figure
- surf(x,y,z)



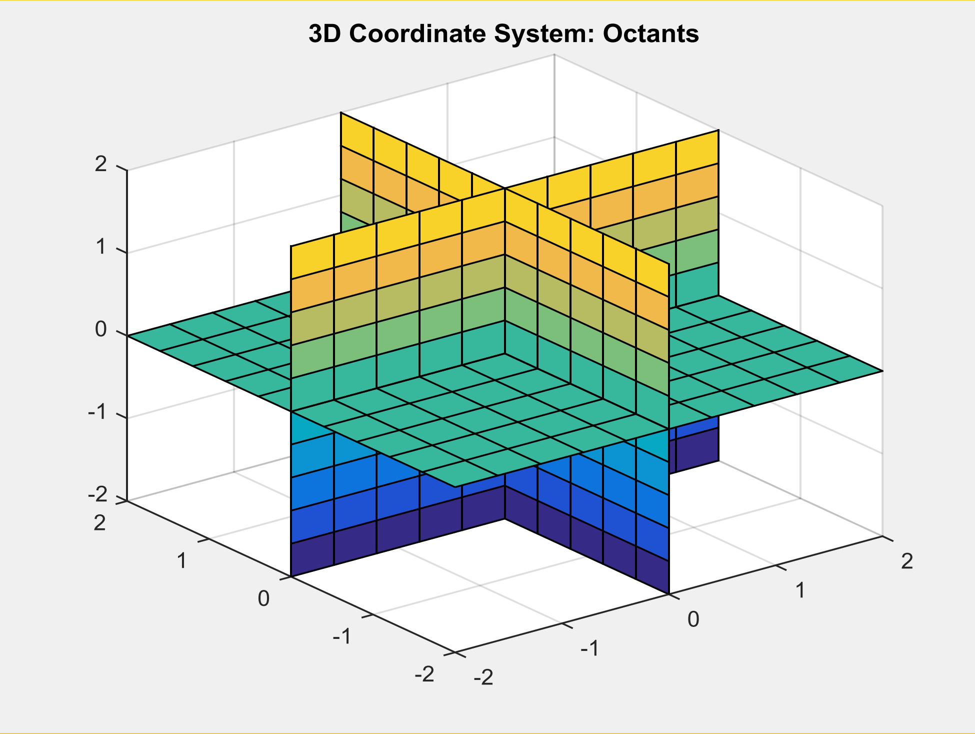
3d Coordinate Systems-Octants

- Octants: A three-dimensional Cartesian system defines a division of space into eight regions or octants.



3D Coordinate System

- %1. xz plane, y=0
- X=-2:.4:2;
- Z=X';
- [x,z]=meshgrid(X,Y);
- y=x.*z.*0+0;
- surf(x,y,z)
- hold on
- %2. xy plane, y=0
- X=-2:.4:2;
- Y=X';
- [x,y]=meshgrid(X,Y);
- z=x.*z.*0+0;
- surf(x,y,z)
- %3. yz plane, x=0
- Y=-2:.4:2;
- Z=Y';
- [y,z]=meshgrid(X,Y);
- x=y.*z.*0+0;
- surf(x,y,z)

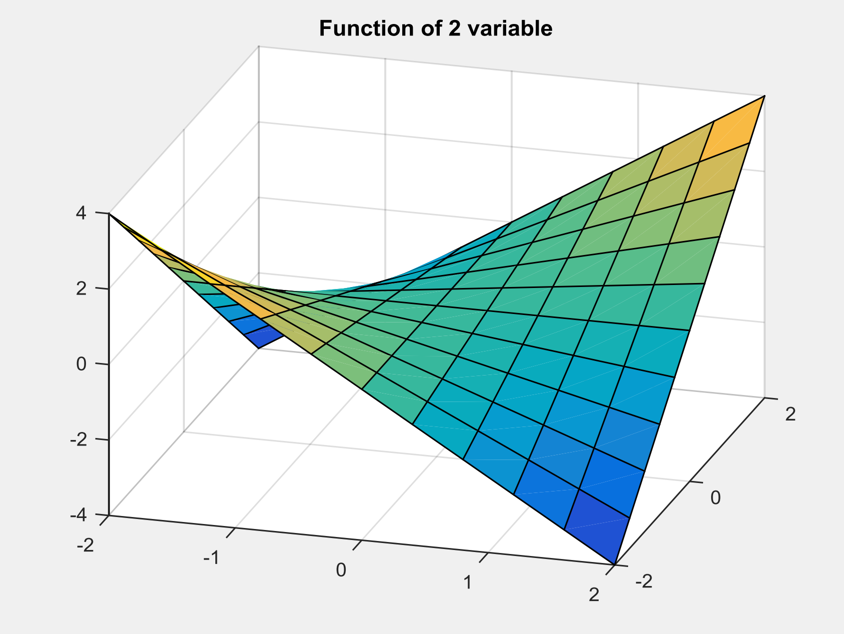


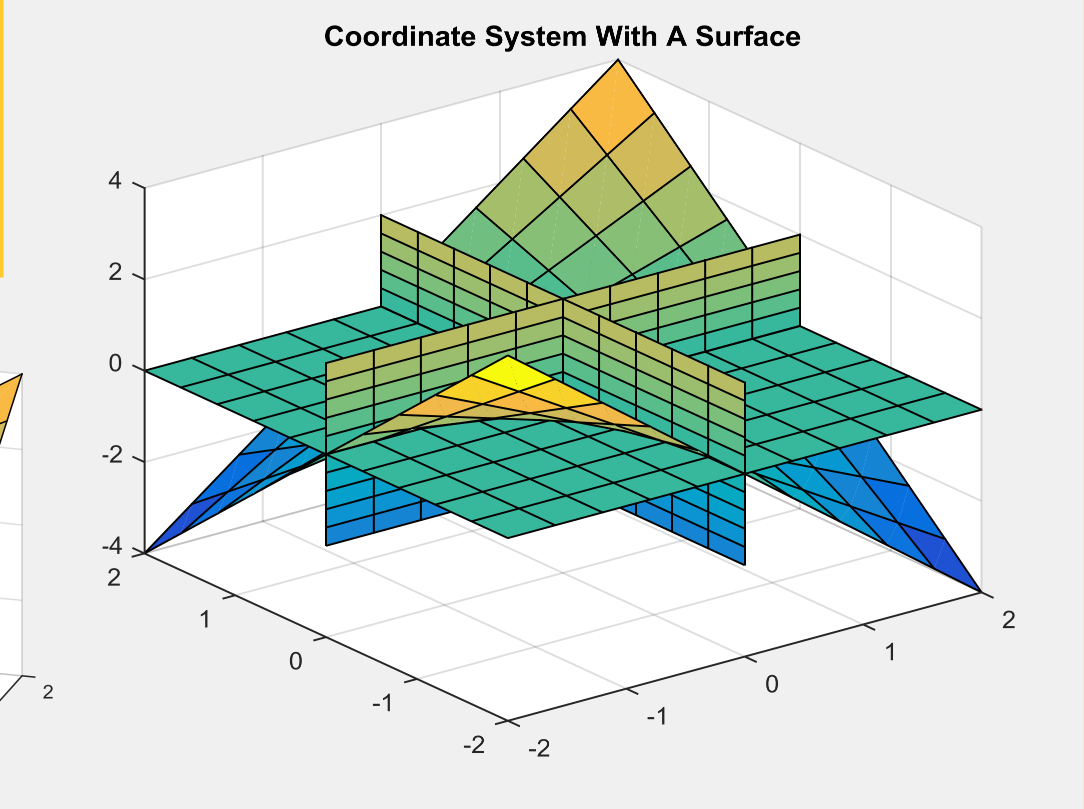
3D Coordinate System with a surface

- X=-2:.4:2;
- Y=X';

[x,y]=meshgrid(X,Y);

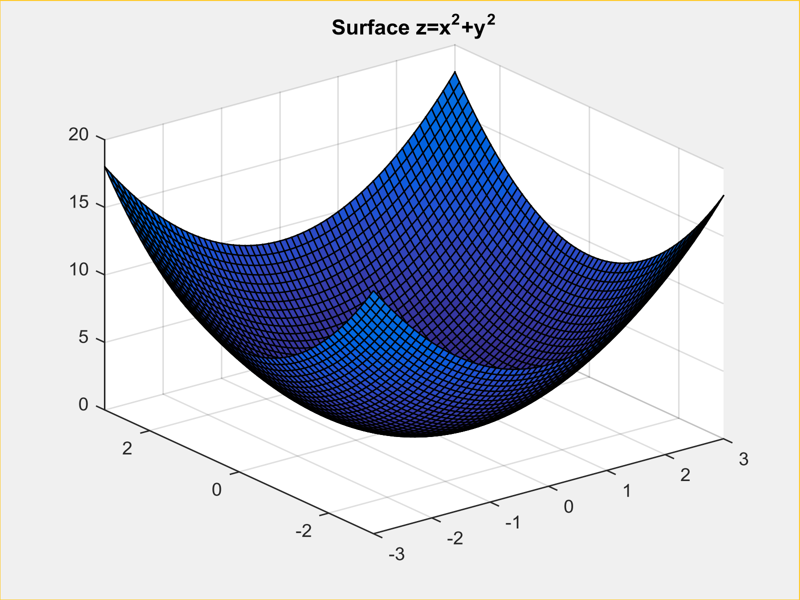
- z=x.*y;
- surf(x,y,z)





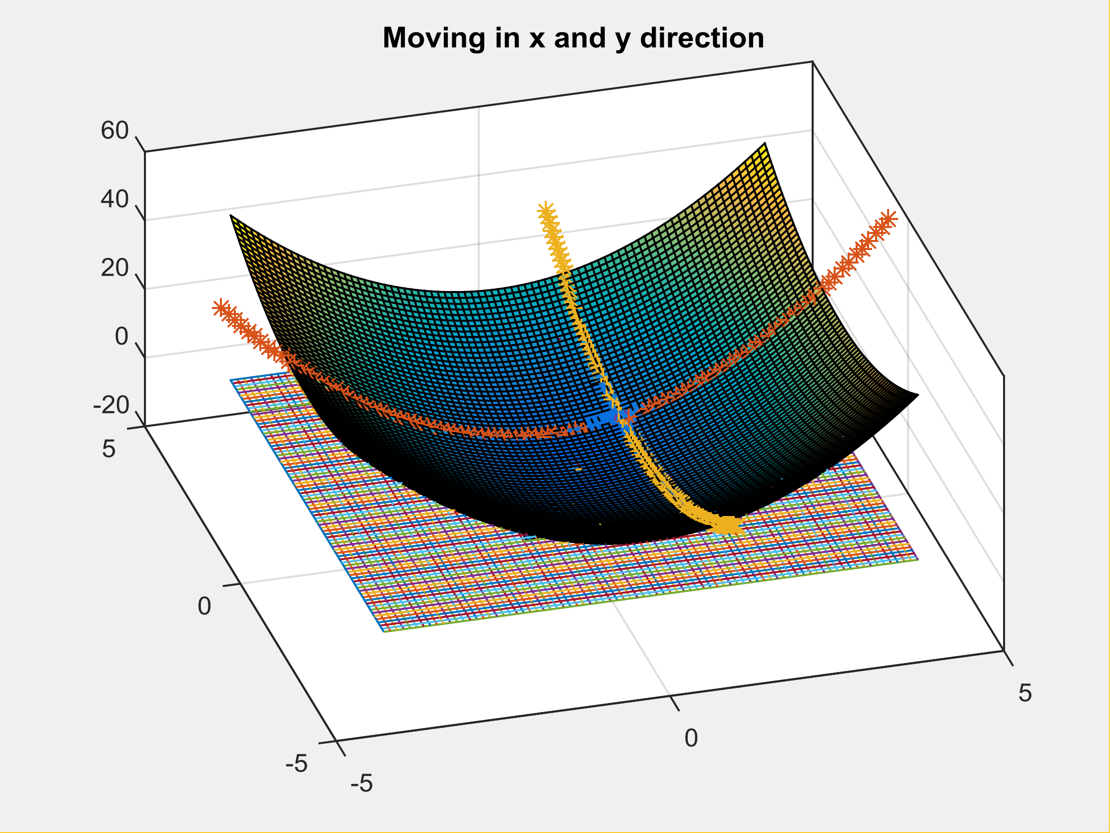
Drawing a surface

- z=x^2+y^2
- X=-7:.1:7
- Y=X'
- [x,y]=meshgrid(X,Y)
- z=x.^2+y.^2
- surf(x,y,z)
- axis([-3 3 -3 3 0 20])



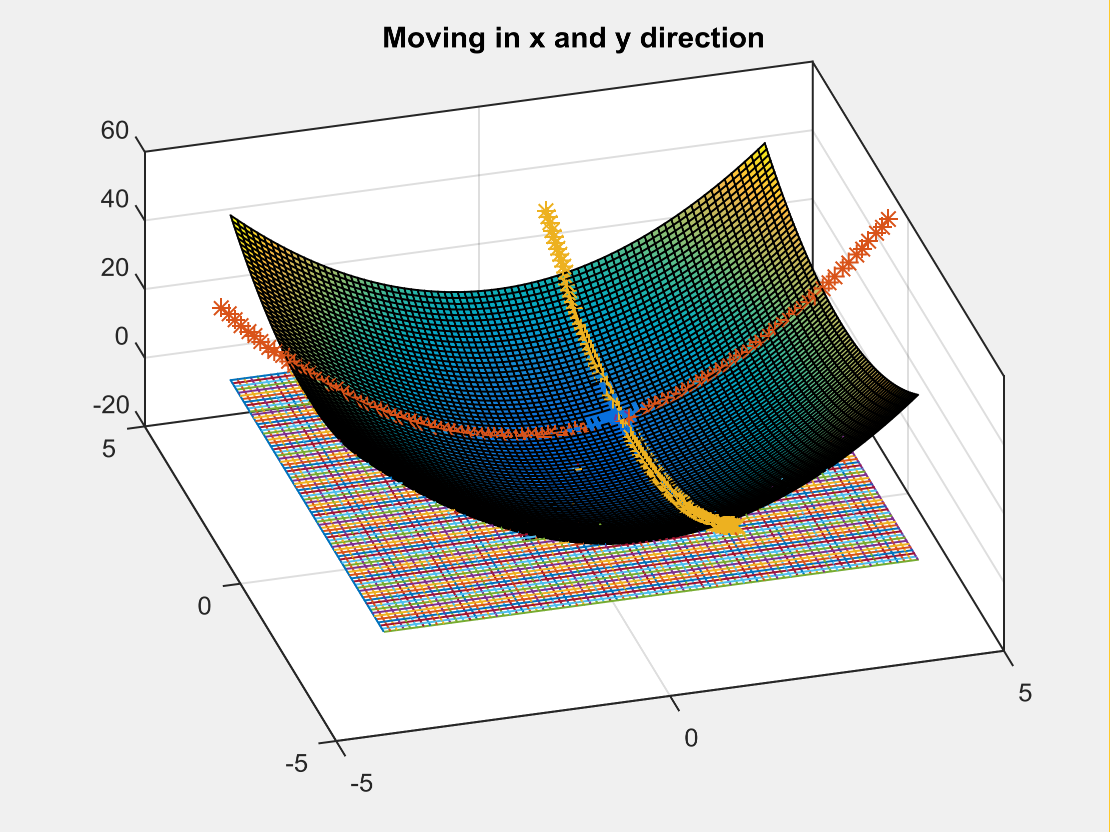
Moving in x-direction of a surface

- X=-4:.1:4;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- z=2.*x.^2+y.^2
- hold on
- surf(x,y,z)
- grid
- %Drawing a function
- %along x axis at y=1
- x=-5:.1:5;
- y=x*0+1;
- z=2.*x.^2+y.^2
- plot3(x,y,z,'*')



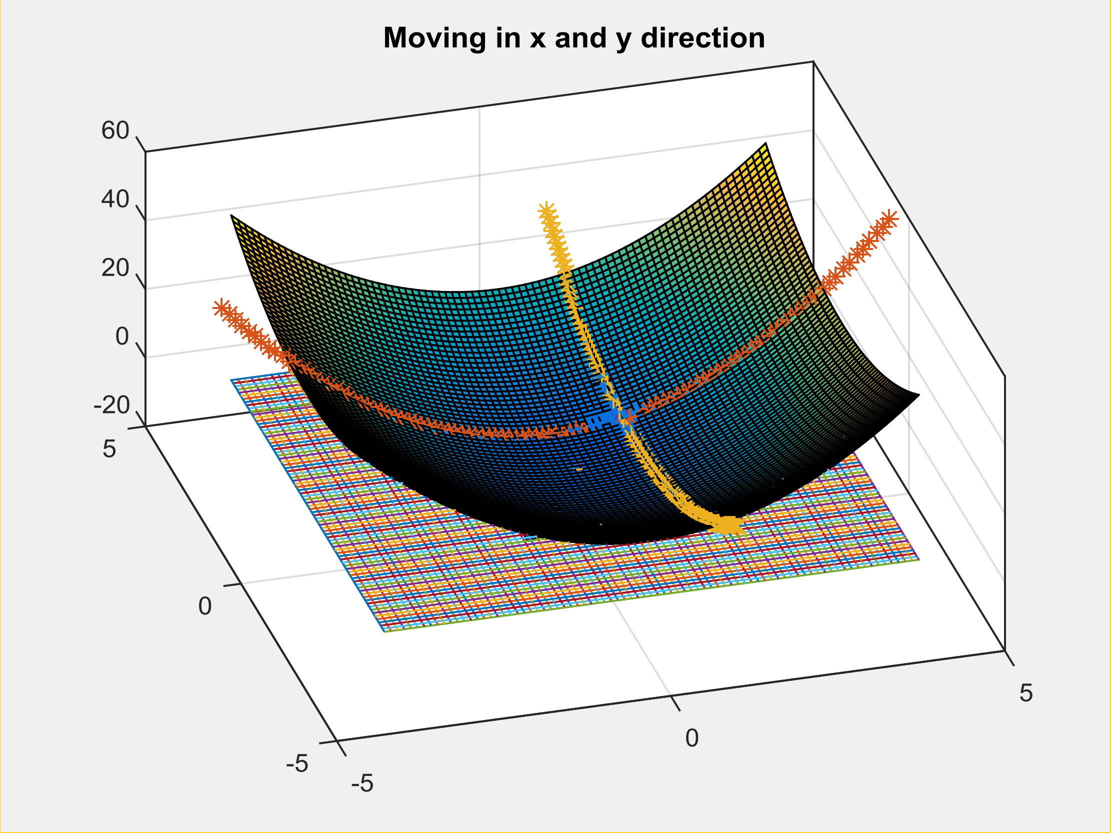
Moving in y-direction of a surface

- X=-4:.1:4;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- z=2.*x.^2+y.^2
- hold on
- surf(x,y,z)
- grid
- %Drawing a function
- %along y axis at x=1
- %y=-5:.1:5;
- x=y*0+1;
- z=2.*x.^2+y.^2
- plot3(x,y,z,'*')



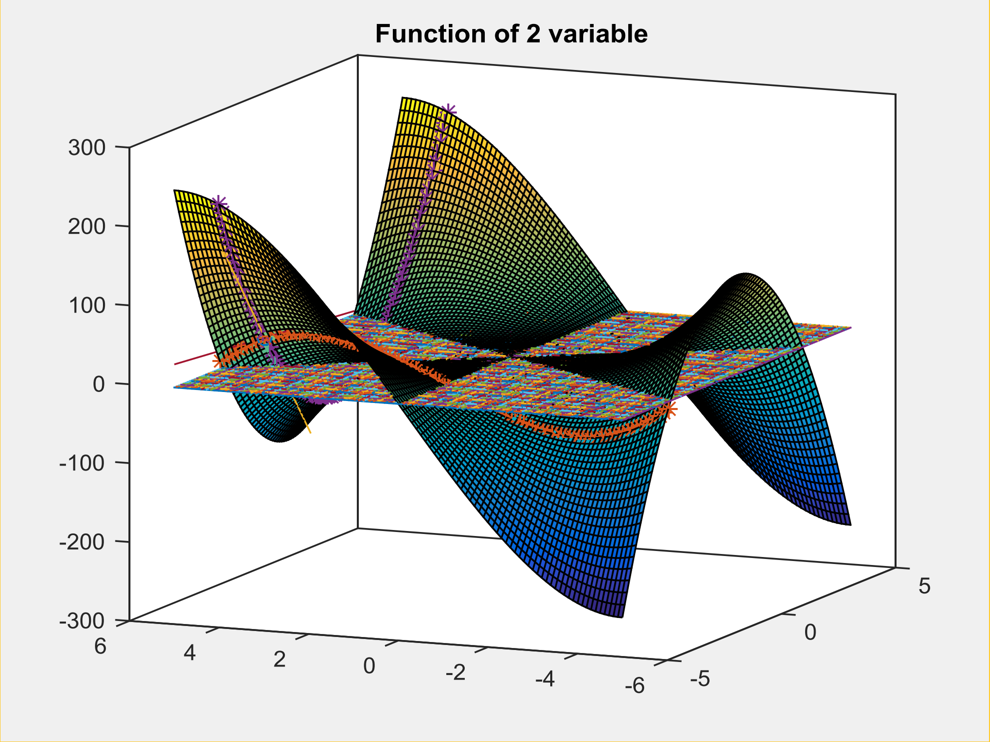
Drawing a Point in a surface

- X=-4:.1:4;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- z=2.*x.^2+y.^2
- hold on
- surf(x,y,z)
- Grid
- % Plotting the point
- x0=1
- y0=1
- z0=2.*x0.^2+y0.^2
- plot3(x0,y0,z0,'o')



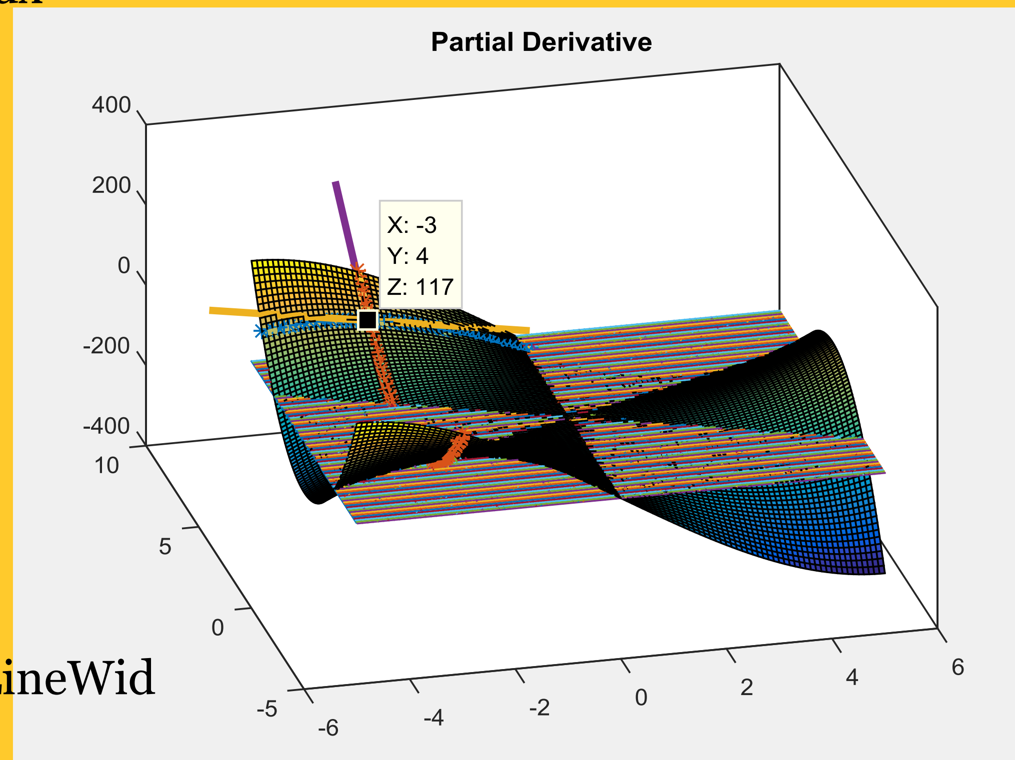
Drawing a surface with Grid

- X=-5:.1:5;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- u = x.^3 - 3.*x.*y.^2;
- hold on
- surf(x,y,u)



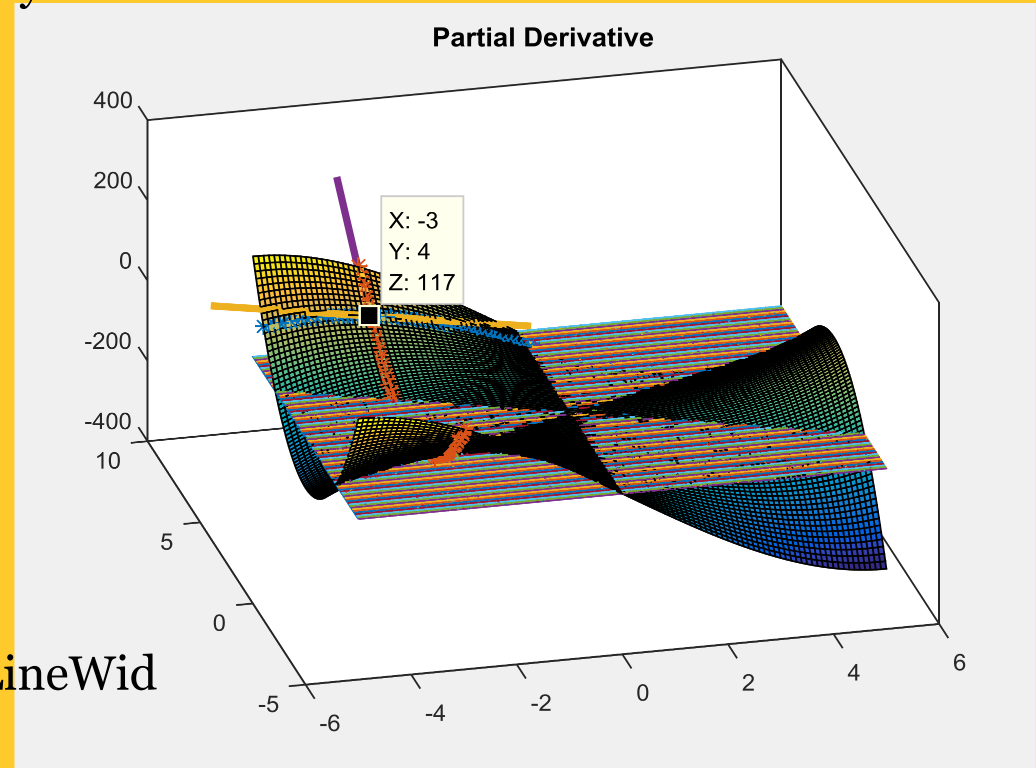
Drawing Partial Derivative at (3,4)


```
%Plotting the partial derivatives dudx
x=-3
y=4
u = x.^3 - 3.*x.*y.^2
m=3*x^2 - 3*y^2
c=u-m*x
x1=x-3
x2=x+3
u1=m*x1+c
u2=m*x2+c
y1=4
y2=4
plot3([x1,x,x2],[y1,y,y2],[u1,u,u2],'LineWidth',3)
grid
```



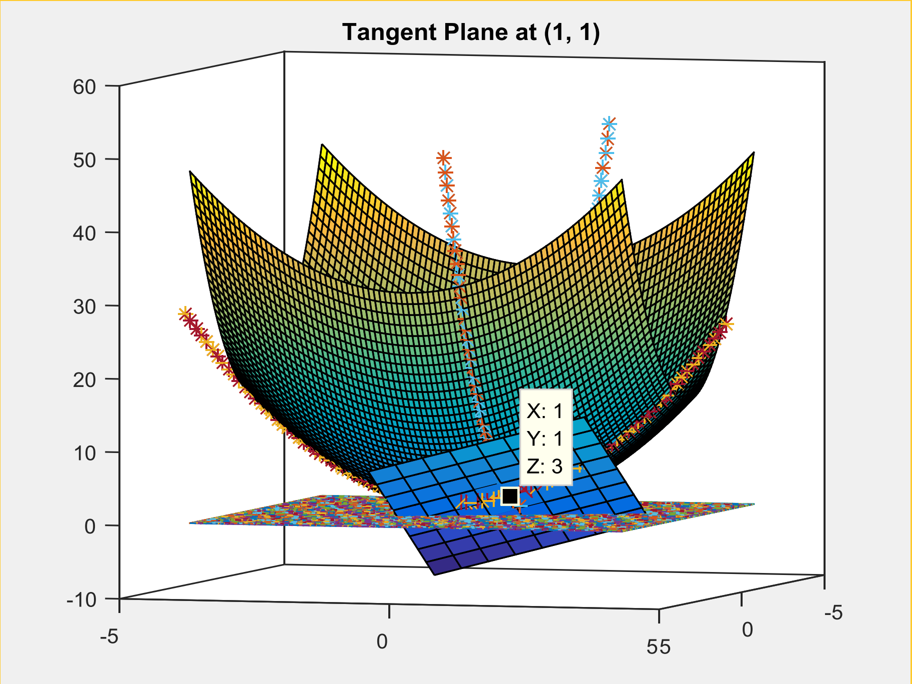
Drawing Partial Derivative at (3,4)


```
%Plotting the partial derivatives dudy
x=-3
y=4
u = x.^3 - 3.*x.*y.^2
m=-6*x*y
c=u-m*y
y1=y-3
y2=y+3
u1=m*y1+c
u2=m*y2+c
x1=-3
x2=-3
plot3([x1,x,x2],[y1,y,y2],[u1,u,u2],'LineWid
th',3)
```



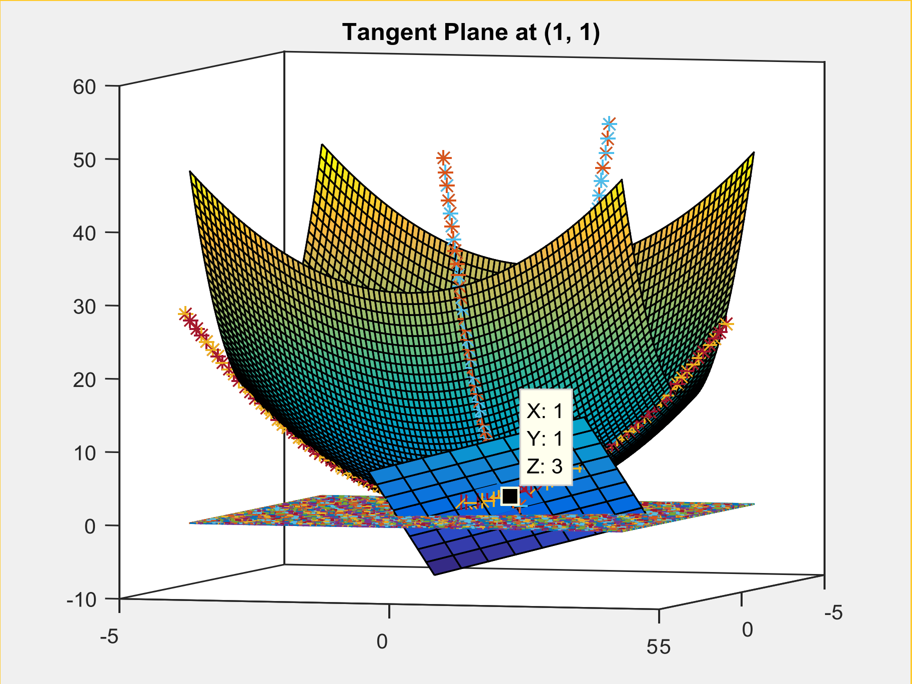
Drawing Tangent Plane


```
X=-4:.1:4;
Y=X';
[x,y]=meshgrid(X,Y);
plot(x,y,y,x)
z=2.*x.^2+y.^2
hold on
surf(x,y,z)
grid
```



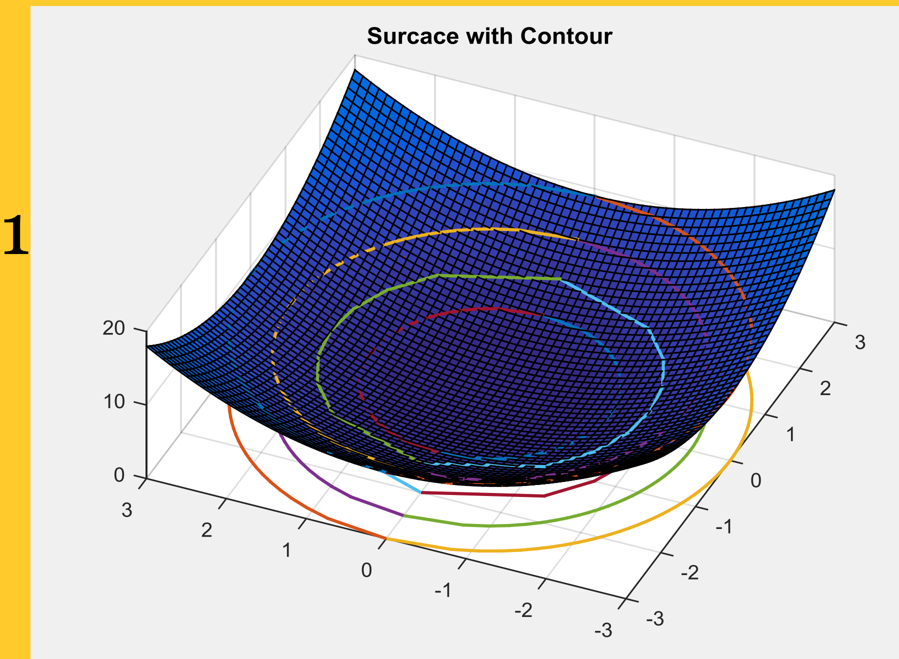
Drawing Tangent Plane


```
% Plotting the point
xo=1
yo=1
zo=2.*xo.^2+yo.^2
dzdx = 4.*x
dzdy=2.*y
plot3(xo,yo,zo,'o')
 %Tangent Plane
X=[xo-2,xo-1.5, xo-1,xo-
.5,xo,xo+.5,xo+1,xo+1.5,xo+2]
Y=[yo-2,yo-1.5, yo-1,yo-
.5,yo,yo+.5,yo+1,yo+1.5,yo+2]'
[x,y]=meshgrid(X,Y)
dzdx = 4.*x
dzdy=2.*y
z=zo+4.*(x-xo)+2.*(y-yo)
surf(x,y,z)
```



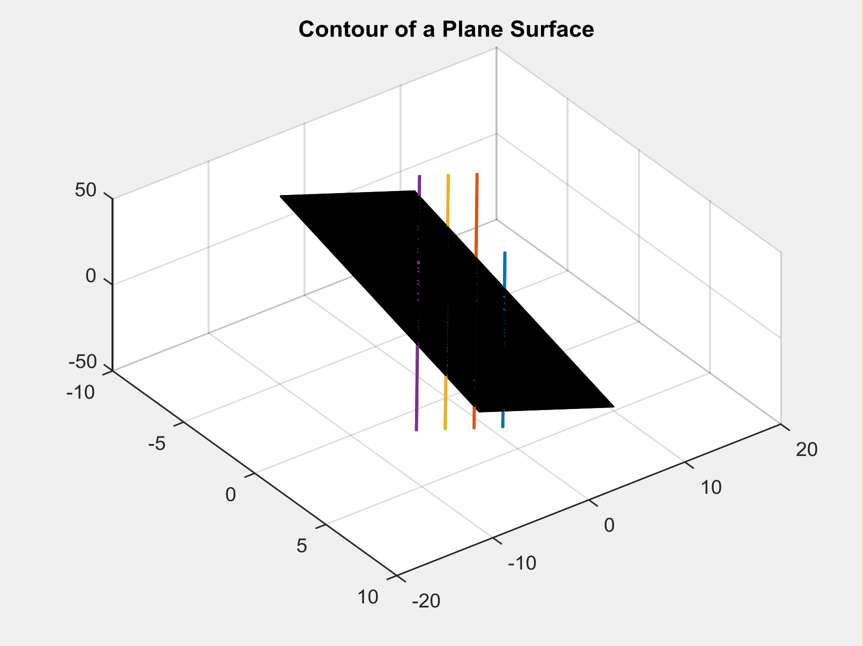
Drawing Contour of a surface

- %1
- x=-3:.1:3
- y=sqrt(9-x.^2)
- z=y.*0+9
- plot3(x,y,z,'LineWidth',1
- %2
- x=-3:.1:3
- y=-sqrt(9-x.^2)
- z=y.*0+9;
- plot3(x,y,z,'LineWidth',1.5)



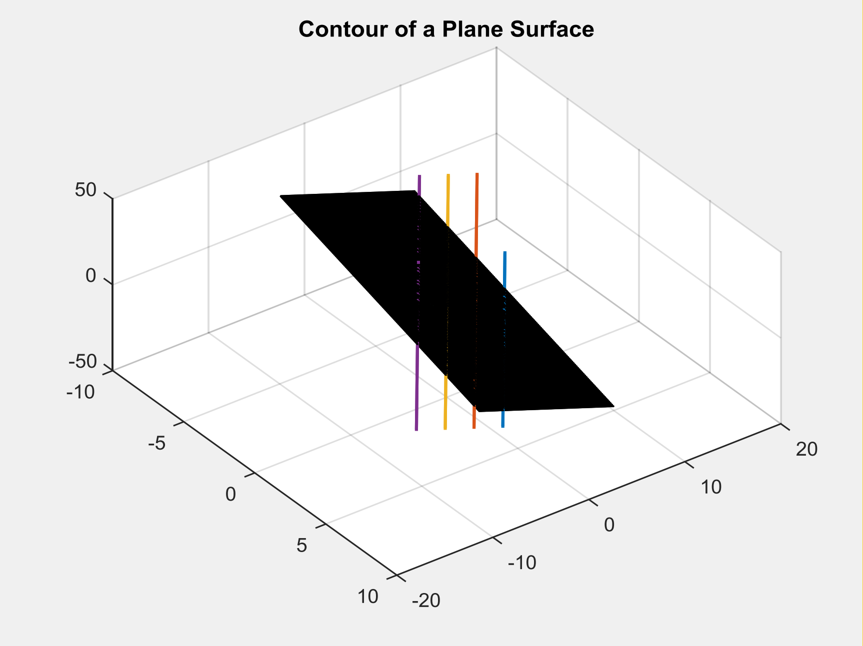
Drawing Contour of a Plane Surface

- %Contour of plane Surface
- X=-7:.1:7
- Y=X'
- [x,y]=meshgrid(X,Y)
- z=6-3*x-2*y
- surf(x,y,z)
-
- hold on
-
- x=-3:.1:8
- y=6-(1.5).*x
- z=y.*0+-6
- plot3(x,y,z,'LineWidth',1.5)
-
- x=-8:.1:8
- y=3-(1.5).*x
- z=y.*0+0
- plot3(x,y,z,'LineWidth',1.5)



Drawing Contour of a Plane Surface

- x=-8:.1:8
- y=-(1.5).*x
- z=y.*0+6
- plot3(x,y,z,'LineWidth',1.5)
-
- x=-8:.1:8
- y=-3-(1.5).*x
- z=y.*0+12
- plot3(x,y,z,'LineWidth',1.5)



Creating a cube in 3d

x	y	z	h
0	0	1	1
2	0	1	1
2	3	1	1
0	3	1	1
0	0	1	1
0	0	0	1
2	0	0	1
2	3	0	1
0	3	0	1
0	3	1	1
2	3	1	1
2	3	0	1
2	0	0	1
2	0	1	1
0	0	1	1
0	0	0	1
0	3	0	1

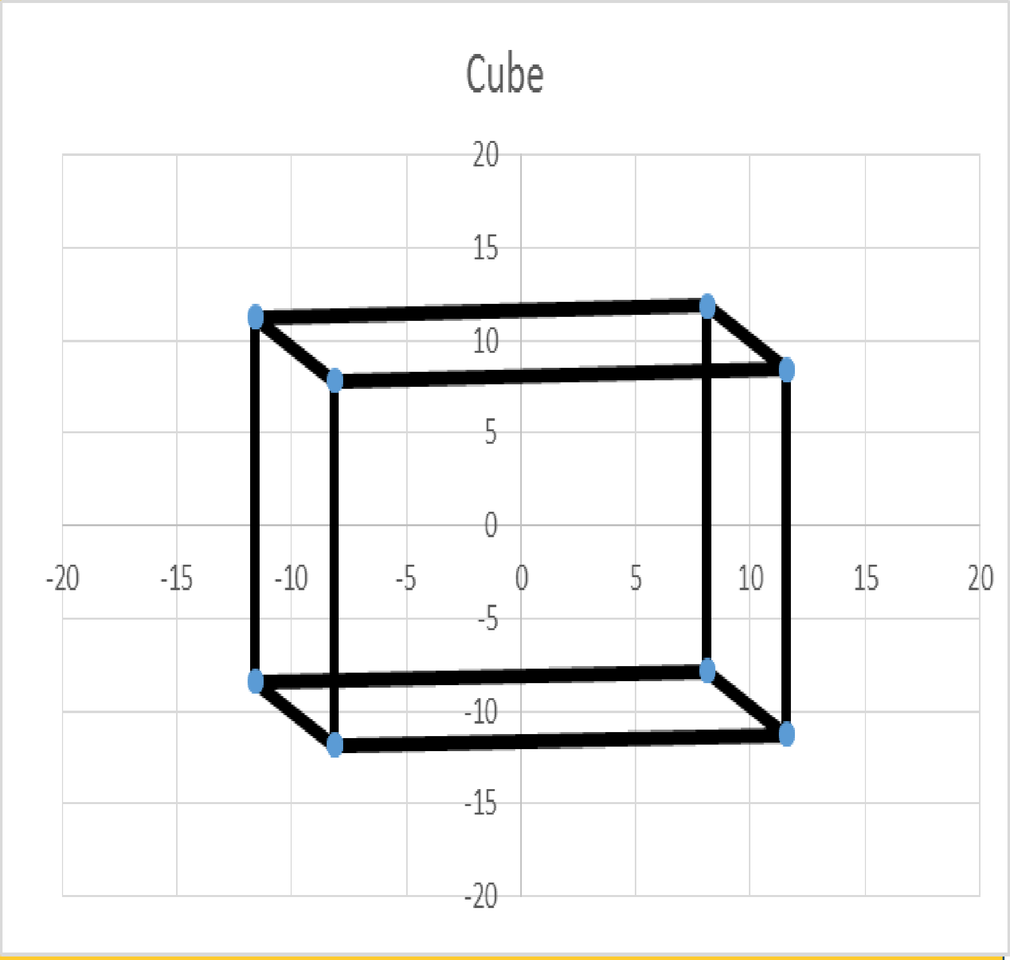
py	py	py	py
=Cos(x)	0	=Sin(x)	0
0	1	0	0
=-sin(x)	0	=Cos(x)	0
0	0	0	1

1	**0**	**0**	**0**
0	**=Cos(y)**	**=sin(y)**	**0**
0	**=-sin(y)**	**=cos(y)**	**0**
0	**0**	**0**	**1**

pz	pz	pz	pz
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	1

Creating a cube in 3d

x	y	z	h
0	0	1	1
2	0	1	1
2	3	1	1
0	3	1	1
0	0	1	1
0	0	0	1
2	0	0	1
2	3	0	1
0	3	0	1
0	3	1	1
2	3	1	1
2	3	0	1
2	0	0	1
2	0	1	1
0	0	1	1
0	0	0	1
0	3	0	1



Creating Sphere

- k =3
- n = 2^k-1
- theta = pi*(-n:2:n)/n
- phi = (pi/2)*(-n:2:n)'/n
- X = cos(phi)*cos(theta)
- Y = cos(phi)*sin(theta)
- Z = sin(phi)*ones(size(theta))
- surf(X,Y,Z)

Creating Sphere

- k = 3
- n = 7

- theta = -3.1416 -2.2440 -1.3464 -0.4488 0.4488 1.3464 2.2440 3.1416

- phi =
- -1.5708
- -1.1220
- -0.6732
- -0.2244
- 0.2244
- 0.6732
- 1.1220
- 1.5708

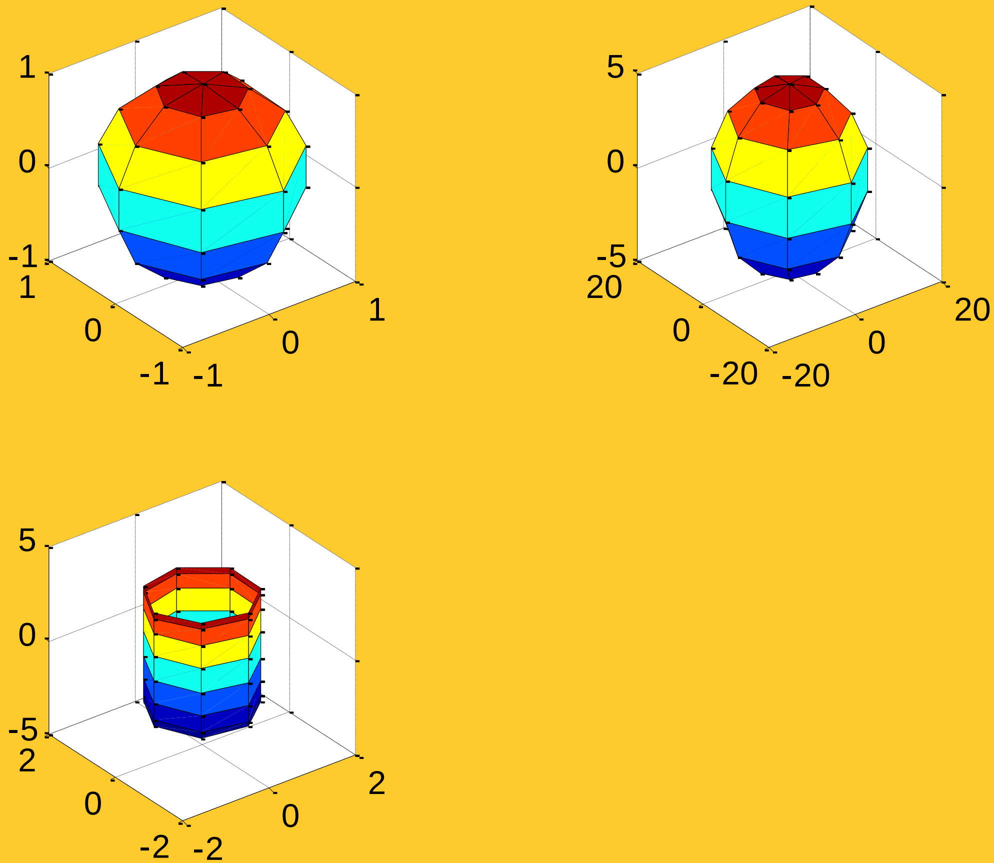
Creating Sphere

- k = 3
- n = 7

- theta = -3.1416 -2.2440 -1.3464 -0.4488 0.4488 1.3464 2.2440 3.1416

- phi =
- -1.5708
- -1.1220
- -0.6732
- -0.2244
- 0.2244
- 0.6732
- 1.1220
- 1.5708

Sphere, Ellipse, Cylinder



Creating Ellipse

- %Ellipse
- subplot(2,2,2)
- k =3
- n = 2^k-1
- r1=3
- r2=5
- theta = pi*(-n:2:n)/n
- phi = (pi/2)*(-n:2:n)'/n
- X = r1*cos(phi)*r2*cos(theta)
- Y = r1*cos(phi)*r2*sin(theta)
- Z = r2*sin(phi)*ones(size(theta))

- surf(X,Y,Z)

Creating Ellipse

- k = 3
- n = 7
- r1 = 3
- r2 = 5
- theta = -3.1416 -2.2440 -1.3464 -0.4488 0.4488 1.3464 2.2440 3.1416
- phi =
- -1.5708
- -1.1220
- -0.6732
- -0.2244
- 0.2244
- 0.6732
- 1.1220
- 1.5708

Creating Cylinder from Super formula


```
%Superformula
 subplot(2,2,3)
k =3
n = 2^k-1

theta = pi*(-n:2:n)/n
phi = (pi/2)*(-n:2:n)/n
a=2
b=2
m=5
n1=2
n2=2
n3=2
```

```
phi1 = (pi/2)*(-n:2:n)'/n
rx1=abs(1/a)*abs(cos(m*theta/4).^n2)+abs(1/b)*abs(sin(m*theta/4).^n3)
rx2=(cos(m*phi/4).^n2)/a+(sin(m*phi/4).^n3)/b
X = rx1*cos(phi1)*rx2'*cos(theta)
Y = rx1*cos(phi1)*rx2'*sin(theta)
Z = 3*sin(phi1)*ones(size(theta))
 surf(X,Y,Z)
axis square
```

Creating Ellipse with Compression Method

Creating All Conic Sections From One Formula

Interpolation

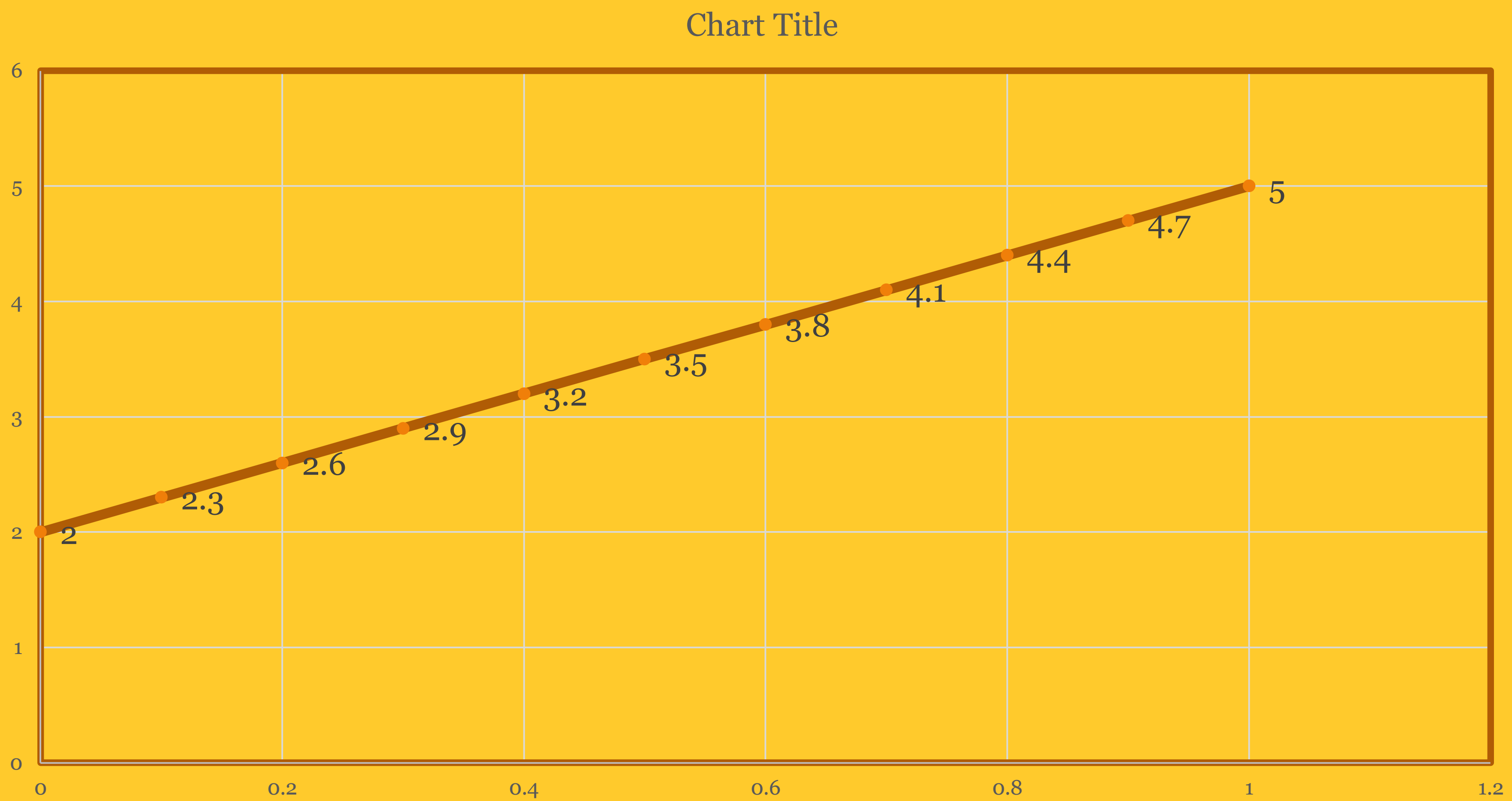
Interpolation:

Interpolation is the process of estimating values between data points

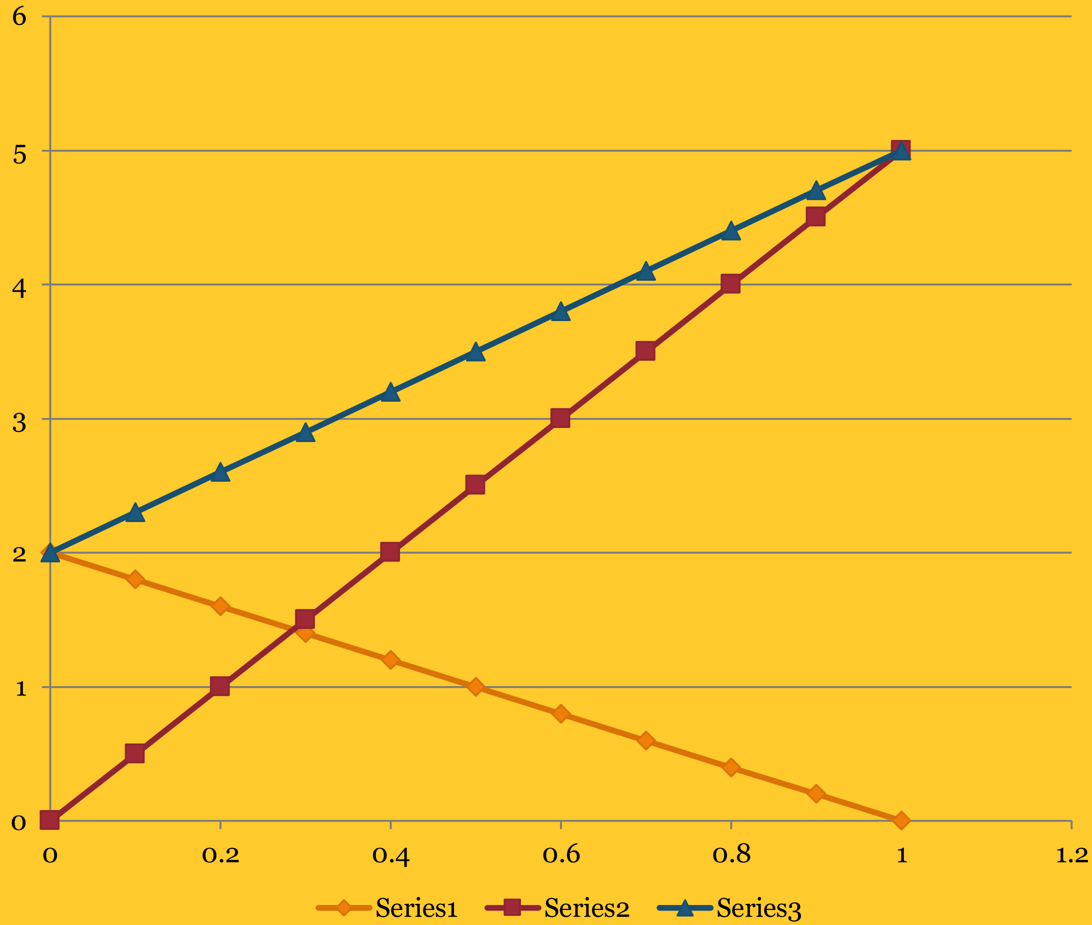
Note: There are many confusion about the objective of interpolation.

It can be a process of finding intermediate points or it can be moving from one point to other points.

Interpolation between two points



Two New Tool
Linear Interpolation and Slider



Interpolation from 2 to 5

t	N11=2*(1-t)	N22=5*t	N=N11+N22
0	2	0	2
0.1	1.8	0.5	2.3
0.2	1.6	1	2.6
0.3	1.4	1.5	2.9
0.4	1.2	2	3.2
0.5	1	2.5	3.5
0.6	0.8	3	3.8
0.7	0.6	3.5	4.1
0.8	0.4	4	4.4
0.9	0.2	4.5	4.7
1	0	5	5

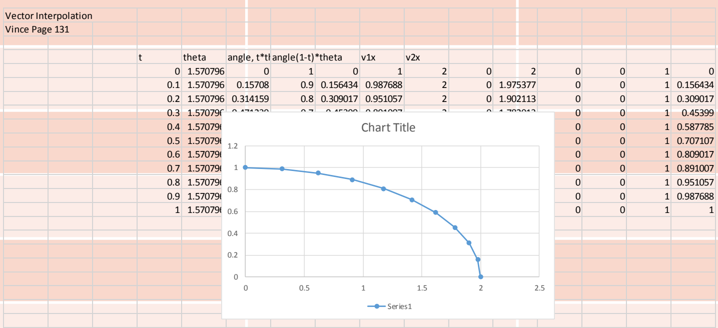
Derivation of interpolation matrix

- Interpolate from n1=2 to n2=5
- We require that at t=0, n=2 and t=1, n=5.
- This can be achieved if t=0, n1=2 and n2=0
- And at t=1, n1=0 and n2=5
- This can be achieved from the formula,
- n=n1+t(n2-n1)
- This can be written as: n=n1+t*n2-t*n1
- or n=n1*(1-t)+n2*t or [1-t t]$\begin{bmatrix} n1 \\ n2 \end{bmatrix}$ or [t 1] $\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} n1 \\ n2 \end{bmatrix}$

Interpolation from 2 to 5

t varies from 0 to 1
Point varies from 2 to 5

t	h	interpolant		From to	n
0	1	-1	1	2	2
0.1	1	1	0	5	2.3
0.2	1				2.6
0.3	1				2.9
0.4	1				3.2
0.5	1				3.5
0.6	1				3.8
0.7	1				4.1
0.8	1				4.4
0.9	1				4.7
1	1				5



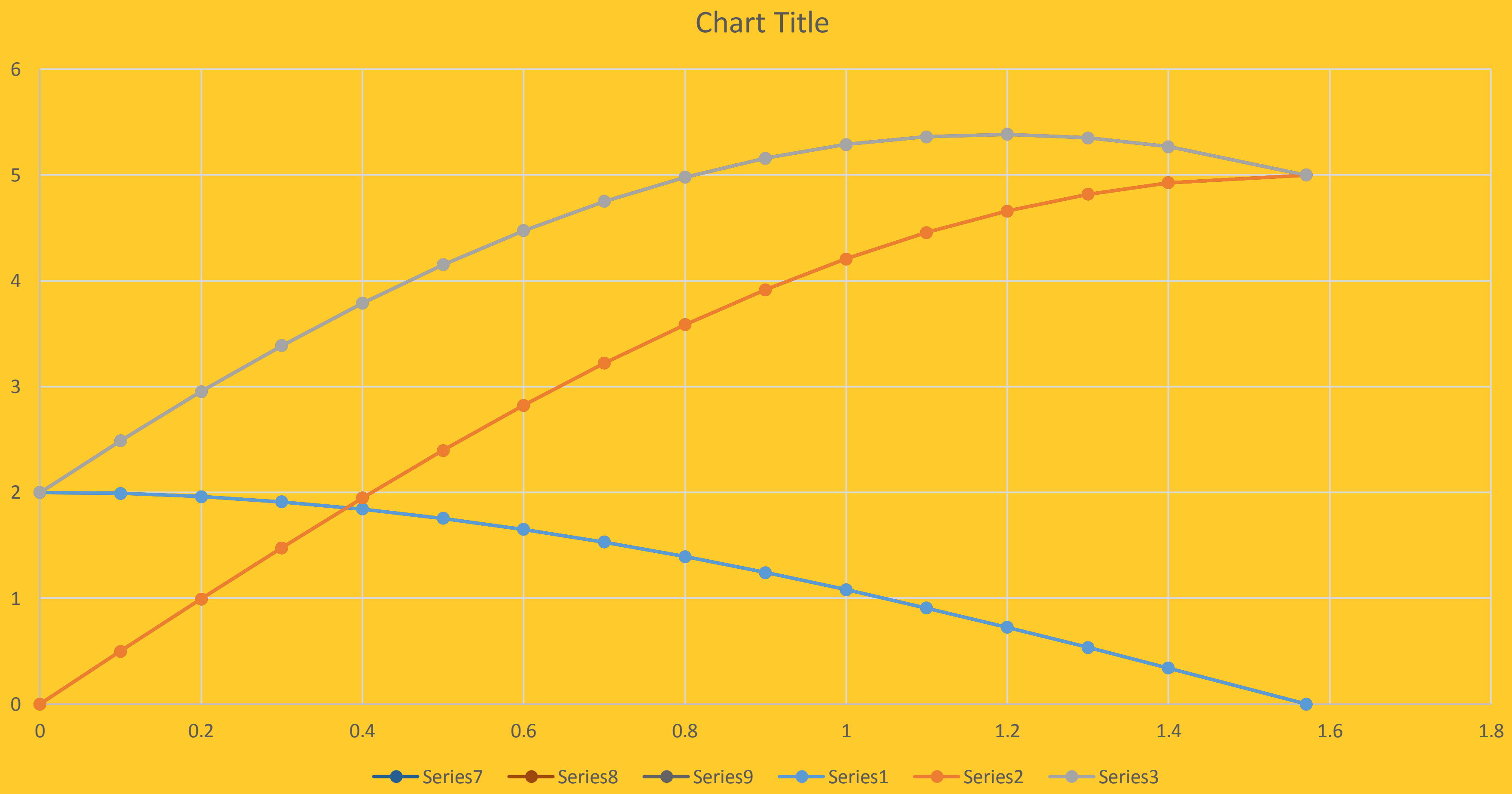
MMULT(MMULT(C4:D14,E4:F5),G4:G5)

Non linear interpolation

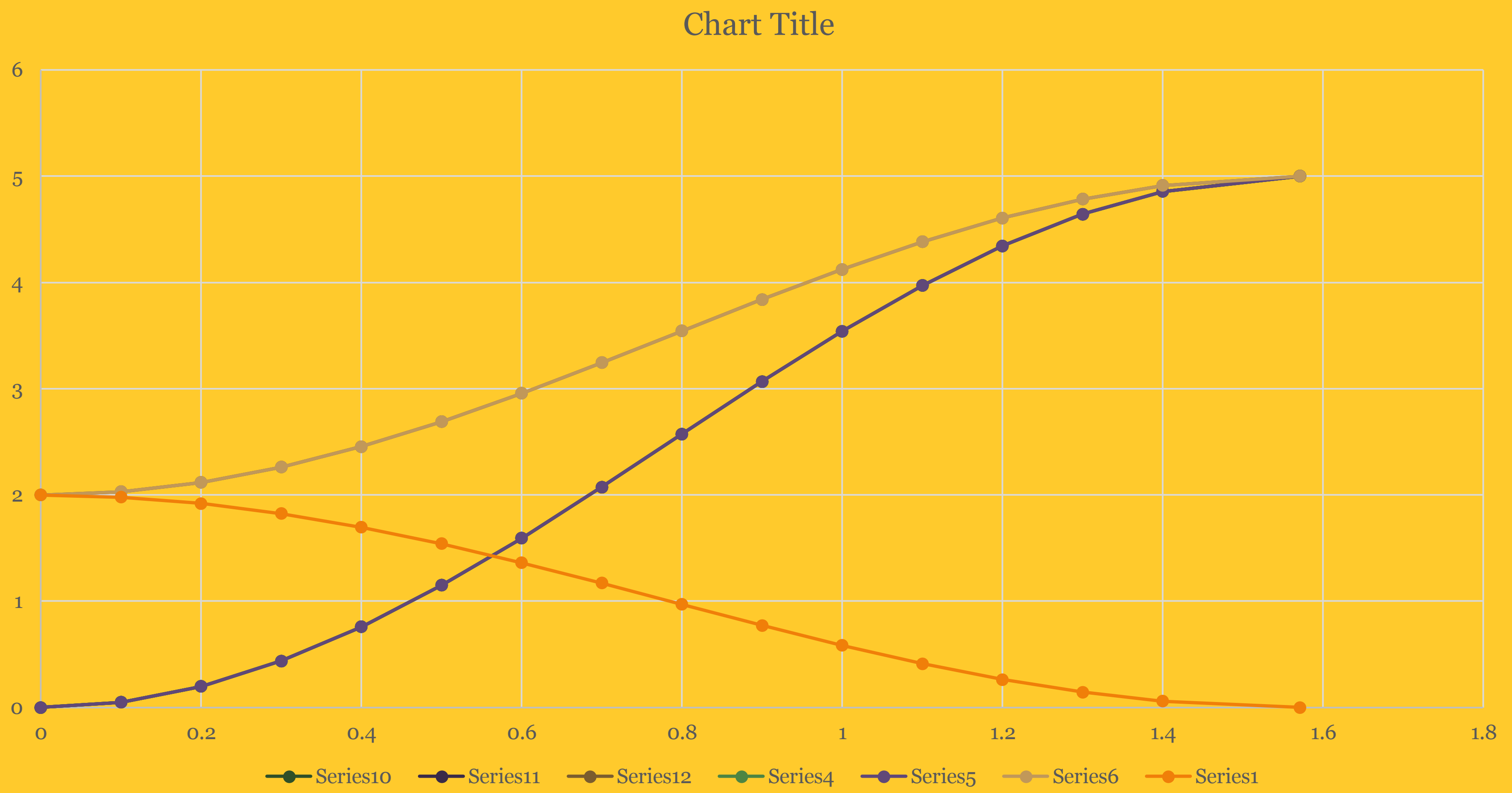
- Linear interpolant ensures equal steps in parameter t gives rise equal steps in the interpolated values.

- Many times it is required that equal steps in t gives unequal steps in the interpolated values.
- This can be achieved by :
  1. trigonometric functions (sin^2 x+cos^2 x=1) as x varies from 0 to pi/2
  2. Polynomial equations: [(1-t)+t]^n=1

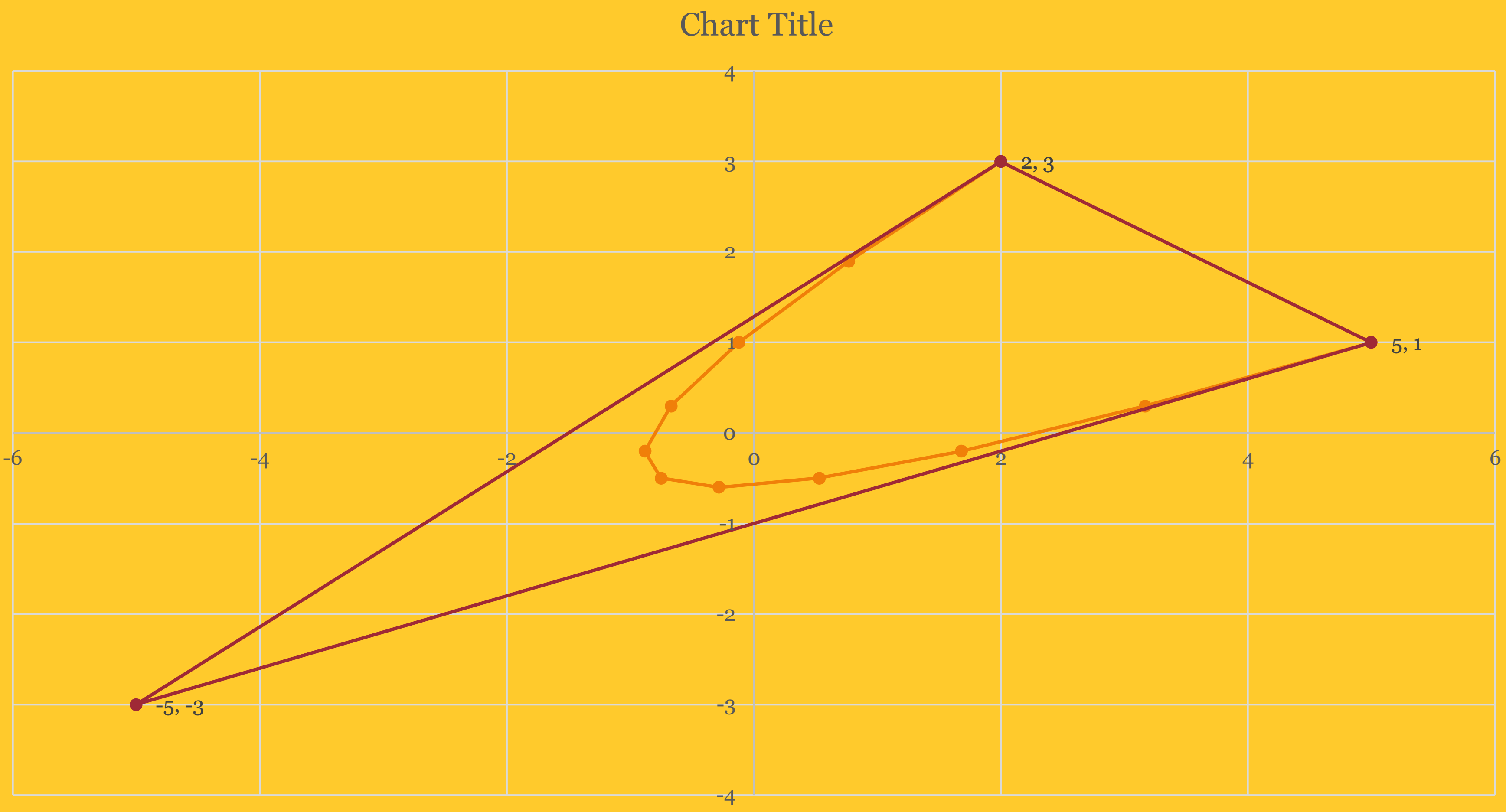
Trigonometric Interpolation



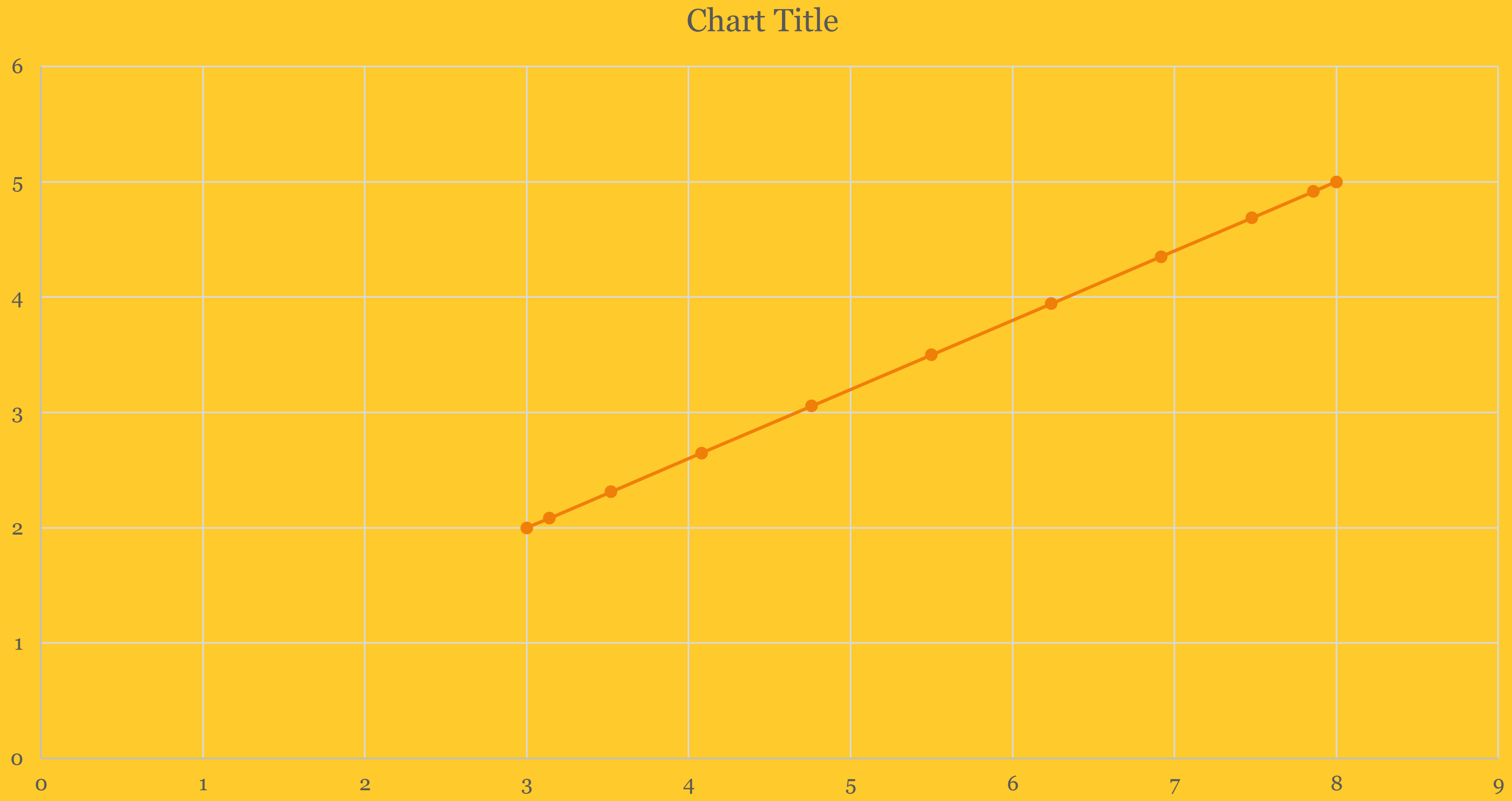
Trigonometric Interpolation



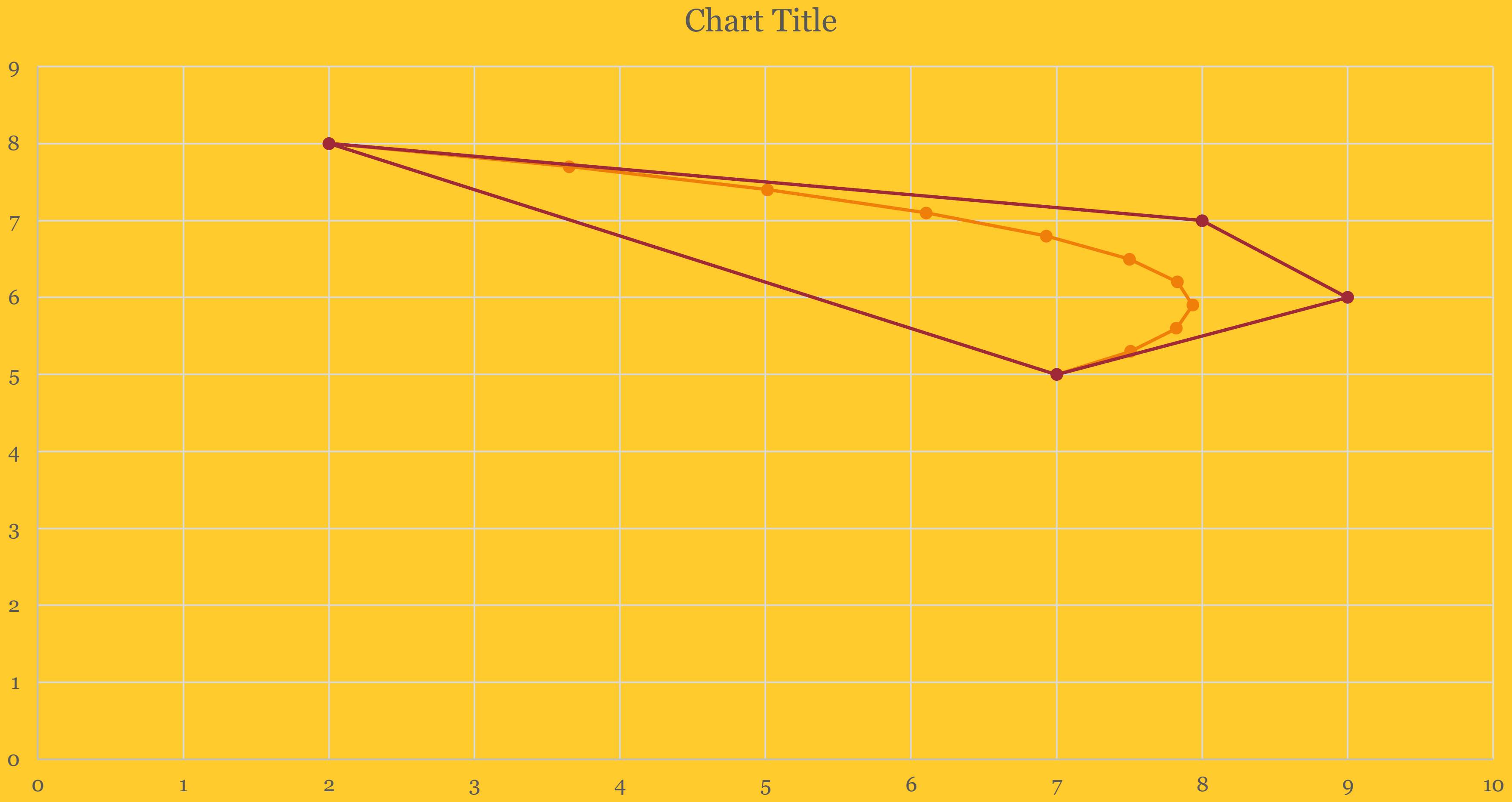
Quadratic Interpolation



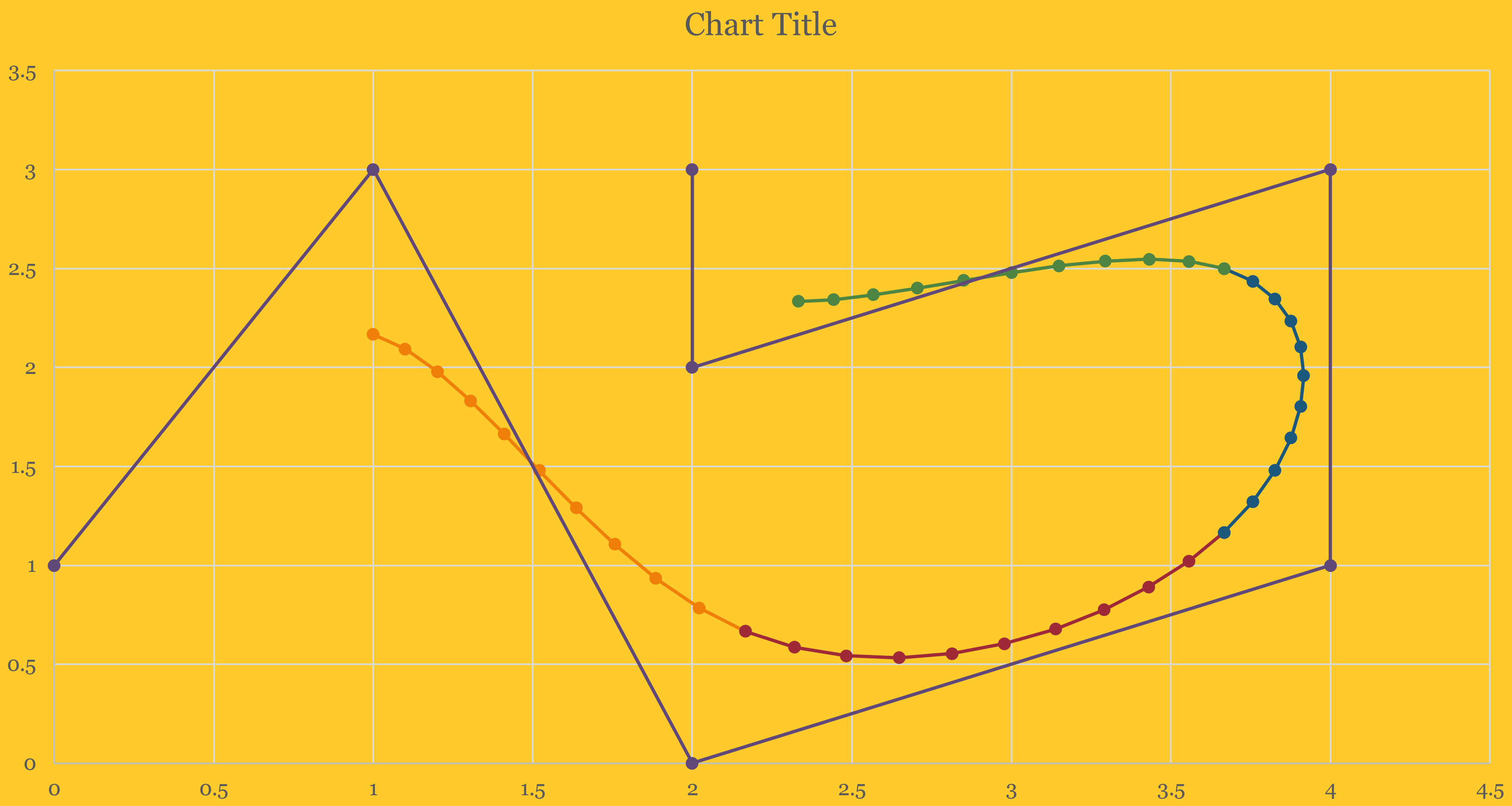
Cubic Interpolation



Bezier Curves



Uniform b-spline



Animation

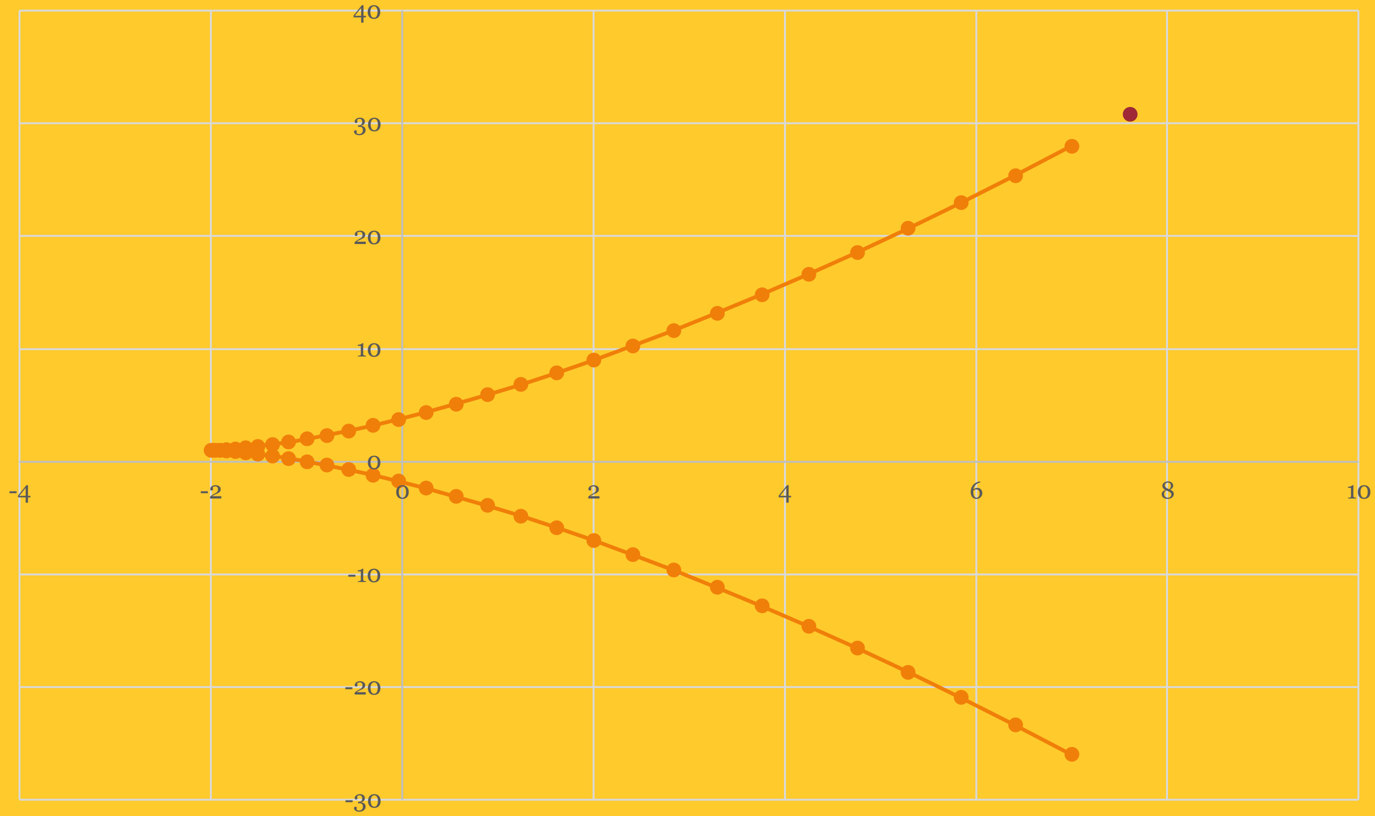
- There are many topics particularly the topics related to dynamic world can be well illustrated with the help of animation.

1) Animating an object movement in a plane: Let an object is moving in a plane whose coordinates are given by the formula – $x=t^2-2$ and $y=t^3+1$. We want to trace the position of the object from $t=-3$ to $t=3$ for increments of 0.1.

Steps for animation

- Step-1: Calculate the values of x and y for different values of t from -3 to 3 with an increment of 0.1.
- Step-2: Plot x and y to create the path of the object.
- Step-3: Insert a slider.
- Step-4: Link the slider value to t
- Step-5: Calculate the corresponding x and y value
- Step-6: Plot the point as object position at time t

Animating an object movement

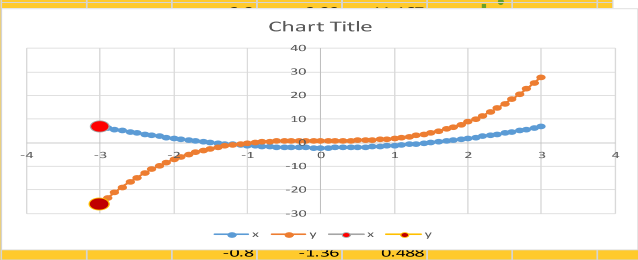


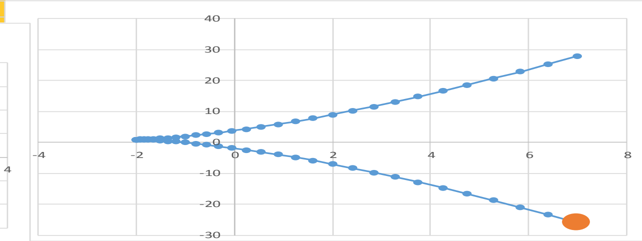
Animation of a moving object

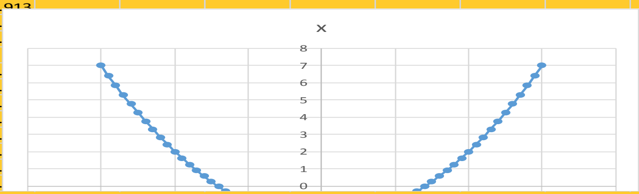
	t	x	y
Animation in excel			
25-05-2016			
DOUBLE DECKER	t	x	y
DELHI TO JAIPUR	-3	7	-26
	-2.9	6.41	-23.389
	-2.8	5.84	-20.952
	-2.7	5.29	-18.683
	-2.6	4.76	-16.576
	-2.5	4.25	-14.625
	-2.4	3.76	-12.824
	-0.8	-1.36	0.488
	-0.7	-1.51	0.657
	-0.6	-1.64	0.784
	-0.5	-1.75	0.875
	-0.4	-1.84	0.936
	-0.3	-1.91	0.973
	-0.2	-1.96	0.992
	-0.1	-1.99	0.999
	1.53E-15	-2	1
	0.1	-1.99	1.001
	0.2	-1.96	1.008
	0.3	-1.91	1.027
	0.4	-1.84	1.064
	0.5	-1.75	1.125
	0.6	-1.64	1.216
	0.7	-1.51	1.343
	0.8	-1.36	1.512
	0.9	-1.19	1.729
	1	-1	2
	1.1	-0.79	2.331
	1.2	-0.56	2.728
	1.3	-0.31	3.197
	1.4	-0.04	3.744
	1.5	0.25	4.375
	1.6	0.56	5.096
	1.7	0.89	5.913
	1.8	1.24	6.
	1.9	1.61	7.
	2	2	
	2.1	2.41	10.
	2.2	2.84	11.
	2.3	3.29	13.
	2.4	3.76	14.
	2.5	4.25	16.
	2.6	4.76	18.
	2.7	5.29	20.
	2.8	5.84	22.

..\cdas sanim

		0
t	x	y
-3	7	-26







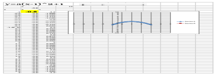
2. Animation of the shape of a graph

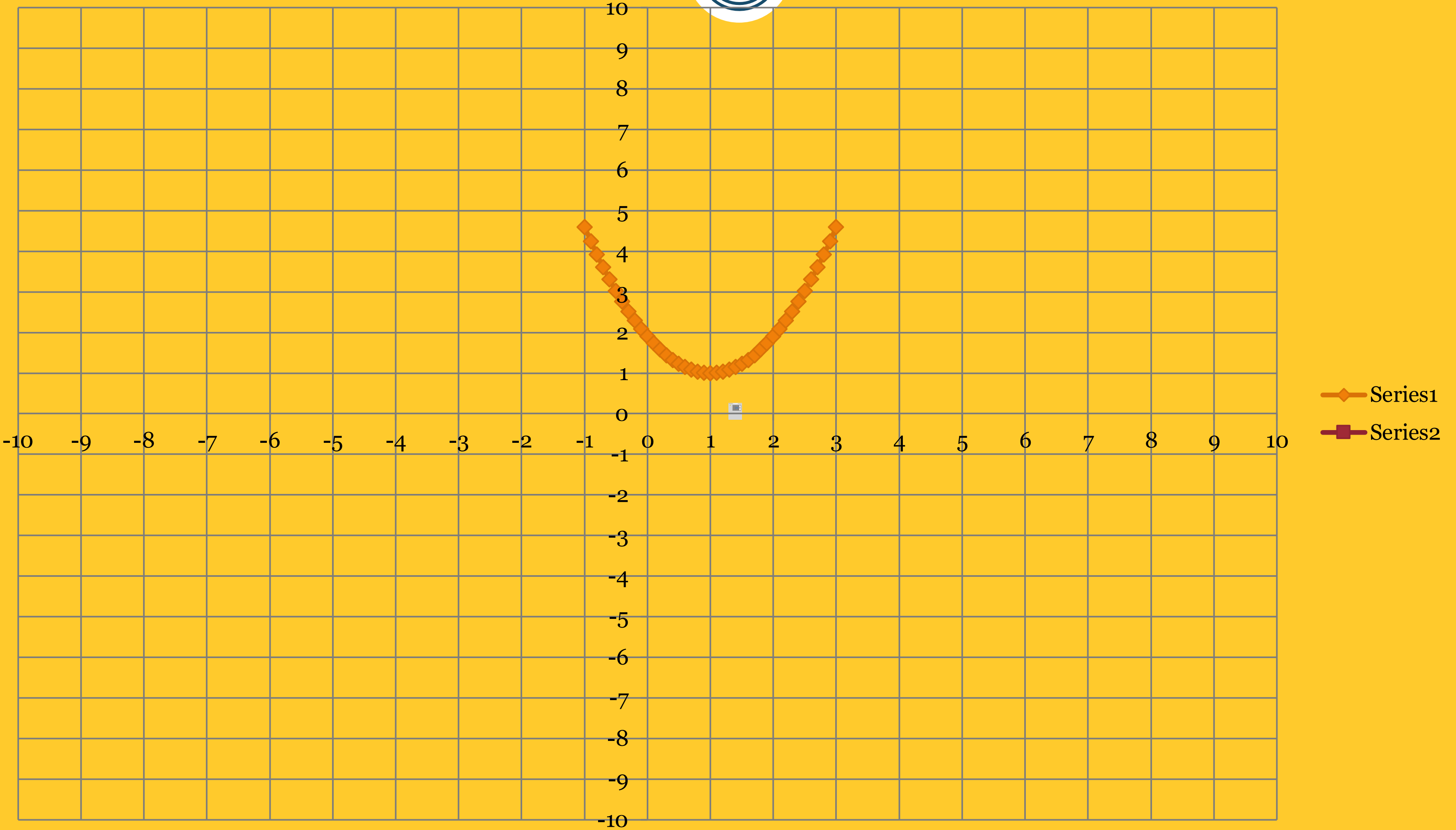
- Here the shape of a graph of a given function is animated. Let the function is given by $y=a(x-1)^2+1$.

- We are required to find the shape of the graph of this function for different values of a.

- For animating this graph we insert a slider whose value is to be linked to the values of a. There is a trick. All the a values are linked to the first value of a.

Animation of a Function

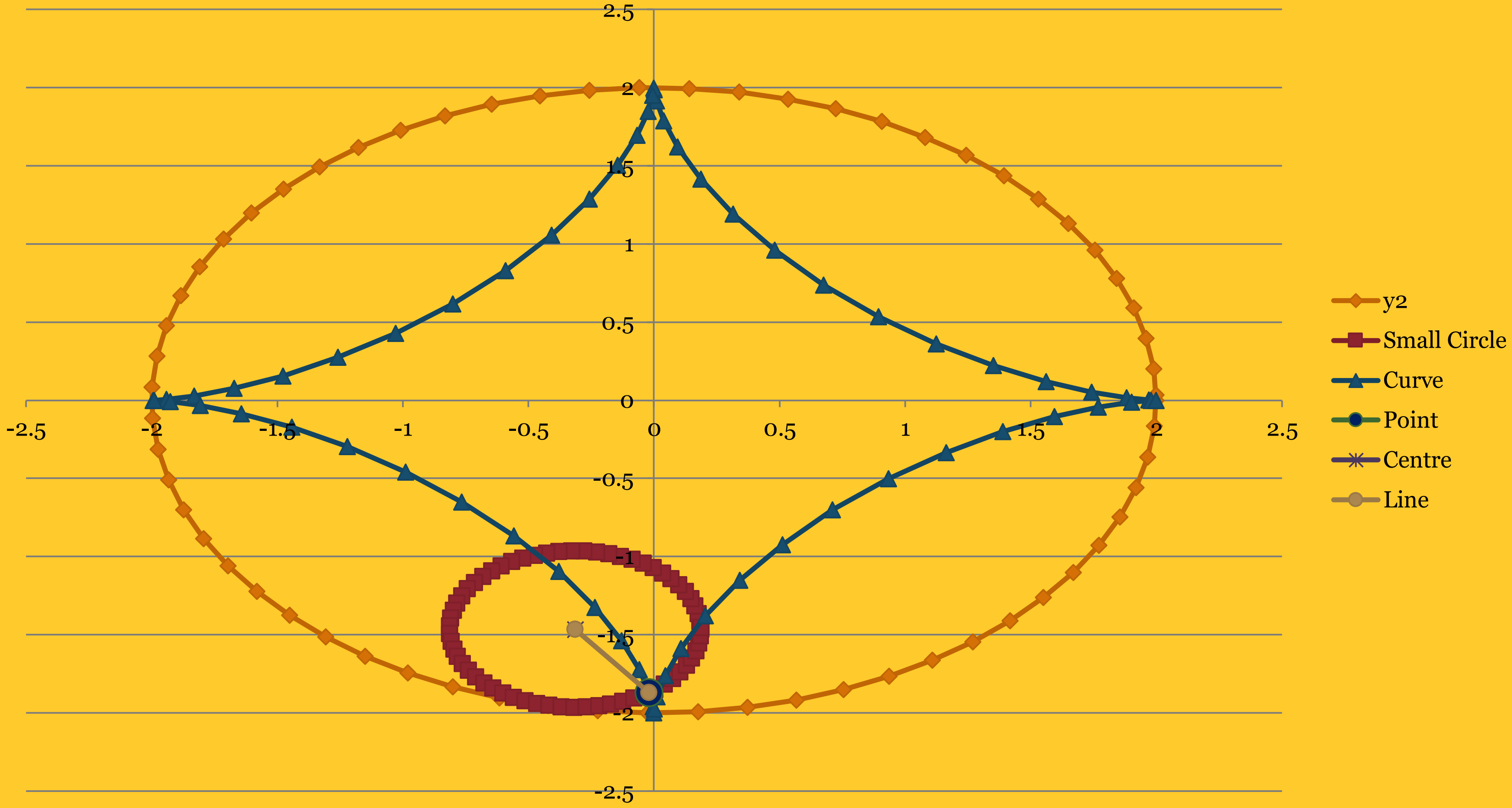




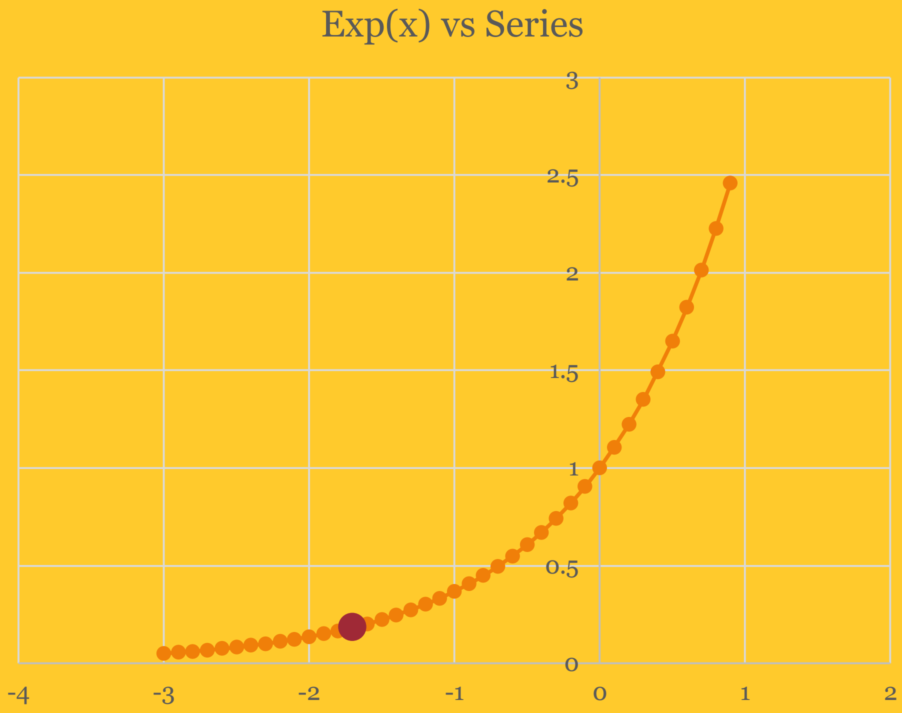
3. Animation of Hypocycloid

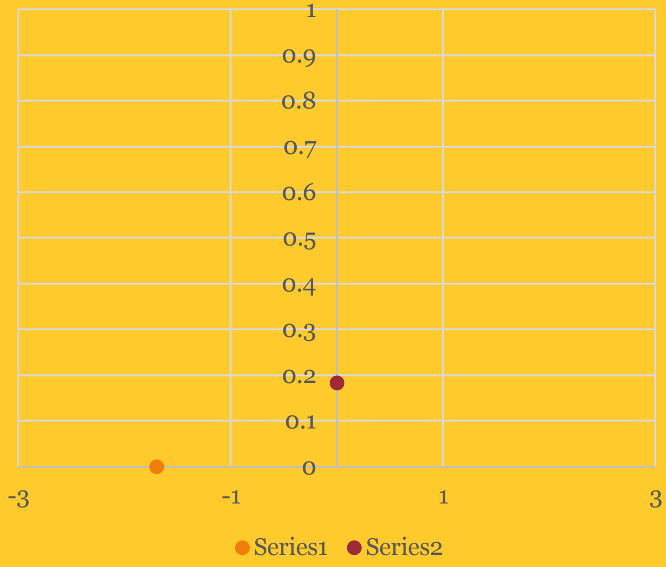
- This is a more complex animation. We want to animate a point on the edge of a small circle which rolls without slipping inside a large circle.
- The trace of the path of the particle is a closed plane curve known as hypocycloid.
- We are required to draw a Large circle, a rotating small circle, the hypocycloid, centre of smaller circle and the point on the edge of the small circle.

Animation of Hypocycloid



Animation of Exponential Series





Most Important and highest used Formulas

1. $$x^2 + y^2 = z^2$$

2. x=r*cos(t) and y=r*sin(t)

Conic Sections

ANALYTICAL VS MATRIX METHODS

Polar Equations

- Any point in the Cartesian coordinates are expressed as (x, y) and the same point in polar coordinate system is expressed as (r, t).
- Hence, x in Cartesian coordinate system can be expressed by

$$x=r*\cos(t)$$

and y can be expressed by

$$y=r*\sin(t).$$

Circle

- The circle is defined by the points whose distances from a fixed point (called origin) is same.
- The circle can be drawn by two methods-

1. By rotating the point with equal spacing
2. By incrementing the point with equal intervals.

- Ex: Draw a circle with radius r=1.
- Then x=1.cos(t)
- y=1.sin(t)

Draw Circle in Excel
Non Uniform Spacing

- Draw Circle with Radius r=1

- x=1.cost

- y=1.sint

- t=0 to 2Π
- Dt=.1

Draw Circle in Excel
Uniform Spacing of Points

- In this, points are equally spaced along the curve. First an initial point is calculated and subsequent points are calculated by adding a small incremental value.

Steps for Drawing a Circle with Uniformly spaced points

Step No.	Xi	Yi
1.	r*cos(ti)	r*sin(ti)
2.	x(i+1)=r*cos(ti+δt)	y(i+1)=r*sin(ti+δt)
Using the sum Angle Formula		
3.	x(i+1)=r(costi.cosδt-sinti.sinδt)	y(i+1)=r(costi.sinδt-cosδt.sinti)
4.	X(i+1)=(xi.cosδt-yi.sinδt)	Y(i+1)=(xi.sinδt-yi.cosδt)

Parabola

- In Cartesian coordinate system, the parabola is represented by $y = \pm\sqrt{4ax}$
- In this equation, y is having two values for each value of x. Hence this equation cannot be represented graphically easily.
- We can draw two curves to draw a parabola- one for (-y) values for each x and one for (+y) values for same x.

Parabola

- Parametric Representation of parabola is given by-

$$x = \tan^2 \phi$$
$$y = \pm\sqrt{a + a\tan\phi}$$

- In many cases, we get parabola as $y=x^2$
- But this is not the standard form of parabola as it is not aligned with the x-axis.

Hyperbola

- The standard form of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- The hyperbola has two separate branches that approach two asymptotes.
- Note: Asymptote is some boundary beyond which a curve will not pass. Beyond origin, x and y are very large compared to 1 in the right hand side of the equation, so it can be approximated as :
- or

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

or

$$y = \frac{b}{a}x$$

$$y = -\frac{b}{a}x$$

Hyperbola

- These equations produces two straight line that pass through origin (c=0) and slope=b/a or –b/a and angle between the lines are arctan(b/a) and –arctan(b/a) to the x-axis.
- The parametric representation is:

  $x=\pm a\sec t$, [h/b]

  $y=\pm b\tan t$, [b/a]
- Another alternative parametric representation-

  $x=a \cosh t$

  $y=b \sinh t$

  $\cosh t=(e^t+e^{-t})/2$

  $\sinh t=(e^t-e^{-t})/2$

as t varies from 0 to ∞ hyperbola is traced out.

Hyperbola

- x_i=a cosh t_i
  y_i=b sinh t_i

- x_{i+1}=a (cosh (t_i+δ_t))
  y_{i+1}=b(sinh (t_i+δ_t))
  or
- x_{i+1}=a (cosht_i .coshδ_t-sinh t_i . sinhδ_t)
  y_{i+1}=b(sinht_i . coshδ_t + cosht_i . sinhδ_t)

- tmin= $\cosh^{-1}$(xmin/a)
  tmax= $\cosh^{-1}$(xmax/a)

- $\cos^{-1} x=\ln(x+\sqrt{x^2-1})$

Equation of a Conic Section

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Can we determine the type graph - whether it is a Line, Circle, Ellipse, Parabola or Hyperbola, by looking at this equation?

Determinant Role in Identification

- If the equation: $ax^2 + bxy + cy^2 + dx + ey + f = 0$
- Then, the major determinant is-
$$\begin{vmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{vmatrix}$$
- The major determinant formed by the coefficients plays a major role in determining when an equation will be a line, an ellipse, a hyperbola or parabola.

Identifying the Type of Conic Section

$$ax^2 + bxy + cy^2 + dx + ey + f = 0\text{........}(1)$$

1. If a, b, c =0, then it is a straight line

   $dx+ey+f=0$

2. If equation 1 can be factorized then it is the equation of two lines

   $(x-1)(y-2)=0$ i.e. $x=1$, $y=2$ lines

3. In other cases either it will be an ellipse or a parabola or a hyperbola
4. So the problem is to ascertain when an equation will be a line or circle or ellipse or parabola or hyperbola

Determinant Role in Identification

$$\begin{vmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{vmatrix}$$

- If Δ=0, then the equation is a line (or pair of lines)
- When major determinant is not 0, then the equation is a Conic Section.
- But is it an Circle, Ellipse, Hyperbola or parabola?

Determinant Role in Identification

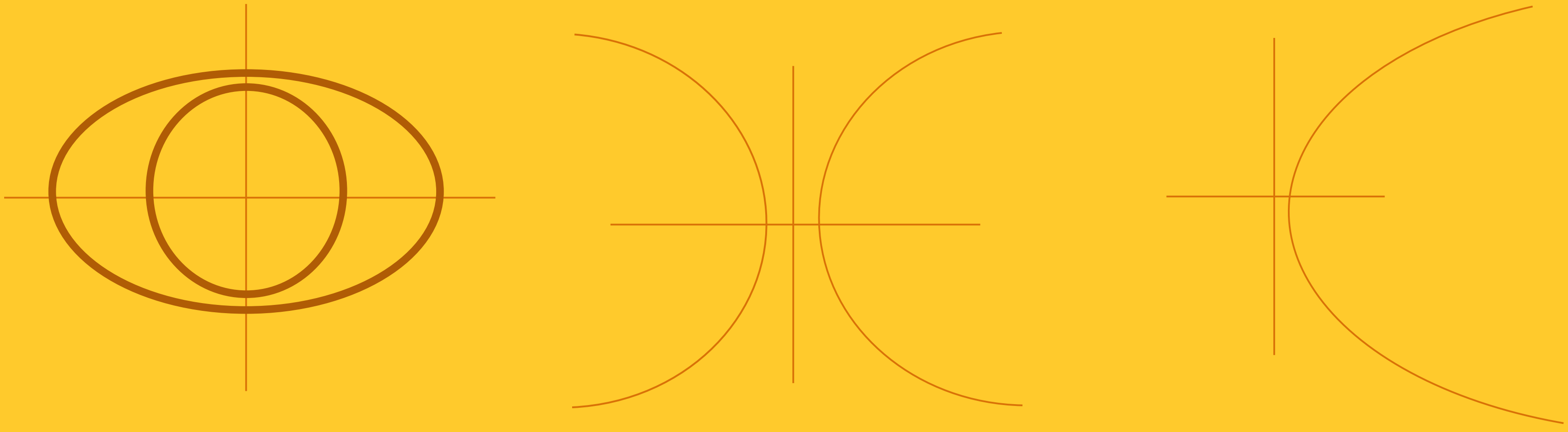
- Minor Determinant determines the type of conic section.
- Minor Determinant= $\begin{vmatrix} a & b/2 \\ b/2 & c \end{vmatrix} = \Delta$
- If $\Delta>0$, Ellipse
- If $\Delta<0$, Hyperbola
- If $\Delta=0$, Parabola

- The conic section is ascertained here but how do we get the standard form of the conic section?

Two types of Conic Sections

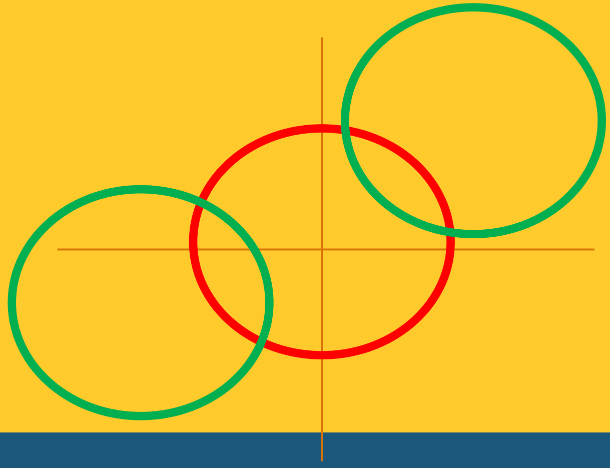
- Central
- Non- Central
- ELLIPSE, CIRCLE, HYPERBOLA- CENTRAL CONIC
- PARABOLA- NON CENTRAL

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$



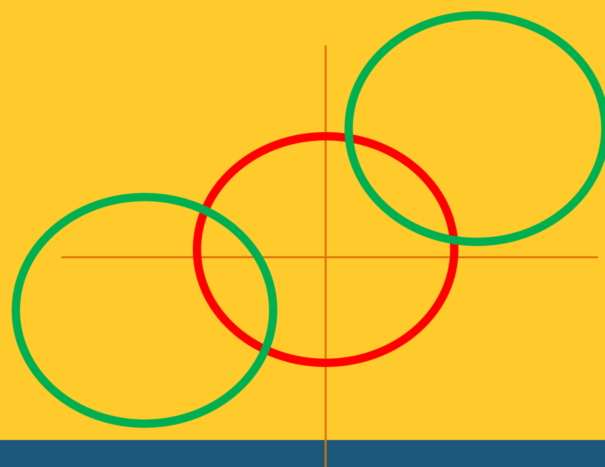
Standard form

- How to transform a Non-Standard equation into a Standard Form?
- In a non standard equation, the x*y term rotates the section in a certain angle and x and y shifts the conic section from origin.



Standard form

- Standard form of a circle= $x^2 + y^2 = r^2$
- Standard form of Ellipse= $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Bringing Back to Standard Form

- In order to transform an equation to its standard form, it is to be rotated back to the standard position and translate the center to the origin.
- The rotation angle can be determined by:

$$t = \frac{1}{2} a\tan(\frac{b}{a-c})$$

Bringing Back to Standard Form

- To determine the value of m and n which is to be translated to x and y direction are calculated from minor determinants-

$$m = \frac{\begin{vmatrix} d & b \\ e & c \end{vmatrix}}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}} \qquad n = \frac{\begin{vmatrix} a & d \\ b & e \end{vmatrix}}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}}$$

Bringing Back to Standard Form

- Rotate back the given equation
- Translate back to origin
- This will remove the x*y, x and y terms from the given equation to get the standard form
- Translation and Rotation Matrix in 2D Homogeneous Plane

$$t_r = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad t_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix}$$

Bringing Back to Standard Form

- If we apply the transformations shown below, we will get the coefficient matrix of the standard equation.

$$t_{ct} = t_t \times t_r \times t_c \times t_r^{-1} \times t_c^{-1}$$

$$t_r = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad t_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix} \quad t_c = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

Ensure whether Standard form corresponds to Original Equation?

- First the standard equation is to be formed
- Then the invariants of standard form and given equation is to be matched.
- 3 invariants do not change when we transform the equations. The invariants are-

1. a+c=a'+c'
2. $\begin{vmatrix} a & b \\ b & c \end{vmatrix} = \begin{vmatrix} a' & b' \\ b' & c' \end{vmatrix}$
3. ΔM=ΔM'

Now how do we draw such conic sections?

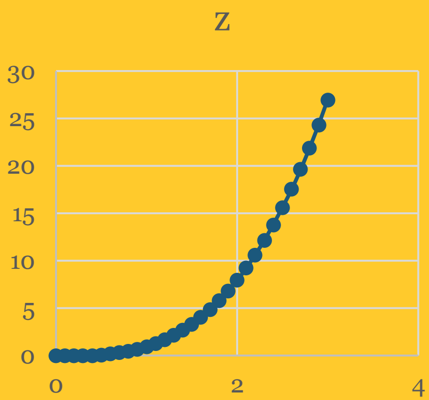
Vector Function

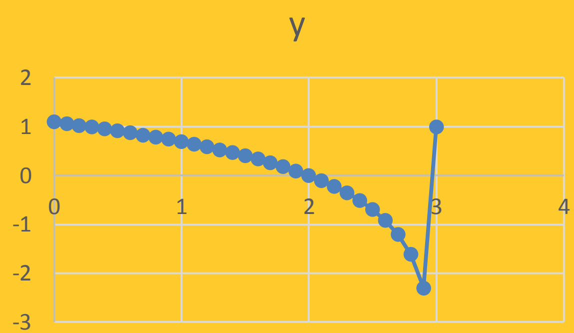
- A vector valued function is a rule that assigns to each element in domain (Reals) an element in range (vectors)
- It is expressed as $r(t)=[f(t),g(t),h(t)]$ or

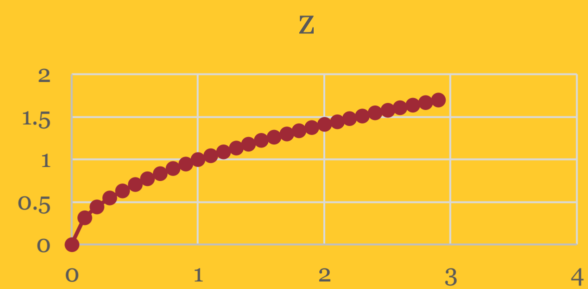
$$r(t)=f(t)i+g(t)j+h(t)k$$

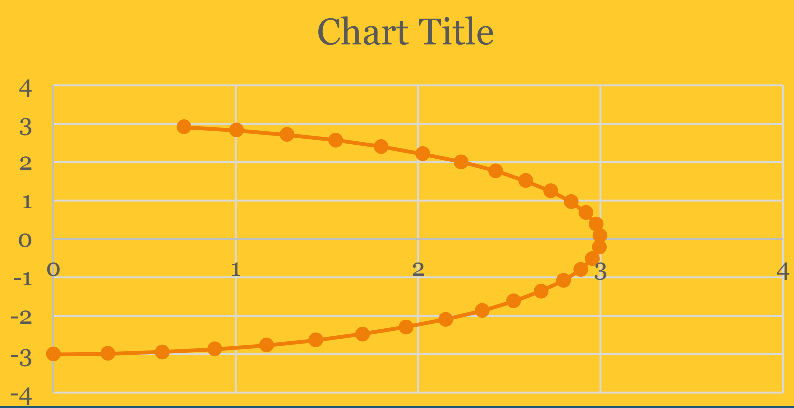
Example of Vector Function

- r=[t^3, ln(3-t), sqrt(t)]



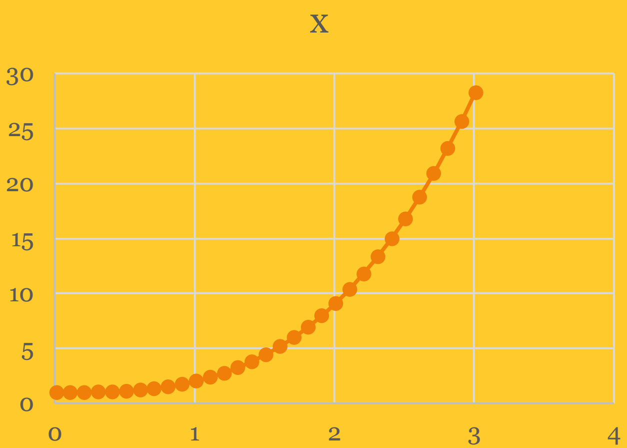


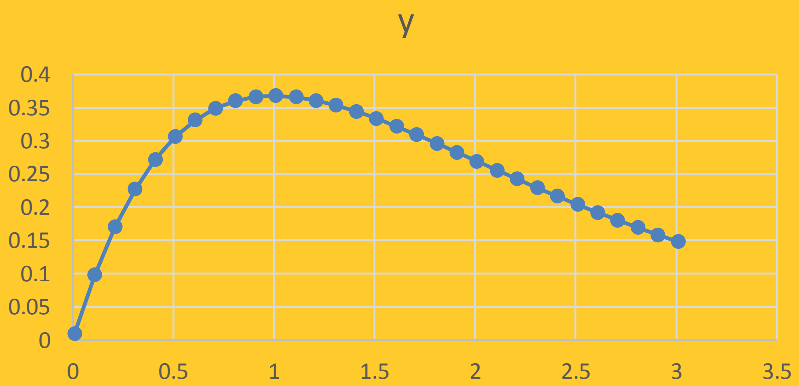


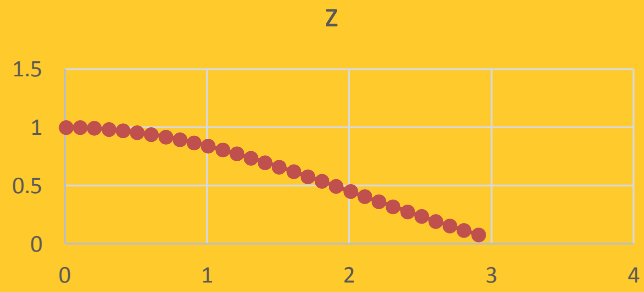


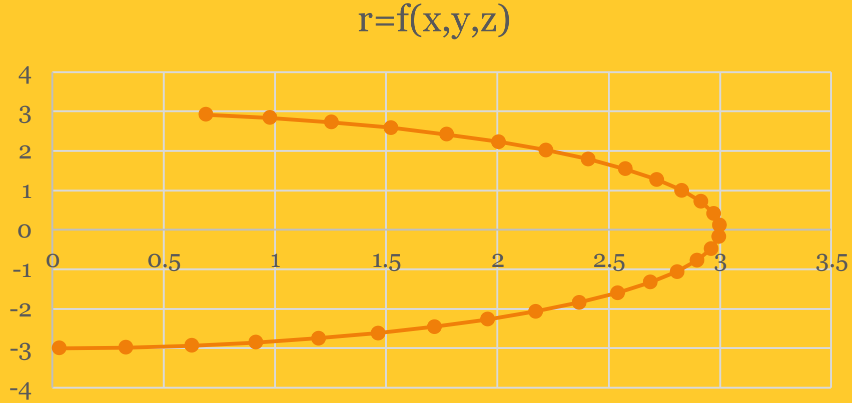
Example of Vector Function

- $R=(1+t^3)i+t*\exp(-t)j+\sin(t)/t\ k$



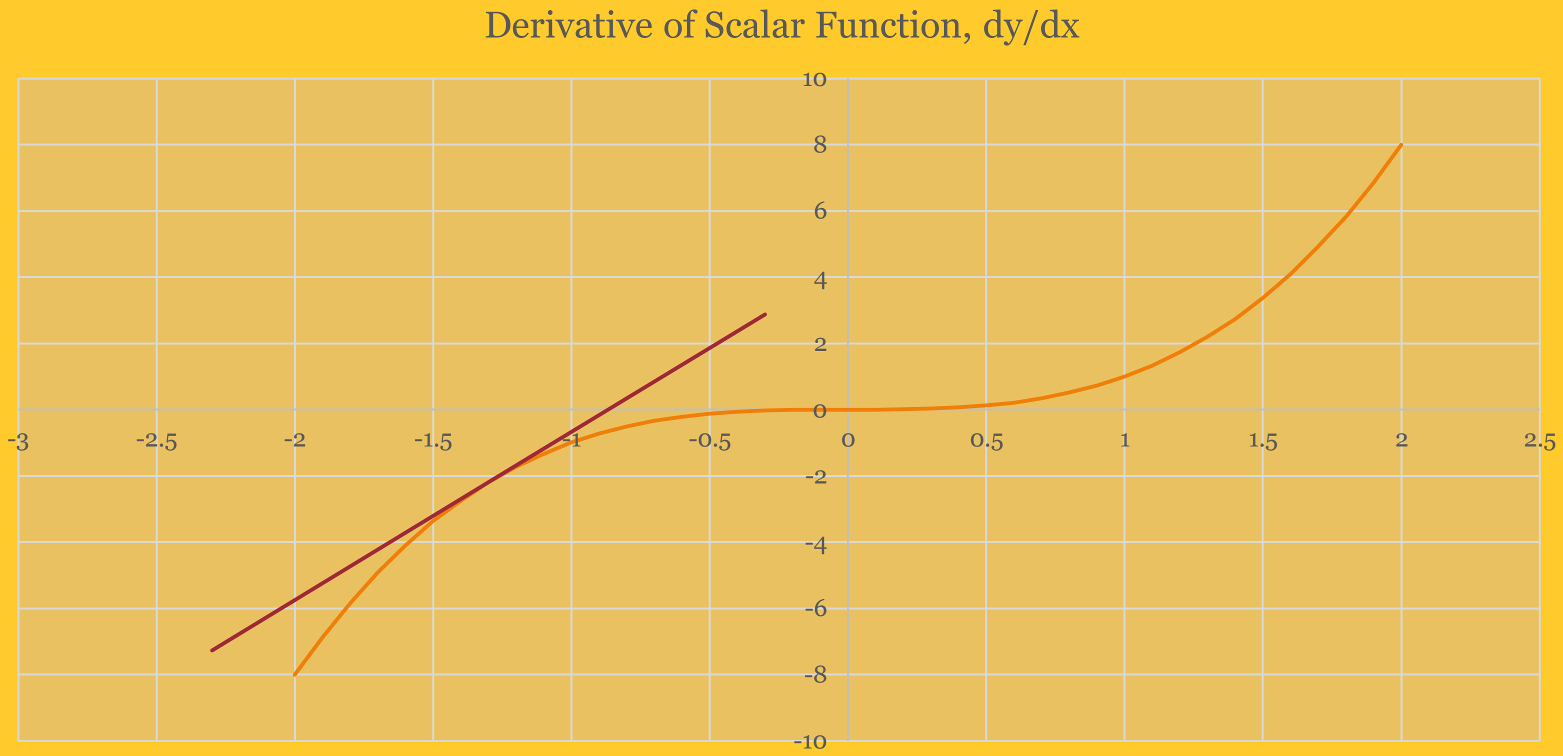






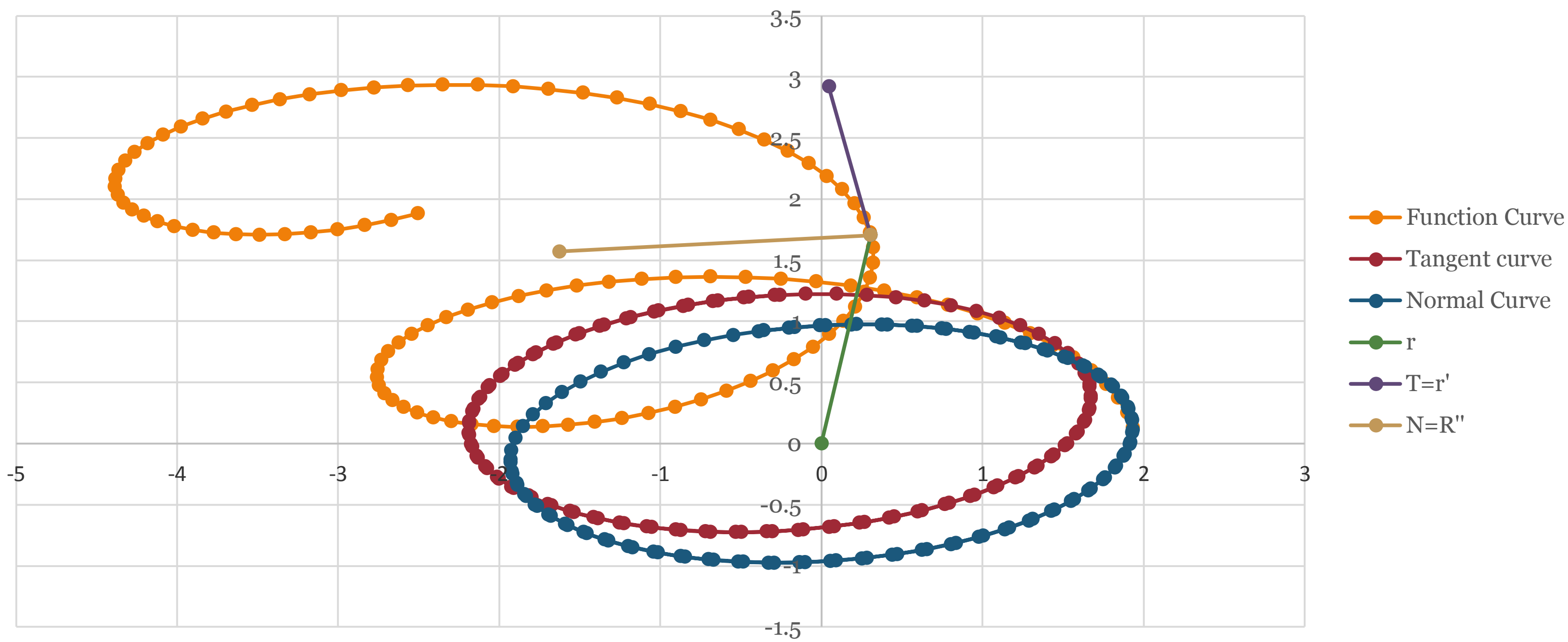
Scalar Function

y=x^3



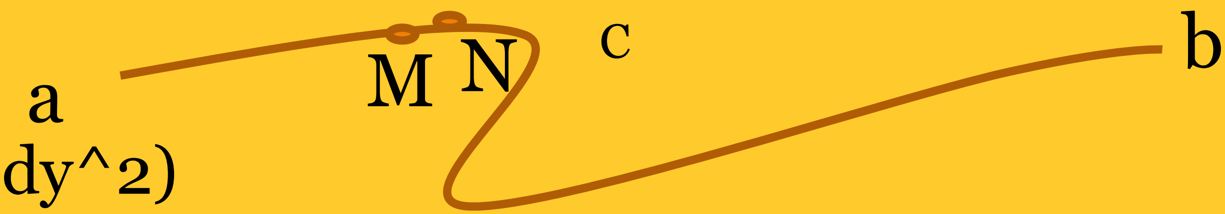
Vector Function

r	2 cos(t)	Sin(t)	t
r'	-2 sin(t)	Cos(t)	1
R’’	-2 cos(t)	-sin(t)	0



Arc Length

- If we take a infinitesimal arc MN, which is equivalent to the cord MN. Let, dx and dy is the increment of x and y for small increment of t, dt,



- $MN=\sqrt{(dx^2+dy^2)}$
- $MN=\sqrt{(((dx^2+dy^2)/dt^2)*dt^2)}$
- $MN=\sqrt{((dx^2/dt^2+dy^2/dt^2)*dt^2)}$
- $MN=\sqrt{((dx/dt)^2+(dy/dt^2))}\ dt$
- $s=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\quad dt$

Arc Length Parametrization of curve

- A parametric representation of a curve with arc length as parameter is called an arc length parametrization of the curve.
- Page-31:Example-4. Find the arc length parametrization of the line x=3t+2, y=2t-1 that has reference point (2,-1) and the same orientation as the original line.
- $s=\int_0^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
- $s=\int_0^t \sqrt{3^2 + 2^2}\, dt = \sqrt{13}\, t$
- $t=s/\sqrt{13}$ Hence, $x= 3s/\sqrt{13} + 2$ and $y=2s/\sqrt{13} - 1$

Curvature-Use of Arc Length Parametrization

Q&A…

ARITHMETIC:

- **Arithmetic** or arithmetics (from the Greek word arithmos, "number") is the oldest and most elementary branch of mathematics. It consists of the study of numbers, especially the properties of the traditional operations between them—addition, subtraction, multiplication and division. (From Wiki)

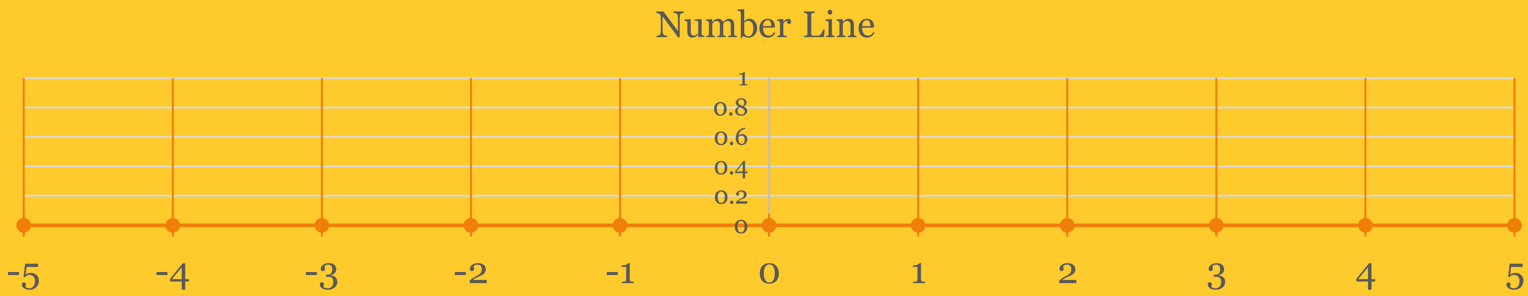
ARITHMETIC:

- FINDS UNKNOWN PARAMETER OF A SYSTEM FROM SOME KNOWN PARAMETERS. GEOMETRICALLY ARITHMETIC DEALS WITH NUMBER LINE

$$A=\pi r^2$$

Radius=10

Area=22/7*10^2=314.

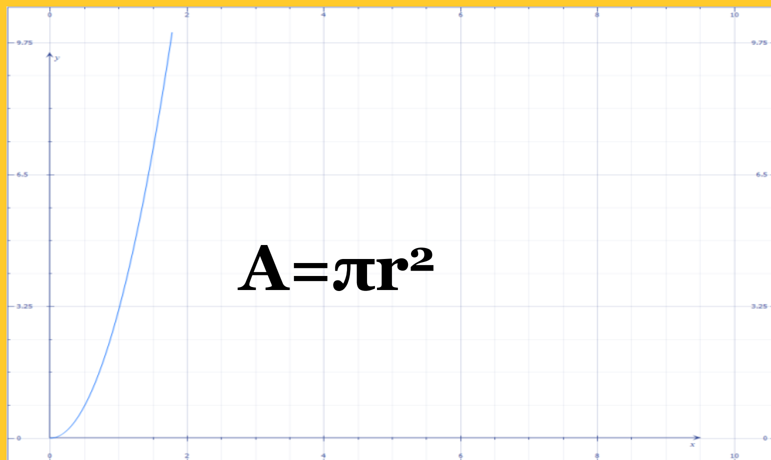


- **Linear system governed by straight lines, so we will deal straight lines first**

- **What is Equation of Straight Lines???**

ALGEBRA

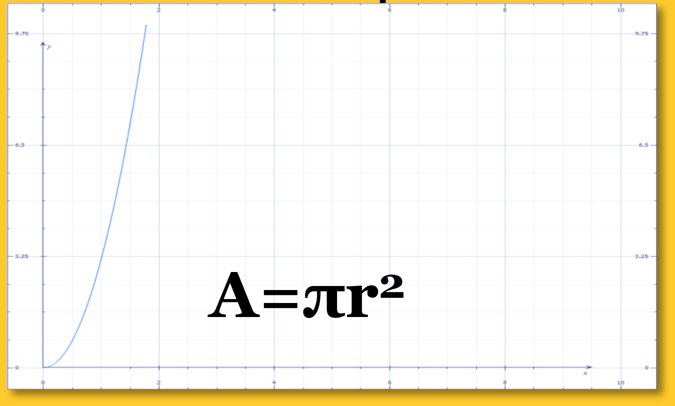
- Finds unknown parameter for a generalised system from the relationship between known and unknown variables



- In algebra, a functional relationship is established among the known and unknown variables and then the unknown variable is calculated. Geometrically it works multi-dimensions

CALCULUS

- Calculus is used to calculate the effect of rate of change of one parameter on other parameter of the system,



- In circle, we want to know the effect of the rate of change on radius on the area.
- Geometrically, in calculus we search for the slope of a

line

Class-XI Math Syllabus

Sl No	Topic	Group
1	Sets	G1
2	Relations and Functions	G1
3	Trigonometric Functions	G1
4	Principle of Mathematical Inductions	G2
5	Complex Numbers and Quadratic Equations	G3
6	Linear Inequalities	G4
7	Permutations and Combinations	G5
8	Binomial Theorems	G5
9	Sequences and Series	G6

Class-XI Math Syllabus

Sl No	Topic	Group
10	Straight Lines	G6
11	Conic Sections	G6
12	Three dimensional Geometries	G1
13	Limits and Derivatives	G7
14	Statistics	G7
15	Probabilities	G7
16	Appendix-1: Infinite Series	G5
17	Appendix-2: Mathematical Modeling	G1

Class-XII Part-1 Math Syllabus

Sl No	Topic	Group
1	Relations and Functions	G1
2	Inverse Trigonometric Functions	G1
3	Matrices	G8
4	Determinants	G8
5	Continuity and Differentiability	G1
6	Application of Derivatives	G1
7	Appendix-1: Proof of Mathematics	G11
8	Appendix-2: Mathematical Modeling	G12

Class-XII Part-2 Math Syllabus

Sl No	Topic	Group
7	Integrals	G1
8	Application of Integrals	G1
9	Differential Equations	G1
10	Vector Algebra	G2
11	Three Dimensional Geometry	G3
12	Linear Programming	G4
13	Probability	G5

Group-1: Set, Functions, Calculus

Sl No	Topics	Class
1	Sets	XI
2	Relations and Functions	XI
3	Relations and Functions	XII
4	Trigonometric Functions	XI
5	Inverse Trigonometric Functions	XII
6	Limits and Derivatives	XI
7	Continuity and Differentiability	XII
8	Application of Derivatives	XII
9	Integration	XII
10	Application of Integrations	XII
11	Differential Equations	XII

Group-2: Mathematical Induction

Sl No	Topics	Class
1	Mathematical Induction	XI

Group-3: Complex Numbers

Sl No	Topics	Class
1	Complex Numbers and Quadratic Equations	XI

Group-4: Linear Inequalities

Sl No	Topics	Class
1	Linear Inequalities	XI

Group-5: Permutations and Combinations

Sl No	Topics	Class
1	Permutations and Combinations	XI
2.	Binomial Theorem	XI
3.	Sequences and Series	XI
4.	Infinite Series	XI APP

Group-6: Coordinate Geometry

Sl No	Topics	Class
1	Straight Lines	XI
2.	Conic Sections	XI
3.	Three Dimensional Geometry	XI
4.	Three Dimensional Geometry	XII P2

Group-7: Statistics and Probability

Sl No	Topics	Class
1	Statistics	XI
2.	Probabilities	XI
3.	Probabilities	XII P2

Group-8: Linear Algebra

Sl No	Topics	Class
1	Matrices	XII
2.	Determinants	XII

Group-9: Vector Algebra

Sl No	Topics	Class
1	Vectors	XII P2

Group-10: Operations Research

Sl No	Topics	Class
1	Linear Programming	XII P2

Group-11: Proof of Mathematics

Sl No	Topics	Class
1	Proof of Mathematics	XI

Group-12: Mathematical Modelling

Sl No	Topics	Class
1	Mathematical Modelling	XI
2.	Mathematical Modelling	XII

Notes on preparation of Number systems

1. This file is started on 20042018
2. This is based on the notes on notebook 5 and typing materials of these notes typed by Aloka Mahato
3. As on 21-04-2018 total slide is 21.
4. Topics covered up to class-V Natural Numbers

21-04-2018

1. Today I will cover whole numbers from class-VI Chapter-2
2. Slide no 39
3. 23-04-2018: Completed up to slide 44

26-04-2018 Venus International New Delhi

1. Total 61 slides
2. Will add these section in my main presentation

13-05-2018

1. Number system starts at slide - 156

Number system

Number system

Class-I	Chapter-2	Natural Number	1 to 9	21
	Chapter-5	Natural Number	10 t0 20	69
	Chapter-8	Natural Numbers	21 to 50	104
	Chapter-11	Natural Numbers	51 to 100	117
	Chapter-3	Addition		51
	Chapter-4	Subtraction		61

Natural Number system: Class-I

Chapter-2	Numbers from 1 to 9		21
	Counting- 1, 2, 3, 4, 5, 6, 6, 7, 8, 9		
	Comparing- More or less		
	Matching		
	Collecting few from many		
Chapter-5	Number	10 to 20	69
	Comparing:		
	Bigger number -	Smaller number	
	Biggest -	Smallest	
	Group of 10		
Chapter-8	Number	from 21 to 50	104
Chapter-11	Numbers	51 to 100	117
Chapter-3	Addition →	One more	51
Chapter-4	Subtraction →	Take away	61
Introduction of	O	by taking out	

Natural Number system: Class-II

Class-II	(1) Ch-2	Counting in Groups	9
	(2) Ch-4	Counting in tens	24
	(3) Ch-8	Tens and ones	56
	(4) Ch-12	Give and take	90

Natural Number system: Class-III

Class-III	-	Numbers up to 1000
	–	Place value
	–	Counting
	–	Comparing –Greatest and smallest
	–	Multiplication
	-	Division

Class-III	Chapter-2	Fun with numbers	13
	Chapter-3	Give and Take	29
	Chapter-6	Fun with give and take	76
	Chapter-9	How many times (Multiplication Table)	122
	Chapter-12	Can we share (Division)	160

Natural Number system: Class-IV

Natural Number system: Class-V

Syllabus of Class-VI

Natural Number system: Class-VI

Class-VI Chapter-1 Knowing our member

1.1/1.2 Comparing numbers – Greatest ↔ Smallest
1.2.1 Arranging the numbers/5732/2357/7235/2537
1.2.2 Ordering the numbers – Biggest to Smallest
Ascending – Descending
1.2.2 Shifting digits. 795->597
1.2.3 Introducing 10000 → 9+1=10, 99+1=100 ……
1.2.4 Place value – ones, tens, hundredths, thousands
7-Ones Place, 87->8 tens place, 7 ones place->8*10+7
1.2.5 Introducing 100000
1.2.6 Larger Numbers
1.2.7 Use of indicator to keep track of large numbers

Natural Number system: Class-VI

Natural Number system: Class-VI

Roman Numerals: I, V, X, L, C, D, M

Roman	**Numbers**
I – 1	C – 100
V – 5	D – 500
X – 10	M – 1000
L – 50	

All the numbers are formed by these 7 numerals

1.5 Roman Numerals

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X
1	5	10	50	100	500	1000			
I	V	X	L	C	D	M			

Natural Number system: Class-VI

Rules of the Roman numerals :

(a) If a symbol is repeated, its value is added as many times as it occurs.
X=10, XX = 20, XXX - 30

(b) Any symbol is not repeated more than three times. But the symbols V=5, L=50, D=500 never repeated.

(c) If a symbol of smaller value is written to the right of a symbol of greater value, its value gets added to the value of greater symbol.
Example-L X V = 50 + 10 + 5 = 65

(d)If a symbol of smaller value is written to the left of a symbol of greater value, its value is subtracted from the value of greater symbol.

Example- IX=10-1, XL=50-10=40

(e) The symbol V, L, D are never written to the left of a symbol of greater value, i.e., V, L, D are never subtracted.

Example- VX=10-5 not correct, LD=500-50 not correct

Natural Number system: Class-VI

Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbe rs	Natural Numbers	Roman Numbe rs
1	I	11	XI	21	XXI
2	II	12	XII	22	XXII
3	III	13	XIII	23	XXIII
4	IV	14	XIV	24	XXIV
5	V	15	XV	25	XXV
6	VI	16	XVI	26	XXVI
7	VII	17	XVII	27	XXVII
8	VIII	18	XVIII	28	XXVIII
9	IX	19	XIX	29	XXI9
10	X	20	XX	30	XXX

Natural Number system: Class-VI

Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers
31	XXXI	41	XLI	51	LI
32	XXXII	42	XLII	52	LII
33	XXXIII	43	XLIII	53	LIII
34	XXXIV	44	XLIV	54	LIV
35	XXXXV	45	XLV	55	LV
36	XXXVI	46	XLVI	56	LVI
37	XXXVII	47	XLVII	57	LVII
38	XXXVIII	48	XLVIII	58	LVIII
39	XXXIX	49	XLIX	59	LIX
40	XL	50	L	60	LX

Natural Number system: Class-VI

Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers
61	I	71	XII	81	XXI
62	II	72	XII	82	XXII
63	III	73	XIII	83	XXIII
64	IV	74	XIV	84	XXIV
65	V	75	XV	85	XXV
66	VI	76	XVI	86	XXVI
67	VII	77	XVII	87	XXVII
68	VIII	78	XVIII	88	XXVIII
69	IX	79	XIX	89	XXI9
70	X	80	XX	90	XXX

Natural Number system: Class-VI

Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers
91	I	110	XII	121	XXI
92	II	112	XII	122	XXII
93	III	113	XIII	123	XXIII
94	IV	114	XIV	124	XXIV
95	V	115	XV	125	XXV
96	VI	116	XVI	126	XXVI
97	VII	117	XVII	127	XXVII
98	VIII	118	XVIII	128	XXVIII
99	IX	119	XIX	129	XXI9
100	X	120	XX	130	XXX

Natural Number system: Class-VI

Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers	Natural Numbers	Roman Numbers
131	*I*	*141*	*XII*	*151*	*XXI*
132	*II*	*142*	*XII*	*152*	*XXII*
133	*III*	*143*	*XIII*	*153*	*XXIII*
134	*IV*	*144*	*XIV*	*154*	*XXIV*
135	*V*	*145*	*XV*	*155*	*XXV*
136	*VI*	*146*	*XVI*	*156*	*XXVI*
137	*VII*	*147*	*XVII*	*157*	*XXVII*
138	*VIII*	*148*	*XVIII*	*158*	*XXVIII*
139	*IX*	*149*	*XIX*	*159*	*XXI9*
140	*X*	*150*	*XX*	*160*	*XXX*

Natural Number system: Class-VI

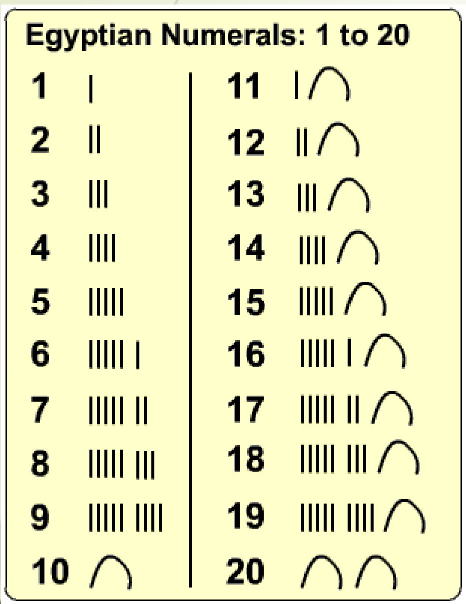
Operation's of Natural Numbers:

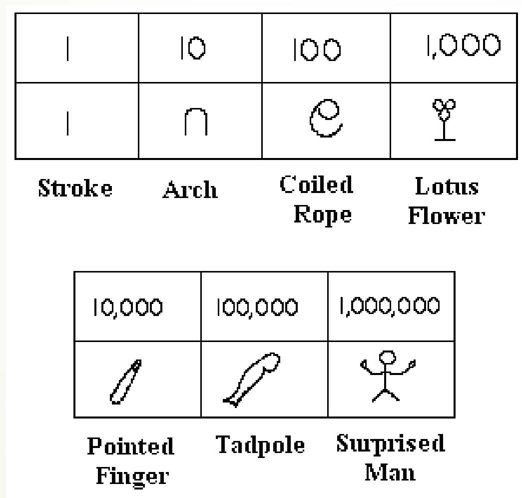
60+9=69
LX+IX=LXIX

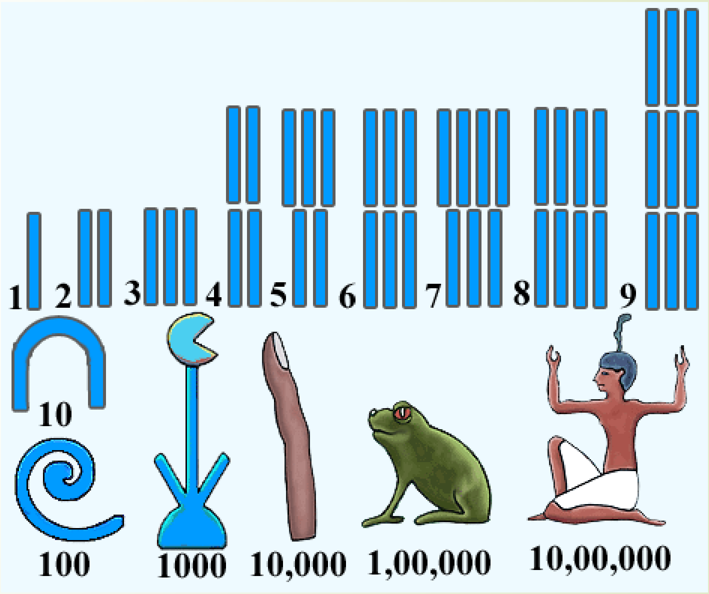
90+8=98

XC+VIII=XCVIII

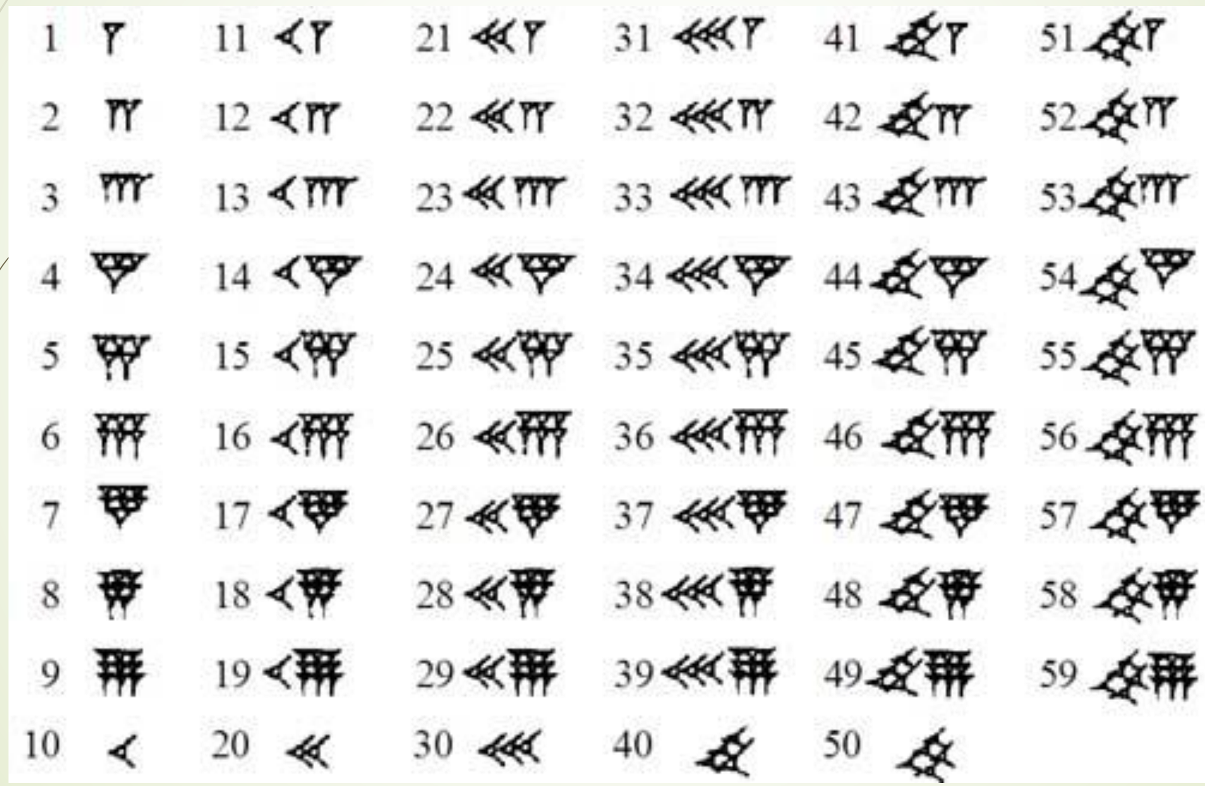
Egyptian Numerals







Babylonian Numerals



Mandarin Numerals-See the Pattern

0	零	líng
1	一	yī
2	二	èr
3	三	sān
4	四	sì
5	五	wǔ
6	六	liù
7	七	qī
8	八	bā
9	九	jiǔ
10	十	shí

11	十一	shí yī
12	十二	shí èr
13	十三	shí sān
14	十四	shí sì
15	十五	shí wǔ
16	十六	shí liù
17	十七	shí qī
18	十八	shí bā
19	十九	shí jiǔ
20	二十	èr shí

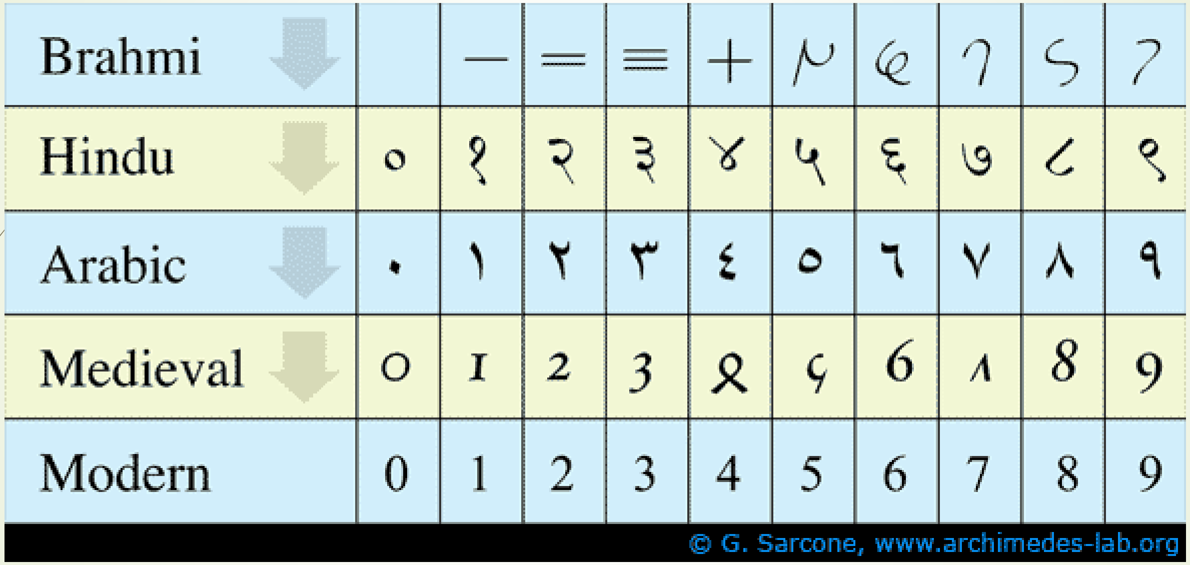
Sanskrit Numerals

०	१	२	३	४
0	1	2	3	4
shuunyá	ekaḥ	dvau	tryaḥ	catvāraḥ

५	६	७	८	९
5	6	7	8	9
pañca	ṣaṭ	sapta	aṣṭa	nava

१०	10 daśa	३०	30 triṃśat
११	11 ekādaśa	३५	35 pañcatriṃśat
१२	12 dvādaśa	४०	40 catvāriṃśat
१३	13 trayodaśa	४५	45 pañcacatvāriṃśat
१४	14 caturdaśa	५०	50 pañcāśat
१५	15 pañcadaśa	५५	55 pañcapañcāśat
१६	16 ṣoḍadaśa	६०	60 ṣaṣṭiḥ
१७	17 saptadaśa	६५	65 pañcaṣaṣṭiḥ
१८	18 aṣṭādaśa	७०	70 saptatiḥ
१९	19 navadaśa	७५	75 pañcasaptatiḥ
२०	20 viṃśatiḥ	८०	80 aśītiḥ
२१	21 ekaviṃśatiḥ	८५	85 pañcaaśītiḥ
२२	22 dvāviṃśatiḥ	९०	90 navatiḥ
२३	23 trayoviṃśatiḥ	९५	95 pañcanavatiḥ
२४	24 caturviṃśatiḥ	१००	100 śatim
२५	25 pañcaviṃśatiḥ		
२९	29 navaviṃśatiḥ		

Different Numerals



Whole Number system: Origin: Class-VI

Origin of Whole Numbers:

Natural numbers are for counting – 1 2 3 4 5 6 7 8 9

Predecessor and Successor

1. Successor : Adding 1 to any number, we get successor of that number

5 → successor → 5 + 1 = 6

2 Predecessor : Subtracting 1 to any number, we get predecessor of that number.

20 → predecessor → 20 – 1 = 19

Question: What is the predecessor to number 1.

1 – 1 = ??

Answer: The number 1 has no predecessor in natural number system. This deficiency has led to the system called whole numbers.

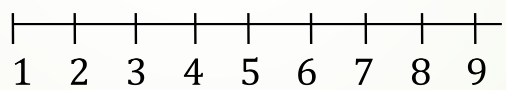
Whole Number system: Class-VI

Whole Numbers

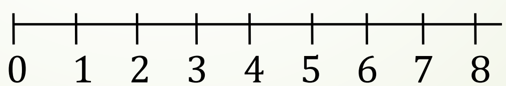
Whole number – The natural numbers along with 0 form the collection of whole numbers.

Whole numbers - 0 1 2 3 4 5 6 7 8 9

Number line - Natural number line



Whole number line



Whole Number system: Class-VI

2.4 Properties of whole numbers for Binary Operations

(a) Closer property - Whole numbers are closed under addition and also under multiplication. Division of whole number is not closed.

(b) Commutative Property of addition and multiplication of whole numbers. Order of addition and multiplication does not matter.
Subtraction and division is not commutative for whole numbers.

(c) Associativity of addition and multiplication of threes whole numbers – Yes
Division and Subtraction do not follow associative rule.
- Sequence matters in associative Law.

(d) Additive identity of whole number = 0

(e) Multiplicative Identity – 1

(f) Inverse identity of 5 , $\frac{1}{5}$

(g) Division by 0 is undefined.

Division is a process of repeated subtraction. The division rule is given below.

$18 \div 6$ = 18 – 6 – 1st Step
= 12 – 6 – 2nd Step
= 6 – 6 – 3rd Step
= 0 Remainder

$18 \div 0$ = 18 – 0 – 1st Step
= 18 – 0 – 2nd Step
= 18 – 0 – 3rd Step
= 18 – 0 – 4th Step and it continues.

So it is said that division by 0 is undefined.

Whole Number system: Class-VI

2.5 Pattern in whole numbers

Arranging numbers in elementary shapes made of dots.

Rule – The shapes can be of any five –

(1) Line
(2) Rectangle
(3) Square
(4) Triangle
(5) Dot

1	•
2	• •
3	• • • ; • • •
4	• • • • ; • • • •
5	• • • • •
6	• • • • • • ; • • • • • • ; • • • • • •
7	• • • • • • •
8	• • • • • • • • ; • • • • • • • •
9	• • • • • • • • •
10	• • • • • • • • • •

Class-VI: Factorization (Whole Numbers)

3.2 Factor: A factor of a number is an exact divisor of that number

Ex-6=1*2*3: Here 1, 2 and 3 are the factors of 6 and 6 is called multiple of 1 , 2 and 3

- 1 is factor of every number
- Every number is a factor of itself
- Every factor of a number is an exact divisor of that number
- Every factor is equal or less than given number

3.3a. Prime numbers: The number whose only factors are 1 and the number itself.
Ex-2 3 5 7 11 13 17 19 23 29 31 37 41..
3.3b. Composite Numbers: The numbers having more than two factors are called composite numbers.
Ex. 4 6 8 9 10 12 ….

Class-VI: Divisibility Rules (Whole Numbers)

3.4 Test of Divisibility (Find the pattern)

a. Divisibility by 2- If ones place is 0 or any even number
b. Divisibility by 3- If digit sum is divisible by 3
c. Divisibility by 4- Is last two digit is divisible by 4
d. Divisibility by 5- If ones place is 0 or 5
e. Divisibility by 6- If it is divisible by both 2 and 3
f. Divisibility by 7- subtract double of the last digit from rest of the number and continue until one digit remains. If it is 0 or 7, then it is divisible by 7
g. Divisibility by 8-If last three digit is divisible by 8
h. Divisibility by 9- If digit sum is divisible by 9
i. Divisibility by 10- If one place is 0

Class-VI: Common Factor and Multiple

3.5 Common Factors: A number which is factor of two numbers

Example-1: 2 is the common factor of 4 and 10
Example- 2: 3 is the common factor of 9 and 81

Prime numbers: The number whose only factors are 1 and the number itself.
Co-Prime: Two numbers which has only 1 as common factor is called co-prime.

Example-Co=prime: 4 and 15, 3 and 10,

Class-VI: Divisibility Rules (Whole Numbers)

3.6 Test of Divisibility

1. If a number is divisible by another number, then the number is divisible by its factors also

2. If a number is divisible by two co-prime numbers, then it is divisible by their product also

3. It two numbers are divisible by a number then their sum also divisible by that number

3.7: Prime Factorization: The process of getting the prime factors of a number are called prime factorization..

Class-VI: Divisibility Rules (Whole Numbers)

Divisibility of Whole numbers: 1/3, 3/1, 5/2, 2/5

Numerator	Denomarator	Quotient	Remainder
1	3	0	1
3	1	3	0
5	2	2	1
2	5	0	2

Class-VI: Integers

6.1 **Integers**

Borrow – Take on loan – 2
Due – To be repaid – 3
Forward – Move ahead + 5
Backward - Go back – 5
Debit – To get + 2
Credit – To give – 2

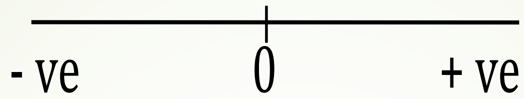
Profit – +ve gain, Loss – -ve gain

Who is where - ve ← 0 → + ve

Who follow whom Follower (Predecessor) – I – Successor

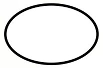
Class-VI: Integers

6.2.1 Number line for integers

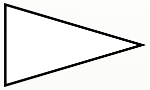


6.2 Geometrical Shape of numbers

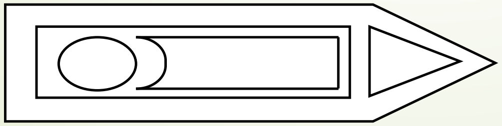
(1) Natural Numbers 

(2) Zero 

(3) Whole numbers 

(4) Negative numbers 

(5) Integers – Collection of all above numbers



Class-VII: Integers

Chapter-1 **Integers**

1.3 Properties of Addition and subtraction of integers

(1) Closure – Sum and difference of two integers is an integer. It follows closure law.

(2) Commutative property –

- *Addition follows commutative law → that is order of addition does not matter*
- *Subtraction do not follow commutative law – For subtraction order of subtraction matters.*

(3) Associative property –
Addition – Associative
Subtraction – Not Associative

(4) Additive identity=0

(5) Additive inverse, 5 = –5

Class-VII: Integers

1.4/1.5 Multiplication properties –

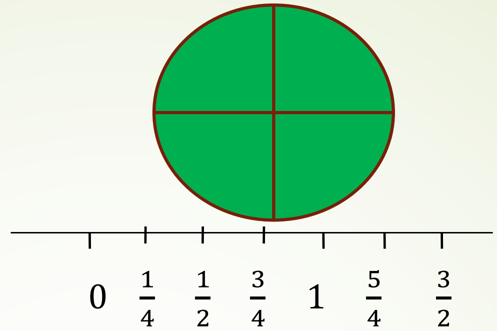
(1) Closure – Yes
(2) Commutative – Yes
(3) Associative – Yes
(4) Distributive property-Yes
(5) Multiplicative identity –1
(6) Multiplicative inverse, 5= $\frac{1}{5}$

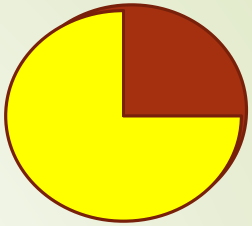
1.6 Division of integers

1.7 Properties- Commutative – No

Class-VI: Fractions

Fraction on number line





Fraction is a part of whole.

Fractional numbers like $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ *are used to represent fraction.*

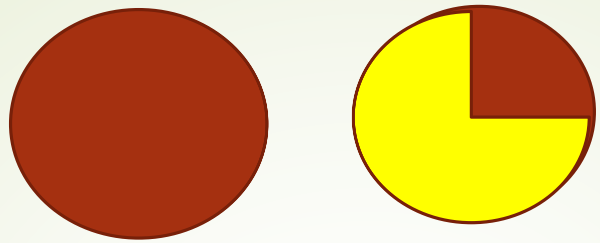
Denominator $\rightarrow$ *its shows number of parts into which whole is divided*
Numerator $\rightarrow$ *its shows number of parts that is considered*

Fraction = $\frac{numerator}{denominator}$

If whole is 8, part is 5, then the fraction = $5/8$

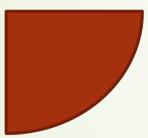
Proper Fraction: In proper fraction, denominator is always greater than numerator and that the value of proper fraction is always greater than 0 and less than 1.

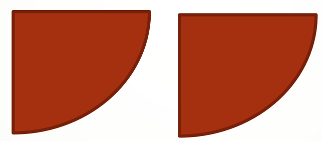
Class-VI: Proper Fractions

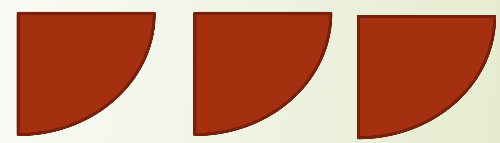


In proper fraction, *denominator is always greater than numerator and that the value of proper fraction is always greater than 0 and less than 1.*

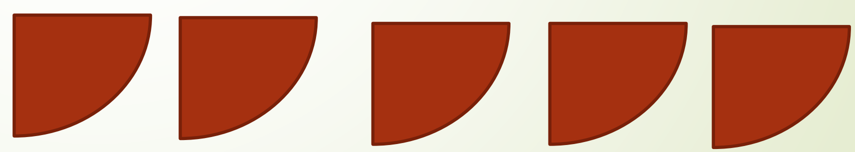
Geometrical Interpretation of Proper Fraction:

$1/4 =$ 

$2/4 =$ 

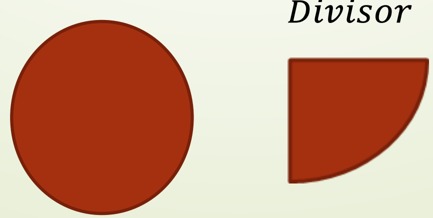
$3/4 =$ 

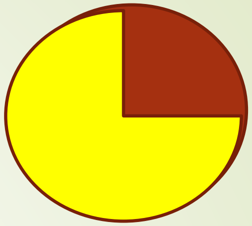
Improper fraction : *When numerator is greater than denominator, then it is improper fraction and its value is greater than 1.*

$5/4 =$ 

Mixed fractions : *A mixed fraction has combination of a whole and a part.*

Mixed fraction – Quotient $\frac{Remainder}{Divisor}$

$1\ \frac{1}{4} =$ 



Class-VI: Fractions

Improper fraction – $\frac{\text{whole x Denominator+ Numerator}}{\text{Denominator}}$

Equivalent fractions - $\frac{1}{2}$, $\frac{4}{8}$, $\frac{2}{4}$ *are equivalent fractions*

Note : Gross product of equivalent fraction is some.

Note : Lowest form or simplest form has no common factor.

Fractions: Class-VI

7.8 Like fraction : Fractions with same denominator, $\frac{1}{15}, \frac{7}{15}$
Unlike fraction – Denominator different

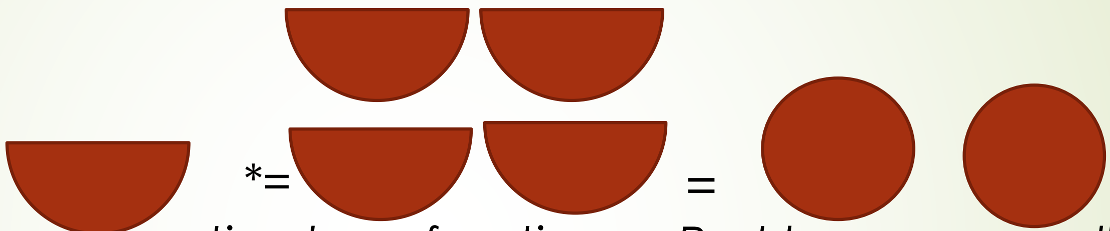
7.9.1 Comparing like fractions: Compare with numerators.

7.9.2 Comparing unlike fractions : Multiply both with LCM of both denominator and compare.

7.10 Addition and subtraction of fractions –
- Like fractions
- fraction that are unlike
- Mixed fractions

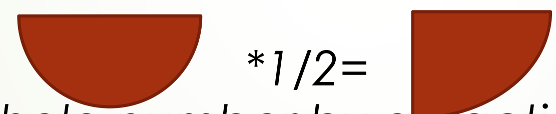
2.3.1 Multiplication of fraction by a whole numbers, Part become bigger

Example: ½*4=2



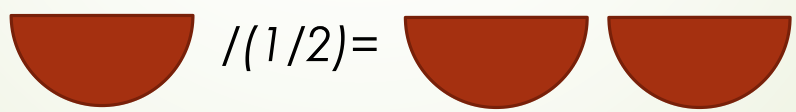
2.3.2 Multiplication of fraction by a fraction → Part become smaller

Example: ½*1/2=1/4,

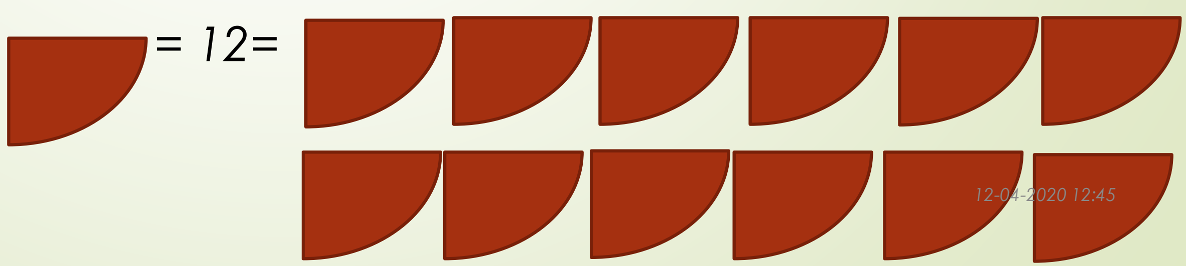


2.4.1 Division of a whole number by a fraction → The number increases

Example-1: 1/(1/2)=2 :

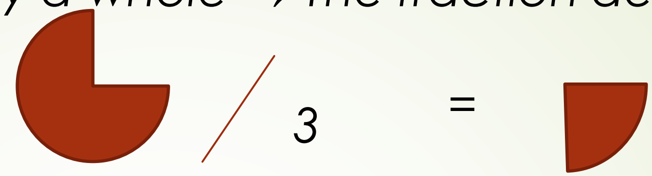


Example-2: 3/(1/4)=12: 3/



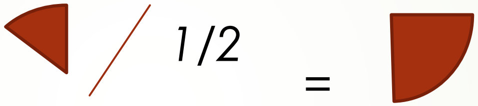
2.4.1 Division of a fraction by a whole → The fraction decreases

Example-1: (3/4)/3=3/12=1/4:



2.4.3 Division of a fraction by a fraction → The fraction increases

Example: (1/8)/(1/2=(1/4) =



Class –VI: Page -164: Chapter-8 : Decimals

Decimals: Decimals can be thought of Fractions with denominator 10

8.2 Tenths → Representing decimals in number line / Fraction as decimal
8.3 Hundredths
8.4/8.5 Comparing decimals/ Using decimals
8.6 Addition of decimals
8.7 Subtraction of decimals

2.5/2.6 Multiplication of decimal numbers → The Number decreases
2.7.1 Division of decimal numbers by 10, 100..→ The number decreases
2.7.2 Division of decimal numbers by whole number→ The number decreases

Class-VI, Page-244: Chapter -12 : Ratio and Proportion:

Two qualities, taller or shorter, heavier or lighter, smaller or longer etc. can be compared by differences or by divisions.

12.2 **Ratio :**
Division gives a better sense. Comparison by division is called ratio.

Example: Cost of Pen=10, cost of pencil = 5, Ratio of cost of pen and pencil=10;5=2:1

12.3 Proportion – Proportion compares two ratios.
Four quantities are involved in proportion- a : b : : c : d where 1st and 4^{th} , a and d, are extreme terms and 2^{nd} and 3^{rd} terms are called middle terms.
Example: Length of actual building and door to the length of replica of building and its door are in same ratio, their proportion is 10:6::1:0.6

Class-VI, Page-244: Chapter -12 : Ratio and Proportion:

12.4 Unitary Method : In unitary method, we find the value of one unit and then value of required number of units is known as unitary method.

Class-VII: Ch-8, Page-153: : Ratio and Proportion:

8.1 Ratio and Proportions
8.2 Equivalent Ratios: It is used to compare two ratios. Convert ratios in like fractions and compare.
8.3.1 Percentage: It is another way of comparing Quantities.
Percentage: Percentage is the numerators of fractions with denominator 100.

Example: Fraction is 320/400, Percentage= 320/400*100/100= 320/400*100%=.8*100%=80%

8.3.2 Converting fractional number to percentage
8.3.3 Converting decimals to percentage
8.3.4 Converting % to fraction or decimal

Class-VII Chapter-9: Rational Number

Rational number – Arise from ratio
+ ve, -ve rational numbers.

Class-VIII, Chapter-1 Rational Numbers

Examples of need of different Numbers:

Natural Numbers, x+2 = 13 or x = 13-2 = 11
If x = natural number, the equation can not be solved.
Now, Whole Numbers, x+5 =5 or x= 0
If x = natural number, the equation can not be solved.
Integer numbers, x+18 = 5, x = -13
If x = natural/ whole number, it can not be solved.
Rational number, 2x = 3, $x = \frac{3}{2}$
If x = Natural/ whole/ Integer, it can not be solved.

Class-VII Chapter-9: Rational Number

Fractional Numbers = ½ , ¾ , 7/9

Ratios = ½, ¾ , 7/9

Rational Numbers = ½ , ¾ , 7/9

What is the difference among them?

In Fractional Numbers and Ratios, the numerators and denominators are natural numbers but in Rational Numbers, the numerator and denominators are integers.

Rational number – Arise from ratio
+ ve, -ve rational numbers.

Rational Numbers are expressed as =p/q, where q#0 and p and q are integers

Properties of Rational Number

1.2.1 Closure - Closed under addition, subtraction, multiplication, Except 0, division is closed

1.2.2 Commutative –

Addition:	Yes
Subtraction:	No
Multiplication:	Yes
Division:	No

1.2.3 Associativity

Addition	-	Yes
Subtraction	-	No
Multiplication	-	Yes
Division	-	No

Properties of Rational Number

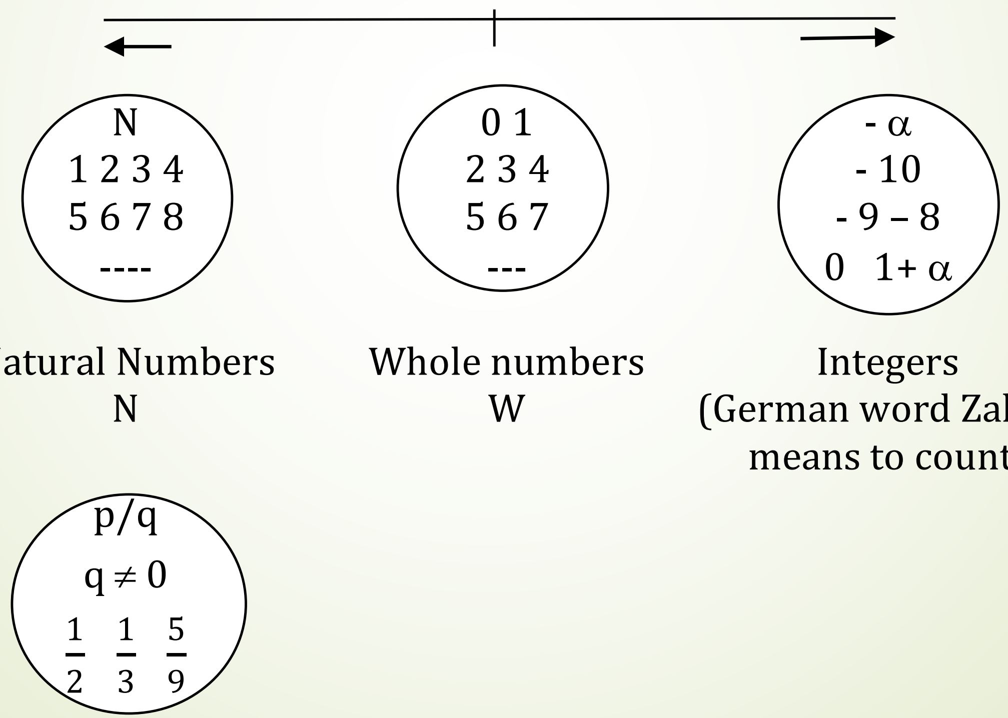
1.2.4 Additive identity: 0
1.2.5 Multiplicative identity: 1
1.2.6 Additive inverse: -x
1.2.7 Multiplicative inverse: $\frac{1}{x}$

Class-IX, Chapter-1: Irrational Numbers

Representation of Rational Numbers on the number line

Number System: Till now, we have studied, Natural Numbers, Whole Numbers, Fractional Numbers, Integers and Rational Numbers and shown geometrically:

1.1



Natural Numbers
N

Whole numbers
W

Integers
(German word Zahlen means to count)

Rational Number
Q
(Quotient)

Irrational Number

Irrational Number

- *A number 'S' is called irrational, if it cannot be written in the form $\frac{P}{Q}$, where p & q are integers and q ≠ 0*
- *There are many irrational numbers - √2, √3, $\sqrt{1.5}$ .101101110111110*
- *When we talk about square root, we always mean positive square root though any number has both positive and negative square roots.*

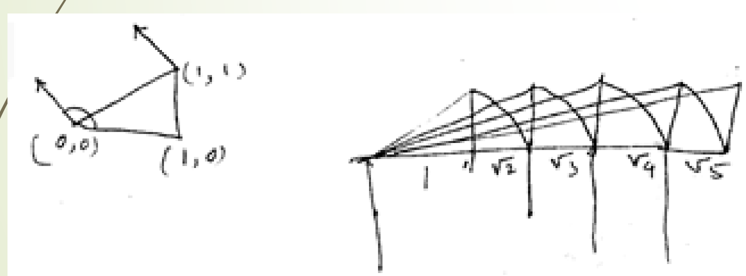
  Ex - $\sqrt{4}$ = 2 or -2, $\sqrt{9}$ = 3 or -3 etc.

History of Irrational Numbers

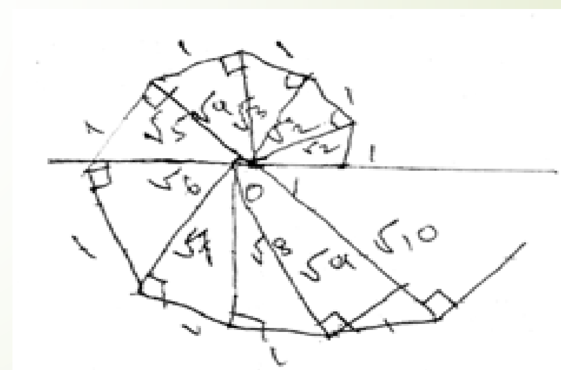
- *Hippacus of croton belong to Pythagoreans in Greece around 400 BC discovered the numbers which is not rational during applying Pythagorous theorem to a right angled triangle of shorter side lengths of 1 unit.*
- *Later (425 BC) Theodorus of Cyrene showed that √3, √5, √6, √7, √10, √11, √12, √13, √14, √17 are also irrationals*
- *π was known to people for thousands of years but Lambert and Legend in the late 1700 ad proved it as irrational*

Irrational Number: Class-IX

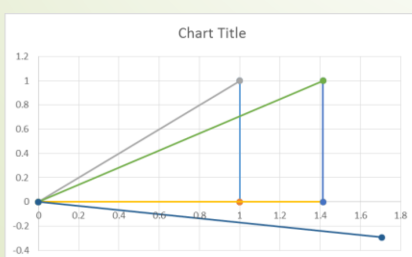
1.2 Spiral of theodorus is based on the Pythagorean theory. It starts with a isosceles right angled triangle with shorter legs of length = 1. So, hypotenuse = $\sqrt{1^2 + 1^2} = \sqrt{2}$. Now with base as hypotenuse we draw a perpendicular with 1 unit and join it from origin. Comparatively it is esy to draw in a number line. The drawing made in excel is shown below. The main problem is to draw the unit perpendicular length.

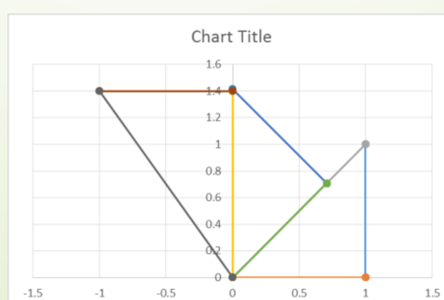


Drawing /Locating irrational numbers in number line



SPIRAL OF THEODORUS





Real Numbers:

Real Numbers and their decimal Expansion-The objective is to distinguish between rational and irrational numbers and their representation on number line.

Ex - $\frac{10}{3}$ = 3.33333 Remainder – 1,1,1,1…. divisor - 3

$\frac{7}{8}$ = 0.875 Remainder - 6, 4, 0, Divisor - 8

$\frac{1}{7}$ = 0.142857 Remainder - 3, 2, 6, 4, 5,1, 3, 2, 6, 4, 5,1, …, Divisor - 7

When we divide an integer by another integer, at least three things happen –

(i) The remainders either become 0 after certain stage or start repeating themselves -

(ii) The number of entries in the repeating string of remainders is less than the divisor

(iii) If the remainders repeat, then we get a repeating block of digits in the quotient.

Irrational Numbers

Case –i : The remainder becomes zero

$\frac{1}{2}$ = .5 , $\frac{7}{8}$ = 0.875

In these cases the decimal expansion terminates or ends after a finite number of steps. Decimal expansion of such numbers are called terminating

Case –ii : The remainder never becomes zero

$\frac{1}{3}$ = 0.3 3 3 3 . . . . . , $\frac{1}{7}$ = 0.142857142857

The usual way of showing repeats in the quotient are $\frac{1}{3}$ =$0.\overline{3}$ or $\frac{1}{7}$ = $0.\overline{142857}$

The bar above the digits indicates the block of digits that repeats.

This type of decimal expansions are called non terminating 'recurring' (Repeating)

Note-1: Decimal expansion of rational numbers have two choices – either they are terminating or non terminating recurring.

Note-2: Any number which is terminating or non terminating recurring in number line can be expressed as rational number in the form of $\frac{p}{q}$, $q \neq 0$

Rational Numbers

Ex-6: Find if 3.142678 is a rational number = $\frac{3142678}{1000000}$

Ex-7: Express $0.\overline{3}$ as rational number, $0.\overline{3}$ = .3333 ….

Let $x = .3333 ….$ Multiply both side by 10,

$10x = .3333 … \times 10$

$10x = 3.3333 …$ $= 3 + .333… = 3 + x$

or $10x - x = 3$

or $9x = 3$

or $x = \frac{1}{3}$ which is rational in the form of $\frac{p}{q}$, $q \neq 0$

Ex-8 $1.2727272 … = 1.\overline{27}$

Let $x = 1.2727272727 ....$ Multiply both side by 100

$100x = 127.272727 ...$

$100x = 126 + 1.272727 ...$

$100x = 126 + x$

or $99x = 126$ or $x = \frac{126}{99} = \frac{14}{11} = \frac{p}{q}, q \neq 0$

So every non terminating recurring decimal expansion of a number can be expressed as rational number $\frac{p}{q}$, $q \neq 0$

Observation: Decimal expansion of rational number is either terminating or non terminating recurring.

Irrational Number

IRRATIONAL NUMBERS:

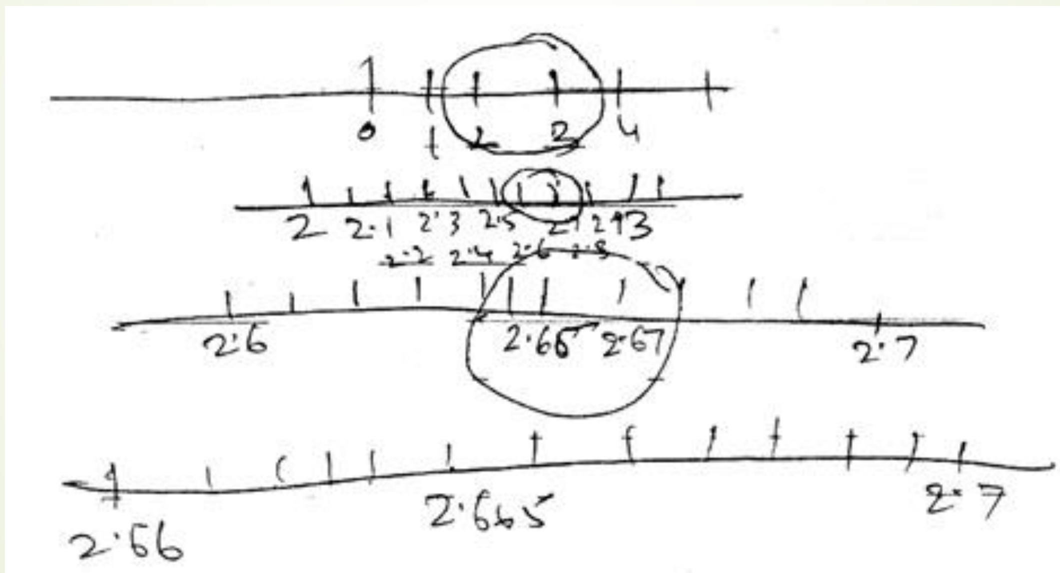
The decimal expansion of an irrational number is non-terminating nonrecurring.
A number whose decimal expansion is non-terminating nonrecurring is irrational.

0.15015001500015 0000 …. *is an irrational number between* $\frac{1}{7}$ *and* $\frac{2}{7}$

Rational number in number line

Representing Real numbers on the number line :

A real number can be located in a number line by magnifying an interval. To locate 2. 665



The process of Successive magnification is used to locate a point in the number line.

Operations on Real Numbers

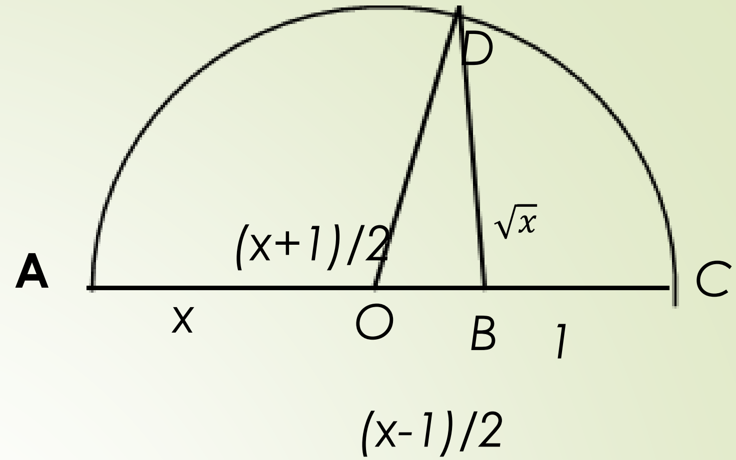
1.5 Operations of on Real numbers:

(i) The Sum or difference of a rational and an irrational number is irrational.

(ii) The product or quotient of a non-zero, rational number with an irrational number is irrational.

(iii) If two irrational numbers are added, subtracted, multiplied or divided, the result may be rational or irrational.

Square root of real number



1.5.b) Finding square root of any + real number geometrically :-

(1) Take AB =x unit in a number line

(2) Make c on extended number line as BC = 1 unit

(3) Take mid point of AC at O

(4) Draw a semi circle with Centre O and radius OC
Draw perpendicular to AC from B to BD
to touch Semi circle at D

(5) OC = OD = OA = (x + 1)/2

(6) OB = $\frac{x+1}{2}$ - 1 = $\frac{x-1}{2}$

(7) $BD^2 = OD^2 - OB^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$

(8) $\frac{x^2+2x+1}{4} - \frac{x^2-2x+1}{4}$

(9) $BD^2 = \frac{4x}{4} = x$

(10) $BD = \sqrt{x}$

Finding Square root of a real number

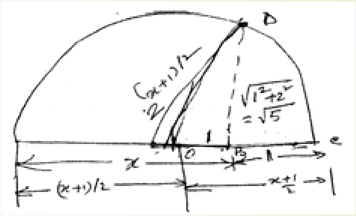
r = (x+1)/2

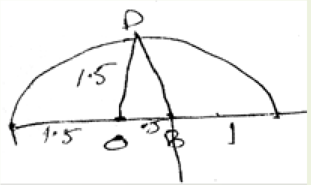
x = 3, r = (3+1)/2 = 2

$ob = \frac{x+1}{2} - 1 = \frac{x-1}{2} = \frac{3-1}{2} = 1$

x = 2

$r = \frac{x+1}{2} = \frac{3}{2} = 1.5$



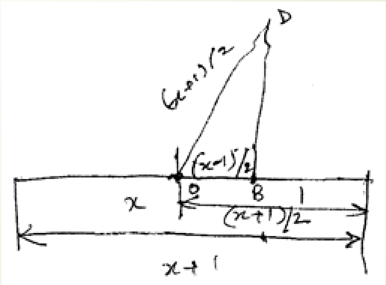


$r = \frac{x+1}{2}$

$OB = \frac{x+1}{2} - 1 = \frac{x+1-2}{2} = \frac{x-1}{2}$

$BD = \sqrt{\left(\frac{x-1}{2}\right)^2 + \left(\frac{x+1}{2}\right)^2}$

$BD = \sqrt{\left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2}$



$= \frac{\sqrt{x^2-2x+1+x^2+2x+1}}{4} \qquad = \frac{\sqrt{2x^2+2}}{4}$

$BD^2 = \frac{4x}{4} = x \qquad BD = \sqrt{x}$

Class –X: Chapter- 1: Real Numbers

Divisibility of Integers

$b\,\overline{)\,a\,(}\;q$ $\qquad r < b$

r

1.1 Two important properties of real numbers
(1) Euclid's division algorithm
(2) Fundamental Theory of Arithmetic

1.2 Euclid's Division algorithm deals with divisibility of integers.
Any positive integer a can be divided by another positive integer b in such a way that it leaves remainder r that is smaller than b

It is helpful in computing HCF
It can be seen that, for each pair of positive numbers a and b, we found whole numbers q and r which satisfy the relation –

$$a = bq + r, \quad 0 \le r \le b$$

Class –X: Chapter- 1: Real Numbers

Divisibility of Integers

$b\overline{)a(}\;q$ $\qquad r < b$

$\underline{\quad}$
r

Let a=6, b=5, then q=1, r=1: MS Excel command-q=Quotient(6,5), r=mod(6,5)

Real Division					
Dividend		Divisor		Quotient	
25	÷	4	=	6.25	
Integer Division				Quotient(	mod(a,b)
				Quotient	Remainder
25	÷	4	=	6	1

Example of finding HFC using Euclid's Division Lemma :

P-5 Ex – 1: Find HCF of 12576 > 4052

$12576 = 4052 \times 3 + 420,\quad r \neq 0$

$4052 = 420 \times 9 + 272,\quad r \neq 0$

$420 = 272 \times 1 + 148,\quad r + 0$

$272 = 148 \times 1 + 124,\quad r \neq 0$

$148 = 124 \times 1 + 24,\quad r \neq 0$

$124 = 24 \times 5 + 4,\quad r \neq 0$

$24 = 4 \times 6 + 0.\quad r = 0$

∴ Divisor at this stage is 4 as remainder = 0 at this point.

4 = HCF (24, 4) = HCF (124, 24) = HCF (148,124)

= HCF (272, 148) = HCF (420, 272)

HCF (4052, 420) = HCF (12576, 4052)

Example of finding HFC using Euclid's Division Lemma :

Finding HCF in Excel			
c	d	Quotient	Remainder
12576	4052	3	420
4052	420	9	272
420	272	1	148
272	148	1	124
148	124	1	24
124	24	5	4
24	4	6	0
	4		

The fundamental theorem of Arithmetic

The fundamental theorem of Arithmetic : It is related to the multiplication of positive integers.

It is seen that every composite number can be expressed as a product of primes in a unique way.

This important fact is the fundamental theorem of arithmetic.

It is used for two major applications -

(i) It is used to prove the irrationality of a number

(ii) It is used to explore when a rational number p/q is terminating and non terminating repeating. This is done by prime factorization of q to reveal the nature of the decimal expansion of p/q.

Prime Numbers are the numbers which has two factors, 1 and the number itself

The fundamental theorem of Arithmetic

Observation:

- *Any natural number can be written as a product of its prime factors.*
- *Every composite number can be written as product of powers of primes.*
- *This leads to the conjecture that every composite number can be written as products of powers of primes.*
- *This is called the fundamental theorem of arithmetic.*
- *The prime factorization of a natural number is unique.*
- *HCF or GCD is the product of smallest power of each common prime factors. 6=2*3, 20=2*2*5=2^{2}* 5, Hence, HCF of 6 and 20 is 2*
- *LCM is the product of greatest power of each common factors. LCM of 6 and 20 is 2^2 * 3*5=60*
- *EXCEL COMMAND, GCD(6,20)=2 and LCM(6,20)=60*

Arithmetic

Square

- If a natural number m can be expressed as n^2 where n is also a natural number, then m is a square number.
- m

m	n
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8

Square Roots

1. Prime Factorization Method
2. Long Division Method

1	$\overline{2}.\overline{00}\ \overline{00}\ \overline{00}$ (1.414
	1
24	100
	96
281	400
	281
2824	11900
	11296
	604

Geometry

Let no one enter who does not know Geometry

Inscription on Plato's Academy at Athens (429-347 BC)

No one will be considered scientifically literate tomorrow who is not familiar with Fractals –

John Archibald Wheeler : **New Scientist** 4 Apr 1985

Syllabus-Class-1

- **Class-I:**
- **Geometry (10 hrs.)**
- Chapter – 1:
- **SHAPES & SPATIAL UNDERSTANDING**
- • Develops and uses vocabulary of spatial relationship (Top, Bottom, On, Under, Inside, Outside, Above, Below, Near, Far, Before, After)

Syllabus-Class-1

- **Class-I:**
- **Geometry (10 hrs.)**
- **SOLIDS AROUND US**
- • Collects objects from the surroundings having different sizes and shapes like pebbles, boxes, balls, cones, pipes, etc.
- • Sorts, Classifies and describes the objects on the basis of shapes, and other observable properties.
- • Observes and describes the way shapes affect movements like rolling and sliding.
- • Sorts 2 - D shapes such as flat objects made of card etc.

Syllabus: Class-II

- **Class-II:**
- **Geometry (13 hrs.)**
- **SHAPES & SPATIAL UNDERSTANDING**
- *3-D and 2-D Shapes*
- • Observes objects in the environment and gets a qualitative feel for their geometrical attributes.
- • Identifies the basic 3-D shapes such as cuboid, cylinder, cone, sphere by their names.
- • Traces the 2-D outlines of 3-D objects.
- • Observes and identifies these 2-D shapes.
- • Identifies 2-D shapes viz., rectangle, square, triangle, circle by their names.

Syllabus: Class-II

- **Class-II:**
- **Geometry (13 hrs.)**
- **SHAPES & SPATIAL UNDERSTANDING**
- • Describes intuitively the properties of these 2-D shapes.
- • Identifies and makes straight lines by folding, straight edged objects, stretched strings and draws free hand and with a ruler.
- • Draws horizontal, vertical and slant lines (free hand).
- • Distinguishes between straight and curved lines.
- • Identifies objects by observing their shadows.

Syllabus: Class-III

- **Class-III:**
- **Geometry (16 hrs.)**
- SHAPES & SPATIAL UNDERSTANDING
- • Creates shapes through paper folding, paper cutting.
- • Identifies 2-D shapes
- • Describes the various 2-D shapes by counting their sides, corners and diagonals.
- • Makes shapes on the dot-grid using straight lines and curves.

Syllabus: Class-III

- **Class-III:**
- **Geometry (16 hrs.)**
- SHAPES & SPATIAL UNDERSTANDING
- • Matches the properties of two 2-D shapes by observing their sides and corners (vertices).
- • Tiles a given region using a tile of a given shape.
- • Distinguishes between shapes that tile and that do not tile.
- • Intuitive idea of a map. Reads simple maps (not necessarily scaled)
- • Draws some 3D-objects

Syllabus: Class-IV

- **Class-IV:**
- Geometry (16 hrs.)
- SHAPES & SPATIAL UNDERSTANDING
- • Draws a circle free hand and with compass.
- • Identifies centre, radius and diameter of a circle.
- • Uses Tangrams to create different shapes.
- • Tiles geometrical shapes: using one or two shapes.

Syllabus: Class-IV

- **Class-IV:**
- Geometry (16 hrs.)
- SHAPES & SPATIAL UNDERSTANDING
- • Chooses a tile among a given number of tiles that can tile a given region both intuitively and experimentally. • Explores intuitively the area and perimeter of simple shapes.
- • Makes 4-faced, 5-faced and 6-faced cubes from given nets especially designed for the same.
- • Explores intuitively the reflections through inkblots, paper cutting and paper folding.
- • Reads and draws 3-D objects, making use of the familiarity with the conventions used in this.
- • Draws intuitively the plan, elevation and side view of simple objects.

Syllabus: Class-V

- **Class-V:**
- Geometry (16 hrs.)
- SHAPES & SPATIAL UNDERSTANDING
- • Gets the feel of perspective while drawing a 3-D object in 2-D.
- • Gets the feel of an angle through observation and paper folding.
- • Identifies right angles in the environment.
- • Classifies angles into right, acute and obtuse angles.
- • Represents right angle, acute angle and obtuse angle by drawing and tracing.
- • Explores intuitively rotations and reflections of familiar 2-D shapes.
- • Explores intuitively symmetry in familiar 3-D shapes.
- • Makes the shapes of cubes, cylinders and cones using nets especially designed for this purpose.

Class-VI

- Geometry
- Transition from Primary to Upper Primary
- Bottom up concepts not top down in teaching mathematics

Syllabus: Class-VI

- **Class-VI:**
- Geometry (65 hrs)

(i) Basic geometrical ideas (2 -D): Introduction to geometry. Its linkage with and reflection in everyday experience.

- Line, line segment, ray.
- Open and closed figures.
- Interior and exterior of closed figures.
- Curvilinear and linear boundaries
- Angle — Vertex, arm, interior and exterior,
- Triangle — vertices, sides, angles, interior and exterior, altitude and median
- Quadrilateral — Sides, vertices, angles, diagonals, adjacent sides and opposite sides (only convex quadrilateral are to be discussed), interior and exterior of a quadrilateral.
- Circle — Centre, radius, diameter, arc, sector, chord, segment, semicircle, circumference, interior and exterior.

Syllabus: Class-VI

- **Class-VI:**
- Geometry (65 hrs)

II. Understanding Elementary Shapes (2-D and 3-D):

• Measure of Line segment

• Measure of angles • Pair of lines – Intersecting and perpendicular lines – Parallel lines

• Types of angles- acute, obtuse, right, straight, reflex, complete and zero angle

• Classification of triangles (on the basis of sides, and of angles)

• Types of quadrilaterals – Trapezium, parallelogram, rectangle, square, rhombus.

• Simple polygons (introduction) (Up to octagons regular as well as non regular).

• Identification of 3-D shapes: Cubes, Cuboids, cylinder, sphere, cone, prism (triangular), pyramid (triangular and square) Identification and locating in the surroundings

• Elements of 3-D figures. (Faces, Edges and vertices)

• Nets for cube, cuboids, cylinders, cones and tetrahedrons.

Syllabus: Class-VI

- **Class-VI:**
- Geometry (65 hrs)

III. Symmetry: (reflection)
  - Observation and identification of 2-D symmetrical objects for reflection symmetry
  - Operation of reflection (taking mirror images) of simple 2-D objects
  - Recognizing reflection symmetry (identifying axes)

IV. Constructions (using Straight edge Scale, protractor, compasses)
  - Drawing of a line segment
  - Construction of circle
  - Perpendicular bisector
  - Construction of angles (using protractor)
  - Angle 60°, 120° (Using Compasses)
  - Angle bisector- making angles of 30°, 45°, 90° etc. (using compasses)
  - Angle equal to a given angle (using compass)
  - Drawing a line perpendicular to a given line from a point a) on the line b) outside the line.

Syllabus: Class-VII

- **Class-VII:**
- Geometry (60 hrs)

1. Understanding shapes:

• Pairs of angles (linear, supplementary, complementary, adjacent, vertically opposite) (verification and simple proof of vertically opposite angles)

• Properties of parallel lines with transversal (alternate, corresponding, interior, exterior angles)

2. Properties of triangles:

• Angle sum property (with notions of proof & verification through paper folding, proofs using property of parallel lines, difference between proof and verification.)

• Exterior angle property

• Sum of two sides of a it's third side

• Pythagoras Theorem (Verification only)

Syllabus: Class-VII

- **Class-VII:**
- Geometry (60 hrs)

3) Symmetry

• Recalling reflection symmetry

• Idea of rotational symmetry, observations of rotational symmetry of 2-D objects. (90, 120, 180)

• Operation of rotation through 90 and 180 of simple figures.

• Examples of figures with both rotation and reflection symmetry (both operations)

• Examples of figures that have reflection and rotation symmetry and vice-versa

4) Representing 3-D in 2-D:

• Drawing 3-D figures in 2-D showing hidden faces.

• Identification and counting of vertices, edges, faces, nets (for cubes, cuboids, and cylinders, cones).

Syllabus: Class-VII

- **Class-VII:**
- Geometry (60 hrs)

• Matching pictures with objects (Identifying names)

• Mapping the space around approximately through visual estimation.

5) Congruence

• Congruence through superposition (examples blades, stamps, etc.)

• Extend congruence to simple geometrical shapes e.g. triangles, circles.

• Criteria of congruence (by verification) SSS, SAS, ASA, RHS

6. Construction (Using scale, protractor, compass)

• Construction of a line parallel to a given line from a point outside it.(Simple proof as remark with the reasoning of alternate angles)

• Construction of simple triangles. Like given three sides, given a side and two angles on it, given two sides and the angle between them.

Syllabus: Class-VIII

- **Class-VIII:**
- Geometry (40 hrs)

1. Understanding shapes:

• Properties of quadrilaterals – Sum of angles of a quadrilateral is equal to 360^o (By verification)

• Properties of parallelogram (By verification)

i. Opposite sides of a parallelogram are equal, Parallelogram are equal,
ii. Opposite angles of a parallelogram are equal
iii. Diagonals of a parallelogram bisect each other. [Why (iv), (v) and (vi) follow from (ii)]
iv. Diagonals of a rectangle are equal and bisect each other.
v. Diagonals of a rhombus bisect each other at right angles. (vi) Diagonals of a square are equal and bisect each other at right angles.

Syllabus: Class-VIII

- **Class-VIII:**
- Geometry (40 hrs)

2) Representing 3-D in 2-D

• Identify and Match pictures with objects [more complicated e.g. nested, joint 2-D and 3-D shapes (not more than 2)].

• Drawing 2-D representation of 3-D objects (Continued and extended)

• Counting vertices, edges & faces & verifying Euler's relation for 3-D figures with flat faces (cubes, cuboids, tetrahedrons, prisms and pyramids)

3) Construction:

Construction of Quadrilaterals:

• Given four sides and one diagonal

• Three sides and two diagonals • Three sides and two included angles • Two adjacent sides and three angles

Syllabus: Class-IX

- **Class-IX**
- Geometry

1. Introduction to Euclid's Geometry (Periods 6)

- History – Euclid and geometry in India. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates, and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem.
- 1. Given two distinct points, there exists one and only one line through them.
- 2. (Prove) Two distinct lines cannot have more than one point in common.

Syllabus: Class-IX

- **Class-IX**
- Geometry

2) Lines and Angles (Periods 10)

- 1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
- 2. (Prove) If two lines intersect, the vertically opposite angles are equal.
- 3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
- 4. (Motivate) Lines, which are parallel to a given line, are parallel.
- 5. (Prove) The sum of the angles of a triangle is 180°.
- 6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Syllabus: Class-IX

- **Class-IX :** Geometry

Triangles (Periods 20)

- 1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
- 2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
- 3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
- 4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle.
- 5. (Prove) The angles opposite to equal sides of a triangle are equal.
- 6. (Motivate) The sides opposite to equal angles of a triangle are equal.
- 7. (Motivate) Triangle inequalities and relation between 'angle and facing side'; inequalities in a triangle.

Syllabus: Class-IX

- **Class-IX**
- Quadrilaterals (Periods 10)
- 1. (Prove) The diagonal divides a parallelogram into two congruent triangles.
- 2. (Motivate) In a parallelogram opposite sides are equal and conversely.
- 3. (Motivate) In a parallelogram opposite angles are equal and conversely.
- 4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.

Syllabus: Class-IX

- **Class-IX**
- Geometry
- 5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely.
- 6. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and (motivate) its converse.

5. Area (Periods 4) Review concept of area, recall area of a rectangle.

- 1. (Prove) Parallelograms on the same base and between the same parallels have the same area.
- 2. (Motivate) Triangles on the same base and between the same parallels are equal in area and its converse.

Syllabus: Class-IX

Circles (Periods 15)

Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter, chord, arc, subtended angle.

- 1. (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse.
- 2. (Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and conversely, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
- 3. (Motivate) There is one and only one circle passing through three given non-collinear points.
- 4. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre(s) and conversely.
- 5. (Prove) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- 6. (Motivate) Angles in the same segment of a circle are equal.

Syllabus: Class-IX

Circles (Periods 15)

Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter, chord, arc, subtended angle.

- 7. (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
- 8. (Motivate) The sum of the either pair of the opposite angles of a cyclic quadrilateral is 180 and its converse.

7. Constructions (Periods 10)

- 1. Construction of bisectors of a line segment and angle, 60°, 90°, 45° angles etc, equilateral triangles.
- 2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle.
- 3. Construction of a triangle of given perimeter and base angles.

Syllabus: Class-X

- **Class-X**
- Triangles (Periods 15)
- Definitions, examples, counter examples of similar triangles.
- 1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
- 2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
- 3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar. 65 Syllabus for Secondary and Higher Secondary Levels
- 4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

Syllabus: Class-X

- **Class-X**
- Triangles (Periods 15)
- 5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.
- 6. (Motivate) If a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
- 7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
- 8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
- 9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right triangle.

Syllabus: Class-X

- **Class-X**

2. Circles (Periods 8)

- Tangents to a circle motivated by chords drawn from points coming closer and closer to the point.
- 1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- 2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.
- 3. Constructions (Periods 8) 1. Division of a line segment in a given ratio (internally). 2. Tangent to a circle from a point outside it.

3. Construction of a triangle similar to a given triangle.

- 1. Division of a line segment in a given ratio (internally).
- 2. Tangent to a circle from a point outside it.
- 3. Construction of a triangle similar to a given triangle.

Syllabus: Class-XI

Class-XI :

UNIT III : COORDINATE GEOMETRY

- **1. Straight Lines (Periods 09)**

Brief recall of 2-D from earlier classes, shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two-point form, intercepts form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.

- **2. Conic Sections (Periods 12)**

Sections of a cone: Circles, ellipse, parabola, hyperbola, a point, a straight line and pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

- **3. Introduction to Three-dimensional Geometry (Periods 08)**

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Syllabus: Class-XII

- **Class-XII**
- **UNIT IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY**
- **1. Vectors (Periods 10)**
- Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection
- of a vector on a line. Vector (cross) product of vectors, scalar triple product.
- **2. Three-dimensional Geometry (Periods 12)**
- Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

Basic Geometrical Ideas

Points
Lines
Angle
Triangle
Quadrilaterals
Rhombus
Trapezium
Pentagon
Polygon
Circle
Ellipse
Parabola
Hyperbola
graphs
Curves

Cube
Sphere
Pyramid
Prism
Cylinder
Ellipsoid
Cone
Cuboid

Basic Geometrical Idea - Point , Lines

- Point – A Point determines a location
- Line Segment – Shortest distance between two point
- A Line – Line segment extended infinitely
- Ray – A ray is a portion of a line. It starts at a point and goes endlessly in one direction.
- Intersecting Lines – If two lines have one common point, they are called intersecting lines.
- Parallel Lines – The lines that do not meet are said to be parallel lines.

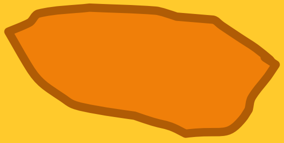
Basic Geometrical Idea - Curve

- Curve – A curve is a line which is not straight

Simple Curve – The curve that does not cross itself is called simple curve.

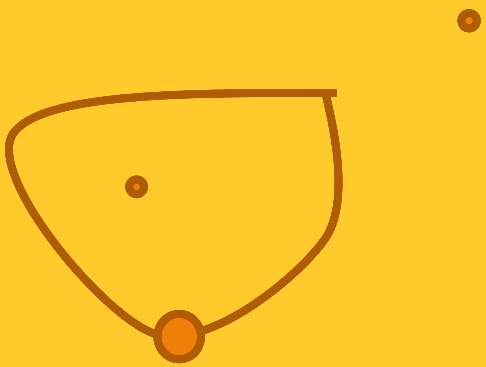
A. Simple Open Curve

B. Simple Closed Curve



Basic Geometrical Idea - Curve

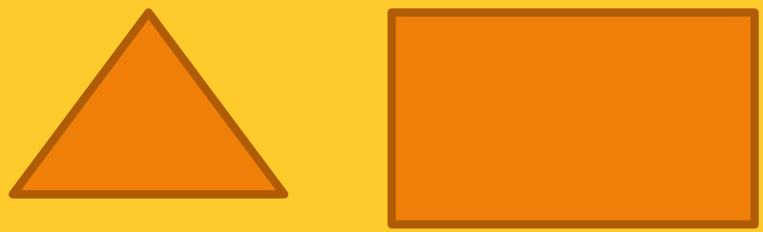
- Parts of a Closed Curve
  a. Interior "Inside" of the Curve
  b. Boundary of the Curve
  c. Exterior of the Curve



- Region – An area interior of a curve together with its boundary is called its region.

Basic Geometrical Idea - Polygon

- Polygon – A polygon is a simple closed figure made up of entirely line segments.
- Poly-Many
- Gon-Angle

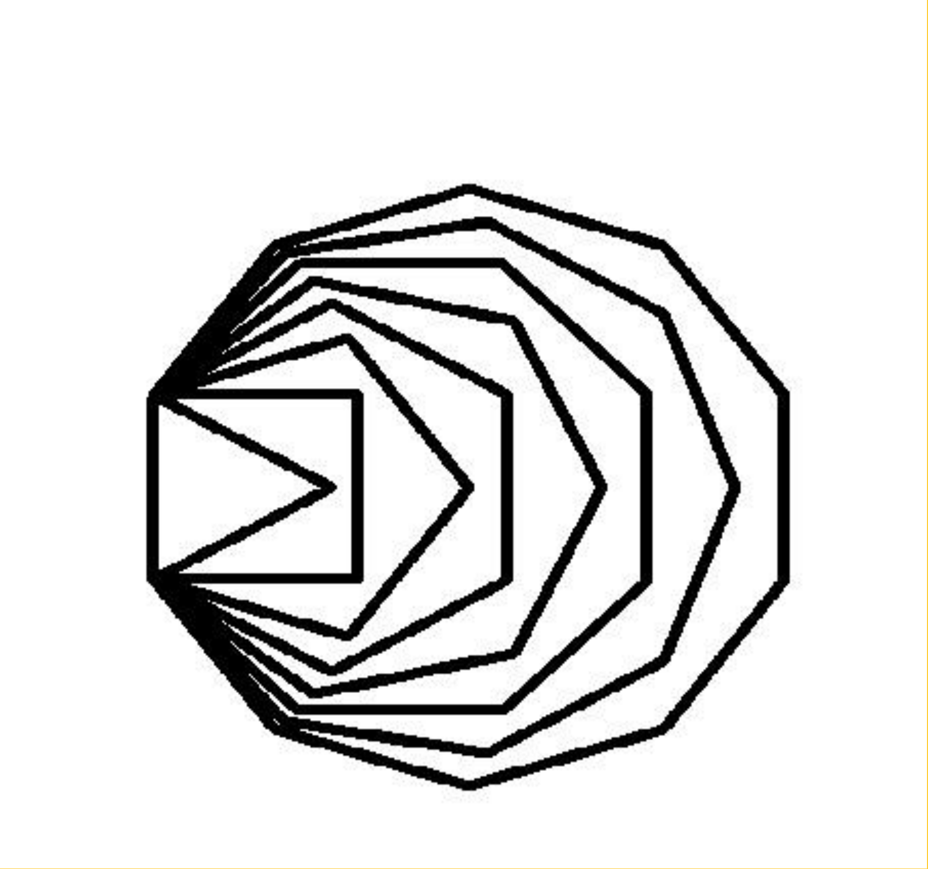


- Parts of a Polygon:
  A. Sides-The line segments forming polygon
  B. Vertices-Meeting point of pair of sides
  C. Diagonals-The line segment joining two non-adjacent vertices
  D. Adjacent sides-Any two sides with common end points
  E. Adjacent vertices-The end points of same sides of a polygon

Creating Regular Polygon

Interior Angle= ((N-2)*180)/N

Exterior Angle= 360/N



Dialog : Repeat n [fd 100 rt 360/n]

Angle

- Angle
- An angle is made up of two rays starting from a common end point.

- Interior
- Boundary
- Exterior

Triangle

- A triangle is a three sided polygon

- Interior
- Boundary
- Exterior

Quadrilaterals

- A four sided polygon is a quadrilaterals.

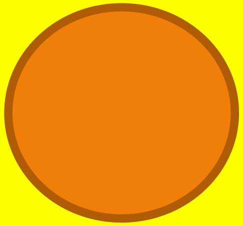
- Parts of a Quadrilaterals:

  (1) Adjacent Sides
  (2) Opposite Sides
  (3) Adjacent Angle
  (4) Opposite Angle

- Interior
- Boundary
- Exterior

Circle

- Circle is a simple closed curve which is not a polygon
- Zerogon?



(Q) Parts of a Circle

(a) Center (C)

(b) Radius (CA)-Distance from Center to any point

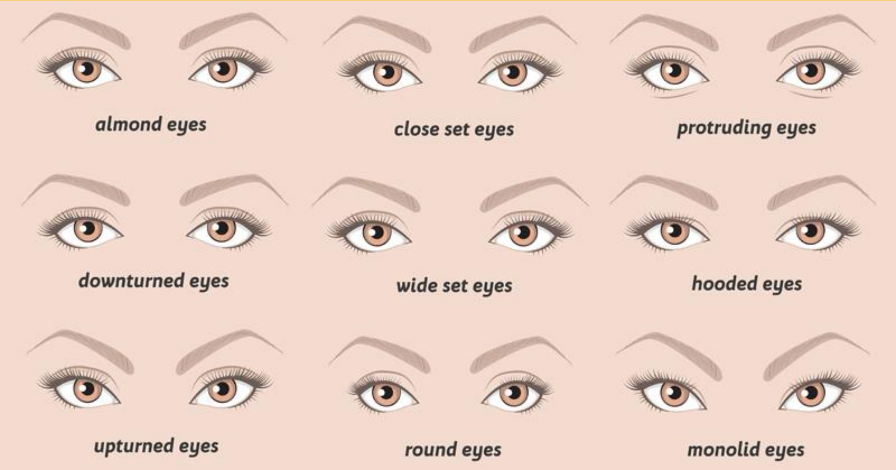
(C) Diameter-Longest cord

(d) Cord- Connecting two points on a circle

(e) Arc-It is a portion of a Circle

Drop

- What is the shape of drop – Monogon?
- What is the shape of eye – Bigon?



Understanding Elementary Shape

- Shape are formed using lines and curves. They have different sizes and measures.
- We will learn how to measure them.

- Measuring Line Segments
  1. **Comparison by observation**
  2. **Comparison by Tracing**
  3. **Comparison by using ruler**
  4. **Comparison by using divider**

Different Types of Angles

1. Right Angle- 90°
2. Straight Angle - 180°
3. Complete Angle - 360°
4. Acute Angle - less than 90°
5. Obtuse Angle - greater than 90°
6. Reflex Angle - Greater than 180°

Measuring Angles

There are three measures of angles:

1. Degree
2. Revolution
3. Radian

Tools for Measuring Angles-Protractor

Triangles

Different types of Triangles:

A. Based on Sides

1. Scalene
2. Isosceles
3. Equilateral

B. Based on Angles:

1. Acute Angled
2. Right Angled
3. Obtuse Angled

Quadrilaterals

Different types of Quadrilaterals

1. Rectangle
2. Square
3. Rhombus
4. Trapezium
5. Parallelogram

Rectangle

- Opposite sides are equal and angles are right angled

Square

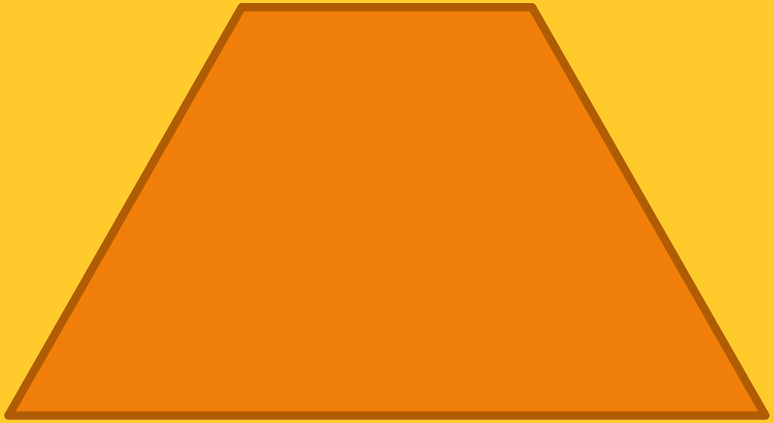
- **All Sides are equal and Angles are right Angle**

Rhombus

- **All sides are equal but angles are not 90°**

Trapezium

- **Two sides are parallel**



Polygons

- **Triangle**
- **Quadrilateral**
- **Pentagon**
- **Hexagon**
- **Octagon**
- **Nonagon**
- **Decagon**

Three Dimensional Shapes

- **Sphere**
- **Cone**
- **Cylinder**
- **Cuboid**
- **Cube**
- **Pyramid**
- **Prism**

Parts of Three Dimensional Objects

- **<u>Vertex</u>** **- The Point where 3 edges meet**
- **<u>Edge</u>** **- The line where two faces meet**
- **Face** **- The flat surface of a cube**

Euler's Formula, $V + F = \text{E} + 2$

Practical Geometry

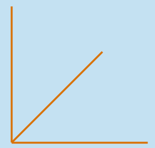
- There are hundreds of shapes that we require to handle every day.
- There are different tools available to construct them.
- Some of the tools are given below:
  1. Ruler
  2. Compass
  3. Divider
  4. Set Square – 2 nos.
     1. 45, 45 90
     2. 30, 60, 90
  5. Protractor

Practical Geometry

Lines and Angles

- Line
- Line Segment
- End Points
- Ray

Angle is formed when two line segment meet at a common point.

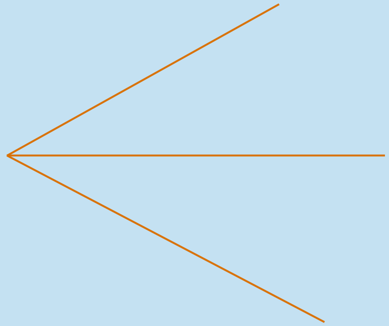


1. Complementary Angles – When the sum of two angles are 90°, then the angles are complementary angle.
2. Supplementary Angles - When the sum of two angles are 180°, then the angles are complementary angle.

Adjacent Angles

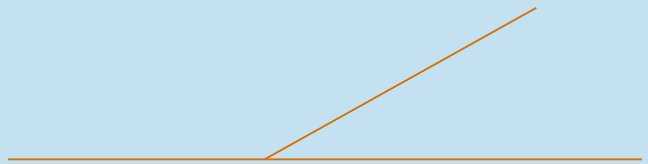
Adjacent Angles:

1. The angles have common Arm
2. The angles have common vertex
3. Non-common arms are on either side of the common arm.



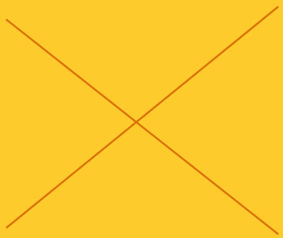
Linear Pair

A linear pair of angle is a pair of a adjacent angles whose non-common sides are opposite rays.



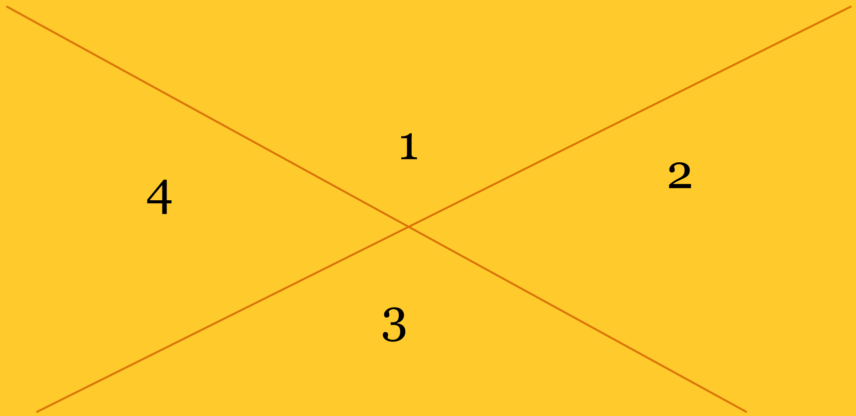
Pair of Lines

- Intersecting Lines-Two lines intersect if they have a point in common



Vertically Opposite Angles

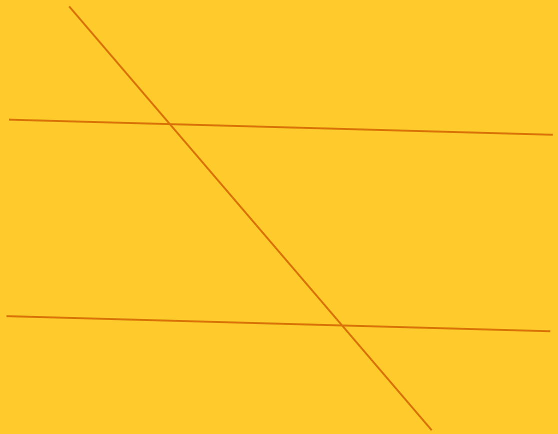
When Two Lines Intersect, they create four angles with two sets of vertically opposite angle



- 1 & 3 and 2 & 4 are Vertically Opposite Angles
- Vertically opposite angles are equal

Transversal

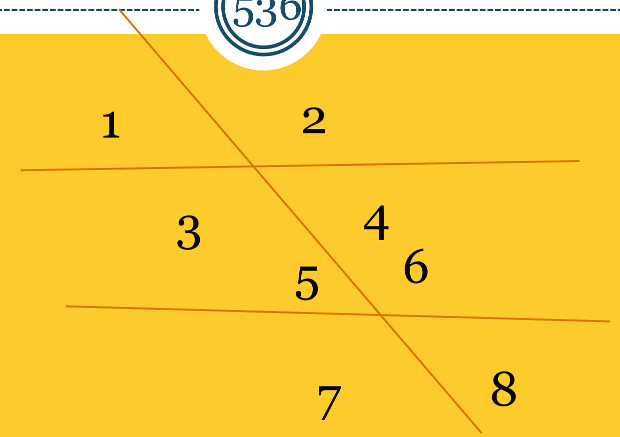
- **A line that intersect two or more lines at distinct points is called a Transversal**



Transversal

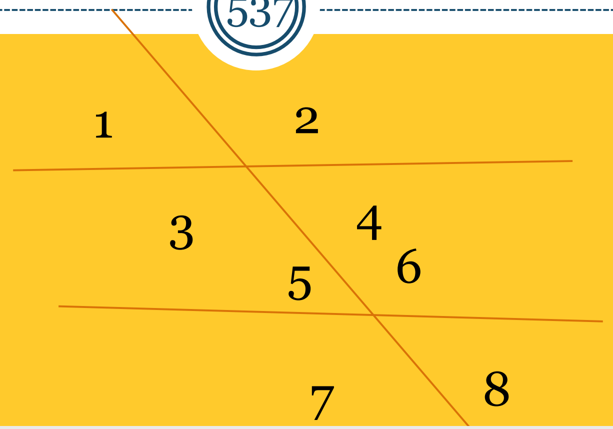
Not Transversal

Angles Made by Transversal



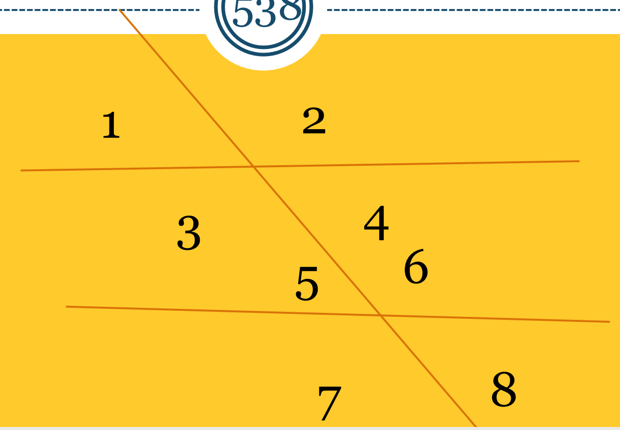
Interior Angles	3 , 4, 5, 6
Exterior Angles	1, 2, 7, 8
Pairs of Alternate Interior Angles	(3, 6), (4, 5)
Pairs of corresponding Angles	(1, 5), (2, 6), (3,7), (4, 8)
Pairs of Alternate exterior angle	(1, 8), (2, 7)
Pairs of interior angles on the same side of the transversal	(3, 5), (4, 6)

Transversal of Parallel Lines



- If two parallel lines are cut by a transversal, each pair of corresponding angles are equal in measure.
- If two parallel lines are cut by a transversal, each pair of alternate interior angles are equal.
- If two lines are cut by a transversal, then each pair of interior angles on the same side of the transversal are supplementary.

Checking for Parallel Lines



- When a transversal cuts two lines, such that pairs of corresponding angles are equal then the lines have to be parallel
- When a transversal cuts two lines such that pairs of alternate interior angles are equal, then the lines have to be parallel.

Note: Carpenter's square and rule is used to measure angles.

The Triangles and its Properties

- Definition of Triangle - Triangle is a simple closed curve made of three line segments.
- Parts of a Triangles:
    1. **Three Vertices**
    2. **Three Angles**
    3. **Three Sides**

- Median of a triangle: The line segment that connects a vertex to the midpoint of the opposite side
- Altitude of a Triangle: The height is the shortest distance from a vertex to the opposite side.

Sum of Angles of A Triangle

- **Sum of the three angles of a triangle is 180°**

Two Special Triangle

- Equilateral Triangle- The triangle whose all the sides are of equal lengths are called equilateral Triangle.
- **Properties of equilateral triangle**
  1. All sided are of same size
  2. Each angle measure 60°
- Isosceles- The triangle whose two sides are of equal lengths are called Isosceles Triangle.
- **Properties of Isosceles triangle**
  1. Two sides have same length
  2. Base angles opposite to the equal sides are equal

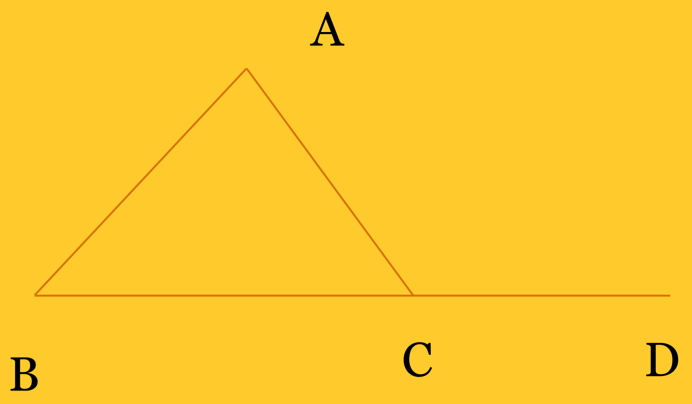
Sum of Lengths

- **The sum of the lengths of two sides of a triangle is greater than third side.**

Right Angle Triangle

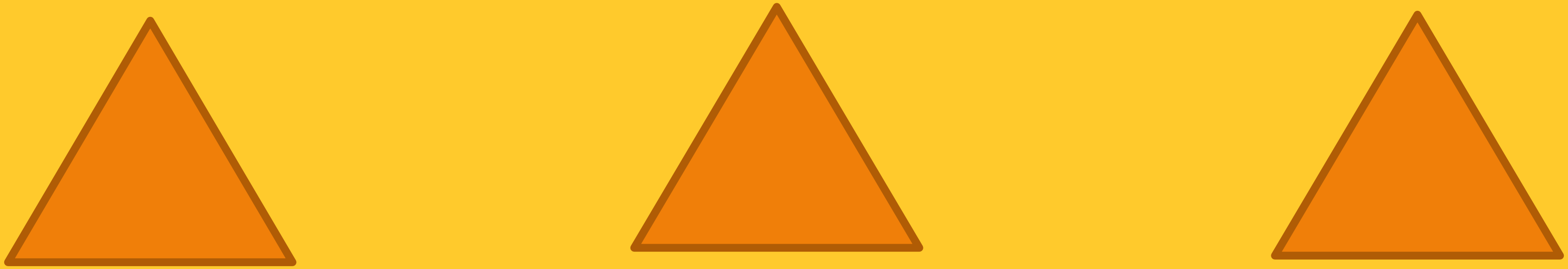
- Pythagoras Property:
- The square of the hypotenuse=sum of the square on the legs
- If the Pythagoras property holds, the triangle must be right-angled

Exterior Angle of a Triangle



1. Angle ACD is the Exterior Angle
2. Angle A and B are called Interior Opposite Angle
3. An exterior of a triangle is equal to the sum of Two Interior Opposite angle

Congruence



Two objects are said to be congruent if the objects are of same size and same shape

Congruence of Line

- **If two lines have same length, they are congruent**
- **It can be said that if length of two lines are same, they are congruent.**

Congruence of Angle

- **If two angles have same measure, they are congruent**
- **It can be said that if length of two lines are same, they are congruent.**

Congruence of Triangle

- **Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.**
- **In two congruent triangles, corresponding vertices, angles and sides are equal.**

Criteria for congruence of Triangles

SSS Congruence Criteria:

- **If three sides of one triangle are equal to the three corresponding sides of another triangle, then the triangles are congruent.**

SAS Congruence Criteria:

- **If two sides and the angle included between them of triangle are equal to two corresponding sides and the angle included between them of another triangle then the triangles are congruent.**

ASA Congruent Criteria :

- **If two angles and the included side of a triangle are equal to two corresponding angles and included side of another triangle then the triangles are congruent.**

Congruence among right angled triangle

- **If hypotenuse and one side of a right angled triangle are respectively equal to the hypotenuse and one side of another right angled triangle, then the triangles are congruent (RHS criteria).**

CPCT

- It is to be noticed that the Corresponding Parts of Congruent Triangles (CPCT) are EQUAL.

Criteria for Congruence of Triangle

- Axiom 7.1 SAS – Two triangles are congruent if two sides and the angle included between them are equal.

- This result can not be proved with the help of previously known results and so it is accepted true as an axioms.

Criteria for Congruence of Triangle

- Theorem 7.1 ASA – Two triangles are congruent if two angles and the included side between them are equal.

Some properties of triangle

- Theorem 7.2: Angles opposite to equal sides of an isosceles triangle are equal
-
- Theorem 7.3: The sides opposite to equal angles of a triangle are equal.
- Theorem 7.4: SSS- If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- Theorem 7.5: RHS - If hypotenuse and one side of a right angled triangle are respectively equal to the hypotenuse and one side of another right angled triangle, then the triangles are congruent(RHS criteria).

Inequalities in a triangle

- Theorem 7.6: If two sides of a triangle are unequal, the angle opposite to the longer side is larger.
- Theorem 7.7: in any triangle, the side opposite to the larger angle is longer.
- Theorem 7.8: The sum of any two sides of a triangle is greater than the third side.

Similarity of Triangles

- **Definition of Congruent: Two objects are said to be congruent if the objects are of same size and same shape**

Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.

- **Definition of Similarity: Two figures having same shape and not necessarily same size are called similar figures.**

Observation

- **All congruent figures are similar but all similar figures are not congruent.**

- **Two polygons of same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion)**

Equiangular Triangles

- Equiangular Triangles: If corresponding angles of two triangles are equal, then they are known as Equiangular Triangles.

- Thales Theorem: The ratios of any two corresponding sides in two equiangular triangles is always the same.

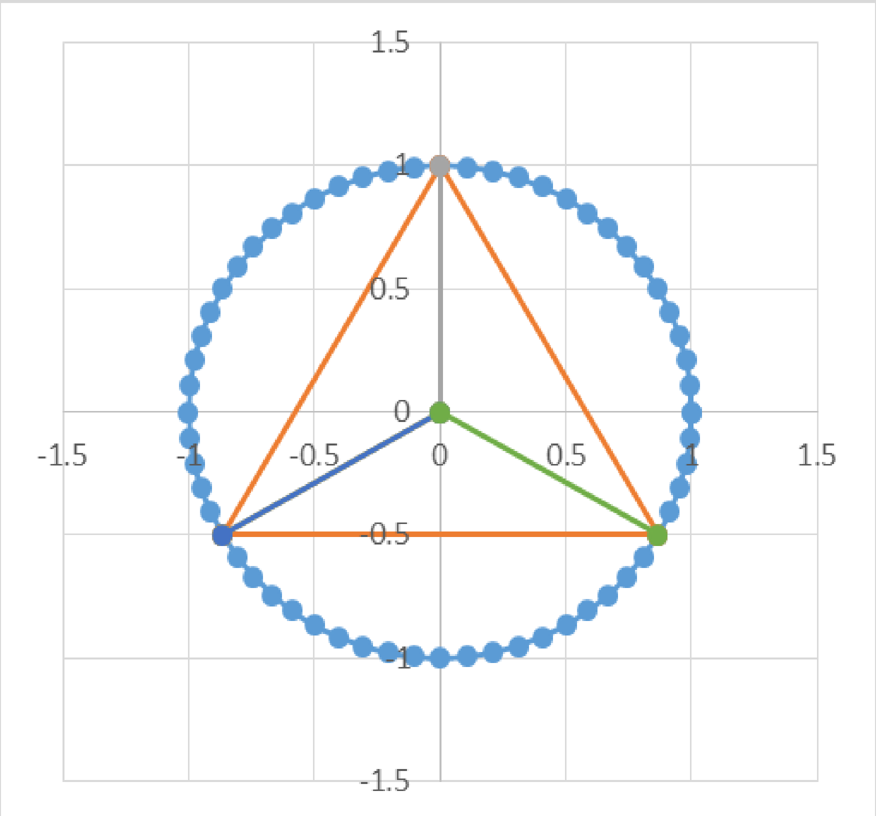
Similarity

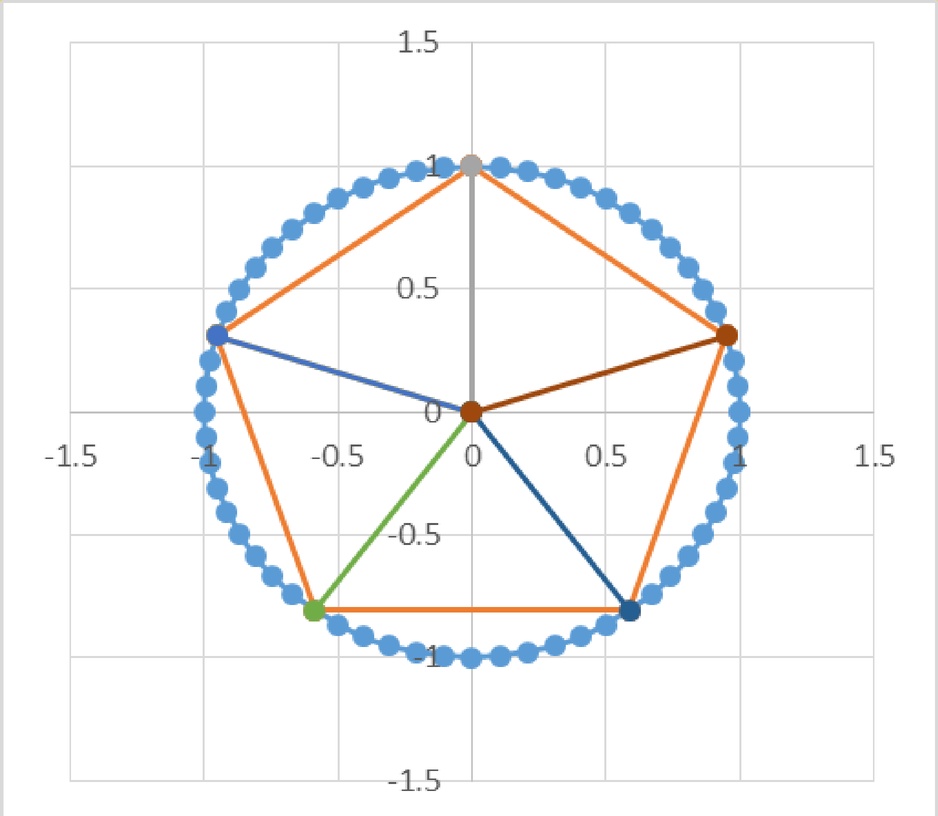
- Theorem 6.1: If a line is drawn parallel to one side of a triangle to intersect the other two sides in the distinct points, the other two sides are divided in the same ratio.
- Theorem 6.2: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Criteria of similarity of triangles

Polygon Pattern

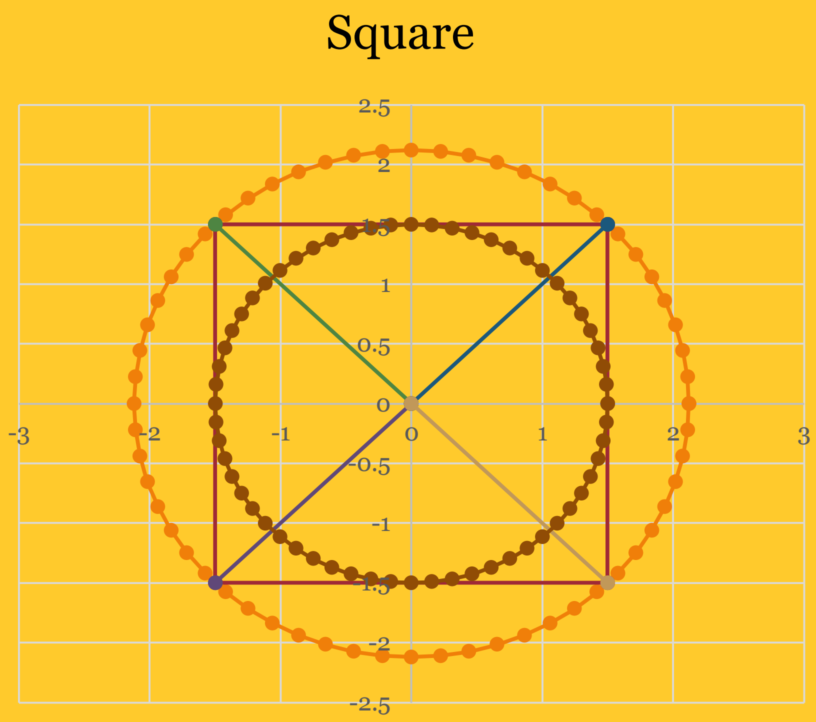
- Exterior Angles = 360/n, n=No of Sides of the polygon

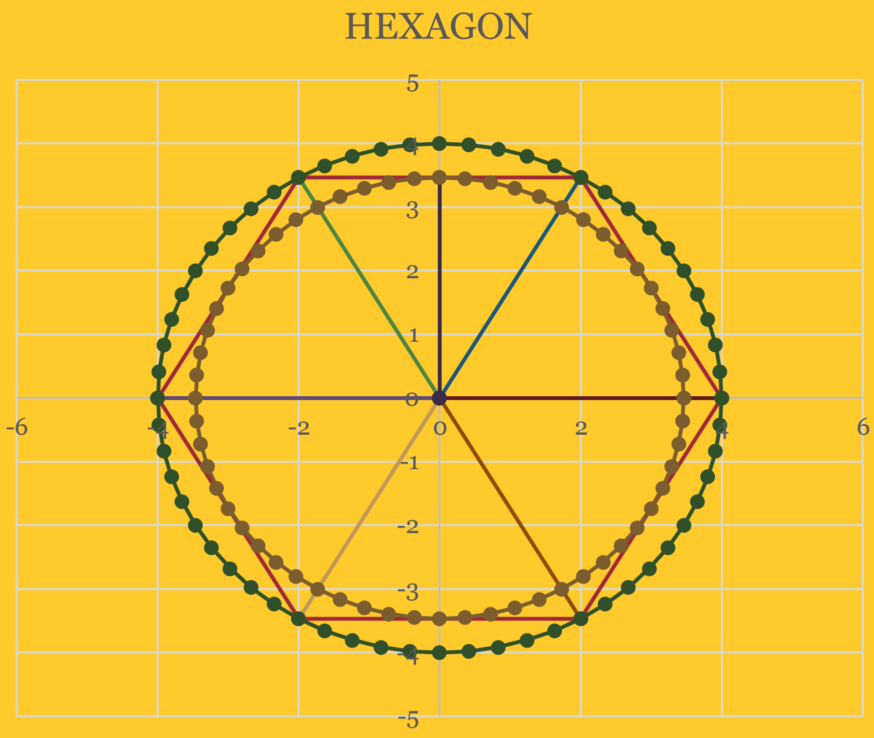




Polygon Pattern

- Angles = 360/n, n=No of Sides of the polygon





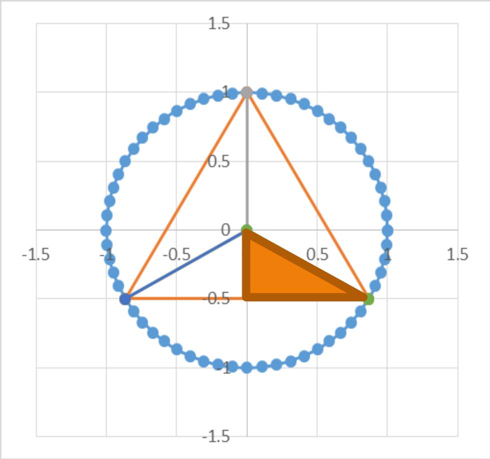
Relationship between n, s, r

Given:

- n= no of side of the polygon
- s =Length of the sides of the polygon

Definitions:

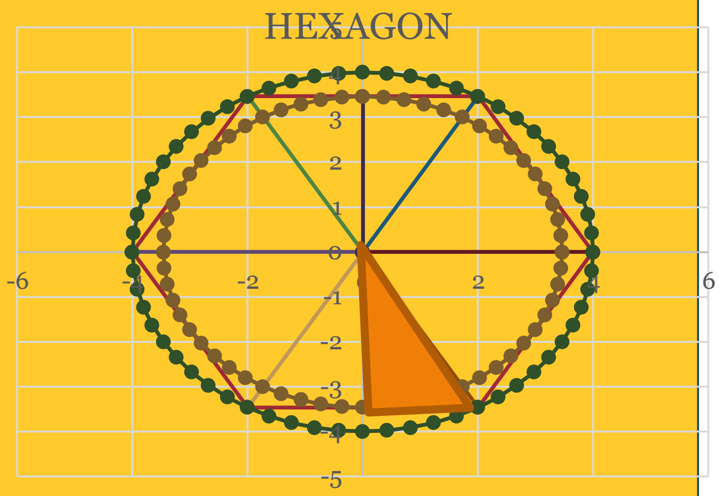
- a=Apothem-It is the line segment from the centre to the mid point of a side
- r= Radius of the circumscribed circle



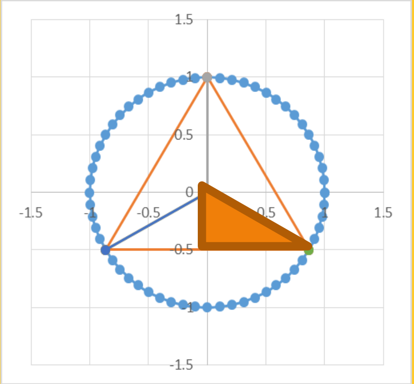
Relationship between n, s, t, r, a

Calculated:

- t=1/2(2*pi()/n)=pi()/n
- r=s/2*cosec(t)
- a=r cos(t)=s/2 cot (t)



- Area of one segment=1/2*s*a
  - =1/2*S*S/2 COT (PI()/N)
- Total Area =n*s^2/4 *cot(pi()/n)



Mensuration Syllabus

- Class-I : Measurements Length, Weight, Time
- Class-II : Measurements Length, Weight, Time
- Class-III : Measurements Length, Weight, Volume, Time
- Class-IV: Measurements Length, Weight, Volume, Time
- Class-V : Measurements Length

Mensuration Syllabus

- Class-VI : Mensuration: Perimeter and Area
- Class-VII : Mensuration: Perimeter and area
- Class-VIII: Mensuration: Area, Volume, Capacity, Surface Area
- Class- IX : Mensuration: Area, Surface Areas, Volumes
- Class-X : Mensuration: Area related to circle, surface areas and volume, converting solids

Class – I: Measurements

Measurement of Length / Weight (Comparison).

- **Longer – Shorter**
- **Longest – Shortest**
- **Taller – Shorter**
- **Tallest – Shortest**
- **Thinner – Thicker**
- **Thickest – Thinnest**
- **Heavier – Lighter**
- **Heaviest – Lightest**

Measurement of Length---

- **Unit – Non standard – span**
- **Unit – Standard – feet**

Chapter – 6 " Measurement – Time"

- **Morning , Noon, Afternoon , Evening , Night , Dawn**
-

Class – II: Measurements

- **Chapter – 13 "The Longest Step"**
- **Length : Measurement Unit**
- ***Hand span***
- ***Fingers***
-
- **Chapter – 3 "Measurement : Weight"**
- **Title : How much can you carry.**
- **Heavier – Lighter**
- **Heaviest – lightest.**
-
- **Chapter – 7 " Measurements : Volume"**
- **Title : Jugs and Mugs**
-
- **Measurements of volume : Cup,**
- **Glass,**
- **Bottle,**
- **Mugs.**

Class – II: Measurements

-
- **Chapter – 9 "Measurement : Time"**
- **Days of the week**
- **Seasons**
- **Months.**

Class – III: Measurements

- Chapter – 4 " Long and Short : Measurement – Length"
- Unit : Non standard – Arm Length, Steps, Match sticks
- Standard - cm, meter, mile.
- Measurement Tools – Scale, Measuring Tape
-
- Chapter – 8 Who is Heavier : Measurement – Weight
- Heavier - Lighter
- Heavier - Lightest
- Unit of measurement - gram, kilogram
- Measurement Tools - Weights, Balances.
-
- Chapter – 11 : Jugs and Mugs : Measurement, Volumes.
- Chapter – Times goes on : Measurement of Time
- Days, weeks, months, seasons, Month wise festivals, calendars, Clock

Class – IV: Measurement

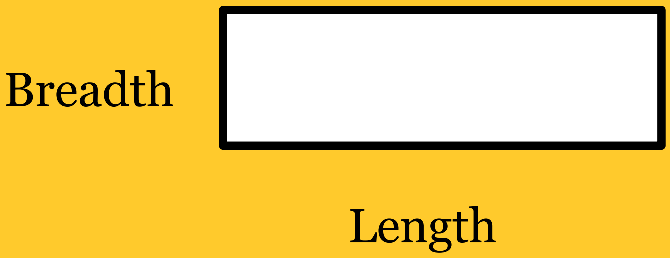
- **Chapter – 2 : Short and Long : Measurement : Length**
- **Chapter – 4 : Tick – Tick – Tick : Measurement : Time.**
- **Chapter – 7 : Jugs and Mugs : Measurement : Volumes**
- **Chapter – 12 : How Heavy? How Light? Measurement : Weight.**
- **Chapter – 13 : Fields and Fences : Measurement : Perimeter and Area.**

Class – V: Measurement

-
- **Length : Area & Perimeter,**
- **Length, weight, volume,**
- **Larger and smaller,**
- **Volume of solids**
- **Time interval.**

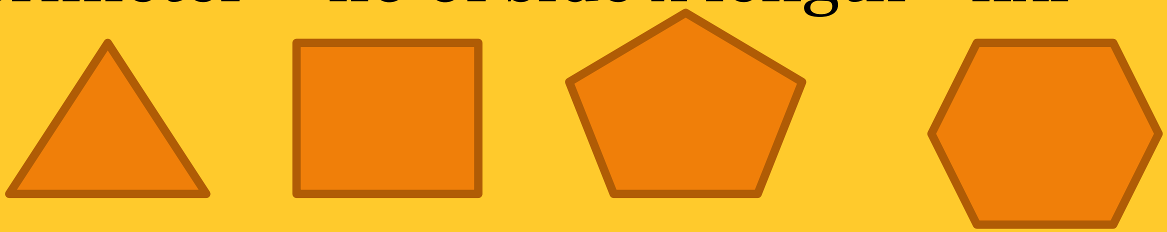
Perimeter of plane figures

- Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.
- 10.2.1.Perimeter of a rectangle
- P= 2 x (Length X Breadth)

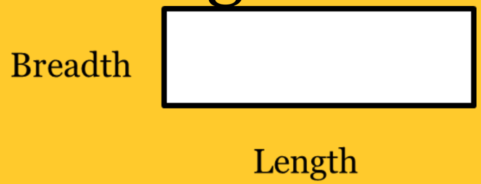


Perimeter of plane figures

- 10.2.2 Perimeter of regular shapes
- Perimeter = no of side x length= nxl

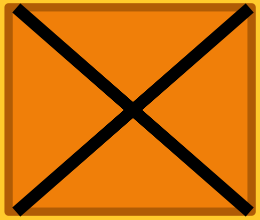


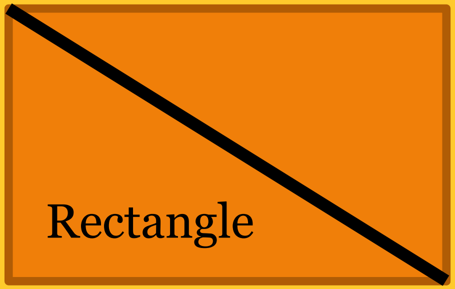
- 10.3 Area- The amount of surface enclosed by a closed figure is called its area.
- 10.3.1. Area of rectangle : Area= Length x breadth=lxb



Perimeter and Area of Squares and rectangles:

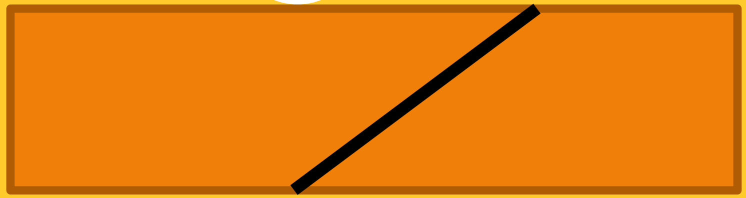
- 11.2.1: Triangles as parts of Rectangle



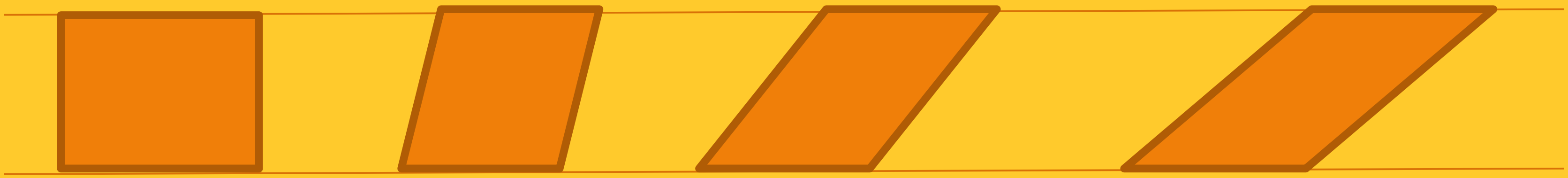


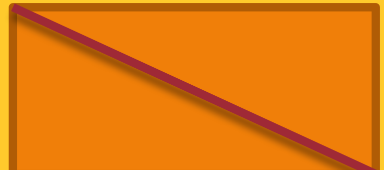
- Congruent parts rectangle:

Class-VII Chapter -11: Mensuration



- 11.2.2: Area of each parallelogram=base x height



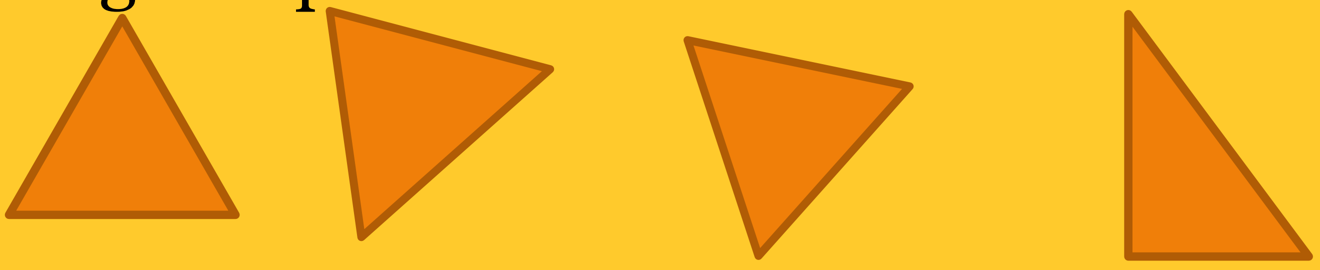
- All these parallelogram has equal area, but different perimeter.
- 11.4 Area of Triangle – ½ x Area of Parallelogram
-  – ½ x base x height

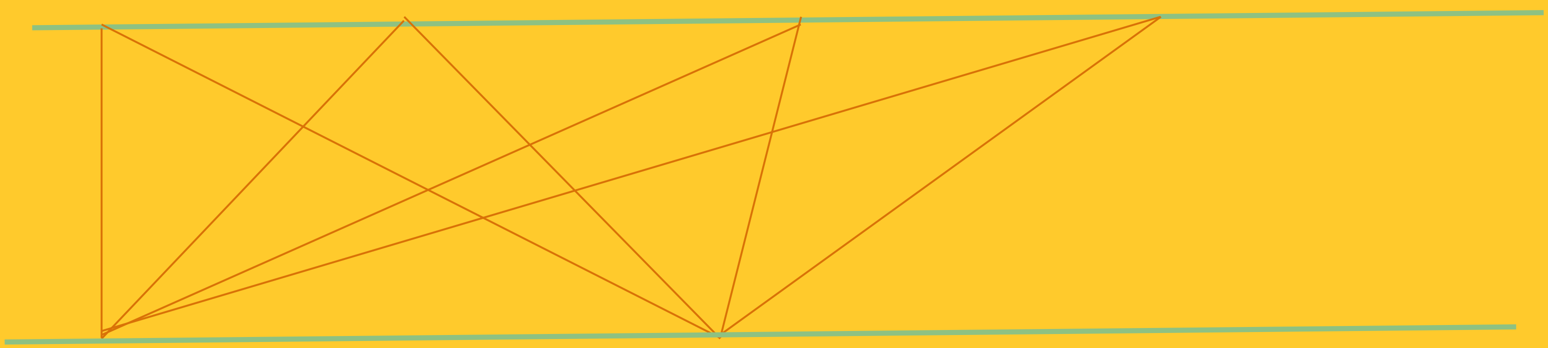
Class-VII Chapter -11: Mensuration

- 11.2.3: Area of each parallelogram=base x height
- 11.4 Area of Triangle – ½ x Area of Parallelogram

  – ½ x base x height

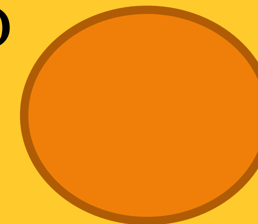
- All congruent triangles are equal in area, but the triangles equal in area need not be congruent.



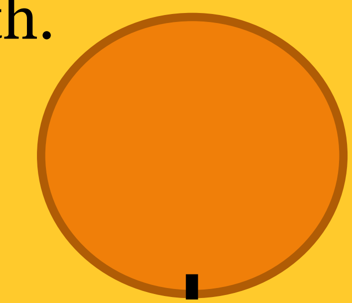


Class-VII Chapter -11

- 11.5 Circles
- 11.5.1 Circumference of a circle – the distance around a circular region is known as its circumference, C=πD



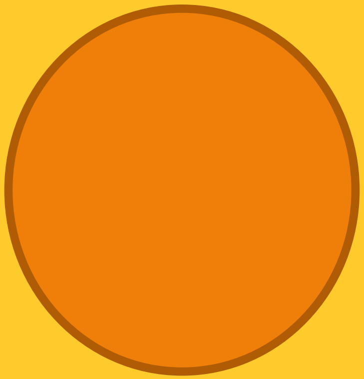
- How to measure the circumference of a circle:
- Mark a point on the edge of the circle. You can wrap a string on the circle up to the point and measure its length.

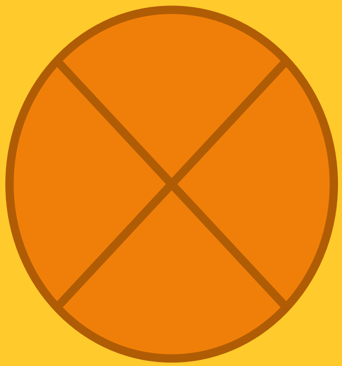


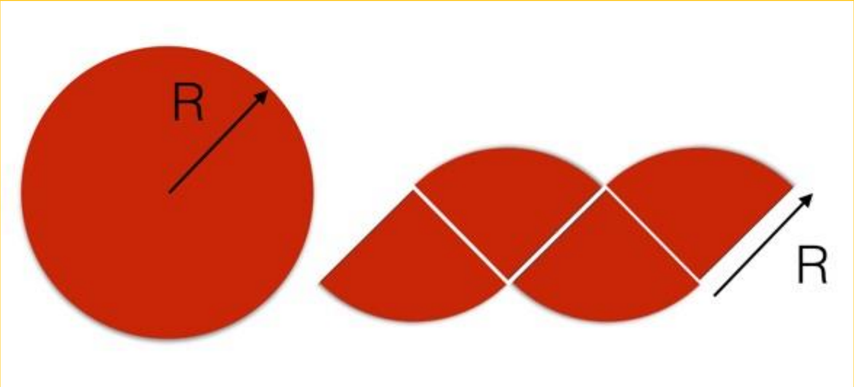
- You can role the circular object and measure the circumference.

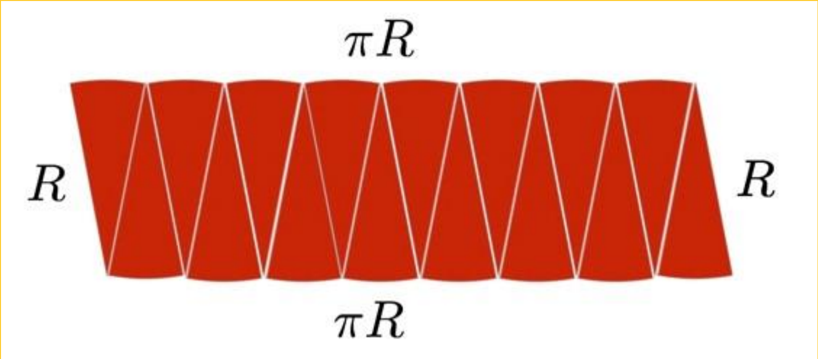
Class-VII Chapter -11

- 11.5 Circles
- 11.5.2 Area of Circle:$2.\pi.R.R = \quad 2.\pi.R\text{^}2$



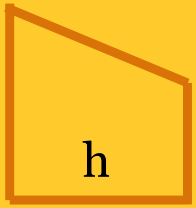






Class-VIII, Chapter-11, P-169

- 11.1/11.2 Perimeter, Circumstances, Area
- 11.3 Area
- 11.3 Area of Trapezium

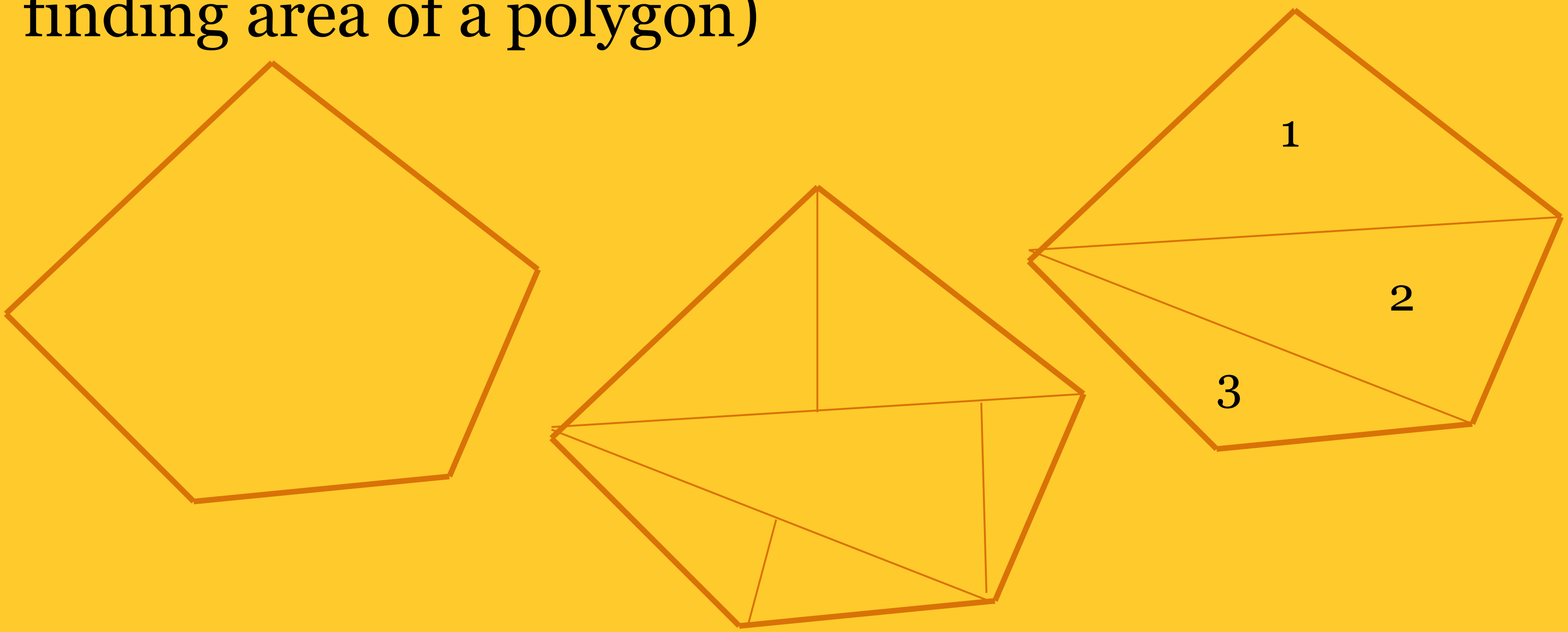


- Area= ½ x h x (Sum of Parallel side)
- 11.4 Area of general quadrilateral (Triangulation)

- Area= ½ x base x height
- Area= ½ x diagonal x (h1 +h2)

Class-VIII, Chapter-11, P-169

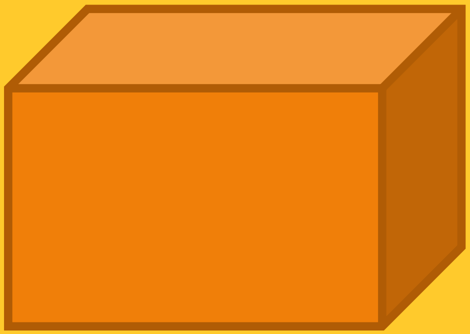
- 11.5 Area of a Polygon (Triangulation Method is used for finding area of a polygon)



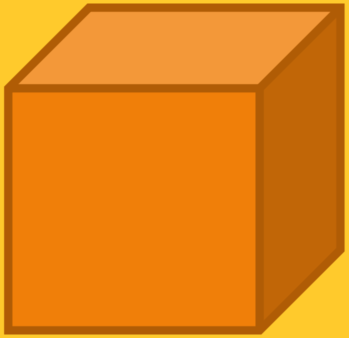
- Calculate area of each triangle and add
- Formula, Area= ½ x base x height

Class-VIII: Solid Shapes

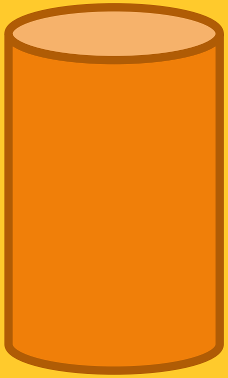
Cuboid



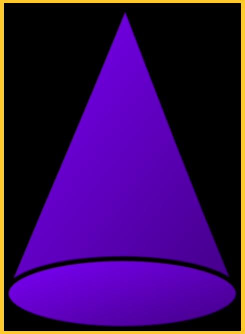
Cube



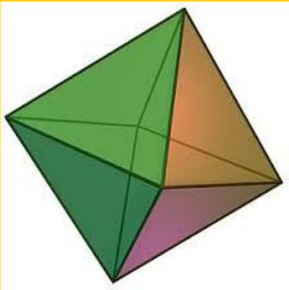
Cylinder



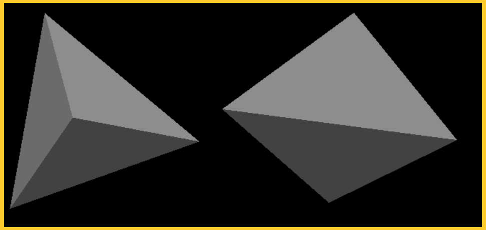
Cone



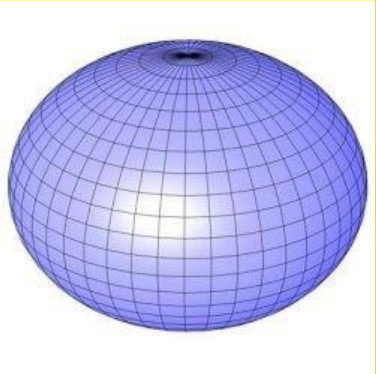
Octahedral



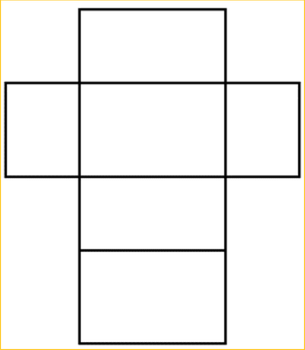
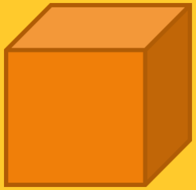
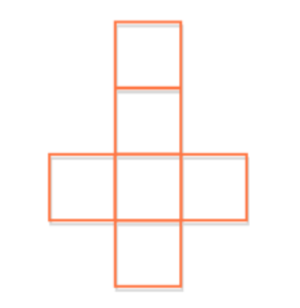
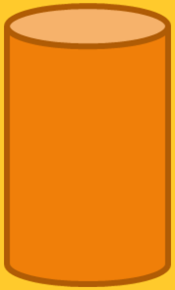
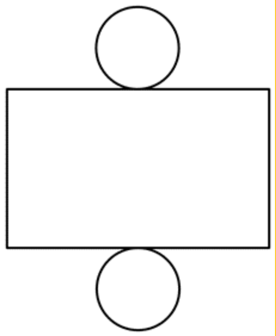
Tetrahedral



Sphere



Surface Area of Solids and Net

11.7 Surface area of Solids:

11.7.1 Cuboid:

Total surface Area=2x(h*l+l*b+bx*h)

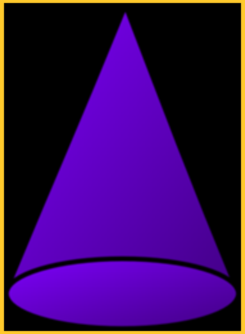
11.7.2: Cube:

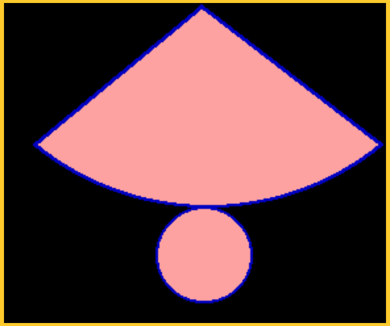
Total Surface Area=6*l^2

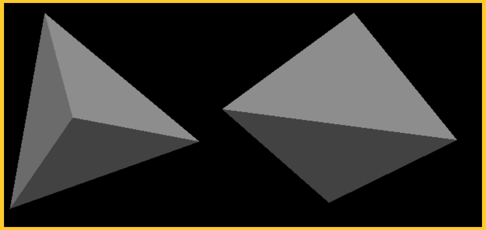
11.7.3: Cylinder:

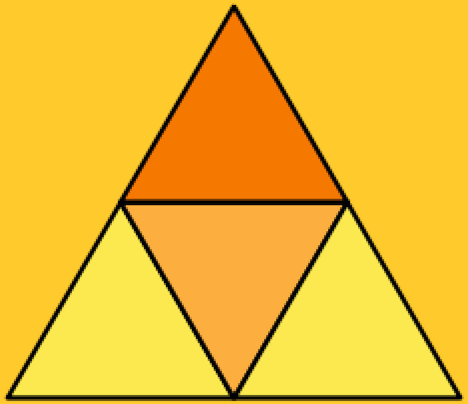
Total Surface Area= $2\pi r^2 + 2\pi r h = 2\pi r(r+h)$

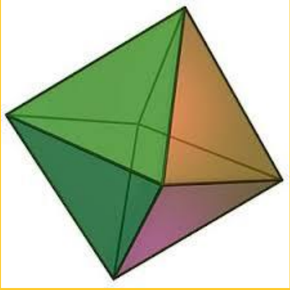
Surface Area Calculation and Net

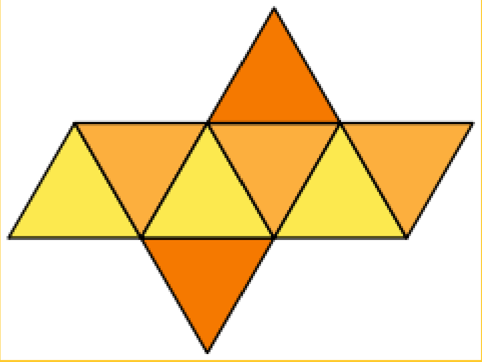


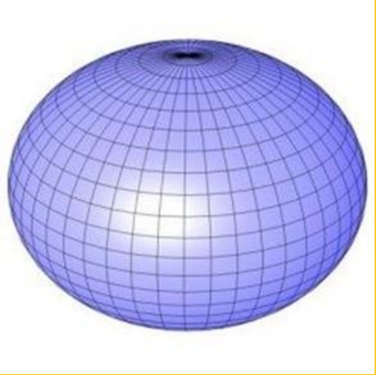


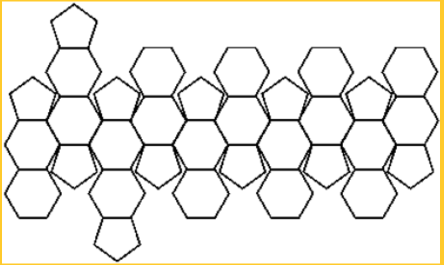












Mensuration

- 11.8 Volume of cube, cuboid and cylinder
- Volume = Volume is the amount of space occupied by a three dimensional objects.
- 11.8.1 Cuboid : Volume =lxbxh
- 11.8.2 Cube : Volume $=l^3$
- 11.8.3 Cylinder : Volume = $\pi r^2 h$
- 11.9 Volume and capacity :

(a) Volume is the amount of space occupied by an object

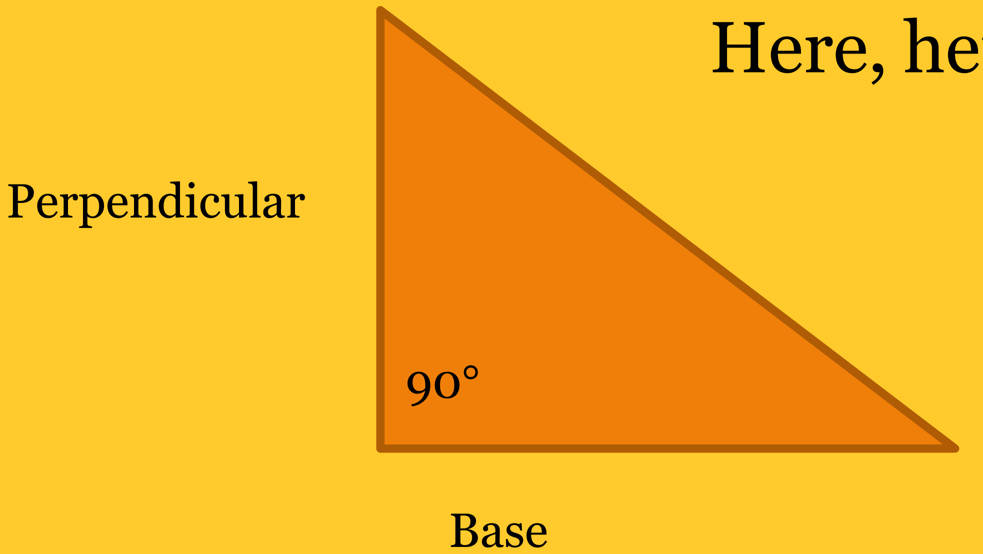
(b) Capacity refers to the quantity that a container holds.

Mensuration – Area of Right Angled Triangle

- 12.1:Finding the area of a triangle : Area= ½ * base* height
- Case-1: When the triangles is a right angled, then,

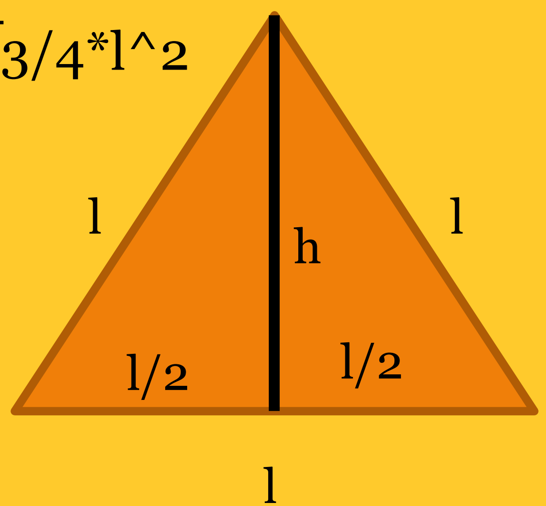
Area= ½ * Base * Perpendicular

Here, height=perpendicular



Area of Equilateral Triangle

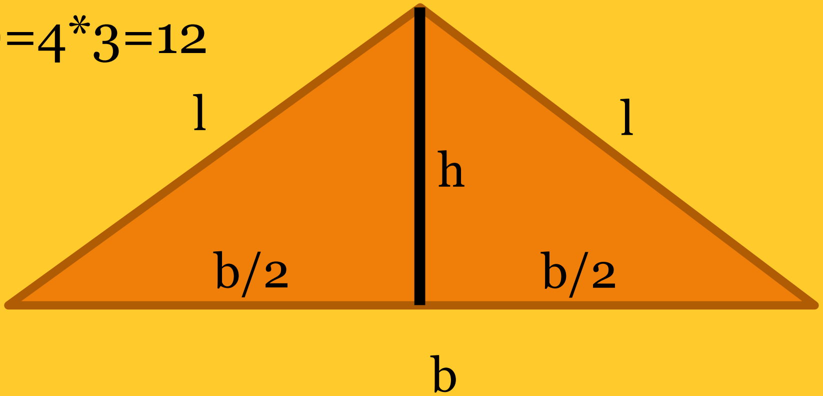
- Case-2: When the triangle is a Equilateral triangle
- Equilateral Triangle-All sides are equal
- Area= ½*Base*Height = ½ * l*h, Here h is to be calculated.
- From Pythogorous Theorem, l^2=h^2+(l/2)^2
- h= √(l²-(l/2)^2=√(3/4*l²)=$\sqrt{3} * \frac{l}{2}$,
- Hence, Area=1/2*l*h=1/2*l*$\sqrt{3} * \frac{l}{2}$= √3/4*l^2
- If l=10, then area=√3*10^2/4=25 √3



Area of Isosceles Triangle

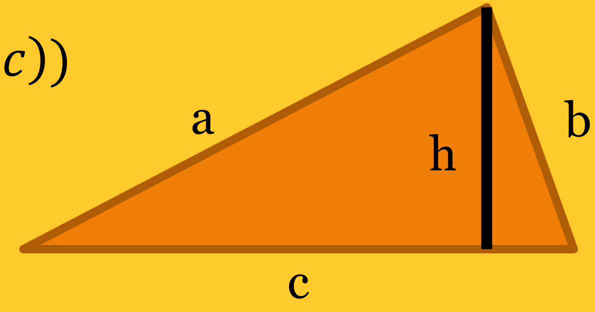
- Case-2: When the triangle is a Isosceles triangle
- Isosceles Triangle-Two sides are equal
- Area= ½*Base*Height = ½ * l*h, Here h is to be calculated.
- From Pythogorous Theorem, l^2=h^2+(b/2)^2
- h= √(l²-(b/2)^2
- Hence, Area=1/2*b*h=1/2*b*√(l²−(b/2)^2)
- If l=5, and b=8, then, Area=1/2*b*h=1/2*b*√(l²−(b/2)^2)

=1/2*8*√(5²−(8/2)^2)= 1/2*8*√(25-16)=4*3=12



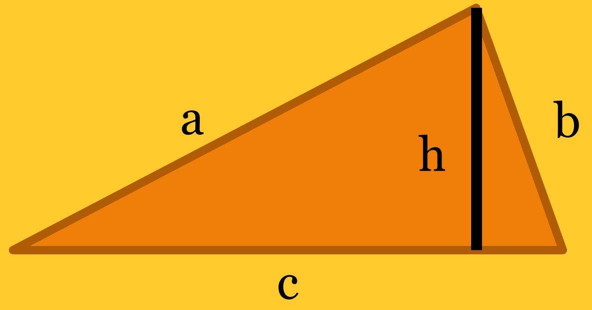
Area of Scalene Triangle

- Case-3: When the triangle is a scelen triangle
- Scelene Triangle-All sides are different
- Area= ½*Base*Height = ½ * l*h,
- Here calculation of h is difficult because a, b and c are different. But if can combine all the length into one parameter, then calculation of area will become easier.
- It is achieved by calculating the perimeter of the triangle.
- Perimeter, s=a+b+c
- And area=$\sqrt{(s(s-a)(s-b)(s-c))}$



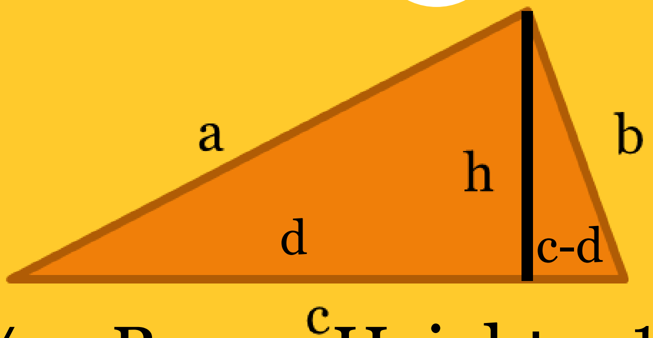
Area of a scalene triangle: Heron's Formula

- 12.2 Finding area of a scalene triangle; Heron's Formula, Finding area when perimeter is known.
- Let, p=a+b+c, where a, b and c are sides of a triangle.
- Then, semi-perimeter=s=p/2=(a+b+c)/2
- Area=√(s(s-a)(s-b)(s-c))
-



- Area= ½ x Base x Height = ½ * c* h

Area of a scalene triangle: Heron's Formula



- Area= ½ x Base x Height = ½ *c* h
- $h^2=a^2-d^2$……………………(i)
- $b^2=h^2+(c-d)^2$
- $a^2-b^2= h^2+d^2-c^2+2cd-d^2-h^2$
- $a^2-b^2= - c^2+2cd$
- or $(a^2-b^2+c^2)/2c)=d$
- Note-Our objective is to reduce no of unknowns

Area of a scalene triangle: Heron's Formula

- $(a^2-b^2+c^2)/2c)=d$
- Putting the value of d in eqn-1,

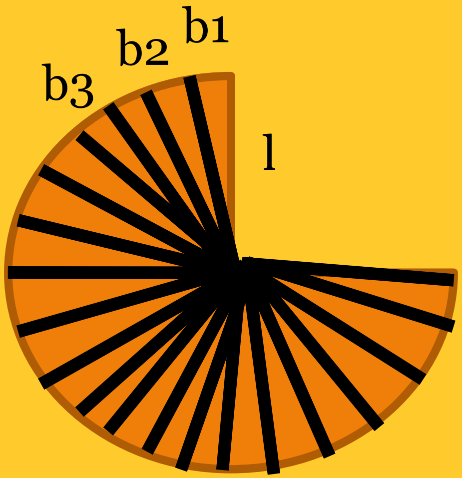
1. $h^2=a^2-d^2$……………………(i)
2. $h^2=a^2-(a^2-b^2+c^2/2c)^2$
3. $h^2=(4c^2a^2-(a^2-b^2+c^2)^2)/4c^2$
4. $h^2=((2ac+a^2-b^2+c^2)(2ac-a^2+b^2-c^2))/4c^2$
5. $h^2=\{(a+c)^2-b^2\}\{b^2-(a-c)^2\}/4c^2$
6. $h^2=(a+c+b)(a+c-b)(b+a-c)(b-a+c)/4c^2$
7. $h^2=p(a+b+c-2b)(a+b+c-2c)(a+b+c-2a)/4c^2$
8. $h^2=p(p-2b)(p-2c)(p-2a)/4c^2$, $s=p/2=(a+b+c)/2$

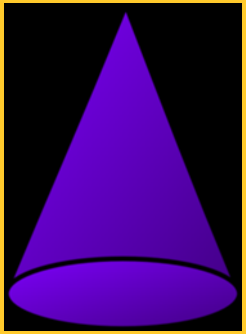
Area of a scalene triangle: Heron's Formula

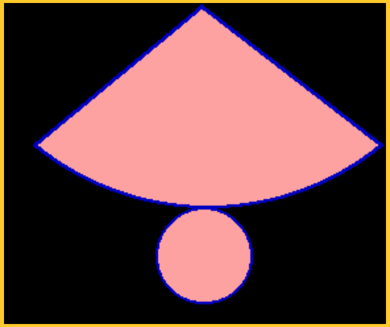
1. $h^2=p(p-2b)(p-2c)(p-2a)/4c^2$, $p=a+b+c=2s$
2. $h=\sqrt{}(p(p-2a)(p-2b)(p-2c) /4c^2)$
3. $h= \sqrt{}(p(p-2a)(p-2b)(p-2c))/2c$
4. Area= ½ c h
5. Area= ½ c{√(p(p-2a)(p-2b)(p-2c)}*1/2c
6. = ¼ √{p(p-2a)(p-2b)(p-2c)}
7. = √{1/16 p(p-2a)(p-2b)(p-2c)
8. = √{p/2 (p-2a)/2)(p-2b)/2)(p-2c)/2)
9. = √{2s/2*(2s-2a)/2*(2s-2b)/2*(2s-2c)/2}
10. = √s(s-a)(s-b)(s-c)

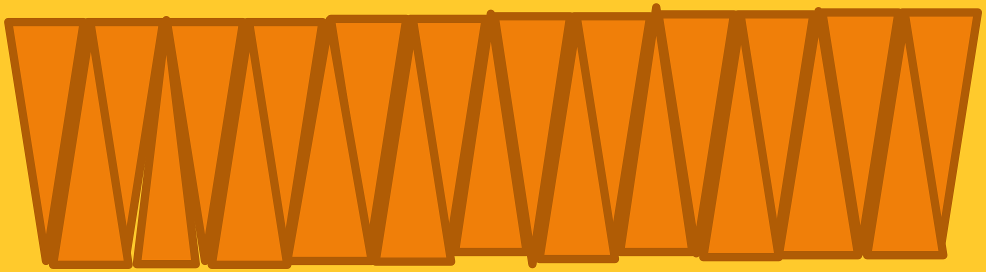
Surface areas of Right Circular Cone

- 13.4 Surface area of right circular cone:
- Surface Area= ½ (lb1+lb2+lb3……..)
- = ½* l (b1+b2+…….)



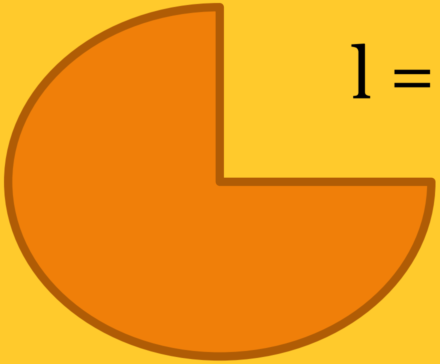


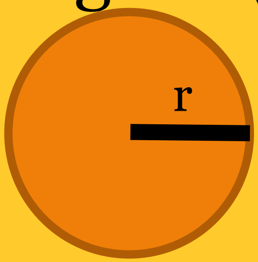


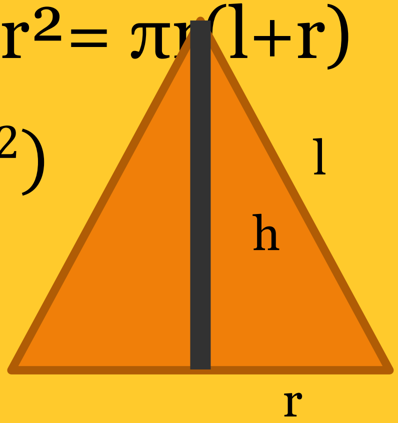


Surface areas of Right Circular Cone

- = ½(l (b1+b2+.......)
- But b1+b2+b3.....is the circumference of the bottom circle = 2πr
- Slant surface area= ½*2πr*l= πrl
- Cover Area = πr^2
- Total surface area= πrl+ πr^2= πr(l+r)
- l = slant height =$\sqrt{(h^2 + r^2)}$

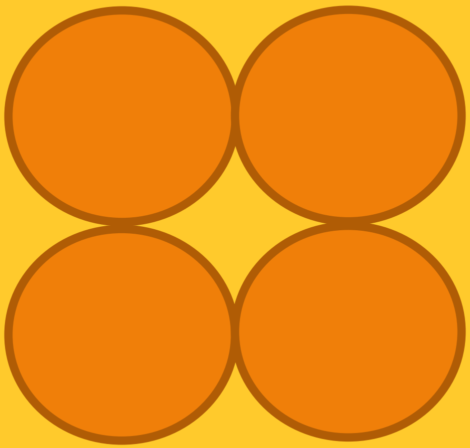
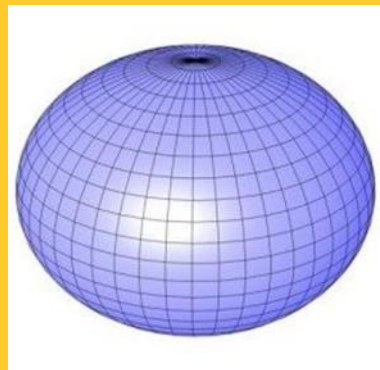
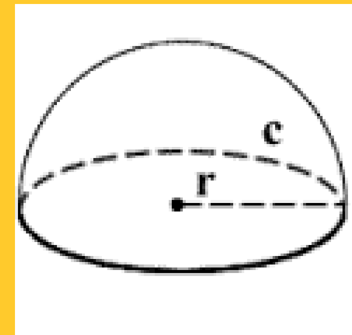


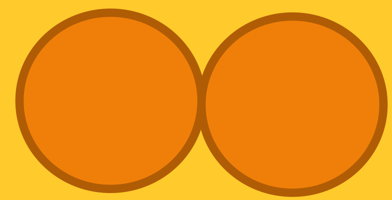




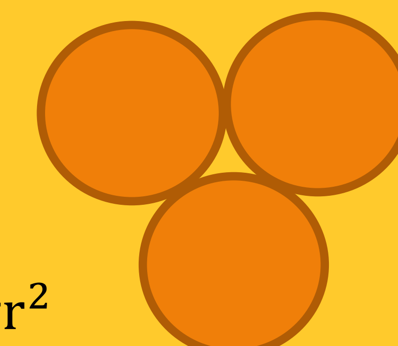
13.5 Surface Area of a sphere

- Surface area of a sphere = $4\ \pi r^2$



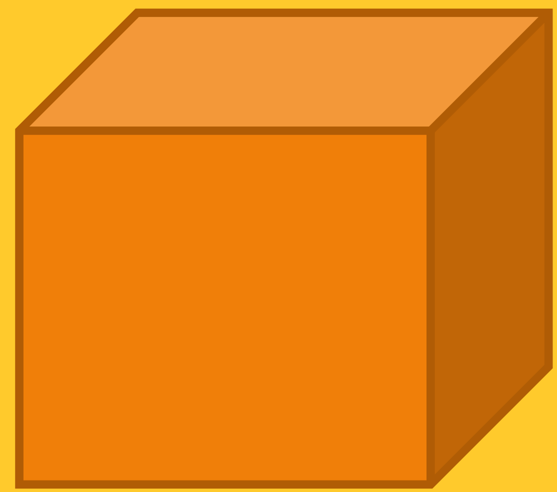
- 13.5 a Surface area of a hemi sphere
- = curved surface area + circular cover
- = $\frac{1}{2} \times 4 \times \pi r^2 = 2\ \pi r^2$
- 13.5 a Surface area of a solid hemi sphere
- = curved surface area + circular cover
- = $\frac{1}{2} \times 4 \times \pi r^2 + \pi r^2 = 2\ \pi r^2 + \pi r^2 = 3\pi r^2$



Volume of a Solid Objects

- 13.6 volume of a cuboid
- Volume in the space occupied by an object
- If L = length, b = breadth and h = height of the cuboid , then volume = length × breadth * height or v = l × b × h.

- Volume of cube = v = L^3



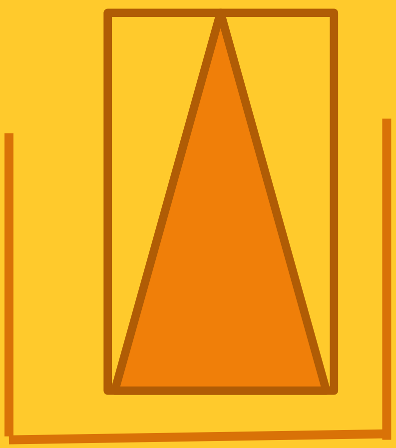
Volume of a Solid Objects

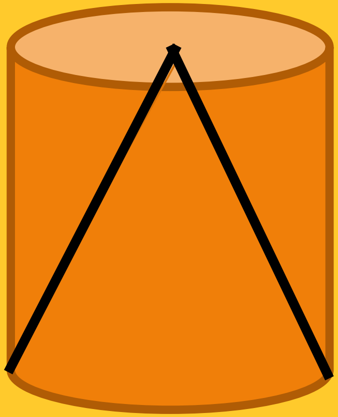
- Volume is the space occupied by a solid.
- Capacity is the volume of the substance that interior of an object can accommodate.

- It is seen that calculating the capacity of a cylinder is very easy from the radius and height of the cylinder.
- This property is used to measure the volume of cone and sphere.

Volume of Cylinder

- 13.7 Volume of a cylinder
- Volume=Base Area*Height=V = πr^2 h
- 13.8 Volume of right circular cone----
- v = ⅓ $\pi r^2 h$



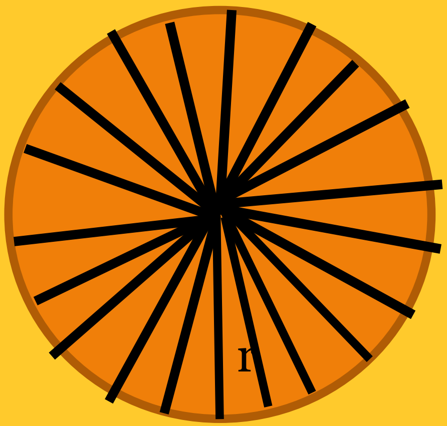


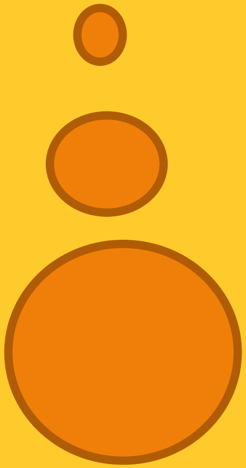
Volume of a sphere

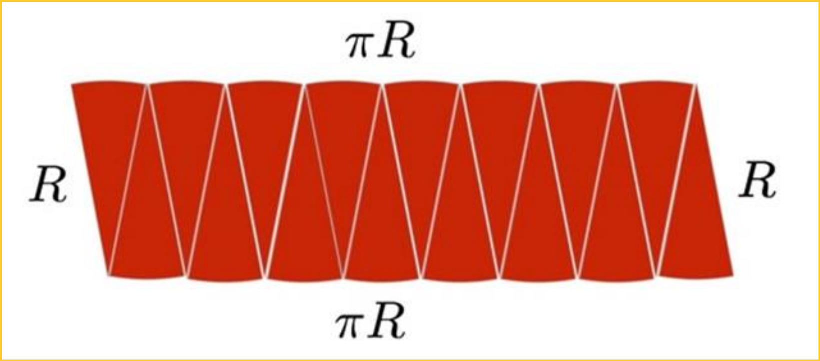
- Volume of a sphere

- $V = 4/3\ \pi r^3$
-
- Volume of a Hemisphere, $v = \frac{2}{3} \pi r^3$

Area and Perimeter of a segments and sectors of a circle







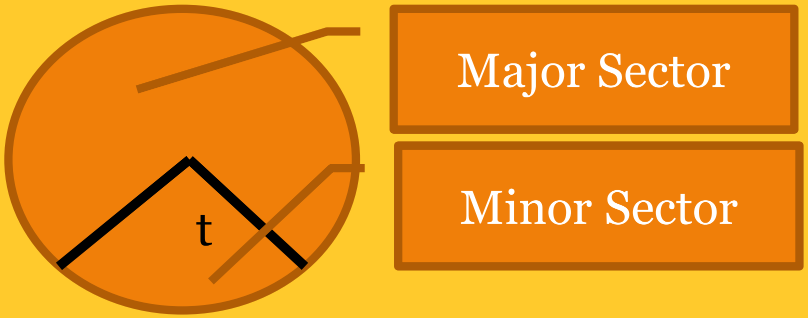
We can easily measure the diameter and circumference of the circle.

It is seen that the ratio of Circumference and Diameter of a circle bears a relationship and is approximately equal to 22/7.

- π= Circumference/2*Radius
- Perimeter = circumference = $2\pi r$
- Area = πr^2
- These information is used to calculate the area of sectors and segments of a circle

Area and Perimeter of a segments and sectors of a circle

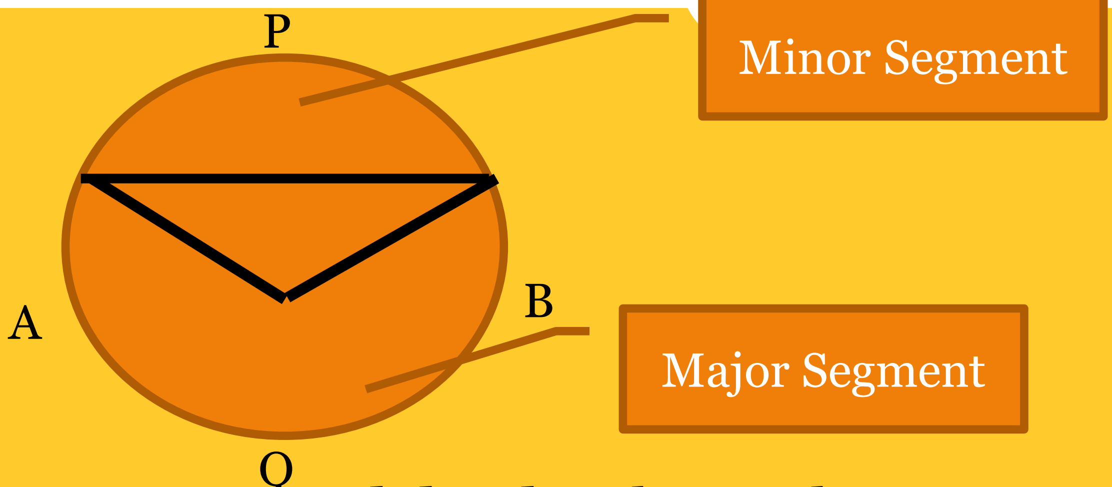
- SECTORS-The part of portion of a circle enclosed by two radius and correspondence arc is called a SECTOR. A circle has two sectors.



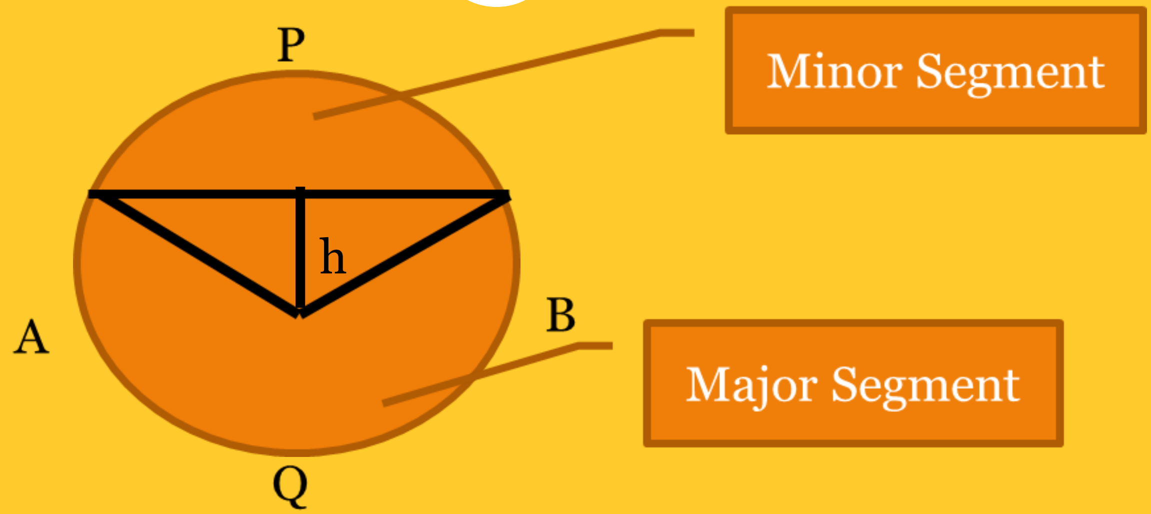
- If the angle is < than π it is minor sector, otherwise it is Major sector.
- Area of a sector of angle, t, Area = $t/360\times \pi r^2$, if t it in degree
- Area = $t * \frac{r^2}{2}$, if t it in radian
- 360
- Length of an arc of sector of an angle, t, Length = t × 2π r, if t is in radian.
- 360
- Length = t/(2 π) * 2 π r = t r, if t is in radian.

Note: Unitary methoods used for calculating area and arc length

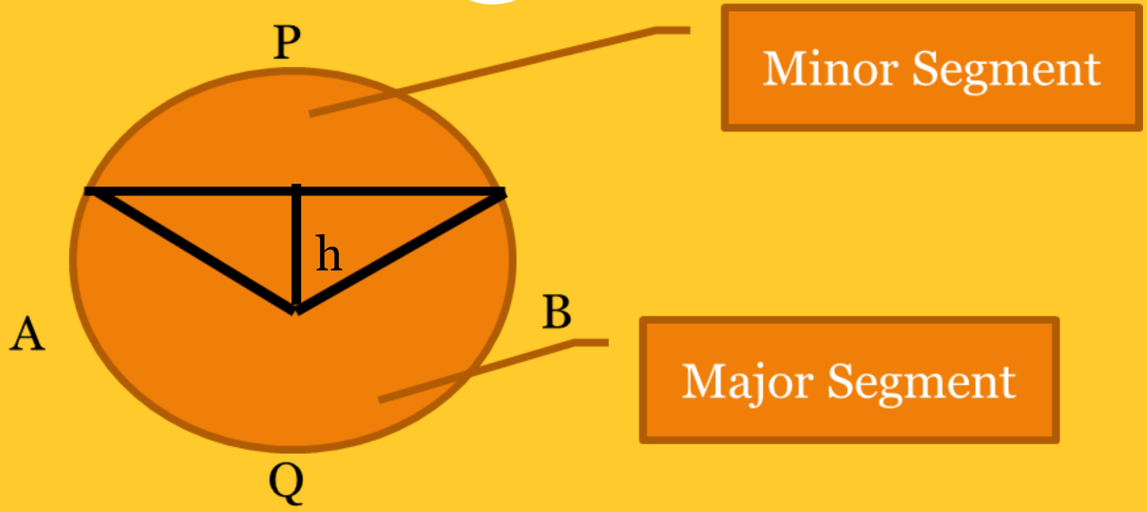
Area and Perimeter of a segments and sectors of a circle



- Segment: A cord divides the circle in two segments.
- Minor segment.
- Major segment.
-
- Minor segment = A P B
- Major segment = A Q B



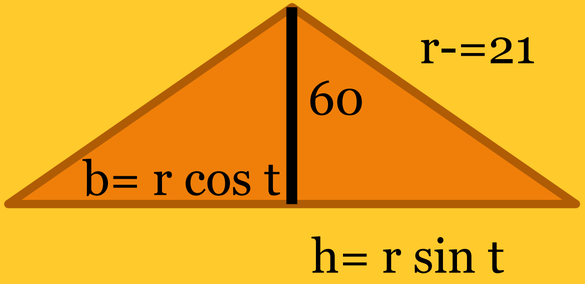
- Area of Minor segment = Area of minor sector – Area of triangle OAB.
- Area of sector A = $\frac{t}{360} \times \pi r^2$
- Area of a triangle = $\frac{1}{2}$ hr ,   h = height of the triangle = r cost $\frac{t}{2}$
- = ½ r. r cos t/2 = ½ r^2 cos t/2
- Therefore, Area of the segment = $\frac{t}{360}$ = πr^2 - ½ r^2 cos t/ 2



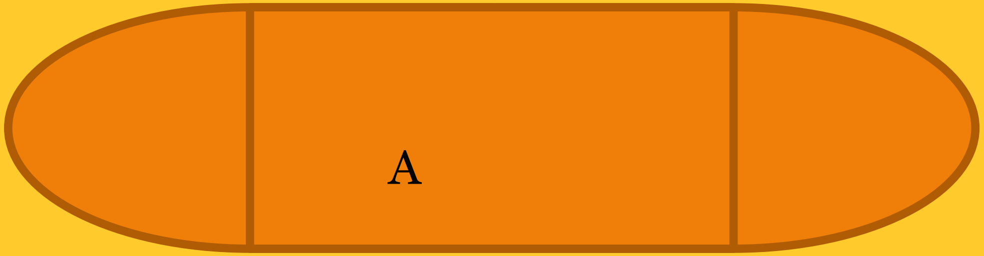
- Therefore, Area of the segment $= \frac{t}{360} \pi r^2 - \frac{1}{2} r^2 \cos t/2$

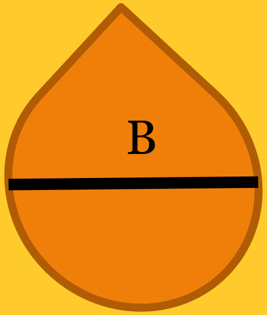
Area of a segment

- <u>Ex- 3</u>
- $t = 120^0$ } area = 461.814 – 110.25
- $r = 21$ cm = 351.569
- Area of a segment
- $r = 21$, $t = 120/2=60^0$
- b = r sin t = 21 sin t = 18.18
- h = r cost = 21 cos t = 10.5
- Area of triangle = ½ 2bh = bh = 190.9
- Area of sector = $t/360 \times \pi r^2$ = 461.81
- Area of the segment = 270.91 or 271.0.



- Surface area of combination of solids:





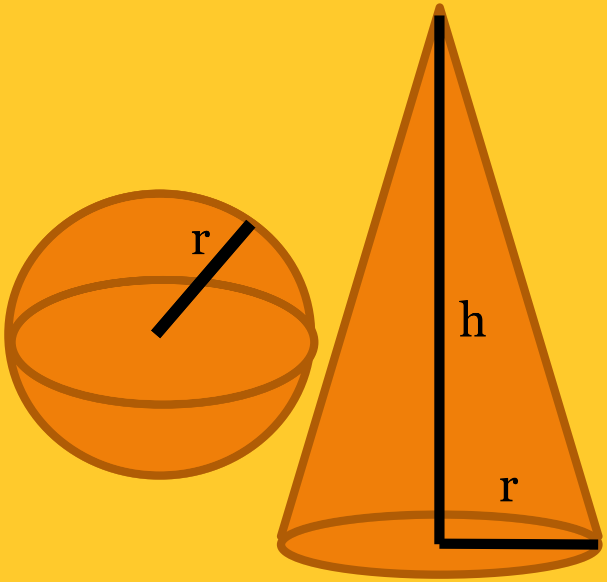
- A. Total Surface Area= Curved surface area of two hemispheres and one cylinder
- B. Total Surface Area= Curved surface area of hemispheres and cone

Conversion of solid from one shape to another

- If the cone of radius r and height h, then volume =1/3 $\pi\ r^2h$.
- Now this cone has been melted and reshaped as a sphere.

So, the volume sphere is same as the cone.

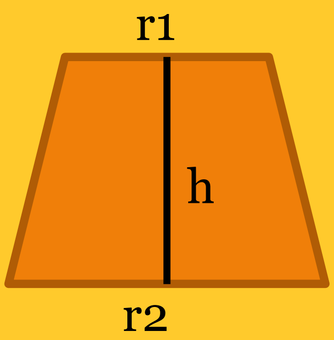
Now, what is the radius of the sphere?
Let, R be the radius of sphere. Then
$1/3\ \pi r^2h = 4/3\ \pi\ R^3$. Hence, R=$\sqrt[3]{(1/4 * r^2h)}$

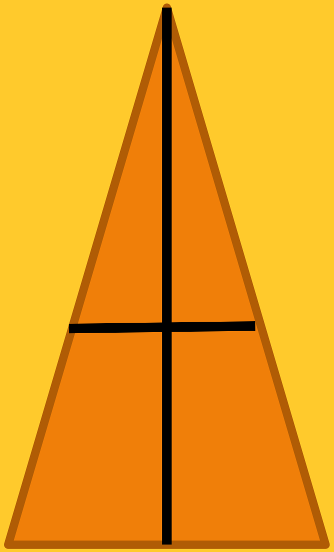


Ex. Let r=6, h=24,

Then, R=$\sqrt[3]{(1/4 * r^2h)}$=$\sqrt[3]{(1/4 * 6^2 * 24)}$=6

Frustum of a cone is formed by removing the upper part of the cone.





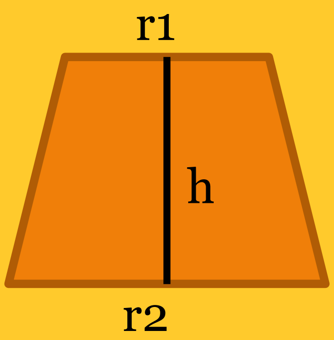
- Area of a frustum :

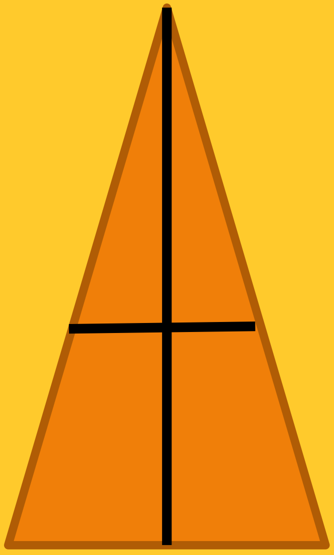
The upper end radius r1 and lower end radius r2 an height of the frustum can be measured directly.

- For finding the area of the frustum, cut it vertically and we will get a section of a circle as shown below----

Solid from removing a part of another solid

Frustum of a cone is formed by removing the upper part of the cone.





- Area of a frustum :

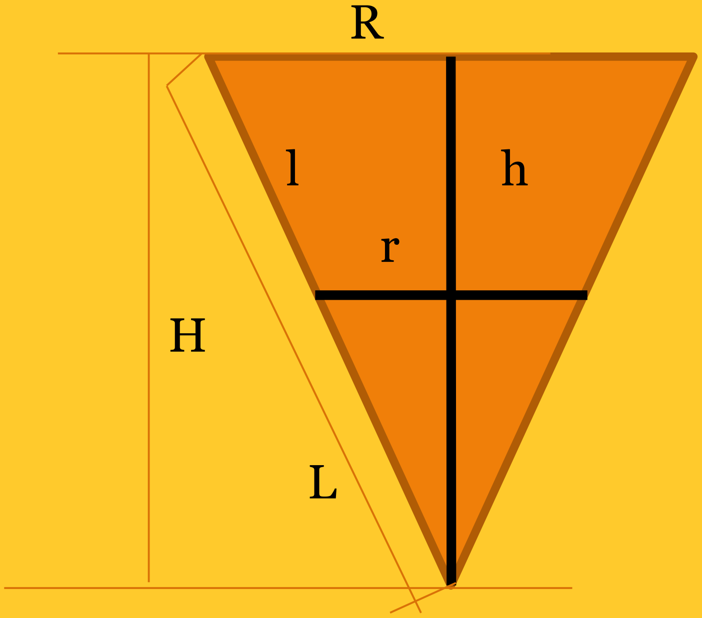
The upper end radius r1 and lower end radius r2 an height of the frustum can be measured directly.

- For finding the area of the frustum, cut it vertically and we will get a section of a circle as shown below----

Surface Area of Frustum of a cone

Now surface area is the Total sector – Small sector

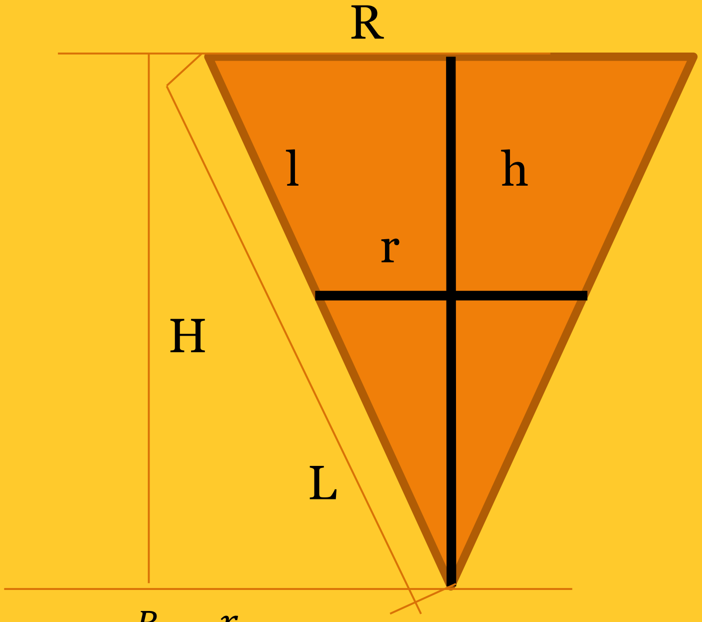
- Height , h = distance between two ends of frustum of cone
- Or its thickness, Slant hight = L
- $\frac{H}{(H-h)} = \frac{L}{(L-l)} = \frac{R}{r}$



- $$\frac{H}{H-h} = \frac{L}{L-l} = \frac{R}{r}$$
- $$\frac{H-h}{H} = \frac{L-l}{L} = \frac{r}{R}$$

Surface Area of Frustum of a cone

- $$\frac{H}{H-h}=\frac{L}{L-l}=\frac{R}{r}$$
- $\frac{H-h}{H}=\frac{L-l}{L}=\frac{r}{R}$
- $1-\frac{h}{H}=1-\frac{l}{L}=\frac{r}{R}$
- $\frac{h}{H}=1-\frac{r}{R}$
- $H=\frac{hR}{R-r}$
- $1-\frac{l}{L}=\frac{r}{R}$
- or $L=\frac{Rl}{R-r}$



From Similarity Property, $\frac{R}{H}=\frac{r}{H-h}$

Or rH=R(H-h)

Or RH-rH=hR

Or H(R-r)=hR

Surface Area of Frustum of a cone

- From right angled triangle: $L^2 = (R-r)^2 + h^2$
- Lateral surface area = $\pi\,(\underline{R+r}) \times h$
- (note : Trapezium formula for area=sum of parallel sides/2*height)

- Total surface area = $\pi(R+r) \times h + \pi r^2 + \pi R^2$
- $= \pi(R+r)h + \pi r^2 + \pi R^2$
-
- Area = $\pi(r + R) + (R- r)^2 + h^2$.
- Ex – 12 $H = 45$
- $R = 28$
- $r = 7$
- Area = $2\,\pi(R+r)\,L + = \pi r^2 + = \pi r^2$
- $= 2\,\pi(R+r)\,(\text{root }(R- r)^2 + H^2)$
- $= 2\,\pi\,(28+7)\,(\text{root}(28-7)^2 + 45^2 + \pi 28^2 + \pi\, 7^2$
- = 8077 ‘ 221
-

Volume of Frustum of a cone

- Volume of frustum of a cone :
- V = volume of total cone – volume of frustum cone
- $= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 (H - h)$
- $\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 H + \frac{1}{3}\pi r^2 h$
-
- $= \frac{\pi}{3}(R^2H - r^2H + r^2 h)$
- $= \frac{\pi}{3}\{ H (R^2 - r^2) + r^2 h\}$
- $= \frac{\pi}{3}\{ (R-r)(R+r)H + r^2 h\}$

From Similarity Property, $\frac{R}{H} = \frac{r}{H-h}$

Or $rH = R(H-h)$

Or $RH - rH = hR$

Or $H(R-r) = hR$

Volume of Frustum of a cone

- $= \frac{\pi}{3} \{ hR(R+r) + r^2 h \}$
- $= \frac{\pi h}{3} \{R(R+r) + r^2\}$
- $= \frac{\pi h}{3} \{ R^2 + rR + r^2 \}$

From Similarity Property, $\frac{R}{H} = \frac{r}{H-h}$

Or rH=R(H-h)

Or RH-rH=hR

Or H(R-r)=hR

Algebra

- Why we require Algebra?

- Algebra is the study of variable. The value of a variable is not fixed. It is represented by letters. It makes description of any system and its parameters very simple.

Syllabus of Algebra

(Class-VI)

1. Introduction of variables through patterns
2. Introduction to unknowns

(Class-VII)

1. Algebraic expressions
2. Identify constant, coefficient and power
3. Arithmetic operation on expression
4. Linear equation of one variable

Syllabus of Algebra

(Class-VIII)

1. Identities
2. Factorization
3. Linear equation in one variable

(Class-IX)

1. Polynomial of one variables
2. Factor and multiples of Polynomials
3. Zeros/ roots of Polynomial
4. Reminder Theorem
5. Factorization

Syllabus of Algebra

(Class-X)

1. Zeros of Polynomial
2. Relationship between zeros and coefficients
3. Division Algorithm
4. Pair of linear equations
5. Algebraic condition for number of solutions
6. Solving linear equations by
    1. Substitutions
    2. Eliminations
    3. Cross Multiplication

Syllabus of Algebra

Class-X

7. Equations reducible to linear equations
8. Quadratic equations – standard form
9. Solution of quadratic equations by –
    1. FACTORIZATION
    2. COMPLETING SQUARE
10. Relation between discriminant and nature of roots
11. Arithmetic Progression
12. Derivation for finding n-th term and sum of n-th term

Syllabus of Algebra

(Class-XI)

1. Principle of mathematical induction
2. Complex numbers
3. Linear inequalities – (a) One Variable (b) Two variables
4. Permutation and combinations
5. Binomial Theorem
6. Sequence and series

   A P, A M, G P, G M, sum of n terms of special series : Σn, Σn^2, Σn^3

Algebra

- Why we require Algebra?

- Algebra is the study of variable. The value of a variable is not fixed. It is represented by letters.

Algebra

Introduction :

- What is Arithmetic?

Arithmetic is the study of numbers and its operations, properties etc.

- What is Geometry?

Geometry is the study of shapes.

- What is Algebra?

Algebra is the study of variables.

Every object follows a pattern and every parameter of any system bears a relationship among themselves. We capture these patterns or relationships through defining some variables and all these studies together called Algebra.

The idea of variables: Area of a square, area = length*length , Perimeter of a square, p=4*l, area of a circle, a=pi*r^2, perimeter of a circle, p=2*pi*r

Algebra

Use of variables in common rule;-

- area=l^2, area=l*b
- Expression with variables, 3+(4+5), 4+36, x+7y, 2+5x, 3x+9y+6
- Using Expression practically:

Example- Him is 3 years younger than her sister. His age is 11. What is his sister's age? Let her sister's age is x.

- Equations, x-3=11
- Solution of an equation: The value of the variable which satisfies the equation is called a solving the equation.

Example: x-3+3=11+3, or x=14

Algebra
Simple Equations

- Example of Equations: $4x+5=65$, $x=15$
- What is equation: An equation is a condition on a variable. In an equation there is an equality sign. LHS=RHS
- Solving an equation: Rules of solving equation
  - (1) Add/subtract/Multiply/Divide both side of equation, equality holds
  - (2) While transposing, change the sign.

Note - Transposing a number is same as adding/ subtracting etc.

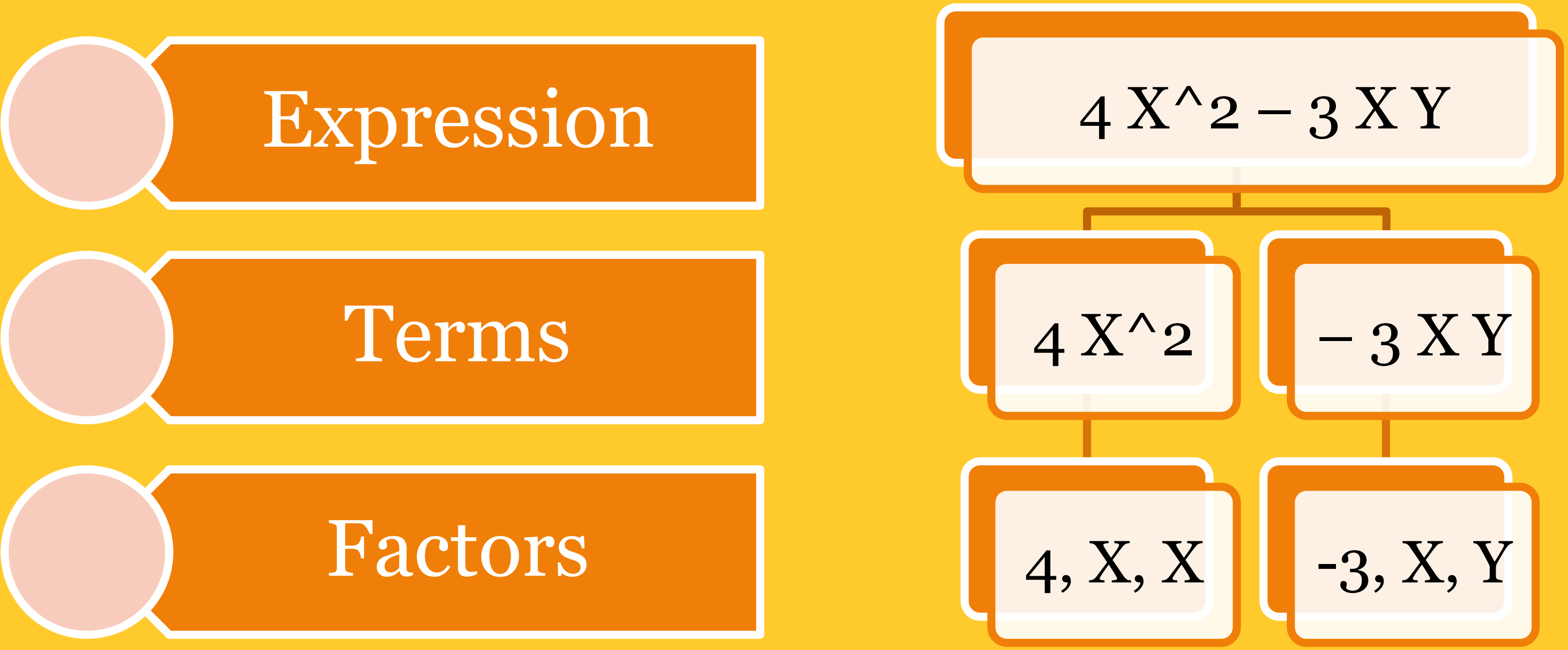
Algebraic Expression

- Expressions are central concept in Algebra.
- How expression formed? Expressions are formed by combining variables and constants.
- Terms of an expression: Each expression has different terms and each term has different parts
- Example: $4x^2+3xy$ - This expression has two terms, first term has been formed by 4,x,x and secondary term is formed by 3,x,y.

  Factors of a term : Each term is made of constants and variables called its factors. The expression, terms and factors are shown in tree diagram-

Algebraic Expression

- The expression, terms and factors one shown in three diagram-



- Coefficients – Numeric factors in a terms is called coefficients.
- Like terms and unlike terms- Terms of an expression that have same algebraic factors are called like terms, otherwise they are unlike terms 12x, 12, 25x, 5y, y, x.
- Monomials, Binomials, Trinomials and polynomials-

Monomials: one term

Binomials : Two unlike terms

Trinomials : Three unlike terms

Linear Equation of one variables

- Expressions: $2x$, $5x-3$, $7x+2y+xy$, $3x^2+7+y^2$
- Equations :$2x=10$, $20x-7=9$, $2y+5/2=37/2$, $62x+10=-2$
- Linear equations: Equations having highest power one.
- Solving equations which have expressions on one side and numbers on the other side:

  Example-1: $2x-3=7$, $2x-3+3=7+3$ or $2x=10$

  $2x/2=10/2$ or $x=5$

- Solving equations having the variable on both sides
- Reducing equations to simpler forms
- Equations reducible to the linear form.

Algebraic Expressions

- Addition and subtraction of Algebraic expressions
- Multiplication of Algebraic Expressions
- Multiplying a monomial by a monomial
  - Multiplying Two Monomial
  - Multiplying Three Monomial
- Multiplying a monomial by a polynomial
  - Multiplying a monomial by a binomial
  - Multiplying a monomial by a trinomial
- Multiplying a polynomial by a polynomial
  - Multiplying a binomial by a binomial
  - Multiplying a binomial by a trinomial
- Identity
- Standard Identity

Factorization of Algebraic Expressions

- Methods of Factorization
    - Method of common factor
    - Regrouping of terms
    - Use of identity
    - Breaking of Middle Terms
- Division of Algebraic Expressions
    - Monomial by monomial
    - Polynomial by monomial
    - Polynomial by polynomial

Polynomials

- Topics-Polynomial Expressions
  - Remainder Theorem
  - Factor Theorem
  - Algebraic Identities
- Polynomial of one variables : Exponent of polynomials are whole numbers.
  $\Sigma x - x^3 + 3x, 3x+5, 2x^2+x+7,$
  Expression $1/x$, t^{-5}, root x are not polynomials
  Degree of Polynomial – is the highest power of x
  Linear Polynomial – Degree is one
  Quadratic Polynomial – Degree is two
- Zeros of a polynomial – Zero of a polynomial p(x) is a number, c, such that p(c) = 0
- Reminder Theorem – Factor, Multiple, dividend=(Divisor X Quotient)+ Remainder

Polynomials

- Factorization of Polynomials – Factorization by splitting middle term
- Algebraic Identities :

  Identity – 1: $(x+y)^2=x^2+2xy+y^2$

  Identity – 2: $(x-y)^2=x^2-2xy+y^2$

  Identity – 3: $x^2-y^2=(x+y)(x-y)$

  Identity-4: $(x+a)(x+b)=x^2+(a+b)x+ab$

Identity-5: $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

Identity-6: $(x+y)^3=x^3+3x^2y+3xy^2+y^3$

Identity-7: $(x-y)^3=x^3-3x^2y-3xy^2+y^3$

Identity-8: $x^3+y^3+z^3=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)-3xyz$

Polynomials

- Polynomials – Linear Polynomials- Degree 1
  Quadratic Polynomials – Degree 2
  Cubic Polynomial – Degree 3

  Ex-1 : $p=x^2-3x-4$, as for $x=-1$ and 4, $p=0$, hence -1 and 4 are called zeros of the polynomial .

- Definition of Zero : A real number k is said to be a zero of a polynomial p(x), if p(k)=0

  Observation : Every zero is related to its co-efficient.

- Geometrical Meaning of the zeros of a polynomial:

  Geometrically, the zero of a polynomial is the x coordinate values of the point where the graph of the function intersect x-axis.

Polynomials

Zeros of different polynomials

1. Linear Polynomial -
2. Quadratic Polynomial
3. Cubic Polynomial

Relationship between zeros and coefficients

- Relationship between zeros and coefficients
- Let α,β,γ,δ be the roots of the equations,
- Linear equations:
- $ax+b=0$, one root- α,

  or $\alpha=-b/a=$-constant term/coefficient of x

Relationship between zeros and coefficients

Quadratic Equations –

$ab^2+bx+c=0$,

Two root = α,β

- **Sum of roots=$\alpha+\beta=-b/a$,=-coefficient of x /coefficient of x^2**
- **Product of roots= $\alpha*\beta=c/a$=constant term /coefficient of x^2**

Relationship between zeros and coefficients
To prove, **$\alpha+\beta=-b/a$ and $\alpha*\beta=c/a$**

Derivation of the formula: When α and β are the zeros of the polynomial, $p= ab^2+bx+c=0$, then $(x-\alpha)$ and $(x-\beta)$ are the factors of p,

Hence, $a x^2+b x+c = k(x-\alpha)(x-\beta)$

$= k\{x^2-x\beta-x\alpha+\alpha\beta\}$

$= k\{x^2-x(\alpha+\beta)+\alpha\beta\}$

$= k x^2 - k(\alpha+\beta) x + k\alpha\beta$

Comparing the terms : $a=k$, $b=-k(\alpha+\beta)$, $c= k\alpha\beta$

$\therefore (\alpha+\beta)=-b/k=-b/a$ as $a=k$

$\therefore \alpha\beta=c/k=c/a$ as $a=k$

Relationship between zeros and coefficients

Ex.-2 , $p=x^2+7x+10$,

$p = (x+2)(x+5)$, ∴ $\alpha = -2$, $\beta = -5$

$\alpha+\beta = -2-5 = -7 = -b/a=-7/1=-7$

$\alpha\beta=-2*-5=10=c/a=10/1=10$

Relationship between zeros and coefficients

For a cubic polynomial, there will be three zeros

ax^3+bx^2+cx+d

$\alpha+\beta+\gamma=-b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha=c/a$

$\alpha\beta\gamma=d/a$

Ex.= $3x^3-5x^2-11x-3$, $\alpha=3$, $\beta=-1$, $\gamma=-1/3$

$\alpha+\beta+\gamma=3+(-1)+(-1/3)=5/3$,

$\alpha\beta+\beta\gamma+\gamma\alpha=3x-1-1\ x\ -1/5+-1/3x3=-11/3=c/a$

$\alpha\beta\gamma=3x-1x-1/3=1=-d/a$

Relationship between zeros and coefficients

For a quartic equation, there will be four zeros

$$ax^4+bx^3+cx^2+dx+e=0$$

$$\alpha+\beta+\gamma+\delta=-b/a$$

$$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\gamma+\alpha\delta+\beta\delta=c/a$$

$$\alpha\beta\gamma+\alpha\gamma\delta+\beta\gamma\delta=-d/a$$

$$\alpha\beta\gamma\delta=e/a$$

- what is the advantage of knowing the relationship between zeros and coefficients?
- From these relationships, there will be **n** unknown and **n** equations. By solving these equations, we can find the roots.

Quadratic Equations

- Quadratic polynomial-ax^2+bx+c

  When thus polynomial is equated to 0, we get a polynomial equations called quadratic equations, $ax^2+bx+c=0$
- Quadratic equations : Standard form $ax^2+bx+c=0$
- Solution of quadratic equations-

  (a) By Factorization

  (b) By Completing squares
- Solution of quadratic equation by Factorization:

  Let $ax^2+bx+c=0$

  Now, if putting α for x, if we get $a\alpha^2+bx+c=0$, then α called the root of the Quadratic equations.

  it is also said that α satisfies QE, roots and zeros of QE are same.

  ex.3 $2x^2-5x+3=0$ or $2x^2-3x-2x-3=0$

  or x(2x-3)-1(2x03)=0

  04 (x-1)(2x-3)=3 or x=1 or x=3/2

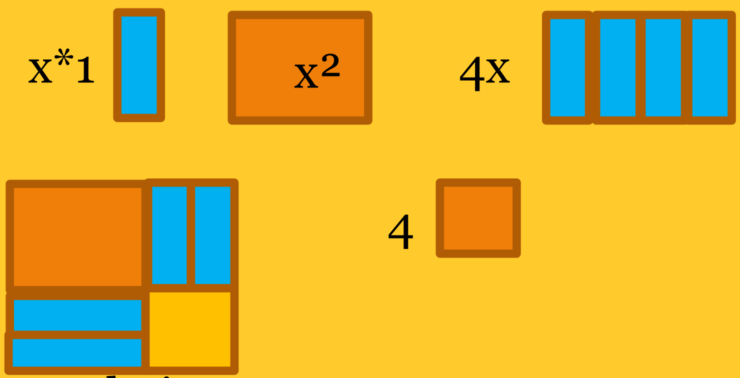
  we have split middle term.

Quadratic Equations

- Solutions of quadratic equations by completing squares, $x^2 + 4x$

  Graphical representation of x^2, x=

Graphical representation of 4x



Ex-1: Express $x^2 + 4x - 5$ by method of completing squares

Now, $x^2 + 4x = (x+2)^2 - 4$

Hence $x^2 + 4x - 5 = (x+2)^2 - 4 - 5 = (x+2)^2 - 9$

Expressing $x^2 + 4x - 5$ as $(x+2)^2 - 9$

So solving $x^2 + 4x - 5 = 0$ can be written as $(x+2)^2 - 9 = 0$. $(x+2)^2 = 9$, so $x + 2 = +/-3$

Hence, x=1 and x=-5.

Now $x^2+4x-5=(x+2)^2-4-5=(x+2)^2-9$

Ex-2 : $9x^2-15x+6=0$

$(3x)^2-2\cdot3x\cdot5/2+(5/2)^2+6=0$

$(3x-5/2)^2-25/4+6=0$

Or $(3x-5/2)^2-1/4=0$

Now $(3x-5/2)^2=1/4$

$3x-5/2=\pm \frac{1}{2}$

or $x=3/3=1$ or $x=2/3$

Ex-3 : $3x^2-5x+2=0$

$x^2-5/3x+2/3=0$

$x^2-2\cdot5/3\ x\cdot1/2+(5/3)^2-(1/2)^2+2/3=0$

$x^2-2\cdot5/3\ x\cdot1/2+(5/3)^2-25/36+2/3=0$

$(x-5/6)^2-1/36=0$

OR $(X-5/6)^2=\pm(1/36)^2$

OR $X=1/6+5/6=1$, $X=2/3$

4.4 General method of completing squares

$ax^2+bx+c=0$

1. To form a square, divide the equation by a,

$ax^2/a+bx/a+c/a=0$

Or $x^2/a+bx/a+c/a=0$

2. By completing square,

$x^2+2.b/a.1/2x+(b/2a)^2-(b/2a)^2+c/a=0$

Or $(x+b/2a)^2+c/2a-(b/2a)^2=0$

Or $(x+b/2a)^2+b^2-4ac/4a^2=0$

Or $(x+b/2a)^2=b^2-4ac/4a^2$

Or $x+b/2a=\pm\sqrt{(b^2-4ac/4a^2)}=\pm\sqrt{(b^2-4ac)}/2a$

Or $x=-b/2a\pm\sqrt{(b^2-4ac)}/2a$

Or $x=(-b\pm\sqrt{(b^2-4ac)})/2a$

If root $b^2=4ac$ is negative, then there will be no real roots

4.5 Nature of roots - $ax^2+bx+c=0$

$x=(-b \pm \sqrt{(b^2-4ac)})/2a$

If b^2-4ac is negative, then there will be no real roots, hence, b^2-4ac determines whether the quadratic equation $ax^2+bx+c=0$ has real roots or not, it is called discriminant of real roots of QE.

Types of roots of QE:

1. If $b^2-4ac > 0$, then 2 distinct real roots
2. If $b^2-4ac = 0$, two equal real roots
3. If $b^2-4ac < 0$, No real roots, two imaginary roots

Complex Number

- Problems faced in algebra to solve quadratic equations led to the development of Imaginary Numbers.

- When imaginary numbers combined with real numbers, complex number is formed.

5.1 Quadratic equations $ax^2+bx+c=0$ roots,

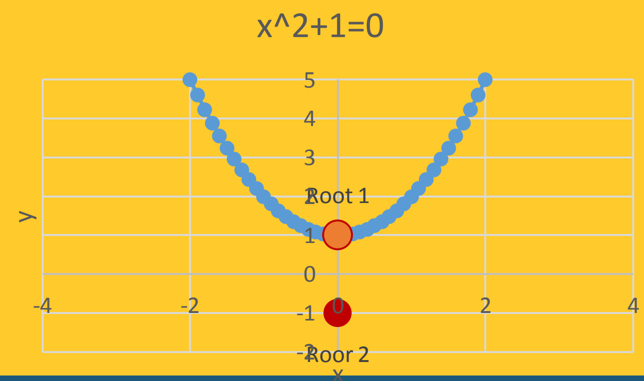
$x= -b+\sqrt{(b^2-4ac)}/2a$, $x=-b- \sqrt{(b^2-4ac)}/2a$

When **Discriminant**, $D=b^2-4ac$ is negative, then there will be no real roots.

Ex. $x^2+1=0$, $x=\pm \sqrt{-1}$

$\sqrt{-1}$ is denoted by i, then $i^2=-1$

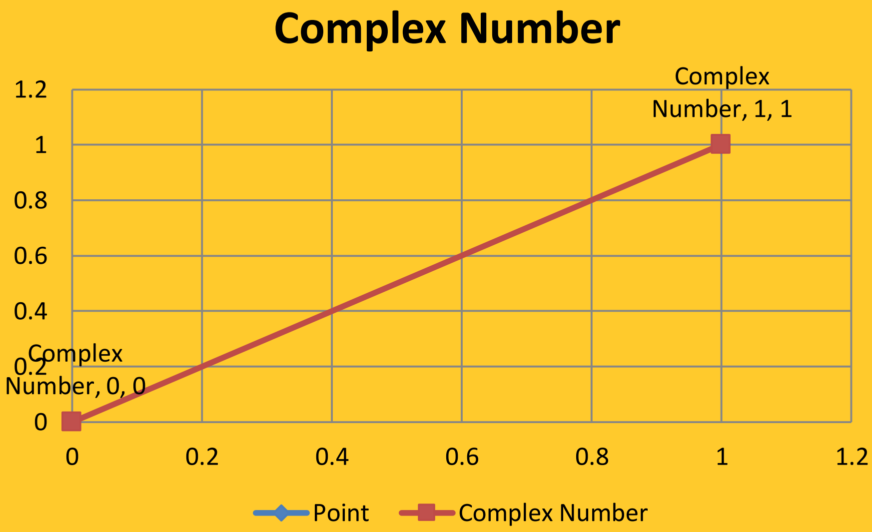
This means that $\pm i$ is a solution of equation $x^2+1=0$



5.2 Complex Numbers :

A real number added with an imaginary number is called a complex number. a=real number,

ib=imaginary number, Complex Number z=a+ib



Complex Number

5.3 Algebra of complex number

Addition of complex number

$Z1=a+ib$, $Z2=c+id$

$Z1+Z2=(a+c) +I (b+d)$

Properties of addition of complex number

(1) Closer Law (2) Commutative Law (3) Associative Law (4) Additive Identity (0+i0), $Z+0=Z$

(5) Additive Inverse (-a-2b), $Z+(-Z)=0$

5.3.2 Subtraction of two complex numbers-

$Z1-Z2=Z1+(-Z2)=Z1-Z2$

5.3.3 Multiplication of two complex numbers

$Z1=a+ib$, $Z2=c+id$, $Z1Z2=(ac-bd)+i(ad+bc)$

Complex Number

Properties – (1) Closer Law

(2) Commutative Law

(3) Associate Law

(4) Multiplicative Identity (1+i0)·

Zx (1+i0)=Z

(5) Multiplication inverse, $Z^{-1}=a/a^2+b^2-ib/\ a^2+b^2$, ∴ $Z \cdot Z^{-1}= Z \cdot 1/Z=1$

(6) Distributive Law:

z1(z2+z3)=z1z2+z1z3

5.3.4 Division of two complex number

Z1=a+ib, Z2=c+id,

$Z1/Z2=Z1.Z2^{-1}$

Power of i

When power is positive integer

$i=\sqrt{-1}$

$i^2=-1$

$i^3=-1\cdot\sqrt{-1}=-i$

$i^4=i^2\cdot i^2=-1\cdot-1=1$

$i^5=i^4\cdot i=1\cdot\sqrt{-1}=i$

$i^6=i^5\cdot i=i\cdot i=i^2=-1$

Power of i

When power is negative integer

$i=\sqrt{-1}$

$i^{-1}=1/i=1/i\cdot i/i=i/-1=-i$

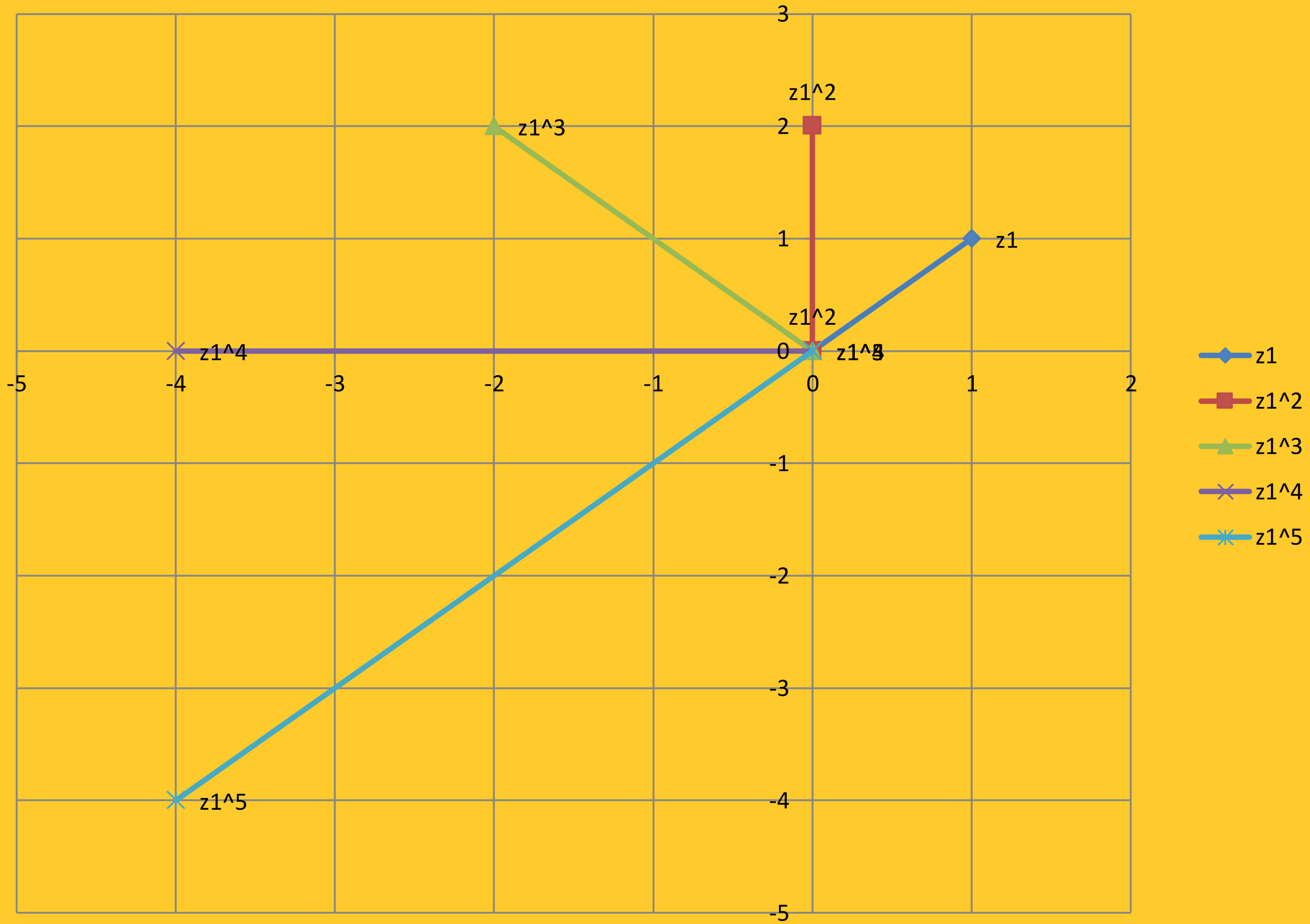
$i^{-2}=1/i^2=1/-1=-1$

$i^{-3}=-1/i^3=1/i^2.1/i=-i\cdot -i=i$

$i^{-4}=1/i^4=1/1=1$

if the power is k, then, $i^{4k}=1$, $i^{4k}+1=i$, $i^{4k}+2=-1$, $i^{4k}+3=-i$

+ve integer Power of z



-ve integer Power of z

Square roots of negative real numbers

The square roots of negative real numbers

We know that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

This rule contradicts when a and b both negative.

Suppose, a=-1, b=-1

Then, $\sqrt{a} \cdot \sqrt{b} = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1 \cdot -1} = \sqrt{1} = 1$,

but we know that $\sqrt{-1} \cdot \sqrt{-1} = i \cdot i = i^2 = -1$

Hence $\sqrt{a} \cdot \sqrt{b} \# \sqrt{ab}$ when a and b is negative number

5.4.a The modules of a complex number z=a+ib

Then modulus of complex number z, denoted by

$|z|=\sqrt{(a^2+b^2)}$

5.4.b Conjugate of a complex number, $\overline{A}$= a+ib

$\overline{z}$= a-ib

Few form by

z. $\overline{z}=z^2$

$|z1z2|=|z1|*|z2|$, $|z1/z2|=|z1|/|z2|$, $\overline{z1 * z2} = \overline{z1} * \overline{z2}$

$\overline{z1+z2} = \overline{z1} + \overline{z2}$, $\overline{z1/z2} = \overline{z1} / \overline{z2}$

- Argand Plane: Argand plane is formed by a horizontal real number line and a vertical imaginary number line with origin at (0,0)

All real part of the complex number is put in x-axis and all imaginary part is put in y-axis

The plane in which all points are assigned a complex number is called argand plane.

- Representation of a point in the Argand Plan:

The modulus of the complex number is the distance of the point and the origin.

Conjugate of complex number

Conjugate of complex number

z=a+ib, $\bar{z}$= a-ib

The conjugate of z=(x,y) is the mirror image of the point (x,y), Geometrically, $\bar{z}$=(x,-y)

- Polar representation of complex numbers:

Magnitude or modules of $z=\sqrt{x^2+y^2}$, t is called angle, argument, amplitude of $z=\tan^{-1} y/x$

If polar coordinate , x=r cos t , y=r sin t

z can be represented as, z=x+iy, z=r cos t +i r sin t

$$z=r\,(\cos t + i \sin t)$$

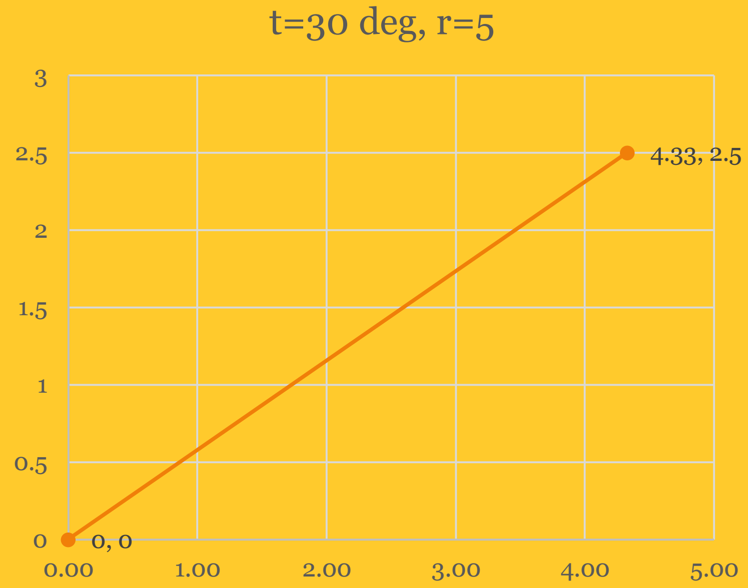
Conjugate of complex number

- Polar representation of complex numbers:

If polar coordinate , x=r cos t , y=r sin t

z can be represented as, z=x+iy, z=r cos t +i r sin t

$$z=r\ (\cos t + i \sin t)$$

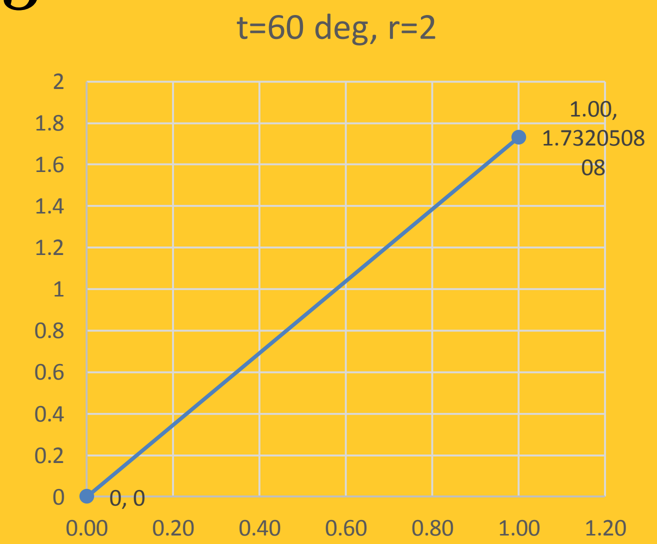


Principal Argument: θ=- π to π, - =excluded

Ex-7: Z=1+iroot 3 is the polar form we require r and θ for this $r=\sqrt{(1^2+\sqrt{3}^2)}=\sqrt{(1+3)}=\sqrt{4}=2$

$\theta=\cos^{-1}(1/2)$ or $t=60°=\pi/3$

$z=\cos(\pi/3)+i\sin(\pi/3)$



5.6 Solution of quadratic equations $ax^2+bx+c=0$

When $D= \sqrt{(b^2-4ac)}$ is less than zero.

Then,

$x=(-b \pm \sqrt{(b^2-4ac)})/2a, = (-b \pm \sqrt{(4ac-b^2)*-1})/2a,$

$x=(-b \pm \sqrt{(4ac -b^2)})\sqrt{-1}/2a,$

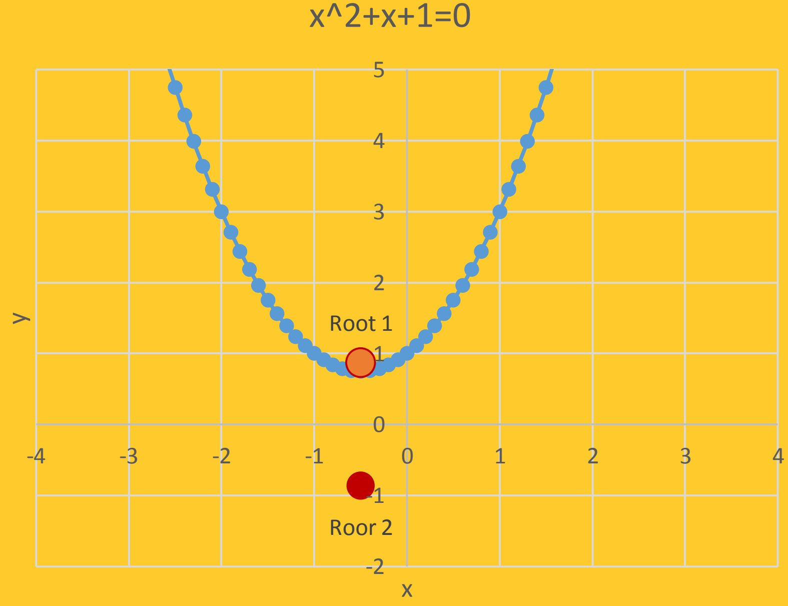
$x=(-b\pm \sqrt{(4ac -b^2)}\, i)/2a$

Ex-10: $x^2+x+1=0$,

Ans: $b^2-4ac=1^2-4\cdot1\cdot1=-3$

$x=(-1\pm\sqrt{-3})/2x1=(-1\pm\sqrt{-3x^*-1})/2=(-1\pm\sqrt{(-3x)^*i})/2$

-0.50	0.87
-0.50	-0.87



Linear inequalities

(Class-XI), Chapter-6

6.1/2: equations - $40x+20y=120$

Inequalities- $40x+20y<120$

$30x<200$

Definition : Any two number or expression are related by the symbol $<, >, \leq, \geq$ form an inequality.

Type of inequality :

(1) Numerical inequality, $5<7$

(2) Literal inequality , $5x<7$

(3) Double inequality, $3<5<9$

(4) Strict inequality, $ax+b<0$

(5) Slack inequality, $ax+by\geq 0$

(6) Linear inequality, $ax+by>5$

(7) Quadratic inequality, $x^2>0$

6.3 Linear inequalities of one variable

$30x<200$

Rule for solving L/E:

Equal numbers may be added to or subtracted from both sides of an inequality without effecting the sign of in equality.

$30x+100<200+100$

$30x-50<200-50$

Both sides of an inequality can be multiplied or divided by the same positive number without affecting the sign of inequality .

If both sides of an inequality be multiplied or divided by an negative number, then the sign of inequality reversed.

Inequalities

Ex.-1 : solve $30x<200$

$30x/30<200/30$

$x<20/3$

∴ $x=0,1,2,3,4,5,6$, when x is neutral number

$x=-\alpha\ldots\ldots,0,1,2,3,4,5,6$ when x is integer number $x<6$

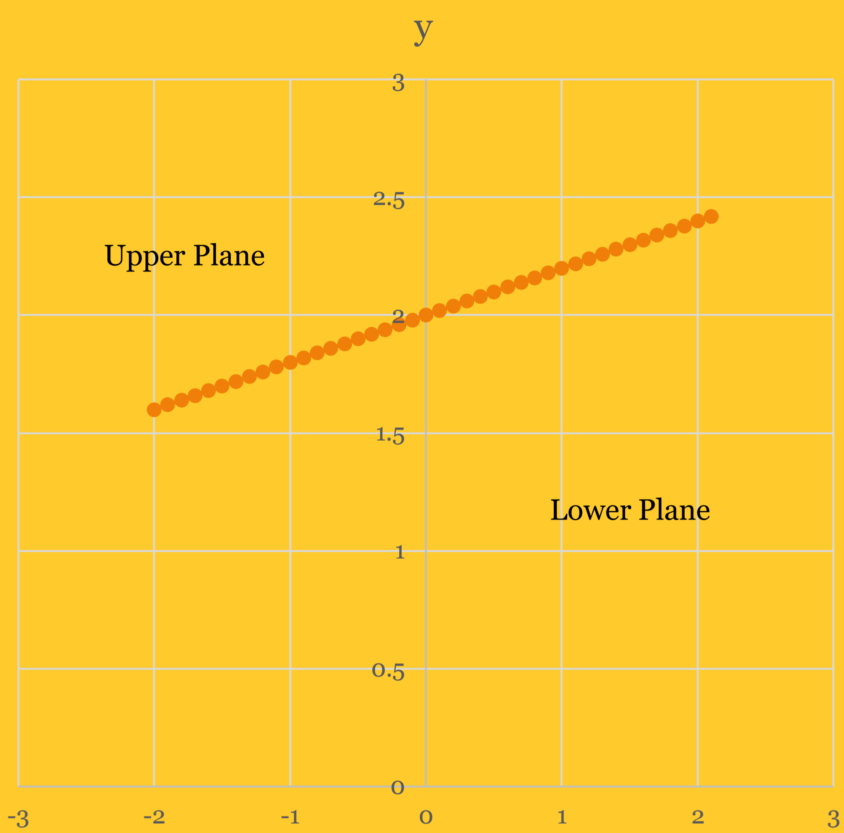
6.4 Graphical solution of Linear Inequalities

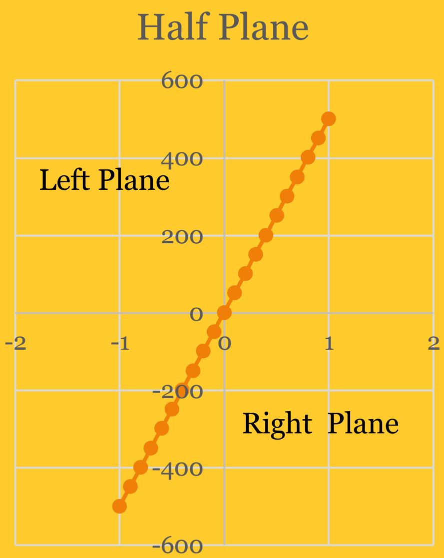
Ex : $ax+by>c$ or $ax+by<c$

Graphically, $ax+by=c$ represent a line

A line divides Cartesian plane into two parts as shown below, this parts are called lower or upper half plane or left or right half planes. There can be three cases in which a point in the plane can exists- (1) Lie on the line (2) Either left or upper half plane (3) Either right or lower half plane

Inequalities





Inequalities

Principle of Mathematical Induction
(Class-XI), Chapter-4

4.1 To establish a fact, we use different reasoning she is late – reasoning

(i) She started late

(ii) She missed the bus

(iii) There is heavy traffic

(iv) The vehicle punctured mid way

Reasoning can be : (i) Deductive

(ii) Inductive

(i) Deductive reasoning borrowed from logic-It is an arrangement expressed in three statements- Examples-1:

Statement (a) Socrates is a man

Statement (b) All men are mortal therefore

Statement (c) Socrates is mortal

Now if statement (a) and (b) are true then truth of statement (c) is established.

Example-2

Statement (a) Eight is divisible by two

Statement (b) Any number divisible by 2 is an even number, therefore

Statement (c) Eight is an even number

- By deductive reasoning, we establish a particular case from few general case.

(2) Inductive reasoning- in contrast to deductive reasoning, inductive reasoning depends on working in each case and developing a conjecture by observing incidents till each and every case is observed.

The word induction means generalization from particular from particular cases or facts.

4.3 The principal of mathematical induction

Let the given statement p(n) is such that

(i) The statement is true for n=1, i.e. P(1) is true,

(ii) IF the statement is true for n=k, then the statement is also true for n=k+1, truth of P(k) implies the truth of P(k+1), then P(n) is true for all material number n.

Trigonometry

- Trigonometry

Why we Study Trigonometry

- Problem in measurement of angles led to the subject trigonometry.

Definition: Trigonometry

- NCERT:

The word trigonometry is derived from the Greek words 'tri' (meaning three), 'gon' (meaning sides) and metron (meaning measurements)

- For me:

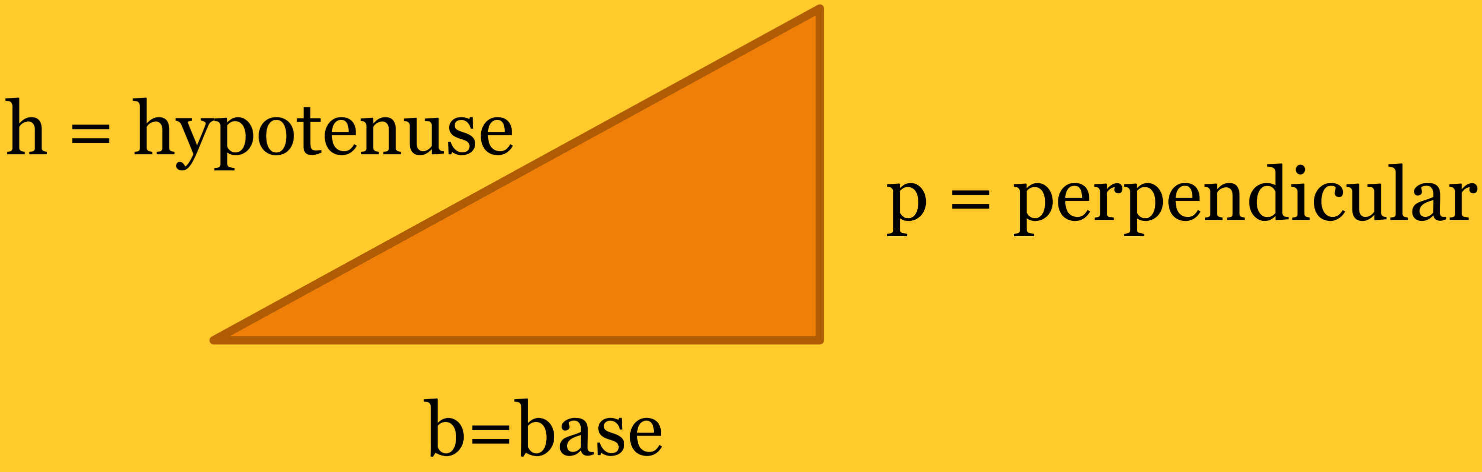
Trigonometry is the study of measurement of three angles of a right angled triangle.

- त्रिकोणमिति

Trigonometry

E2,3

The study of angle and ratio of sides of a right angled triangle

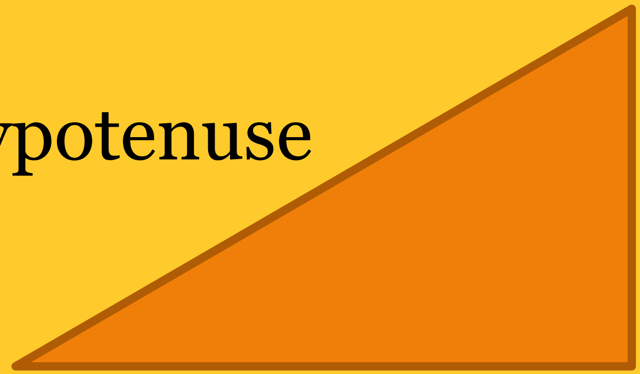


Trigonometry

E2,3

The study of angle and ratio of sides of a right angled triangle

h = hypotenuse

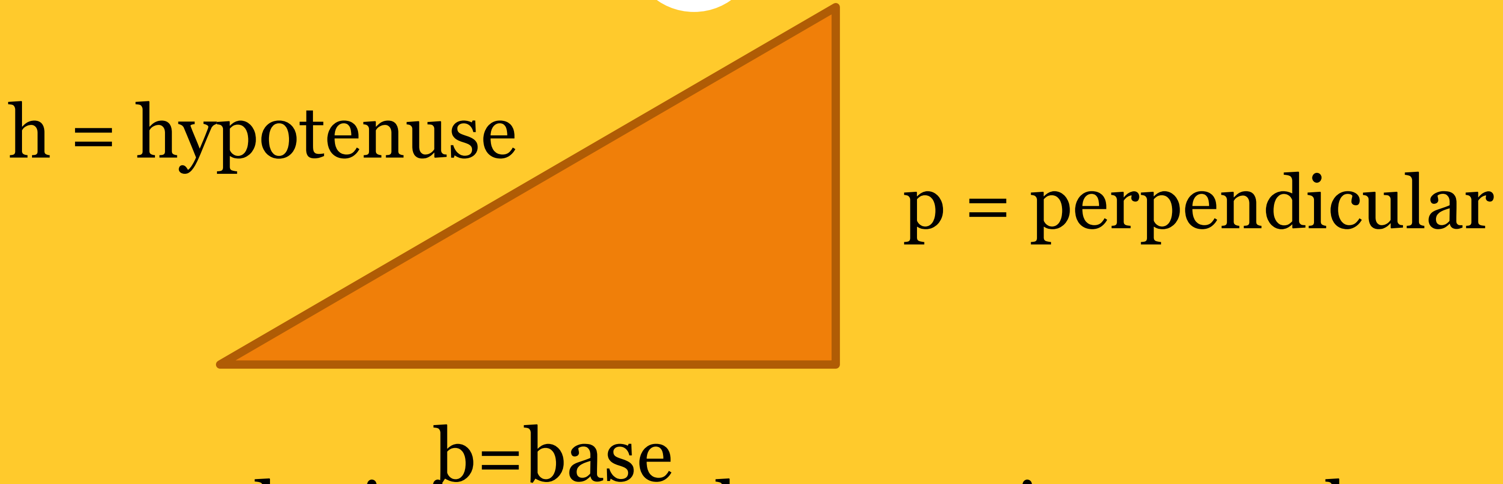


p = perpendicular

b=base

Define Ratios: the quantitative relation between two amounts showing the number of times one value contains or is contained within the other. Maximum and minimum values a ratios can take are –inf to +inf

Trigonometry



Maximum and minimum values a ratios can take are –inf to +inf

So when trigonometric ratios takes –inf to +inf, angles vary from –inf to +inf

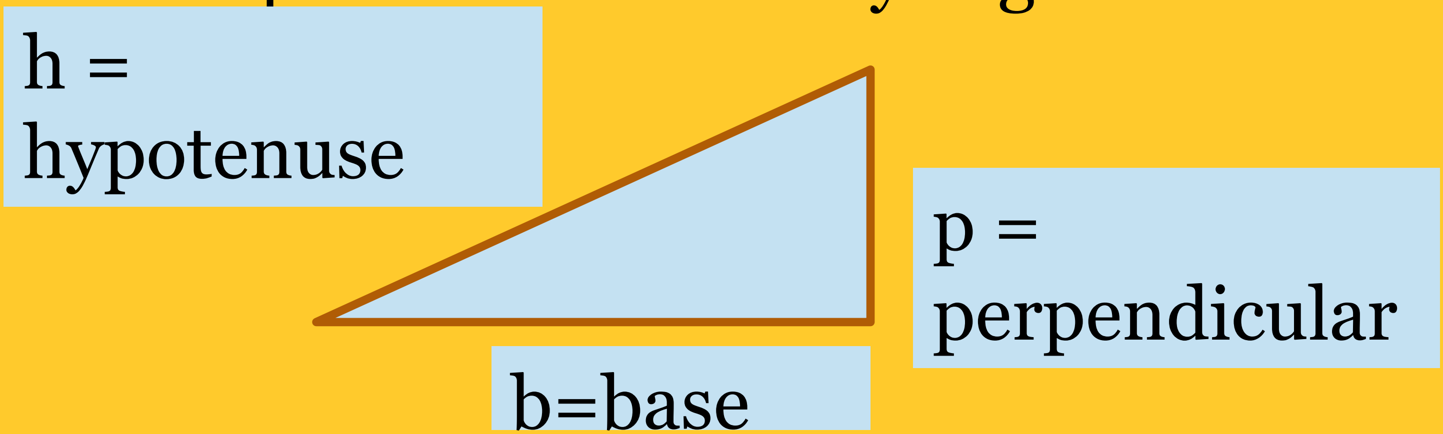
Question: Angle is a physical object, then how a physical object can be negative?

Trigonometry

- Definition of Trigonometry:

Trigonometry is the study of relationship between the ratios of the sides of a right angled triangle to its angles.

- These relationships are establishes by trigonometric ratios.



sin(x)=p/h, cos(x)=b/h, tan(x)=p/b

Trigonometric Ratios

- Trigonometric ratios of some specific angles (0, 30, 45, 60, 90)

Case-1: Trigonometric Ratios of 0 degree:

b=h=1, p=0,

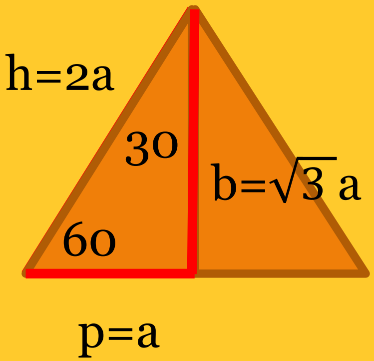
For 0 degree:
h=b =1, p=0

sin(x)=p/h, cos(x)=b/h, tan(x)=p/b

sin(0)=0/1=0, cos(0)=1/1=1, tan(0)=0/1=0

Trigonometric Ratios

- Trigonometric Ratios of 30 degree: Let the side of a equilateral triangle be 2a. Then drawing a line from a vertex to the base divides the triangle two right angled triangle. Then the base of one triangle is a and perpendicular is $\sqrt{(2a)^2 - a^\wedge 2}$, $\sqrt{3}$ a

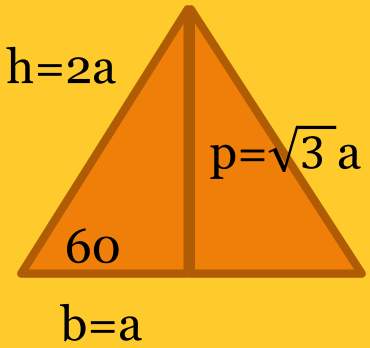


For 30 degree:
h=2a, b =$\sqrt{3}$a, p=a

sin(30)=p/h=a/2a=1/2,
cos(30)=b/h=$\sqrt{3}$ a/2a=$\sqrt{3}$ /2,
tan(30)=p/b=a/$\sqrt{3}$ a=1/$\sqrt{3}$

Trigonometric Ratios

- Trigonometric Ratios of 60 degree: Let the side of a equilateral triangle be 2a. Then drawing a line from a vertex to the base divides the triangle two right angled triangle. Then the base of one triangle is a and perpendicular is $\sqrt{(2a)^2 - a\hat{}2}$, $\sqrt{3}$ a

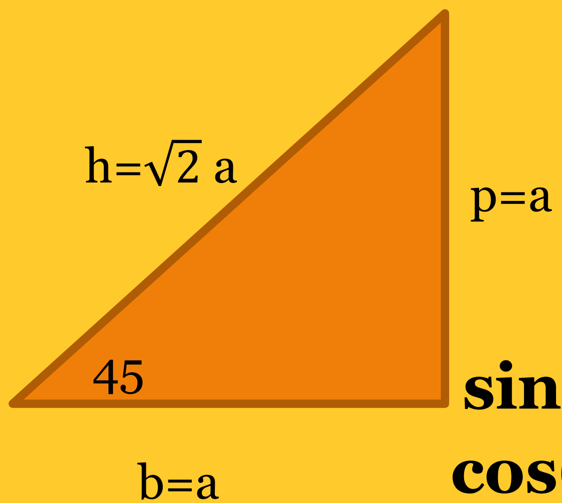


For 60 degree:
h=2 a, p =$\sqrt{3}$ a, b=a

sin(60)=p/h=$\sqrt{3}$ a/2a=$\sqrt{3}$ /2,
cos(60)=b/h= a/2a=1/2,
tan(60)=p/b=$\sqrt{3}$ a/a=$\sqrt{3}$

Trigonometric Ratios

- Trigonometric Ratios of 45 degree: Consider right angle triangle whose other angles are 45 degree and base=a and perpendicular=a, then hypotenuse= $\sqrt{a^2 + a^2} = \sqrt{2}$ a



For 45 degree:
h=$\sqrt{2}$ a, p = a, b=a

sin(45)=p/h=a/$\sqrt{2}$ a=1/$\sqrt{2}$,
cos(45)=b/h= a/$\sqrt{2}$ a=1/$\sqrt{2}$,
tan(45)=p/b=a/a=1

Trigonometric Ratios

- Trigonometric ratios of some specific angles (0, 30, 45, 60, 90)

For 90 degree:
h=1, p = 1, b=0

- Trigonometric Ratios of 90 degree:
- p=h=1, b=0,

sin(x)=p/h, cos(x)=b/h, tan(x)=p/b
sin(90)=1/1=1, cos(90)=0/1=0,
tan(90)=1/0=undefined

Trigonometric Functions

- $y=\sin(x)$
- $y=\cos(x)$
- $y=\tan(x)$

- Why study trigonometric functions?

Trigonometric Functions

Why we required to study trigonometric functions?

- For 0, 30, 45, 60 and 90 degrees, we have calculated trigonometric ratios but in doing so we have used different methods of calculation.
- For efficient calculations, we should develop a method through which we can calculate trigonometric ratios of any angle. This is achieved by trigonometric functions.
- To understand the concepts behind trigonometric functions, we are required to get clarity about few related topics and their definitions.

What is angle

- We are going to calculate the trigonometric ratios of different angles, so let us define angle first.
- Geometry: Definition of angle – Meeting of two line segment at a point forms an angle.
- Trigonometry: Definition of angle: Angle is a measure of rotation of a given ray about its initial point.
- The original ray is called initial side and final position of the ray after rotation is called the terminal side. The point of rotation is called vertex.
- Positive angle – When rotation is anticlockwise
- Negative angle – When rotation is clockwise

Measure of an Angle

- There are three approaches to measure angles:
    - Revolution
    - Degree
    - Radian

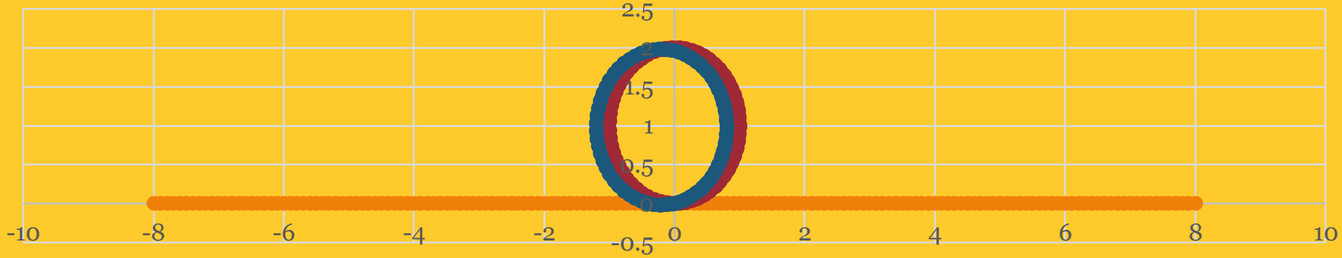
1. Revolution: Complete rotation of initial side. Used for measuring large angles.

2. Degree: It is considered that 1 revolution is 360 degree and 1 degree is equal to $1/360^{th}$ of a revolution

3. Radian: An angle subtended at the center by an arc of length equal to the radius of the circle.

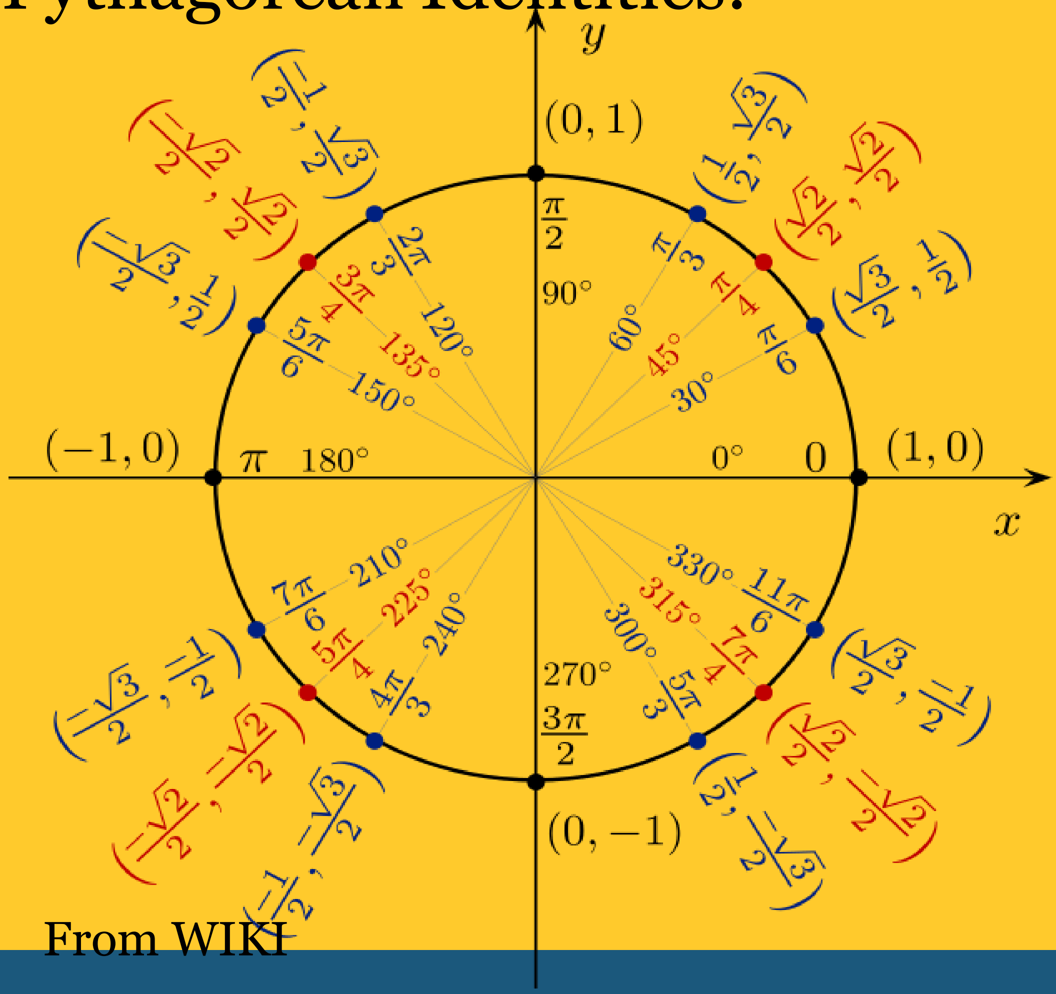
Relationship: Radian and Real Numbers

- If we rope a number line from zero in the anti clockwise direction along a circle, then every positive real number will corresponds a radian measure and conversely.
- If we rope a number line from zero in the clockwise direction along a circle, then every negative real number will corresponds a radian measure and conversely.
- Observation: From the above facts, it can be concluded that radian measures and real numbers can be considered as one and the same.



Trigonometry

Pythagorean Identities:

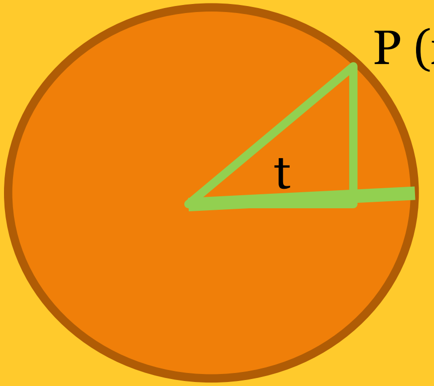


From WIKI

Degree	Radian	Approx
0	0	0.0
45.0	0.785398	0.8
90.0	1.570796	1.6
135.0	2.356194	2.4
180.0	3.141593	3.1
225.0	3.926991	3.9
270.0	4.712389	4.7
315.0	5.497787	5.5
360.0	6.283185	6.3

Trigonometric Functions

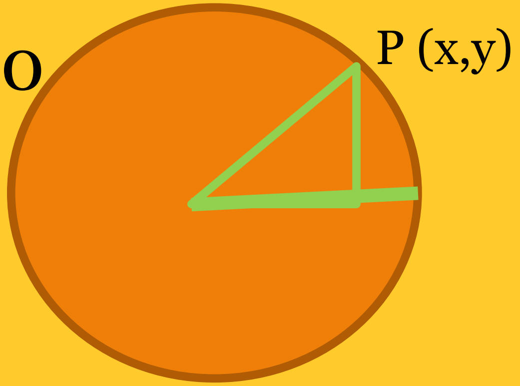
- Consider a unit circle.
- Angle t
- x=cos(t)
- y=sin(t)
- t = length of the arc subtended by the angle



sin(x)=p/h, cos(x)=b/h, tan(x)=p/b

Trigonometric Functions

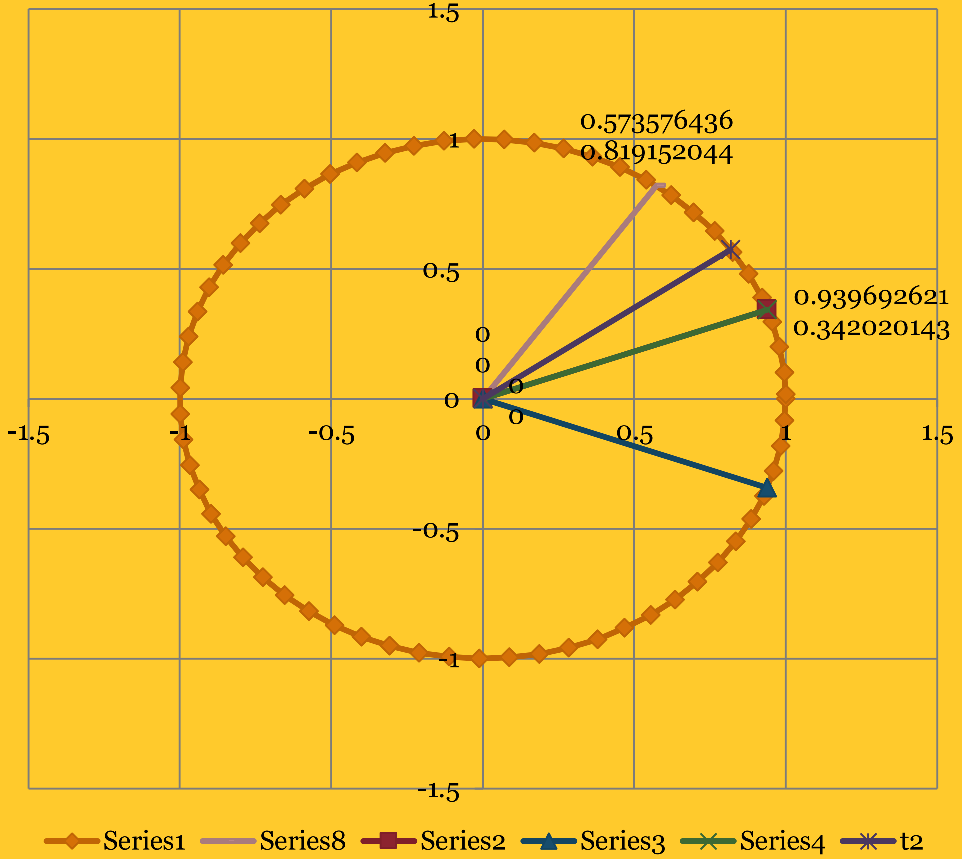
- The value of hypotenuse is always 1 and the values of base and perpendicular varies from -1 to 1.
- For sin (t) and cos(t), the perpendicular and base is divided by the hypotenuse and there is no problem because hypotenuse is always 1 and sin(t) and cos(t) can take the values from -1 to 1.
- For tan(t)=p/b and at 90 degree base=0 and tan(t) is undefined.



sin(x)=p/h, cos(x)=b/h, tan(x)=p/b

E2,3

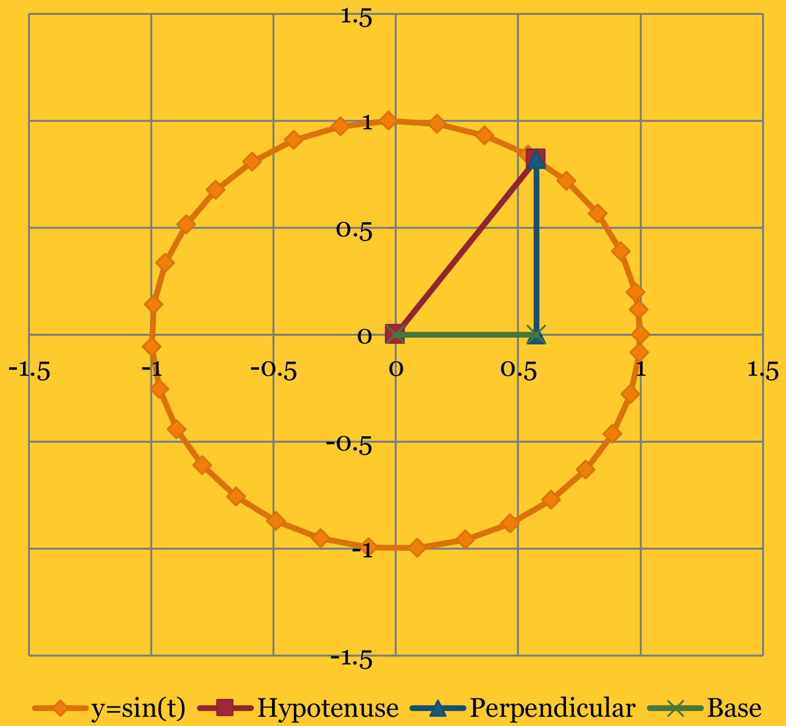
The study of angle and sides of a right angle triangle

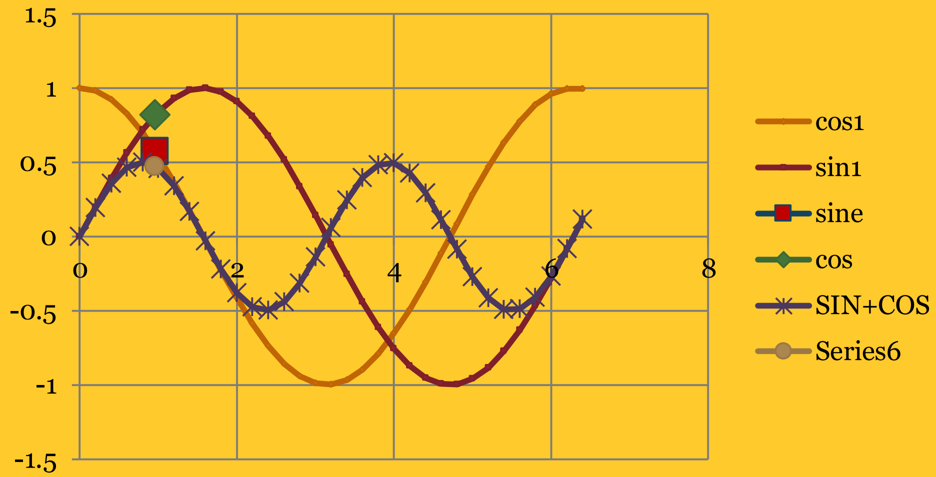


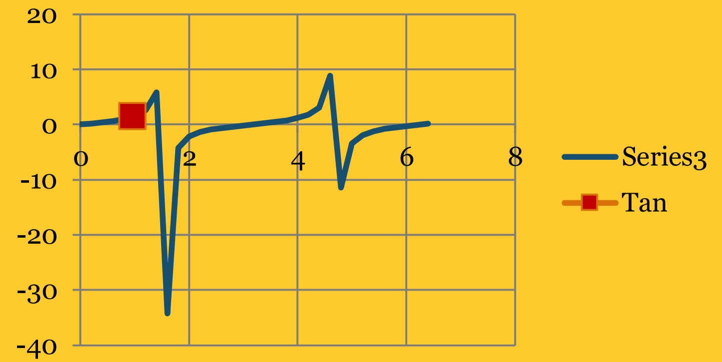
Model of Trigonometric Functions

E2,3

The study of angles and sides of a right angle triangle







Trigonometry

Pythagorean Identities:

- The basic relationship between the sine and the cosine is the Pythagorean trigonometric identity:

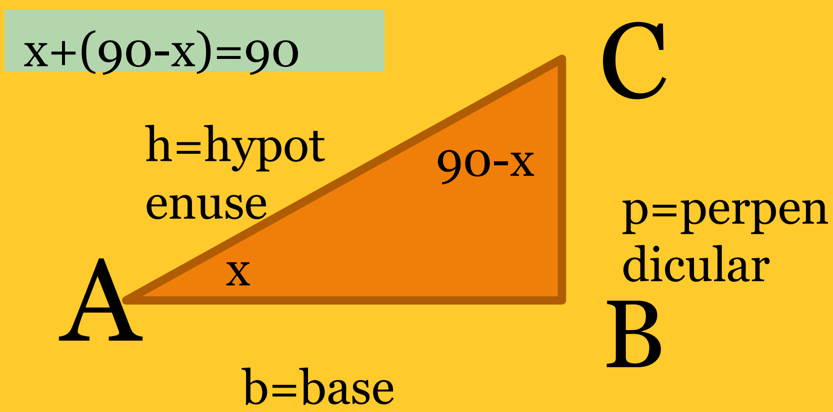
$$\cos^2(t)+\sin^2(t)=1$$

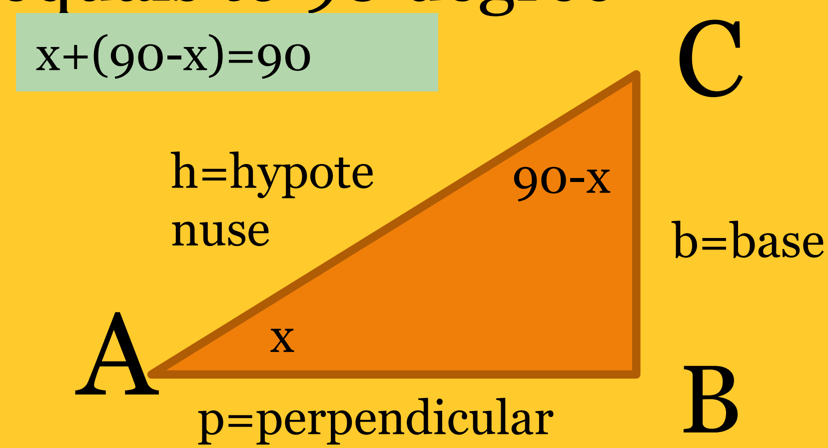
Observations:

- What is the origin of 360 Degree
- Forget Degree – Get Acquainted with Radians

Trigonometric Ratios of Complementary Angles

- Complementary Angles – Two angles are said to be complementary if their sum equals to 90 degree





sin(x)=p/h, cos(x)=b/h, tan(x)=p/b
sin(90-x)=b/h, cos(90-x)=p/h, tan(90-x)=b/p
sin(90-x)=b/h=cos(x), cos(90-x)=p/h=sin(x),
tan(90-x)=b/p=cot(t)

Trigonometric Identities

- What is identities?

Trigonometric Identities

- What is identity: An equation is called an identity if the equation is true for all values of variables involved.

- Trigonometric identities are the equations involving trigonometric ratios that are true for all values of the angles involved.

Trigonometry Identities
Reciprocal Identities

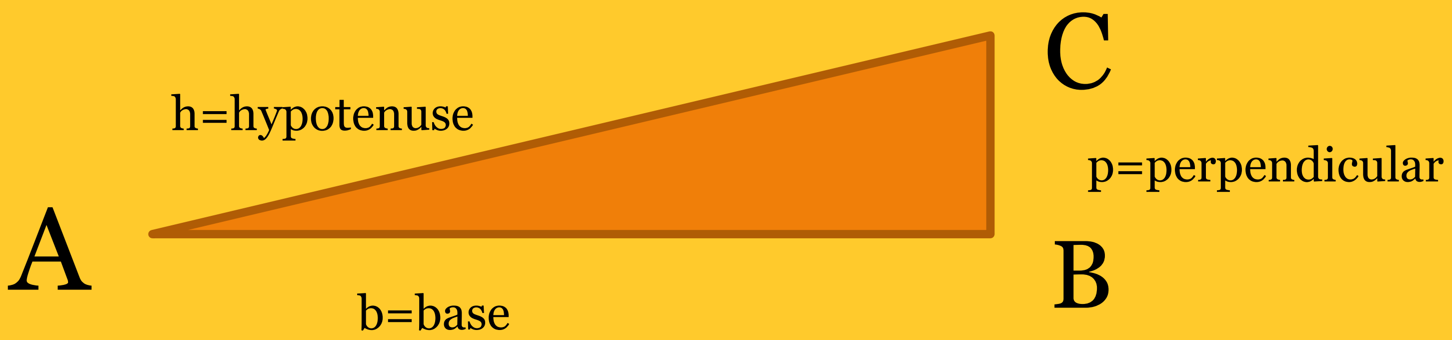
- $\text{cosec x} = \frac{1}{\sin x}$

Trigonometry Identities Quotient Identities

- $\tan x = \frac{\sin x}{\cos x}$

Trigonometry Identities
Pythagorean Identities

- From Pythagoras Theorem: Base^2+Perpendicular^2=Hypotenuse^2
- Dividing both side by Hypotenuse^2, we get,
- $$\frac{\text{Base}^2}{\text{Hypotenuse}^2} + \frac{\text{Perpendicular}^2}{\text{Hypotenuse}^2} = 1$$
- cos^2(x)+sin^2(x)=1



Even Odd Identities

- s(-x)=-s(x)
- c(-x)=c(x)
- t(-x)=-t(x)
- cs(-x)=-cs(x)
- sec(-x)=sec(x)
- cot(-x)=-cot(x)

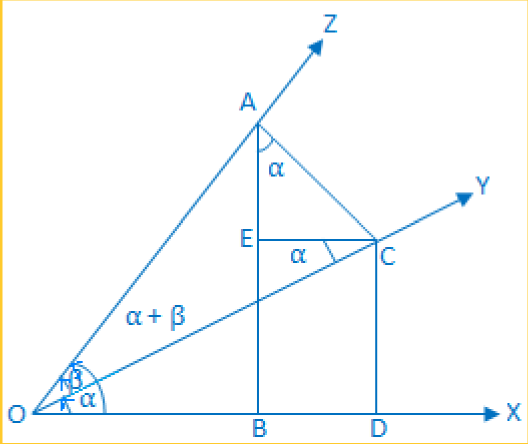
Trigonometric Functions of sum and difference of two angles

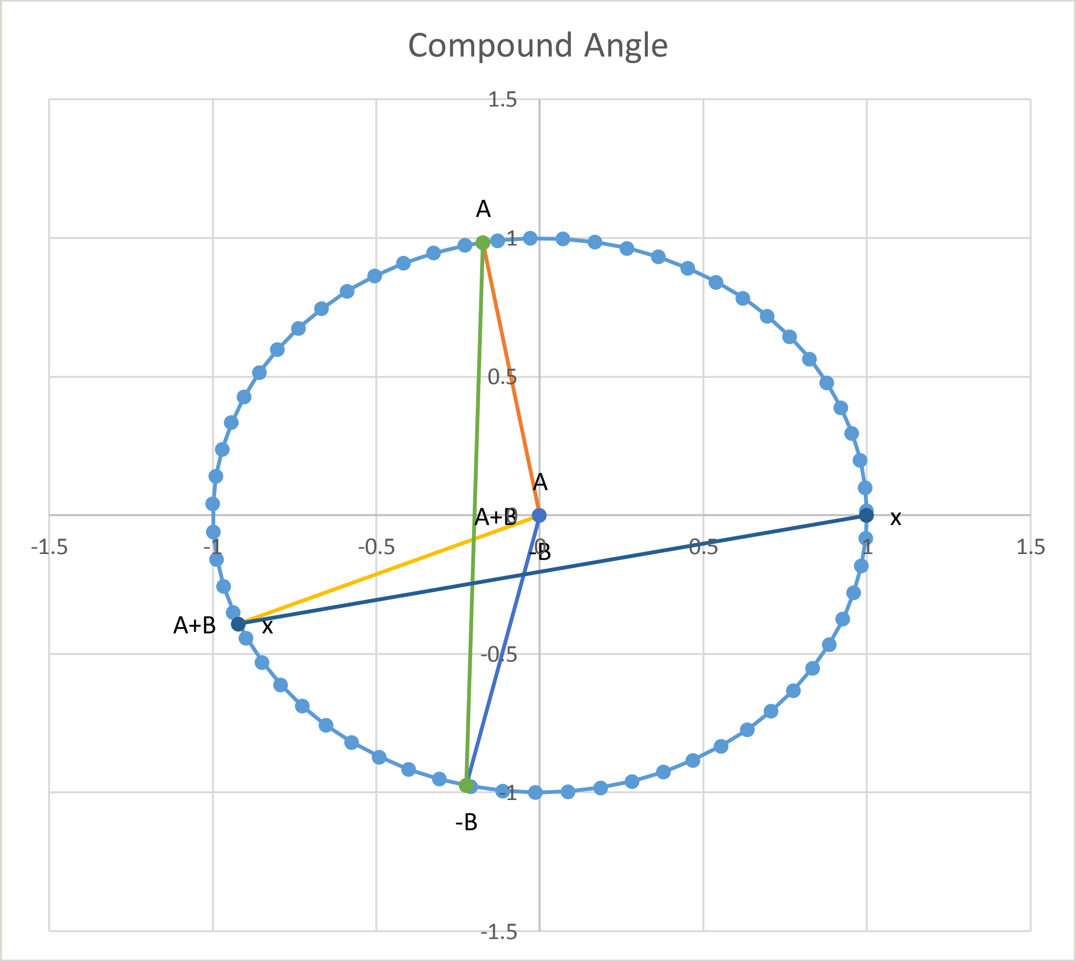
- $\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$
- $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
- $\cos(pi/2-y)=\cos(pi/2)\cos(y)+\sin(pi/2)\sin(y)=\sin(y)$
- $\cos(pi/2+y)=\cos(pi/2)\cos(y)+\sin(pi/2)\sin(y)=-\sin(y)$
- $\cos(pi-y)=\cos(pi)\cos(y)+\sin(pi)\sin(y)=-\cos(y)$
- $\cos(pi+y)=\cos(pi)\cos(y)+\sin(pi)\sin(y)=-\cos(y)$
- $\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$
- $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
- $\sin(pi/2-y)=\sin(pi/2)\cos(y)-\cos(pi/2)\sin(y)=\cos(y)$
- $\sin(pi/2-y)=\sin(pi/2)\cos(y)-\cos(pi/2)\sin(y)=\cos(y)$

Compound Angle identities

We equate lengths

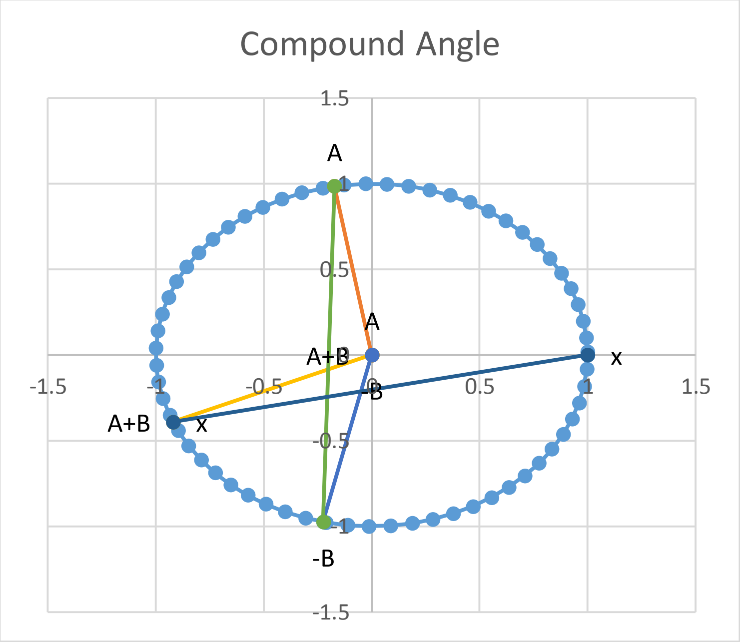
A-B,XA+B





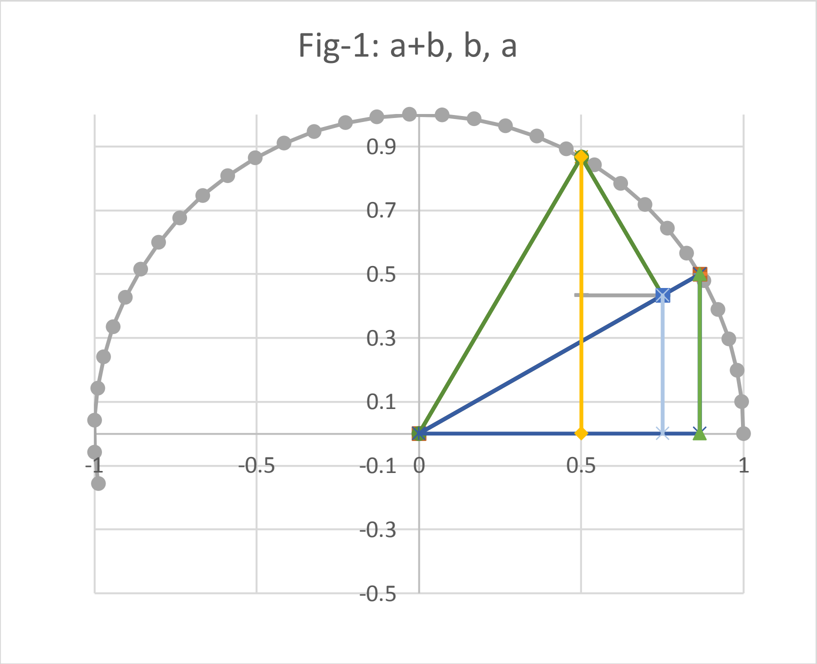
Compound Angle identities

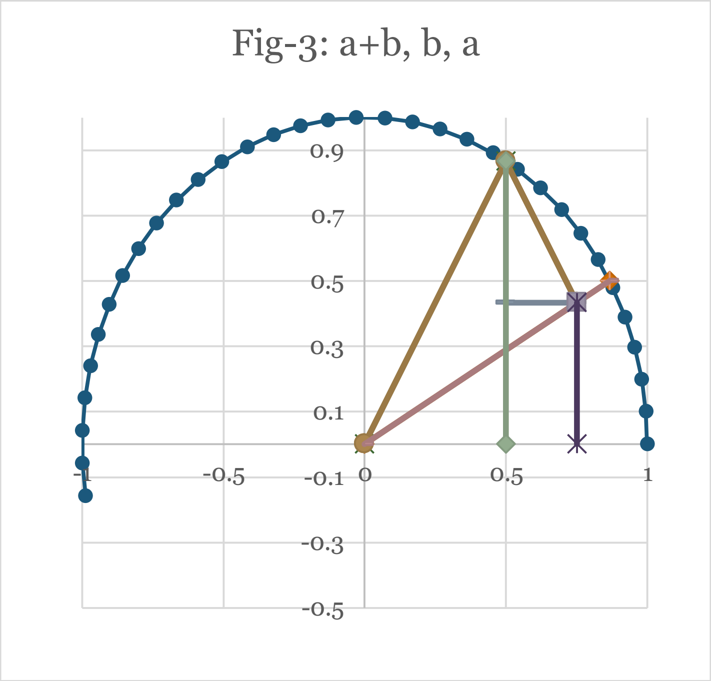
From the length calculation, we can calculate the identities but it does not reveal the true relationship of the identity elements. A much simpler process can be used for this.

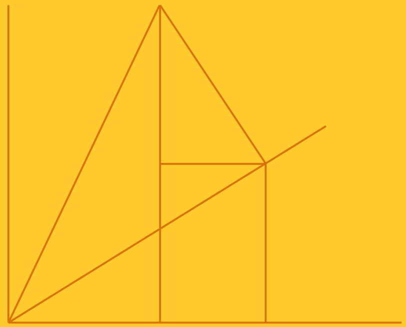


$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

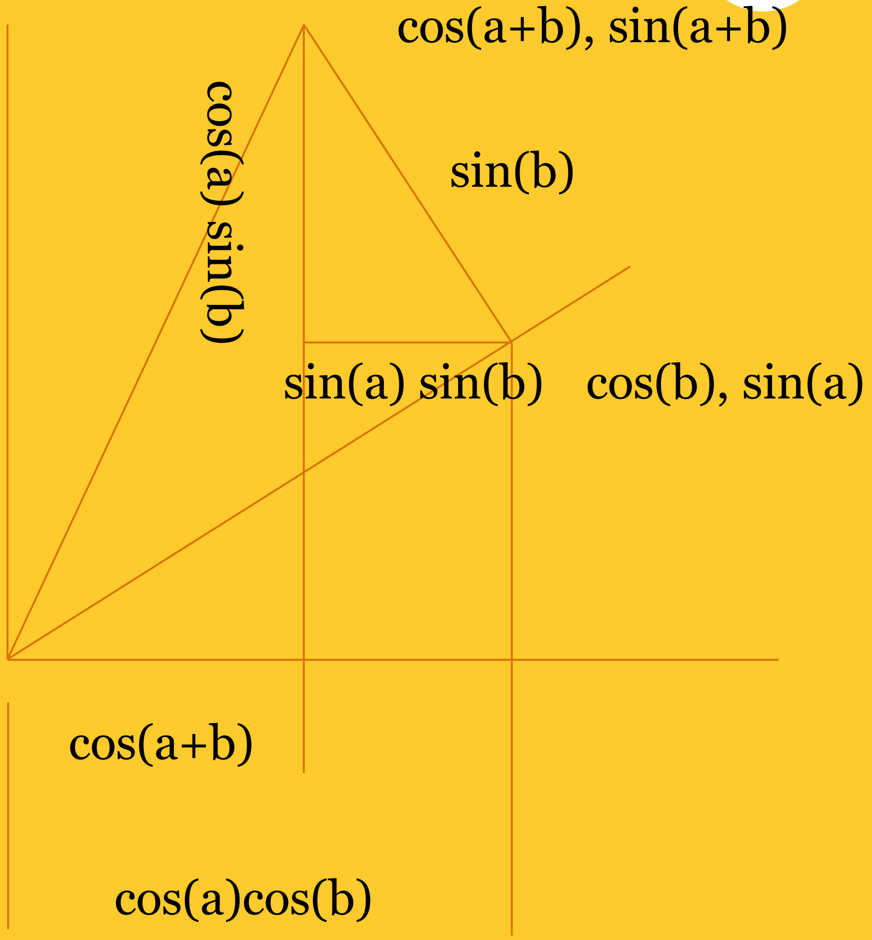
Compound Angle identities

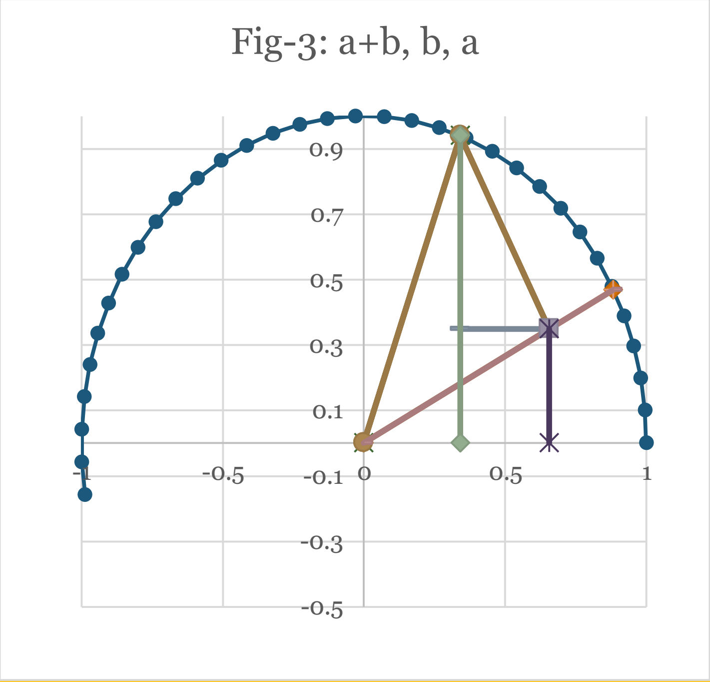




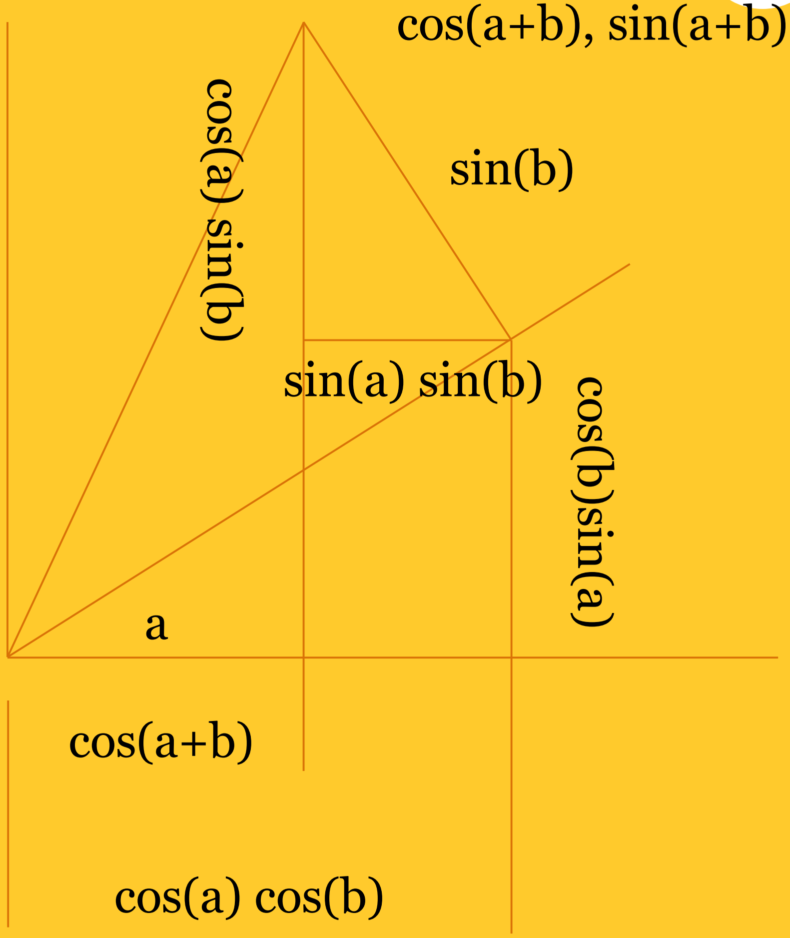


Compound Angle identities





Compound Angle identities

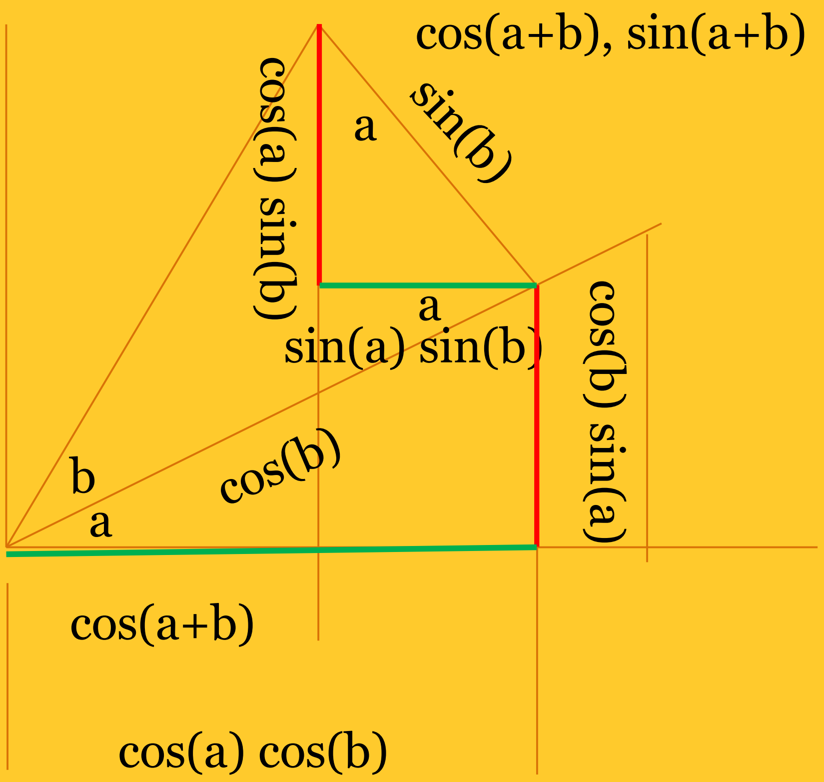


tan(a+b)=sin(a+b)/cos(a+b)
=sin(a)cos(b)+cos(a)sin(b)/
cos(a)cos(b)+sin(a)sin(b)

=(sin(a)cos(b)+cos(a)sin(b))/
cos(a)cos(b)/
(cos(a)cos(b)+sin(a)sin(b))/
cos(a)cos(y)

=tan(a)+tan(b))/(1+tan(a)tan
(b))

Compound Angle identities



cos(a+b)=cos(a)cos(b)-sin(a)sin(b)
When a=b,
cos(a+a)=cos(a)cos(a)-sin(a)sin(a)
cos(2a)=cos^2(a)-sin^2(a)

sin(a+b)=sin(a)cos(b)+cos(a)sin(b),

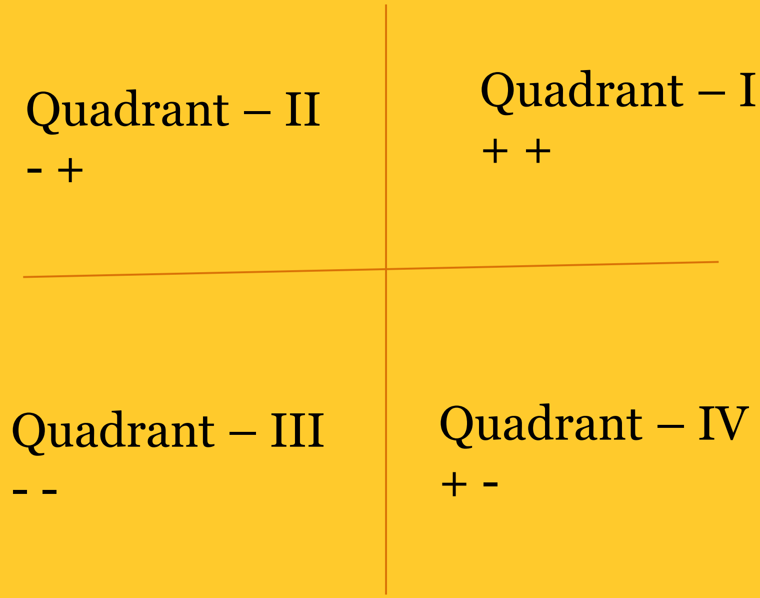
When a=b,
sin(a+a)=sin(a)cos(a)+cos(a)sin(a)
sin(2a)=2 sin(a)cos(a)

Remember the geometry, not the formula

- What is the requirement of coordinate geometry when geometry is known to us for centuries?

Coordinate Geometry

- Coordinate System



Coordinate Geometry

- Rene Descartes – La Geometry 1637
- Geometry and Algebra

Main advantages:

- Locating a point
- Plotting a point
- Distance between points
- Dividing a line segment (Section formula)
- Finding Area
- Finding Angle of inclination
- Defining a line
- Distance of a point from a line
- Conic sections
- All ideas in 2d can be extended to 3d

Coordinate Geometry

- Point (x, y)

- Plot(3,5)

Coordinate Geometry

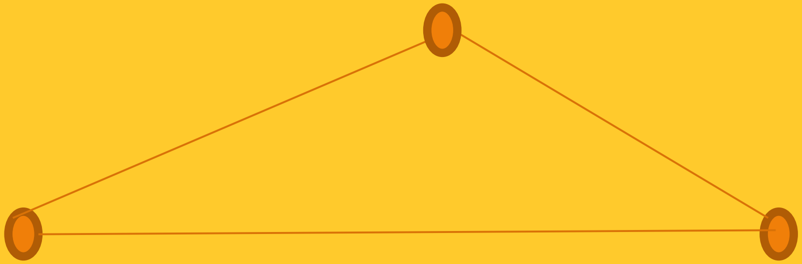
- Distance Between 2 Points:

Distance = sqrt((x2-x1)^2+(y2-y1)^2)

Origin of Distance Formula: Pythagorus theorem,
Base^2+perpendicular^2=hypoteneus^2
Basis: Coordinates provide distance

Coordinate Geometry

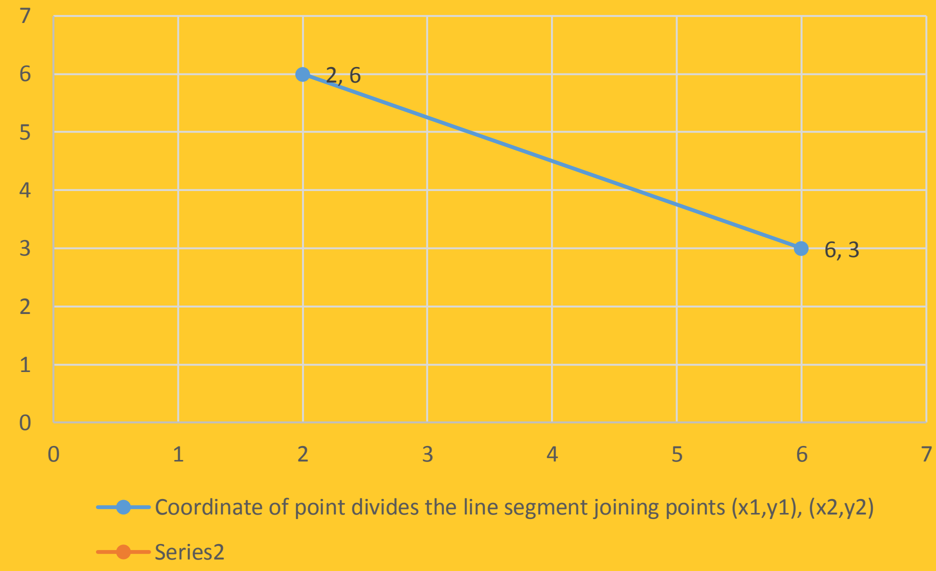
- Exercise: Plot the points and calculate the distances:
- P(3,2), Q(-2,-3), R(2,3)
- PQ=7.07
- QR=7.21
- PR=1.42



Distance between two points

Exercise:

a = (2, 6)
b=(6, 3)
ab =(4, -3)
DISTANCE=5



How can you establish that length is 5 unit?

Section Formula

- Coordinate of point divides the line segment joining points (x1,y1), (x2,y2) – Internally at a ratio of m:n

rx=(mx2+nx1)/(m+n)

ry=(my2+ny1)/(m+n)

Exercise:

Section Formula

- Coordinate of point divides the line segment joining points (x1,y1), (x2,y2) – Externally

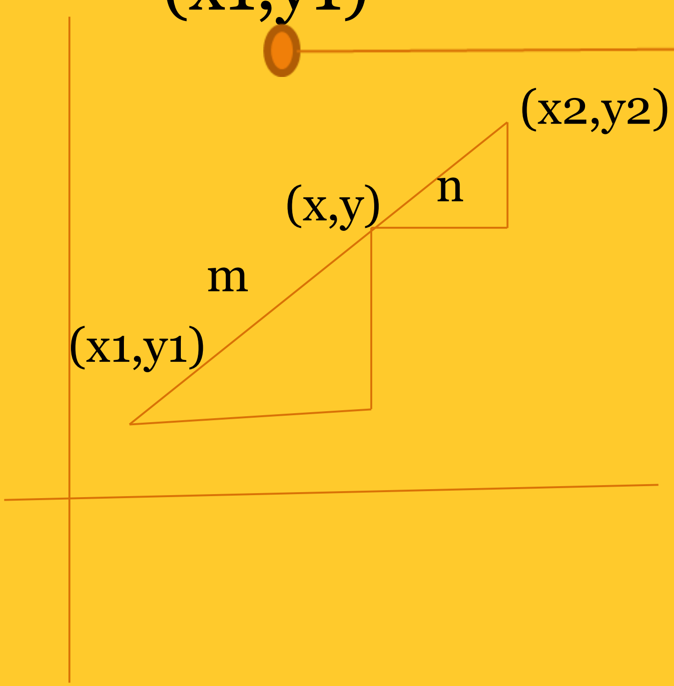
$rx=(mx2-nx1)/(m-n)$
$ry=(my2-ny1)/(m-n)$

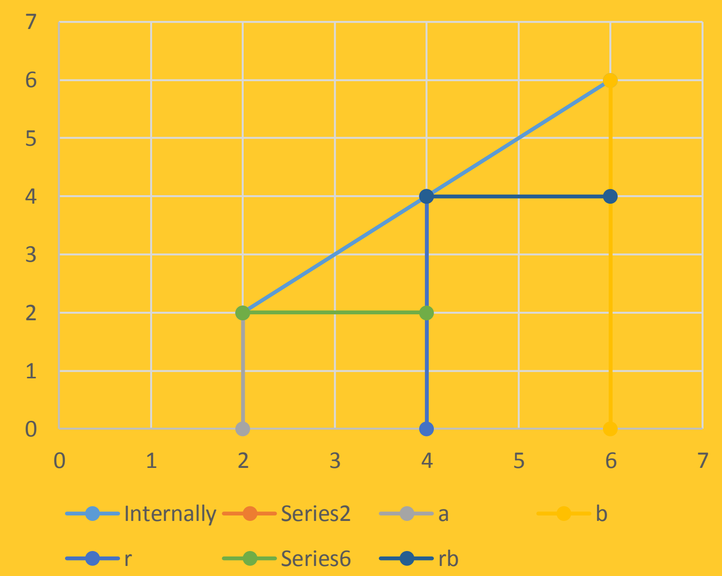
Basis of section formula:
From similarity theorem and distance formula,

Derivation of Section Formula

$rx=(mx2+nx1)/(m+n)$

$ry=(my2+ny1)/(m+n)$





Area from Coordinates(2d)

Area Bounded by three points:

11	9
6	16
7	5

Area=1/2*(x1*(y2-y3)+x2*(y3-y1)+x3*(y1-y2))

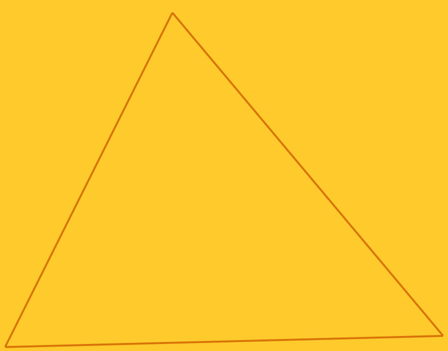
Basis of Area Formula:
Area of Trapezium=Sum of parallel sides times distance between them

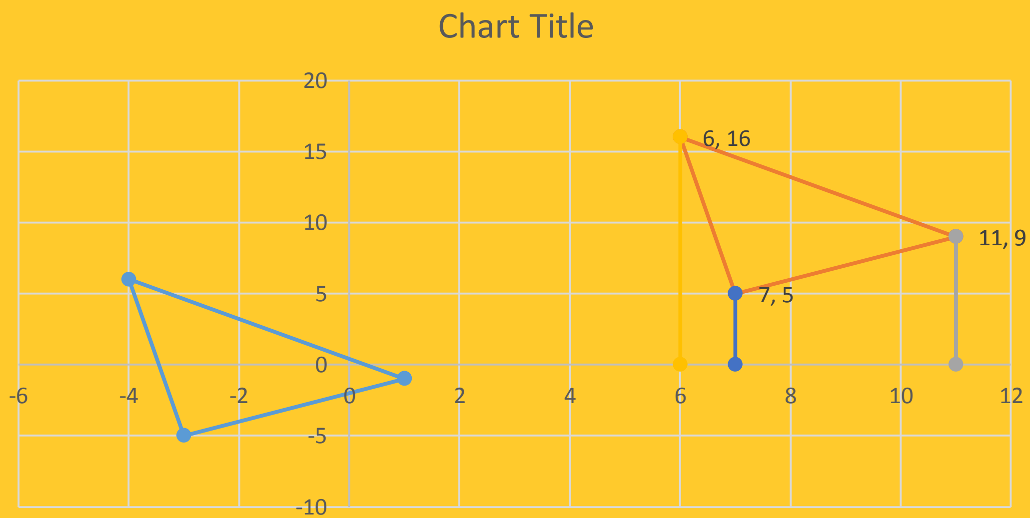
Area from Coordinates(2d)

Area Bounded by three points:

Ex- Points: (1,-1), (-4,6), (-3,-5)

Area=1/2*(x1*(y2-y3)+x2*(y3-y1)+x3*(y1-y2))



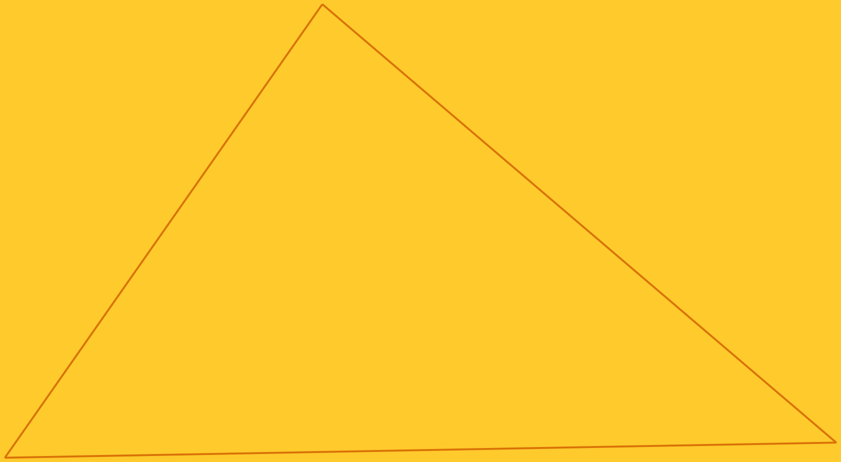


Origin of Area Formula:
Area of Trapezium=Sum of parallel sides times distance between them

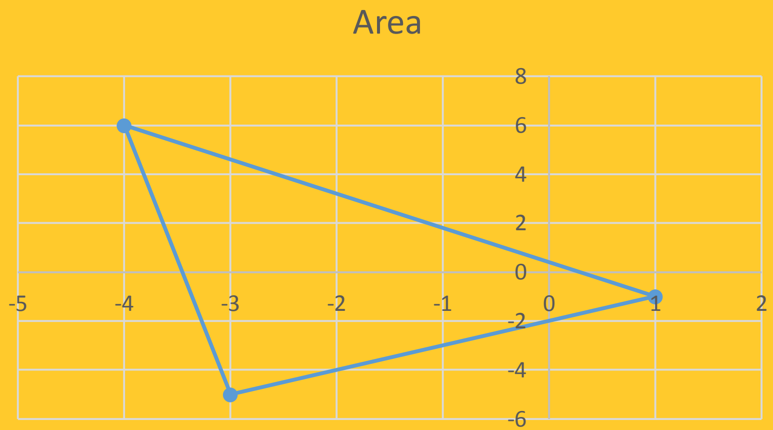
Area from Coordinates(3d)

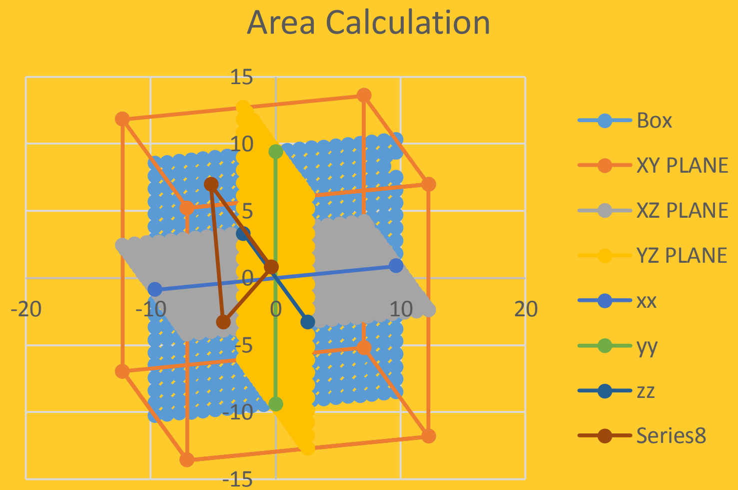
Area Bounded by three points in 3d:

area=1/2*(z1_*(x2_*y3_-x3_*y2_)-z2_*(x1_*y3_-x3_*y1_)+z3_*(x1_*y2_-y1_*x2_))



Area from Coordinates (3d)





Syllabus: Class-XI

Class-XI :

- **1. Straight Lines (Periods 09)**
- Slope of a line and angle between two lines.
- Various forms of equations of a line:
- parallel to axes,
- point-slope form,
- slope-intercept form,
- two-point form, intercepts form and normal form.
- General equation of a line.
- Equation of family of lines passing through the point of intersection of two lines.
- Distance of a point from a line.

Syllabus: Class-XI

Class-XI :

UNIT III : COORDINATE GEOMETRY

- **2. Conic Sections (Periods 12)**
- Sections of a cone:
- Circles,
- ellipse,
- parabola,
- hyperbola,
- a point,
- a straight line
- pair of intersecting lines as a degenerated case of a conic section.
- Standard equations and simple properties of parabola, ellipse and hyperbola.
- Standard equation of a circle.

Syllabus: Class-XI

Class-XI :

UNIT III : COORDINATE GEOMETRY

- **3. Introduction to Three-dimensional Geometry (Periods 08)**
- Coordinate axes and coordinate planes in three dimensions.
- Coordinates of a point.
- Distance between two points
- Section formula.

Inclination

Inclination of a line is measured by the angle it make in positive direction with x-axis.

Angle of inclination - The angle made by a line with positive x-axis is called the inclination of a line

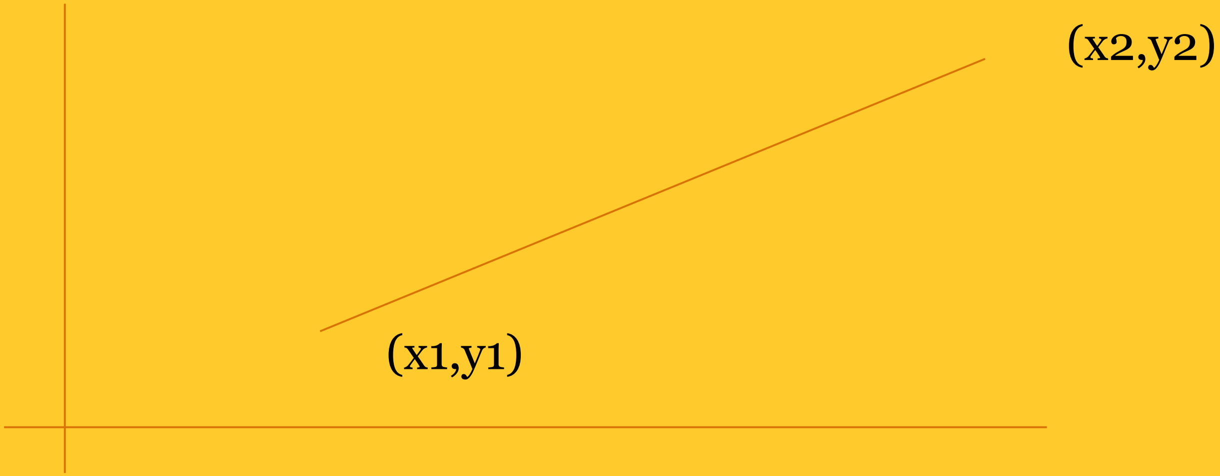
Slope

Slope of a Line:

If t is the angle of inclination of a line, then the slope of the line or gradient is tan (t)

Slope – When 2 points given

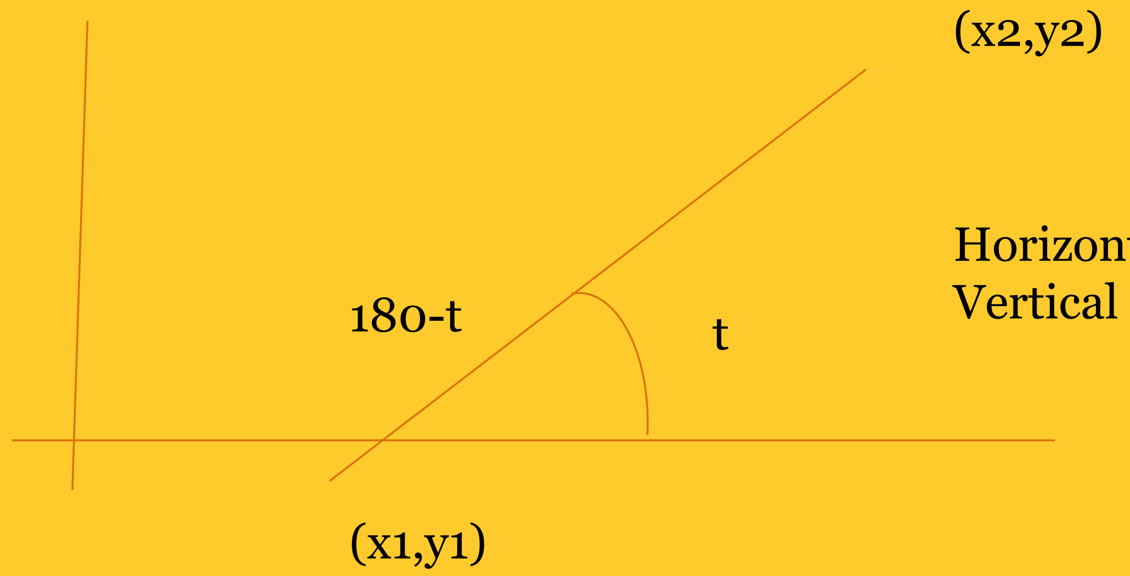
$$m=\tan(t)=(y2-y1)/(x2-x1)$$



Note: Coordinate Points helped in finding inclination of line

Slope

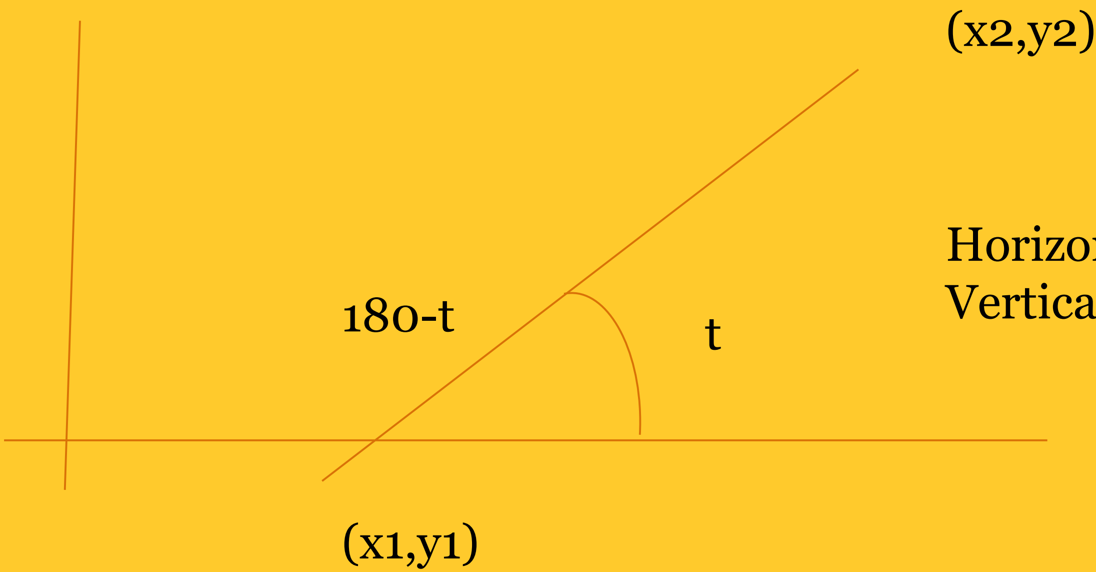
Slope=gradient=m=tan(t)=(y2-y1)/(x2-x1)
A line form two supplementary angle with x-axis.



Horizontal line – m=0
Vertical line – m=undefined

Slope – When inclination is more than 90 degree

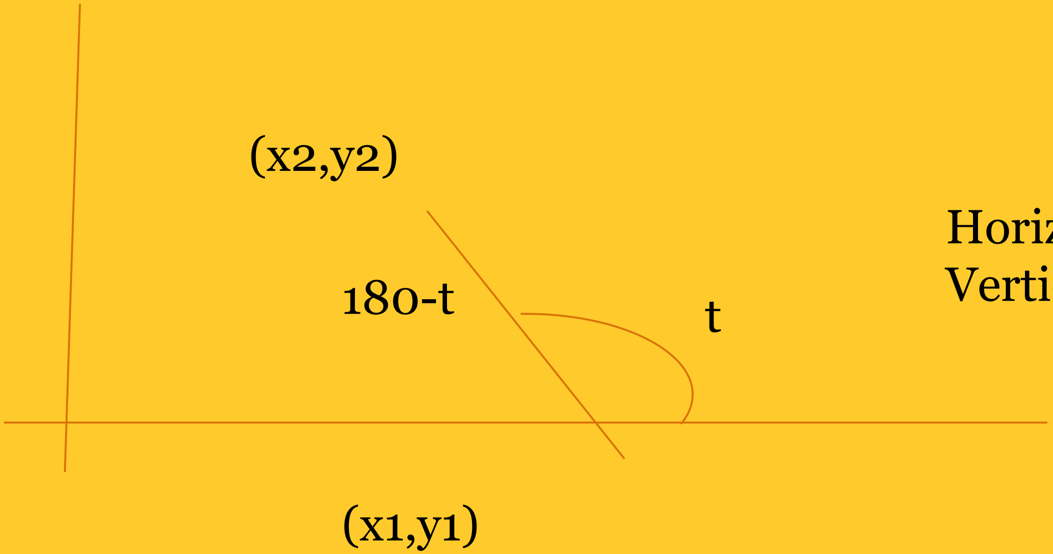
Angle< 90: Slope=gradient=m=tan(t)=(y2-y1)/(x2-x1)
A line form two supplementary angle with x-axis.



Horizontal line – m=0
Vertical line – m=undefined

Slope – When inclination is more than 90 degree

Angle >90: Slope=gradient=m=tan(t)=(y2-y1)/(x1-x2)



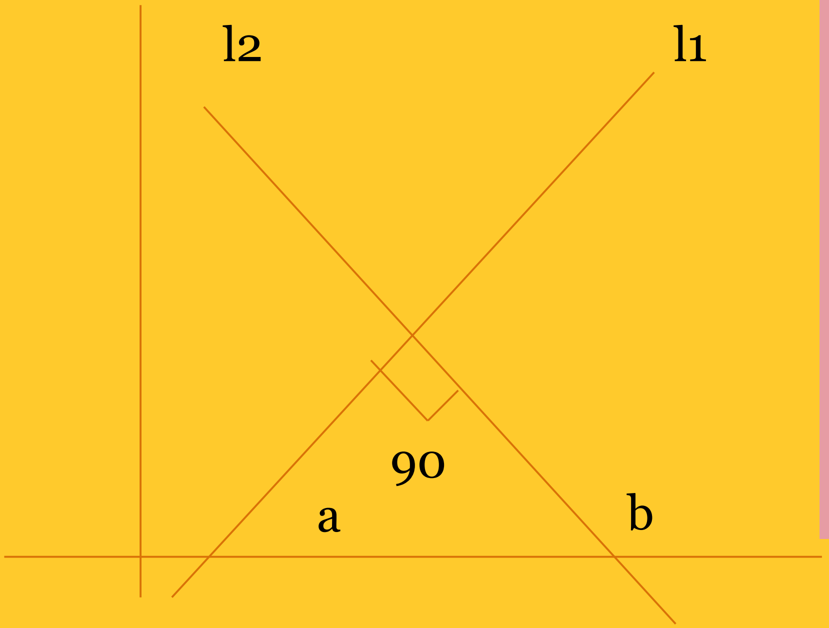
Horizontal line – m=0
Vertical line – m=undefined

Condition of Parallelism

$$M_1 = M_2$$

Condition of Perpendicularity

$$M_1=-1/M_2$$

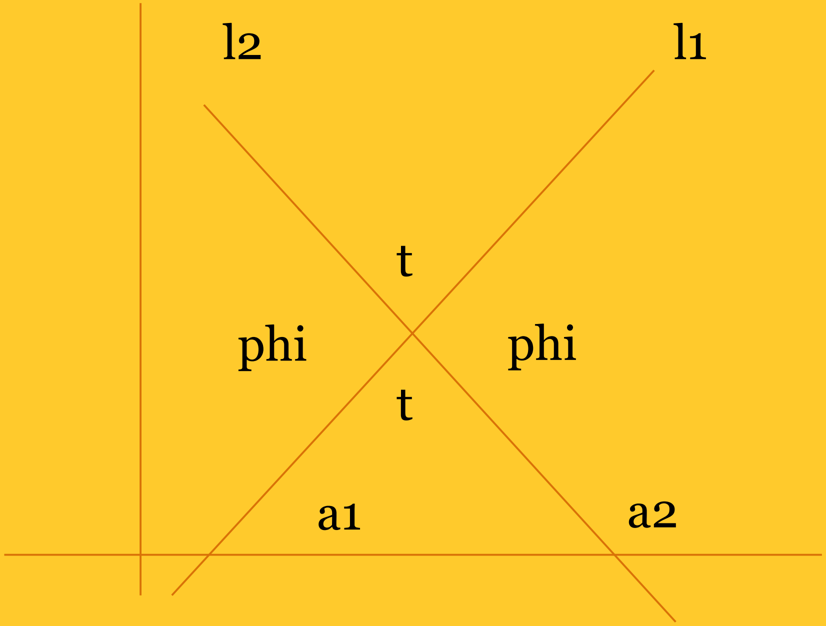


From Exterior angle theorem:
$b=a+90$
$\tan b = \tan (a+90)$
$\tan(b)=\tan(a+90)=-\cot(a)$
$=-1/\tan(a)$
$m_1=\tan(a)$
$m_2=\tan(b)$
$m_2=-1/m_1$

Angle t and between two intersecting lines

When two lines l1 and l2 intersect each other, they make two vertically opposite angles, t and phi

From Geometry, we know that
Exterior angle = sum of opp Angles.
Hence, t=(a2-a1)



Let a1 and a2 are the inclinations of the lines l1 and l2. Then t=(a2-a1)
tan(t)=tan(a2-a1)
From trigonometry, we get,

tan(t)=tan(a2-a1)= $\frac{\tan(a2)-\tan(a1)}{1+\tan(a2)\tan(a1)}$

tan(t)= $\frac{m2-m1}{1+m2\, m1}$

Again, tan(phi)=tan(180-t)=-tan(t)

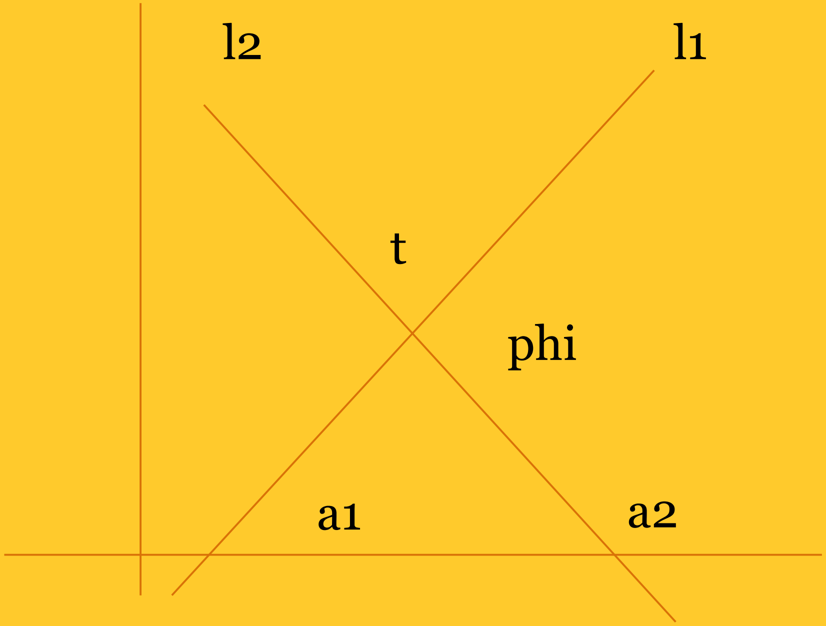
tan(phi)= - $\frac{m2-m1}{1+m2\, m1}$

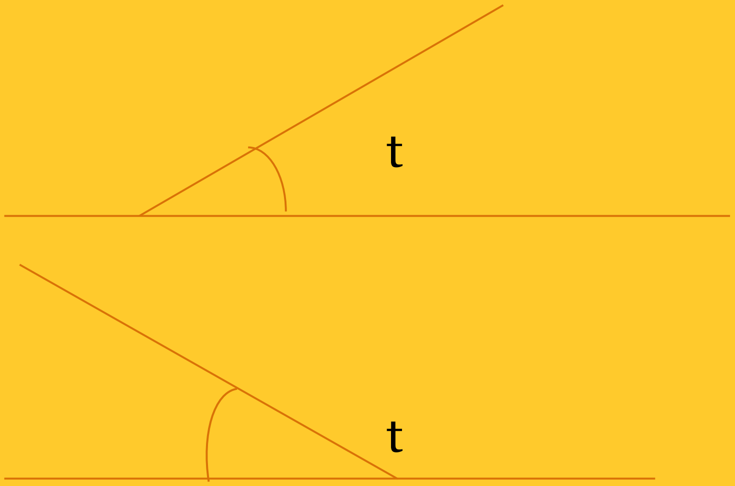
Angle t and between two intersecting lines

When two lines l1 and l2 intersect each other, they make two vertically opposite angles, t and phi

If (m2-m1)/(1+m1*m2) is positive, then tan(t) is positive and t is acute angle and phi is obtuse angle.

If (m2-m1)/(1+m1*m2) is negative then tan (t) is negative and t is obtuse and phi is acute

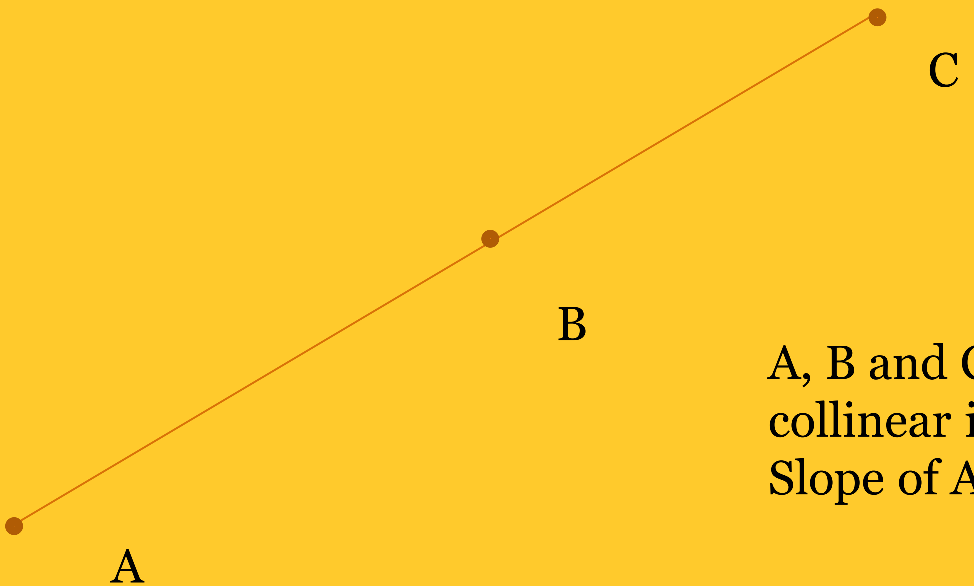




Collinearity of Three Points

Collinearity means points lie in same line.

Three points will be collinear if and only if the slope the points are same



A, B and C points are collinear iff,
Slope of AB = Slope of BC

Basis: Laws of Parallel lines
Area is zero then 3 points are collinear

Points are Coplanar in 2d

- In 2d, points are in same plane

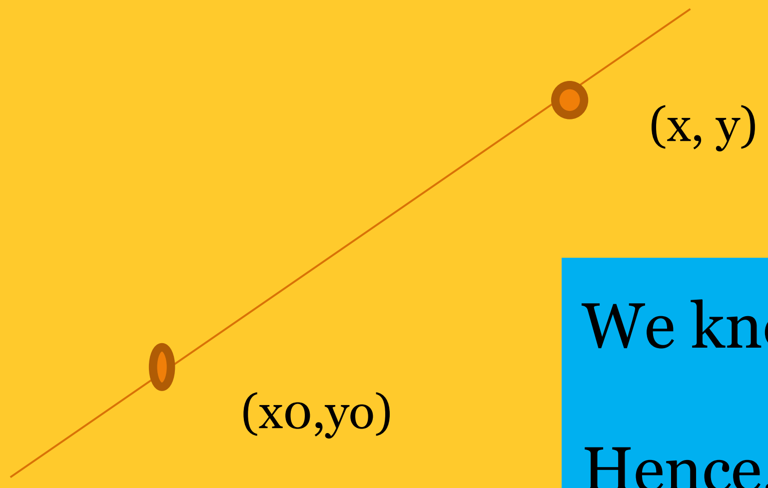
Various Forms of Equation of Lines

- We are required to establish a relation between x and y with the given parameters:
- Different forms of equation of Lines

1. Point Slope Form
2. Two Point Form
3. Slope Intercept form
4. Intercept Form
5. Normal Form
6. Vertical Line
7. Horizontal Line

1. Point Slope Form

- When a point $(x0,y0)$ and slope (m) of the line given, then we can form a equation of that line as,



We know, $m=\frac{(y-y0)}{(x-x0)}$

Hence, the equation of line is:

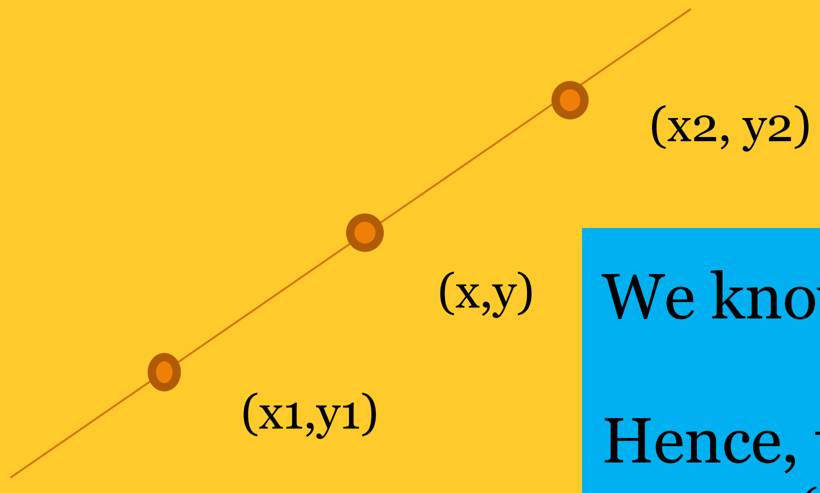
$y-y0=m(x-x0)$

$y=y0+m(x-x0)$

Ex-Draw a line – Given point (3,5) and m=5

2. Two Point Form

- When two points (x1,y1) and (x2,y2) of the line given, then we can form a equation of that line as,



We know, m= $\frac{(y2-y1)}{(x2-x1)} = \frac{(y-y1)}{(x-x1)}$

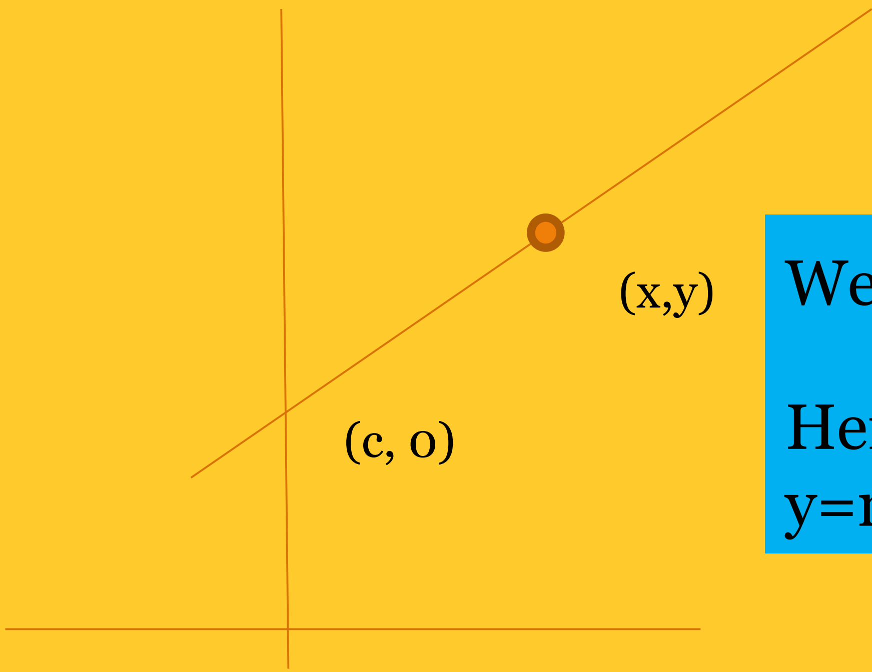
Hence, the equation of line is:

y=y1+($\frac{(y2-y1)}{(x2-x1)}$ (x-x1)

Ex-Draw a line – Given point1 (3,5) and Point2 (9, 5)

3. Slope Intercept forms

- When slope (m) and intercept © of the line given, then we can form a equation of that line as,



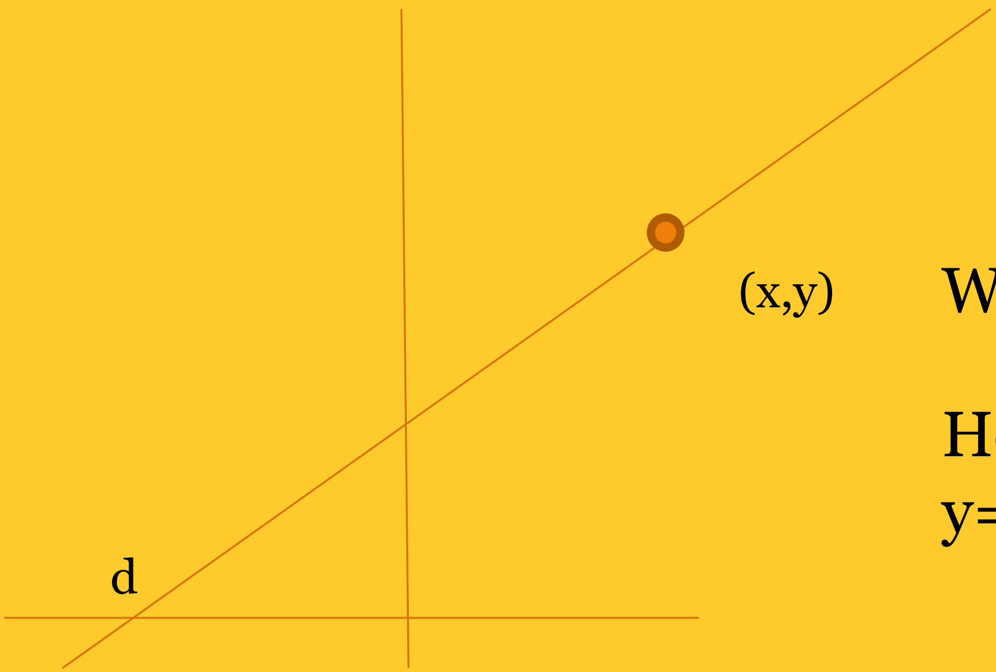
We know, m=$\frac{(y-c)}{(x-0)}$

Hence, the equation of line is:
y=mx+c

Ex-Draw a line – Given Intercept 5 and m=5

3a. Slope x-Intercept forms

- When slope (m) and intercept © of the line given, then we can form a equation of that line as,



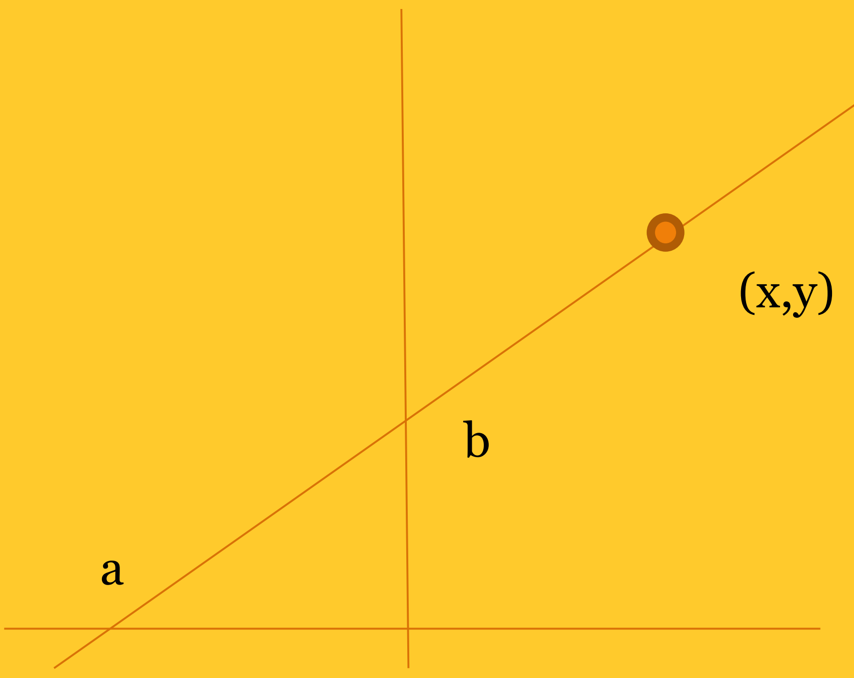
We know, $m=\frac{(y-0)}{(x-d)}$

Hence, the equation of line is:
y=m(x-d)

Ex-Draw a line – Given x intercept 3 and m=5

4. Intercept Forms

- When x-intercept (a) and y-intercept (b) of the line given, then we can form a equation of that line as,



We know, m=$\frac{(y-0)}{(x-a)} = \frac{(b-0)}{(0-a)}$-

ay=b(x-a)

-ay=bx-ab

=bx+ay=ab(Dividing both side by ab we get:

$\frac{x}{a}+\frac{y}{b} = 1$

Hence, the equation of line is:

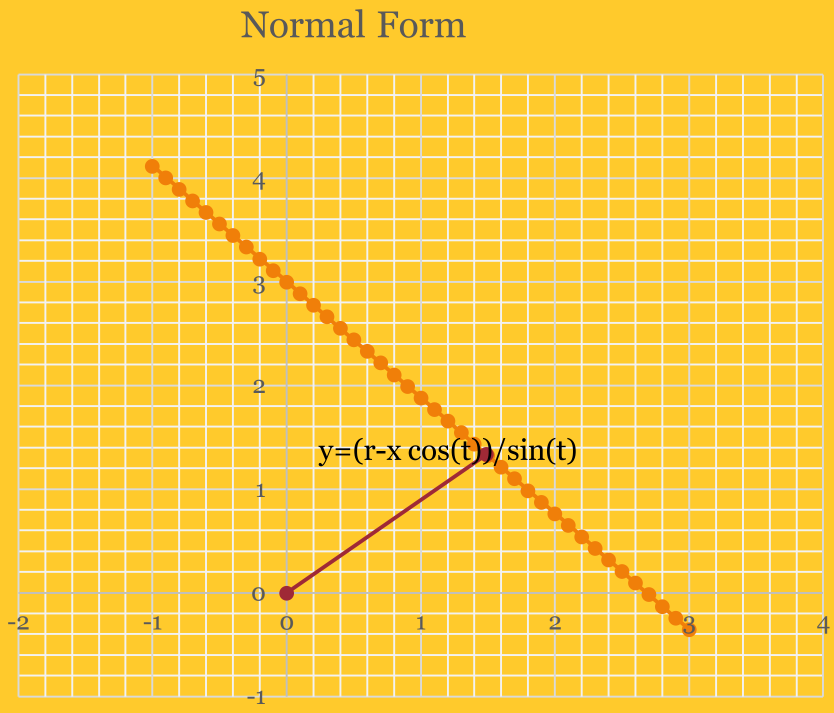
$\frac{x}{a}+\frac{y}{b} = 1$

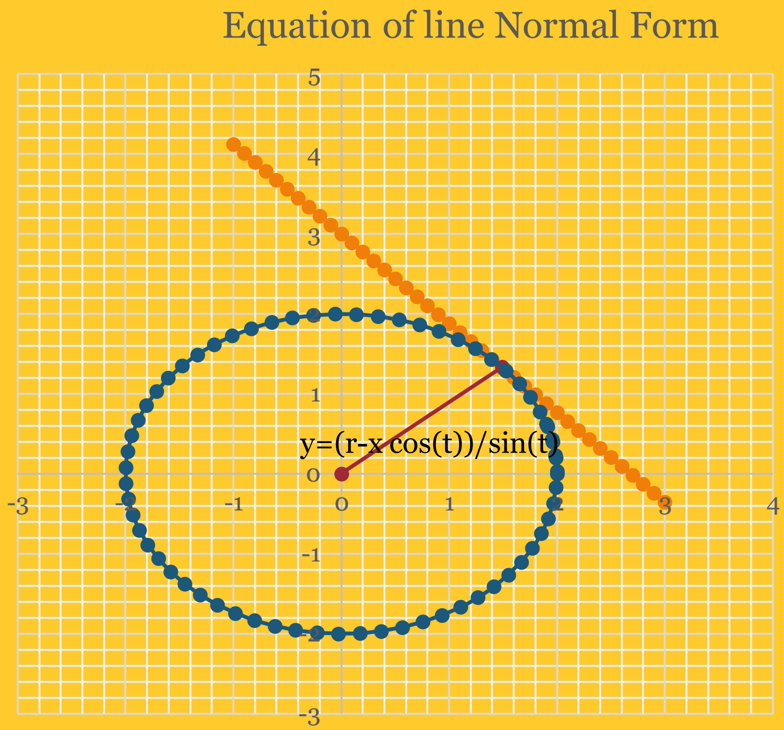
Ex-Draw a line – Given x intercept 3 and y intercept =5

5. Normal Form

The equation of line when the following two parameters given

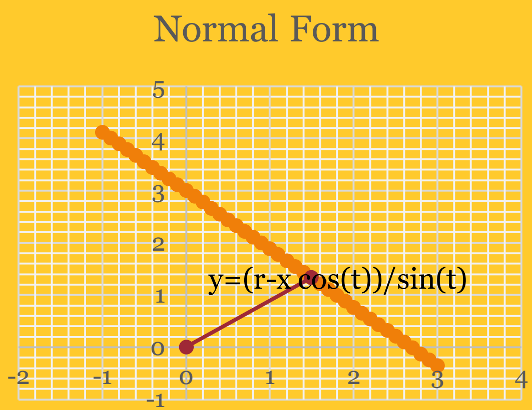
1) Length of the Perpendicular (Normal) from the origin (p)
2) Angle of normal with x-axis (t)

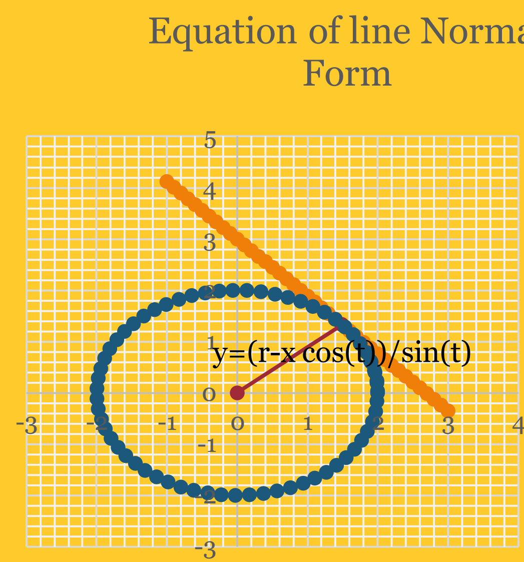




5a. Normal Form

As the given line is perpendicular to the normal, the Slope of the given line is perpendicular to normal=-1/slope of the normal=-1/tan(t)=-cos(t)/sin(t)
Again, the slope of the given line = (y-p sin(t))/(x-p cos(t))
Hence, x cos(t)-p cos ^2(t)=y sin(t)= p sin^2(t)
x cos(t)+y sin (t)=p cos^2(t)+p sin^2(t) =p(cos^2(t)+sin^2(t)=p
Hence, x cos(t)+y sin (t)=p
Or y=(p-x cos(t))/sin(t)





General Equation of Line

- The general equation of line is: A x +B y +C=0

Note: The general form can be reduced to various forms by replacing A, B and C with different parameters.

1. Slope-Intercept form,

y=mx+c or y = (-A/B) x + (– C/B), m=-A/B, c=-C/B

2. Intercept form,

x/a+y/b=1 or x/-C/A+y/-C/B=1, a=-C/A, b=-C/B

3. Normal Form

x cos (t)+y sin(t)=p, A/cos(t)=B/sin(t)=-C/p

Converting General Form

3. Normal Form= $x \cos(t)+y \sin(t)=p$
   Standard Form = $A x + B y + C = 0$

(First convert p in terms of coefficients A, B and C)

$A/\cos(t)=B/\sin(t)=-C/p$ (Ratio of coefficients are same as lines are parallel)

$\cos(t)=- A p / C$, $\sin(t)=-B p / C$

$\cos^2(t)+\sin^2(t)=(Ap/C)^2+(Bp/C)^2=1$

$A^2 p^2/C^2+B^2 p^2/C^2=1$, $p^2(A^2/C^2+B^2/C^2)=1$, or $p^2(A^2+B^2)/C^2=1$

$p^2=C^2/(A^2+B^2)$ or $p=\pm(C/\sqrt{A^2+B^2})$

Now, $\cos(t)=- A p / C = -A*\pm C/\sqrt{A^2+B^2}/C=\pm A/\sqrt{A^2+B^2}$

Similarly, Now, $\sin(t)=- B p / C = -B*\pm C/\sqrt{A^2+B^2}/C=\pm B/\sqrt{A^2+B^2}$

Hence,

$\pm A/\sqrt{A^2+B^2}\, x + -B/\sqrt{A^2+B^2}\, y =\pm(C/\sqrt{A^2+B^2})$

A pencil of Straight Lines

- The collection of lines passing through one point A(x1 y1) is called a pencil of lines through a point

- The point A (x0, y0) is called the vertex of the pencil

Equation of a Pencil of Straight Lines

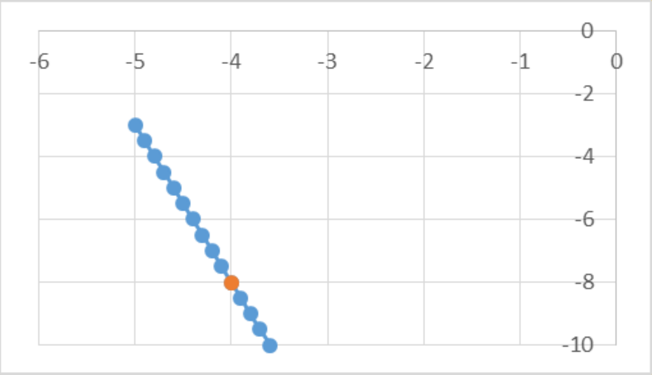
The equation of the pencil is y=y0+m(x-x0)

	x0	y0
If the vertex is	-4	-8

Then the equation of line is (y-y0)=m(x-x0)

y=8+m(x-(-4))

If m=-5,



Equation of the pencil containing two intersecting lines

L_1	$A_1x+B_1y+C_1=0$	$5x+2y+3=0$
L_2	$A_2x+B_2y+C_2=0$	$3x+5y-2=0$

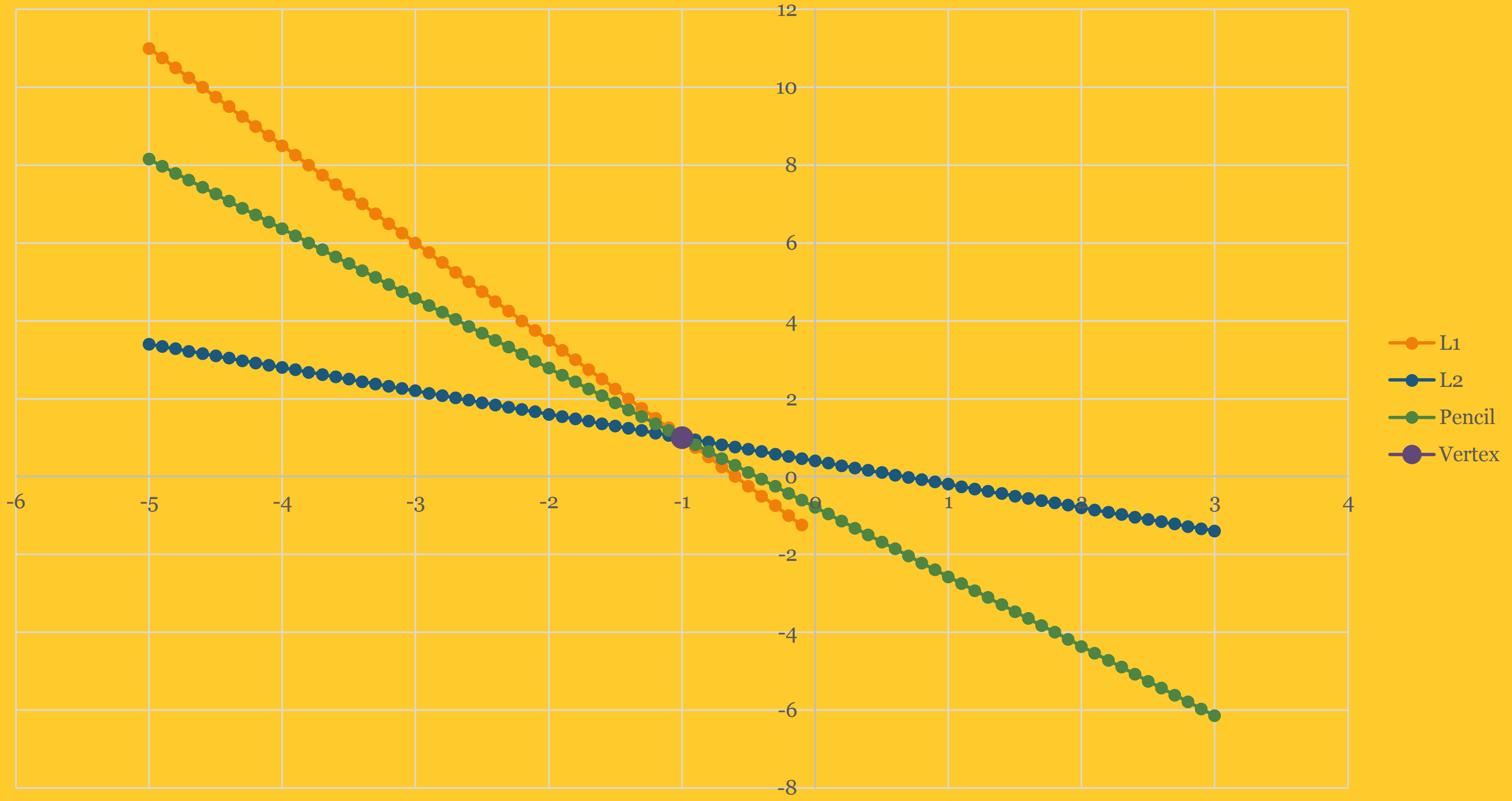
Both the line belong to same pencil. So their vertex is same. We are required to find the equation of the pencil of lines containing L_1 and L_2.

The equation of pencil of straight line is

$m_1(A_1x+B_1y+C_1)+m_2(A_2x+B_2y+C_2)=0$ where m_1 and m_2 are any real number.

The equation can also be represented as
$(A_1x+B_1y+C_1)+m(A_2x+B_2y+C_2)=0$

Equation of the pencil containing two intersecting lines



Equation of a line passing through a point and parallel to given line

- The equation of line passing through a given point and parallel to a given straight line

Given point x_0 y_0

Straight line $y=ax+c$

The lines are parallel, hence their slopes are same

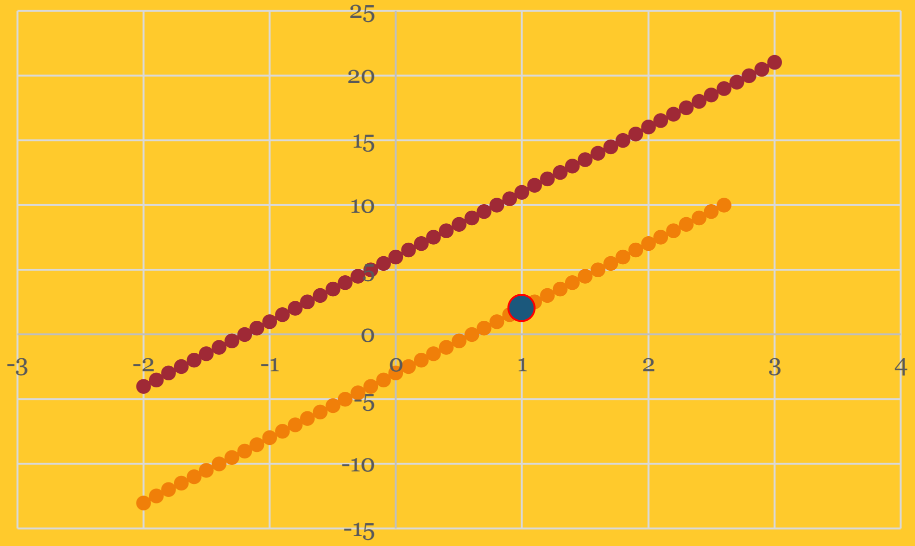
$$a=(y-y_0)/(x-x_0)$$

$$y-y_0=a\,(x-x_0)$$

$$y=y_0+a(x-x_0)$$

Equation of a new line

- The equation of line passing through a given point and parallel to a given straight line



The equation of line passing through a given point and perpendicular to a given straight line

The equation of line passing through a given point and perpendicular to a given straight line

Given point $x0=2$ $y0=1$

Straight line $y=mx+c$

$y=5x+6$

The lines are perpendicular, hence the slope of the line m is -1/a

$m=(y-y0)/(x-x0)$

$m=-1/a$

$y-y0=m(x-x0)$

$y-y0=-1/a(x-x0)$

$a y-ay0=-(x-x0)$

$y=(a y0-(x-x0))/a$

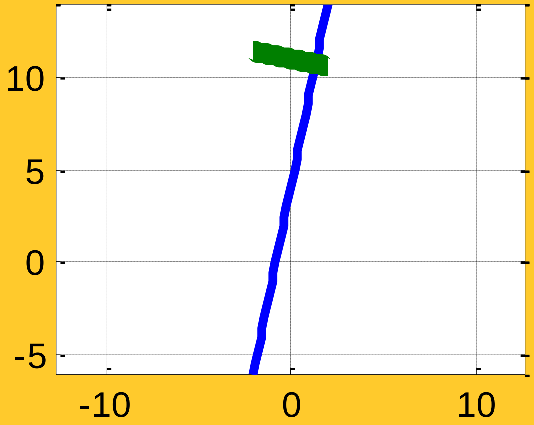
$y=(2*5-(x-1))/5$

$y=(10-(x-1))/5$

$y=2-x/5+1/5$

$y=(11-x)/5$

Equation of a perpendicular line



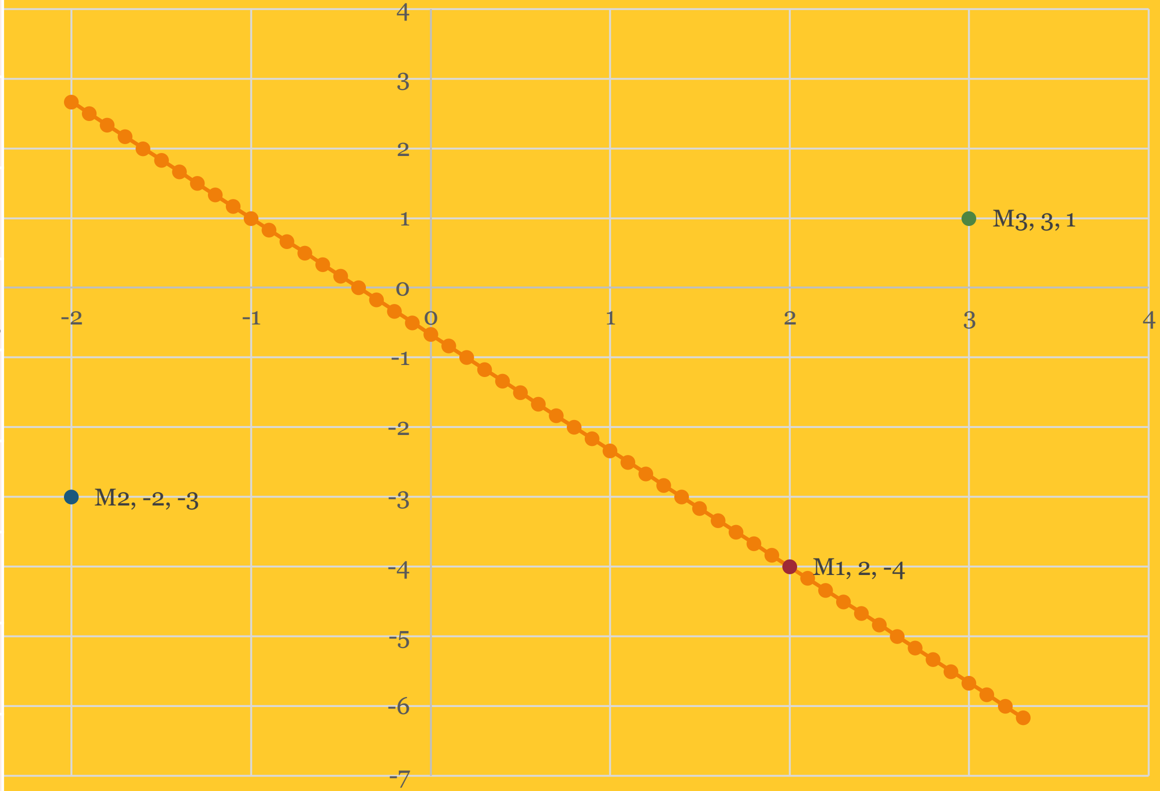
Mutual Position of Straight Line and a pair of points

- The mutual position of two points M1(x1 y1) and M2(x2 y2) and a straight line Ax+By+C=0 is determined from the following characteristics:
- Put the values of M1 and M2 in the given equation:

1. If the values have same sign, then they lie on same side of the line
2. If the values have opposite sign, then they lie on different side of the line
3. If the values are 0, then they lie on the line

Mutual Position of Straight Line and a pair of points

	x1	y1
M1	‘2	-4
	x2	y2
M2	-2	-3
	x3	y3
M3	3	1
Line	5x+3y+2	
M1	0	
M2	-17	
M3	20	



Perpendicular Distance from a Point to a Line

- This is a great problem because it uses all these things that we have learned so far:

  - Distance Formula
  - Slope of Parallel lines
  - Perpendicular Lines
  - Different forms of straight lines
  - Pencil of Straight Lines

Perpendicular Distance from a Point to a Line

- This is a great problem because it uses all these things that we have learned so far:

The distance from a point (m, n) to the line $Ax + By + C = 0$ is given by:

$$d = \frac{|Am + Bn + C|}{\sqrt{A^2 + B^2}}$$

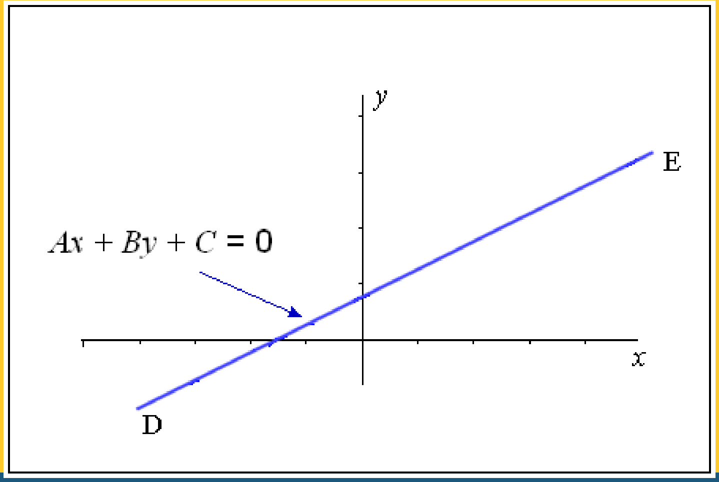
Perpendicular Distance from a Point to a Line

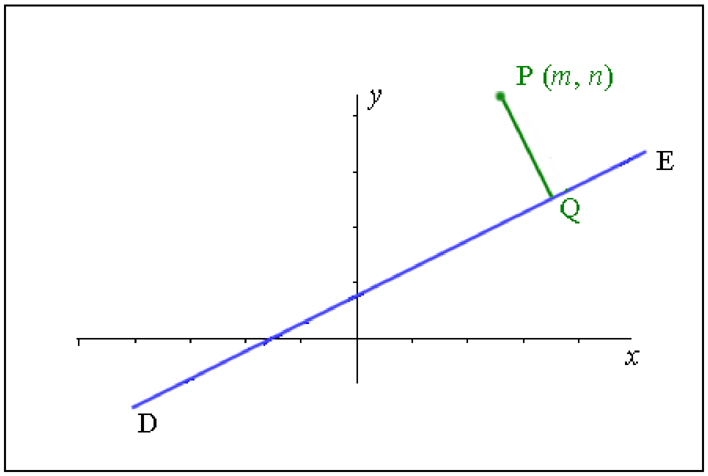
- The coordinate of the point are -

The Point on the Line which is closest to (m, n) has coordinates(x=$\frac{B(Bm-An)-BC}{A^2+B^\wedge 2}$, y=$\frac{A(-Bm+An)-AC}{A^2+B^\wedge 2}$

Perpendicular Distance from a Point to a Line

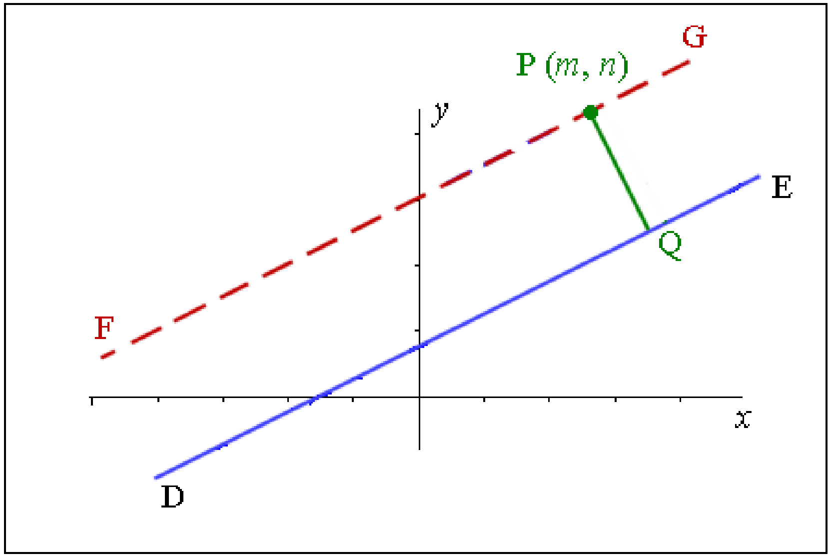
- Let's start with the line $Ax + By + C = 0$ and label it DE. It has slope –A/B.
- We have a point P with coordinates (m, n). We wish to find the perpendicular distance from the point P to the line (that is, distance PQ.





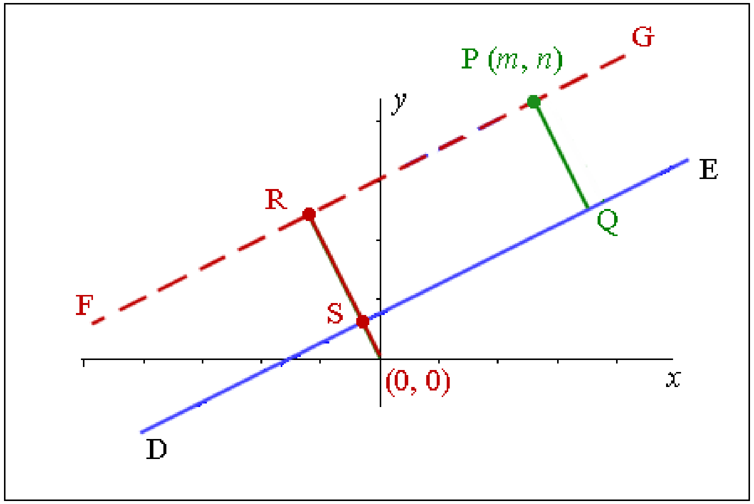
Perpendicular Distance from a Point to a Line

- We now do a trick to make things easier for ourselves (the algebra is really horrible otherwise). We construct a line parallel to DE through (m, n). This line will also have slope -B/A, since it is parallel to DE. We will call this line FG.



Perpendicular Distance from a Point to a Line

- Now we construct another line parallel to PQ passing through the origin.
- This line will have m=B/A, because it is perpendicular to DE.
- Let's call it line RS. We extend it to the origin (0,0).
- We will find the distance RS, which is equal to the distance PQ that we wanted at the start.



Perpendicular Distance from a Point to a Line

- Since FG passes through (m, n) and has slope −A/B, its equation is y-n=-A/B(x-m) or $y=(-Ax+Am+Bn)/B$.
- Line RS has equation $y=B/A\,x$.
- Line FG intersects with line RS when $B/A\,x=(-Ax+Am+Bn)/A$
- Solving this gives us $x=A(Am+Bn)/(A^2+B^2)$
- So after substituting this back into $y=B/A\,x$, *we find that point R is* $A(Am+Bn)\,)/(A^2+B^2),\ B(Am+Bn)/\ (A^2+B^2)$
- *Point S is the intersection of the lines* $y=B/A\,x$ *and* $Ax + By + C = 0$,
- *which can be written* $y=-(Ax+C)/B$.

Perpendicular Distance from a Point to a Line

- This occurs when (that is, we are solving them simultaneously)
- $(Ax+C)/B = B/Ax$
- Solving for x gives $x = -AC/(A^2+B^2)$
- Finding y by substituting back into
- $y = B/A\,x$ gives $y = B/A(-AC/(A^2+B^2) = -BC/(A^2+B^2)$
- So S is the point
- $-AC/(A^2+B^2), \; -BC/(A^2+B^2)$

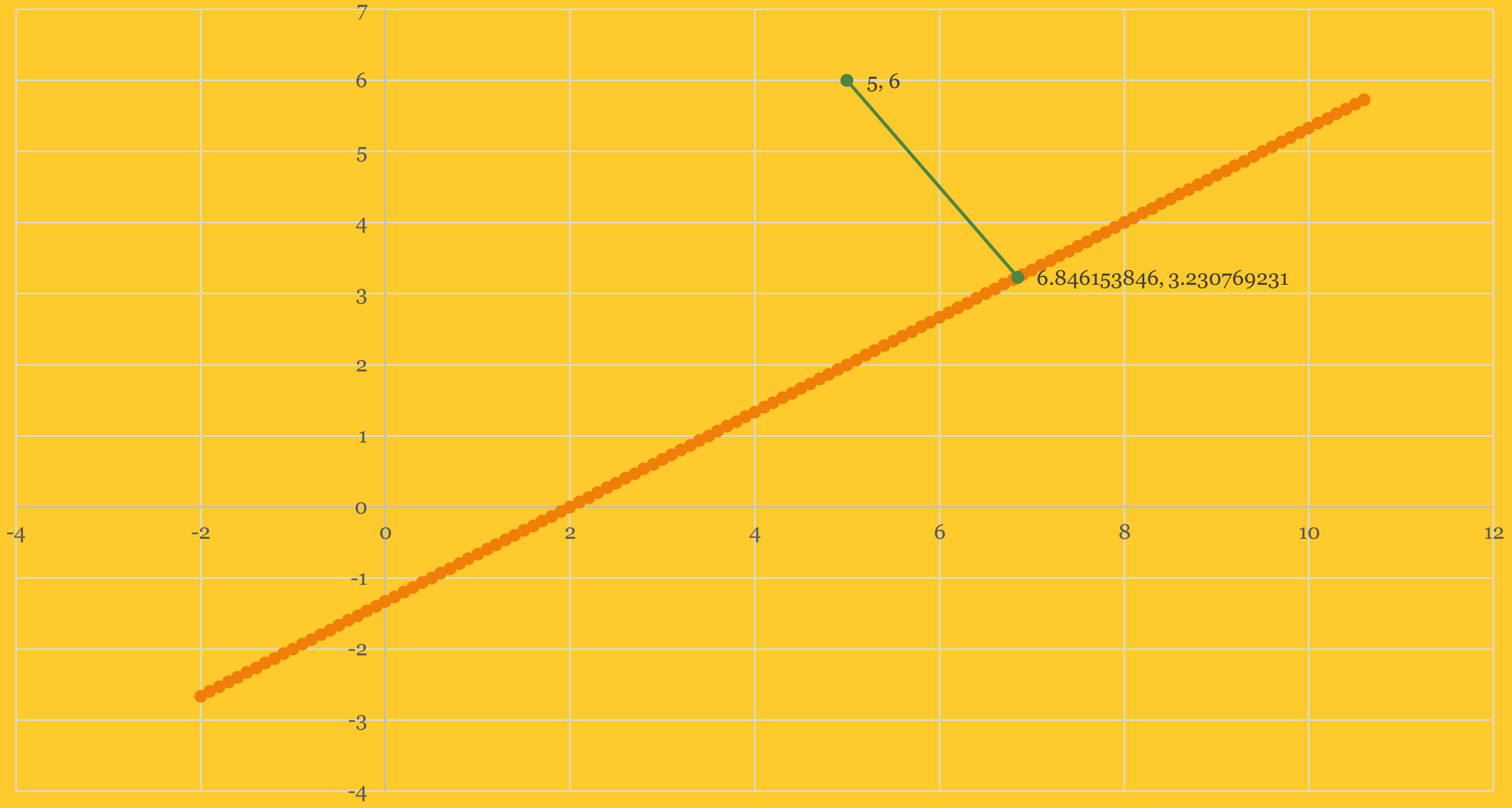
Perpendicular Distance from a Point to a Line

- The distance RS, using the distance formula, $d=\sqrt{(x2-x1)2+(y2-y1)2}$ is
- $d=\sqrt{(-AC/(A^2+B^2)-A(Am+Bn)/(A^2+B^2))^2+(-BC/(A^2+B^2)-B(Am+Bn)/ (A^2+B^2))^2}$
- $= \sqrt{([\{\{-A(Am+Bn+C)\}^2+\{-B(Am+Bn+C)\}^2\}/(A^2+B^2)^2]}$ $=\sqrt{(A^2+B^2)(Am+Bn+C)^2/(A^2+B^2)^2}$
- $=\sqrt{(Am+Bn+C)2/(A^2+B^2)}$
- $=|Am+Bn+C|/\sqrt{(A^2+B^2)}$
- The absolute value sign is necessary since distance must be a positive value, and certain combinations of *A*, *m* , *B*, *n* and *C* can produce a negative number in the numerator.
- So the distance from the point (m, n) to the line $Ax + By + C = 0$ is given by:
- $=|Am+Bn+C|/\sqrt{(A^2+B^2)}$

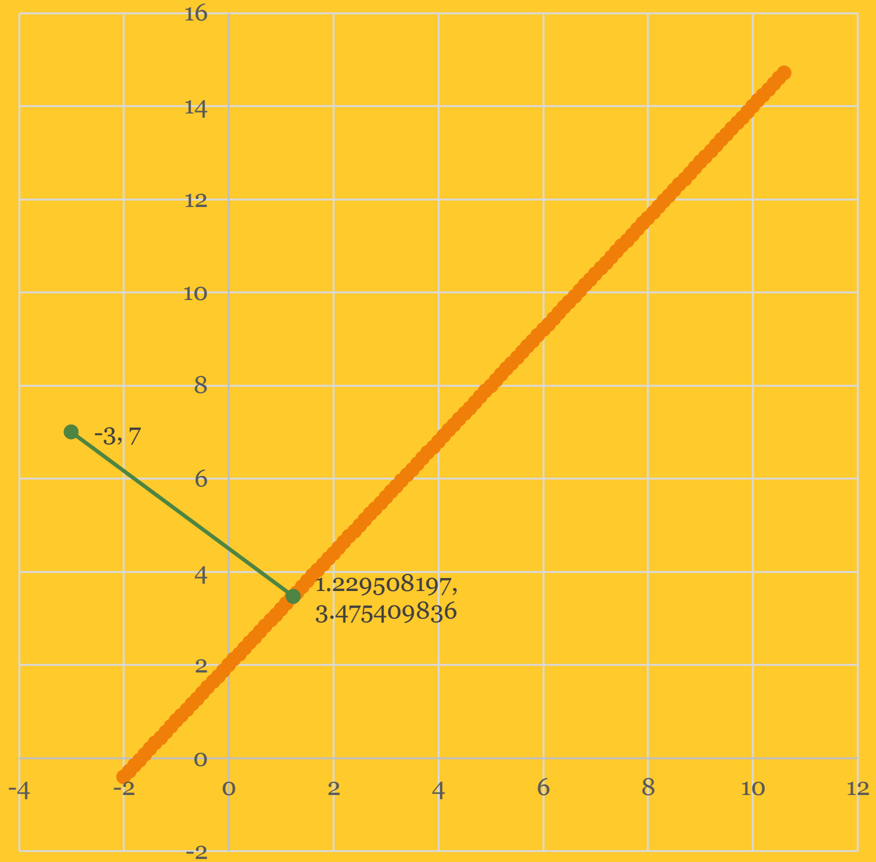
Perpendicular Distance from a Point to a Line

- Example-1: Find the perpendicular distance from =the point (5, 6) to the line -2x+3y+4=0
- d=|Am+Bn+C|/√ (A^2+B^2)
- d=|-2*5+3*6+4)|/√ (-2^2+3^2)
- d= |-10+18+4|/√ (4+9)
- d= 12/√13=3.328

Distance from the point (5, 6) to the line -2x+3y+4=0



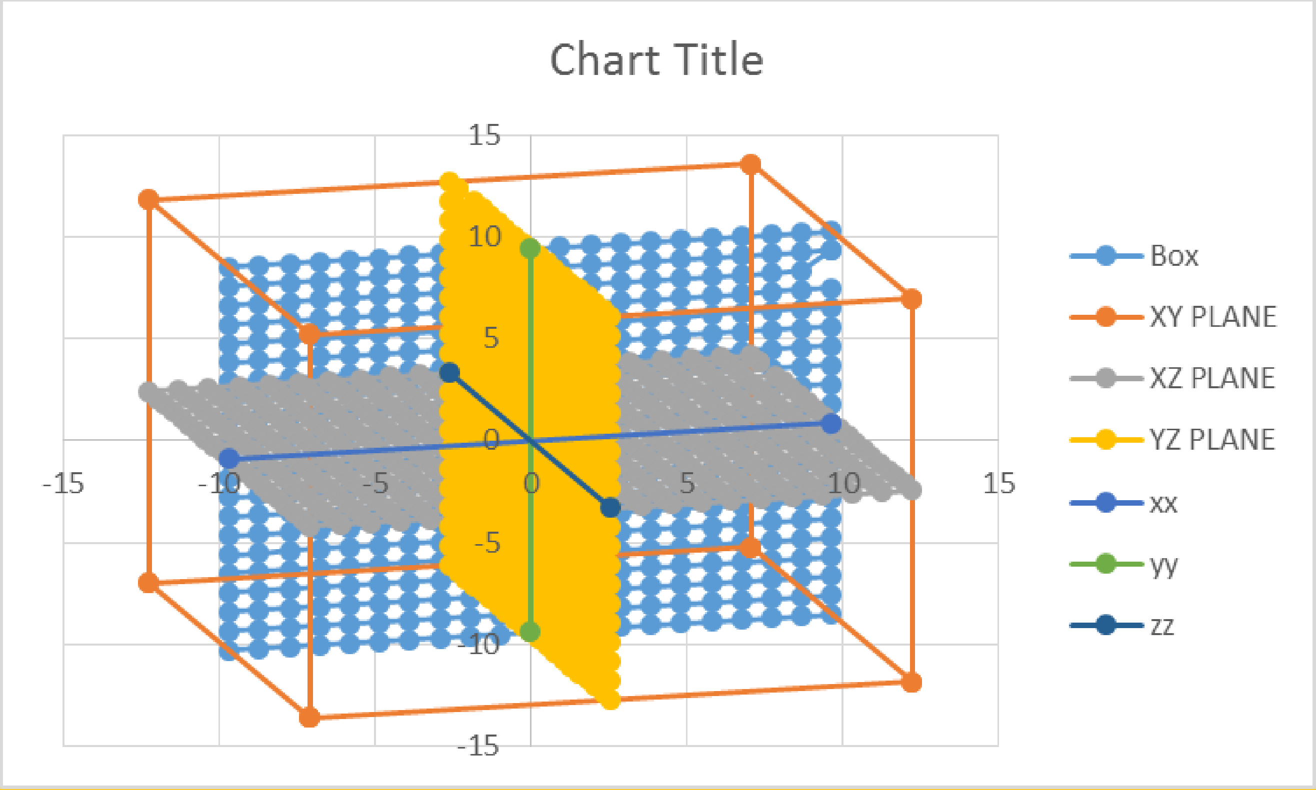
Distance from the point (-3, 7) to the line 6x-5y+10=0



- Three Dimensional Geometry

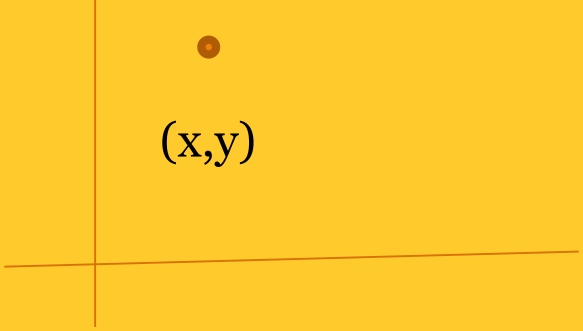
Coordinate Axis and Coordinate Planes

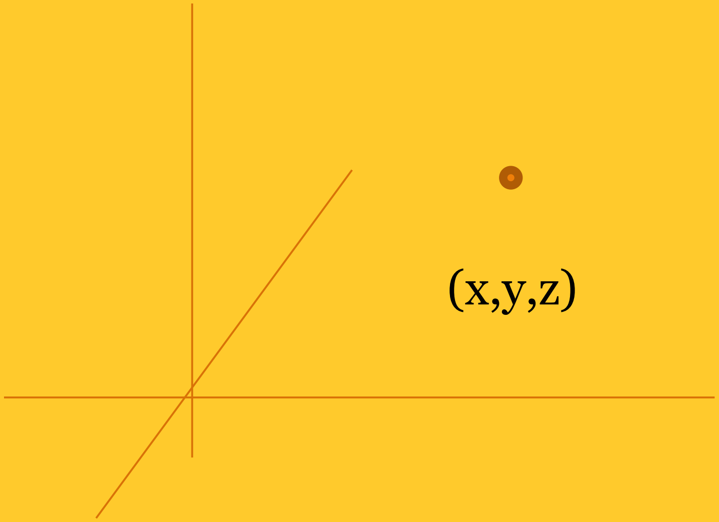
- The three coordinate plane divides the space into eight parts known as OCTANTS.



Three Dimensional Geometry-Point

- To locate a position of a point in a plane, we require two intersecting perpendicular lines in the plane called coordinate axis and the two numbers are called coordinates. Similarly we require three coordinate axis to locate a position of a point in 3 dimensions.





Distance between two points in 3d

- $P1=(x1, y1, z1)$
- $P2=(x2, y2, z2)$

- Distance
- $=\text{sqrt}((x2-x1)^2+(y2-y1)^2+(z2-z1)^2)$

Section Formula-Internally

- $x=(mx_2+nx_1)/(m+n)$
- $y=(my_2+ny_1)/(m+n)$
- $z=(mz_2+nz_1)/(m+n)$

Section Formula-Externally

- $x=(mx_2-nx_1)/(m-n)$

- $y=(my_2-ny_1)/(m-n)$

- $z=(mz_2-nz_1)/(m-n)$

What is Vector?

Why the study of vector is at is Vector?

Vector Algebra

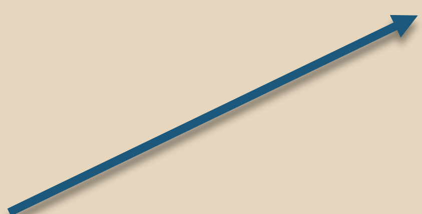
- Locating a point in 2d as well as 3d.
- Distance between two point,
- Unit lengths
- Equation of lines and planes
- Angle between two lines
- Areas of a triangle
- Normal to a plane
- Section formulas
- Projection of a line

Vector Algebra

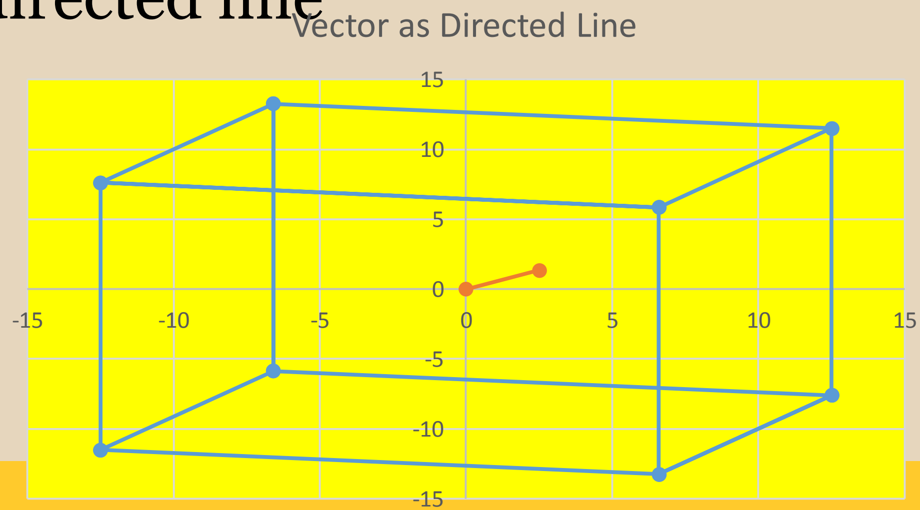
What is Vector?

A vector is the quantity that has magnitude and direction.

Directed Line Segment-A directed Line segment has magnitude and Direction. Hence, geometrically, any vector can be represented as a directed line



Vector



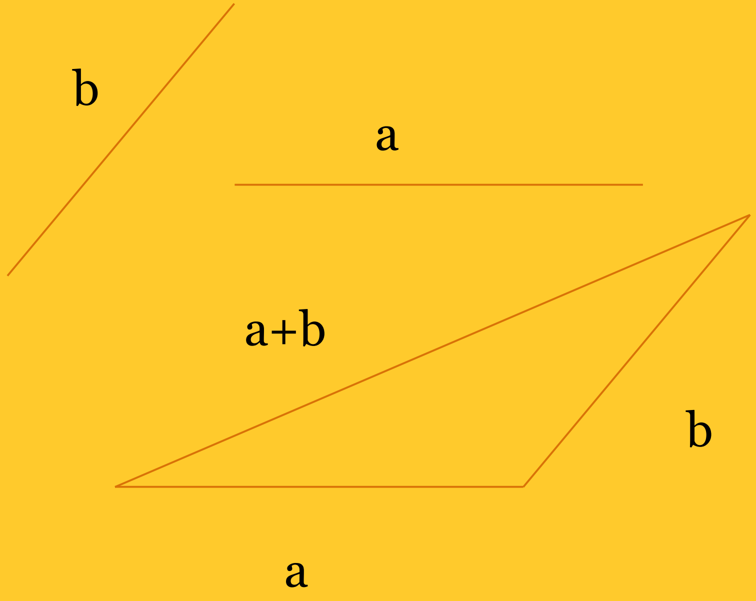
Type of vectors

- Zero Vector-initial and terminal point coincide
- Unit vector-A vector whose magnitude is unity,
- Co-initial Vectors-Vectors having same initial point
- Collinear Vectors-If they are parallel to the same line
- Equal Vectors-Same magnitude and Direction
- Negative Vector- Same magnitude but opposite direction

Addition of Vectors

Triangle law of vector addition:

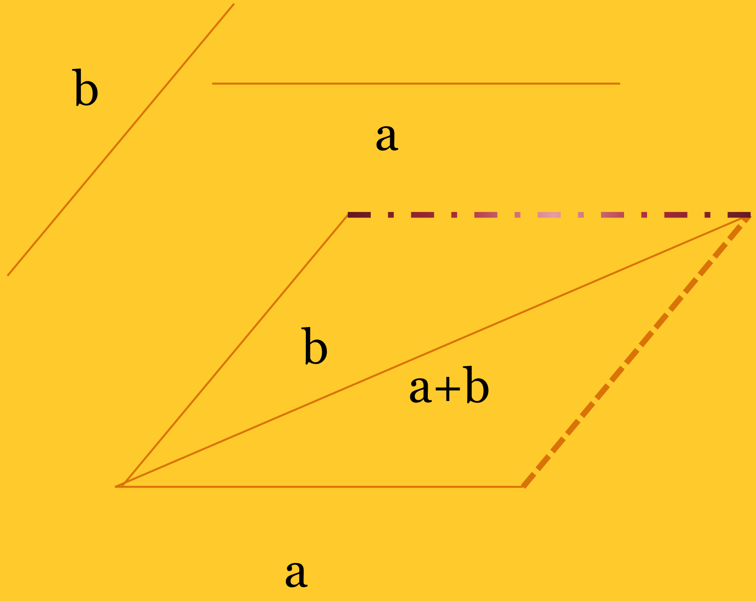
For Vector Addition, Place initial point of one vector to the terminal point of the other. It is known as triangle law of vector addition.



Addition of Vectors

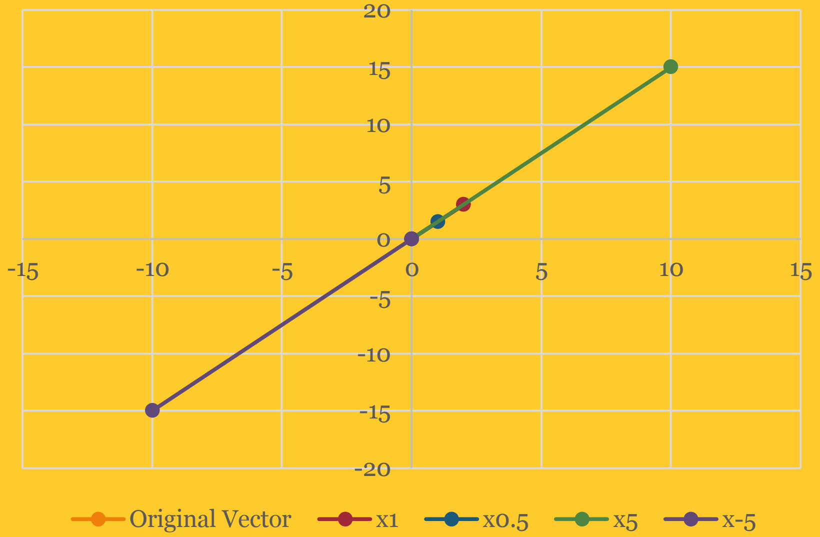
Parallelogram law of vector addition:

For Vector Addition, Place initial points of both the vectors and complete the parallelogram. The diagonal represents the resultant vector.



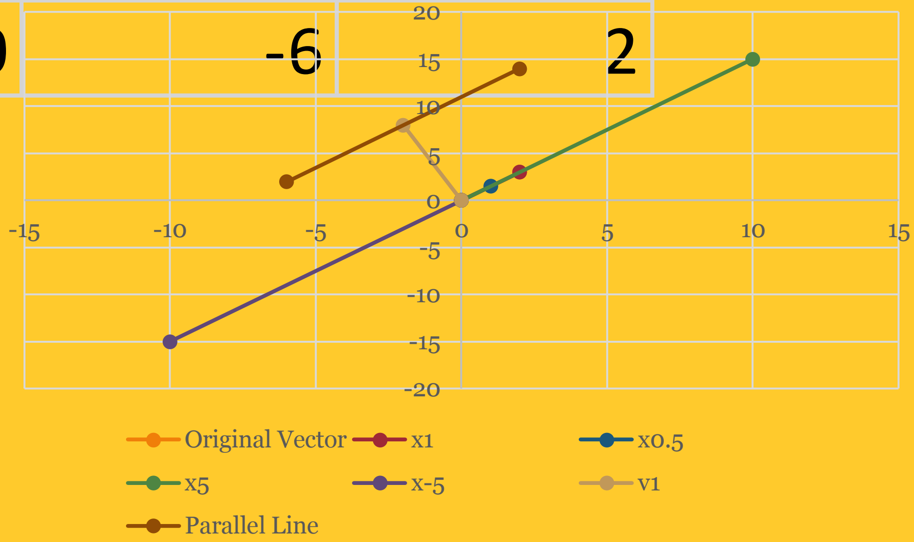
Multiplication of a vector by a scalar

Scalar	x	y	0	0
1	2	3	2	3
0	2	3	0	0
0.5	2	3	1	1.5
0	2	3	0	0
5	2	3	10	15
0	2	3	0	0
-5	2	3	-10	-15
0	2	3	0	0



Use of vector addition and scalar multiplication Draw a Line from a vector parallel to other vector

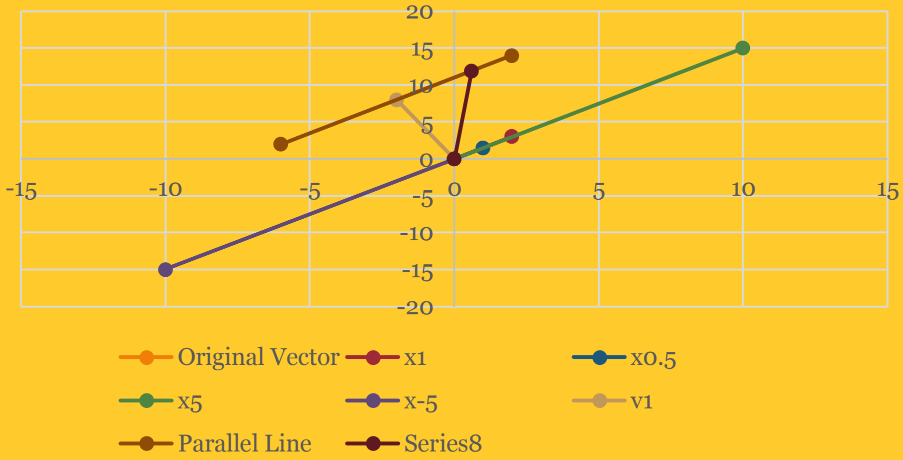
		Vector	
		0	0
v1		2	3
-2	8	2	14
0	0	-6	2



Draw a vector from a vector parallel to other vector

		Vector	
		0	0
v1		2	3
-2	8	2	14
0	0	-6	2

$$r=a+\lambda\, b$$

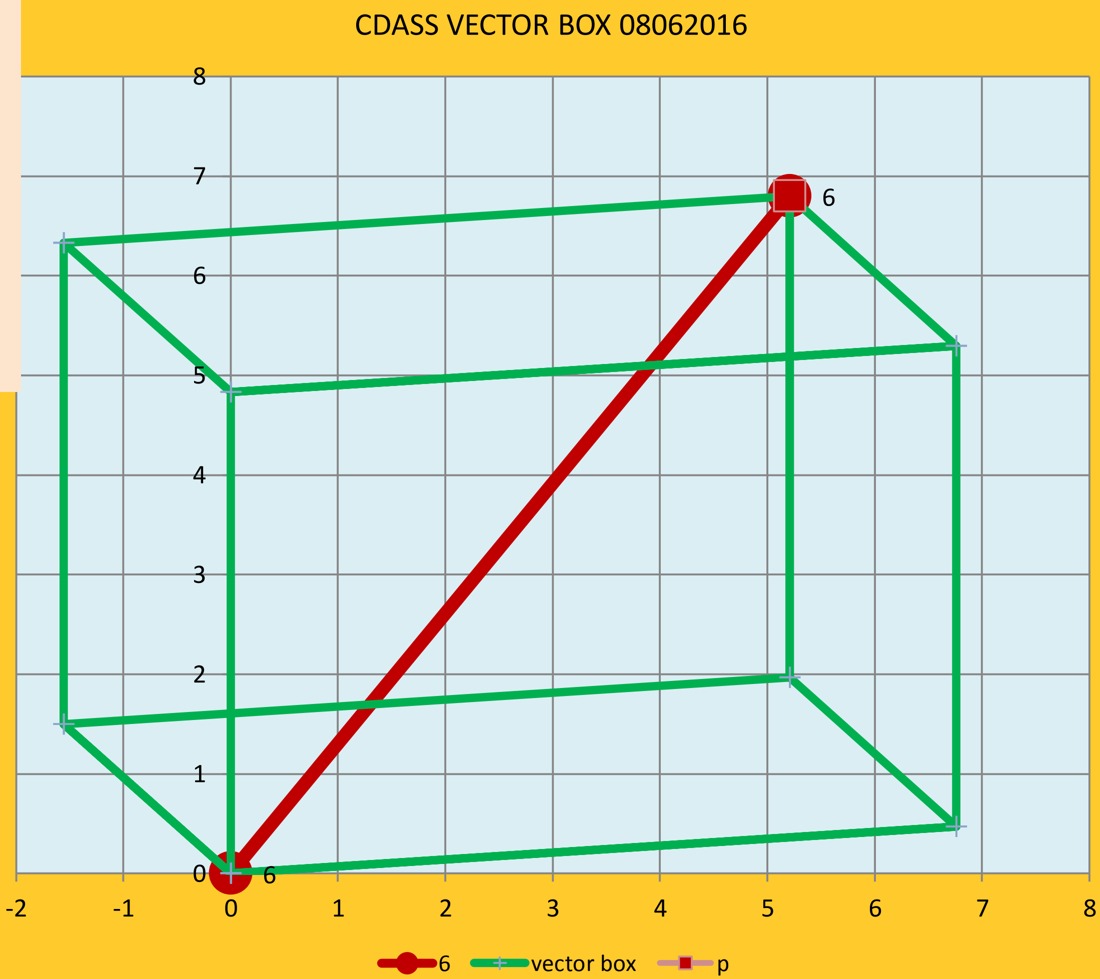


Components of a Vectors

VECTOR OP=[7, 5, 6]
COMPONENT OF VECTOR OP:
X_COMPONENT=[7,0,0]
Y_COMPONENT=[0,5, 0]
Z_COMPONENT=[0,0,6]

|OP|=sqrt(7^2+5^2+6^2)

Position Vector [5, 6, 7]



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Data for representation of a vector

0
1
2
3
0
4
5
6
7
3
2
6
5
1
0
4
7

vector box	0	0	0
vector box	7	0	0
vector box	7	5	0
vector box	0	5	0
vector box	0	0	0
vector box	0	0	6
vector box	7	0	6
vector box	7	5	6
vector box	0	5	6
vector box	0	5	0
vector box	7	5	0
vector box	7	5	6
vector box	7	0	6
vector box	7	0	0
vector box	0	0	0
vector box	0	0	6
vector box	0	5	6

VECTOR OP=[7, 5, 6]

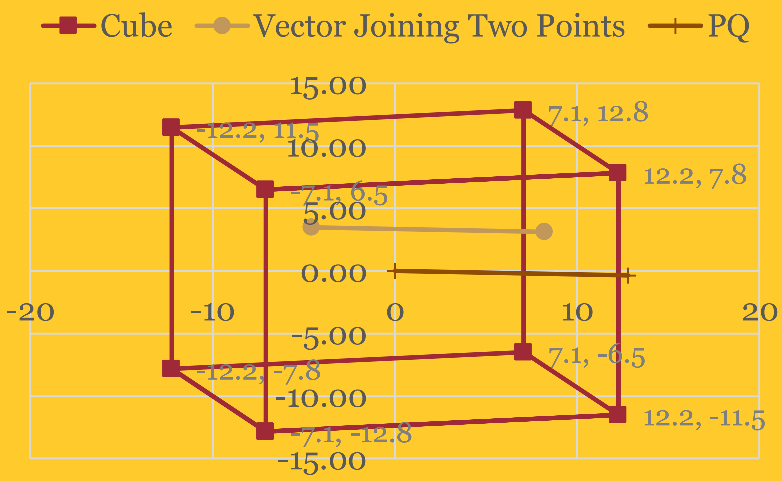
Slider

		17
	x	7
		15
	y	5
		16
	z	6

Vector Joining Two Points

- P=[x1, y1 z1] Q=[x2, y2,z3]
- PQ=P2-P1=[X2-X1, Y2-Y1, Z2-Z1]
- |PQ|=SQRT((X2-X1)^2+(Y2-Y1)^2+(Z2-Z1)^2)

P	-3	2	6
Q	9	3	1
PQ	12	1	-5
	0	0	0



Section Formula

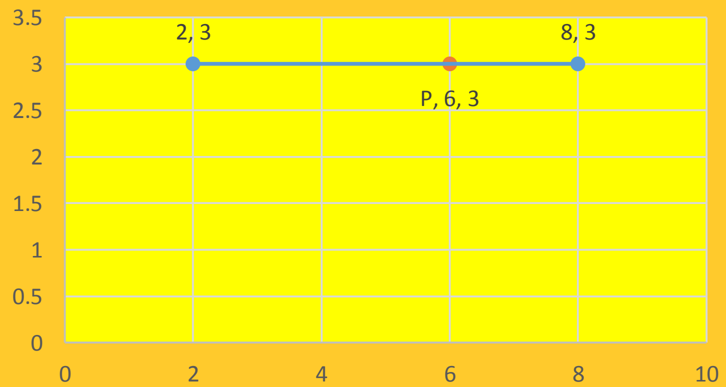
A "ratio" is just a comparison between two different things.
Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is **15 to 20**.

In **mathematics**, two variables are **proportional** if a change in one is always accompanied by a change in the other, and if the changes are always related by use of a constant multiplier. The constant is called the coefficient of proportionality or proportionality constant.

Section Formula

- The **section formula** tells us the coordinates of the point which divides a given line segment into two parts such that their lengths are in the ratio m:n.

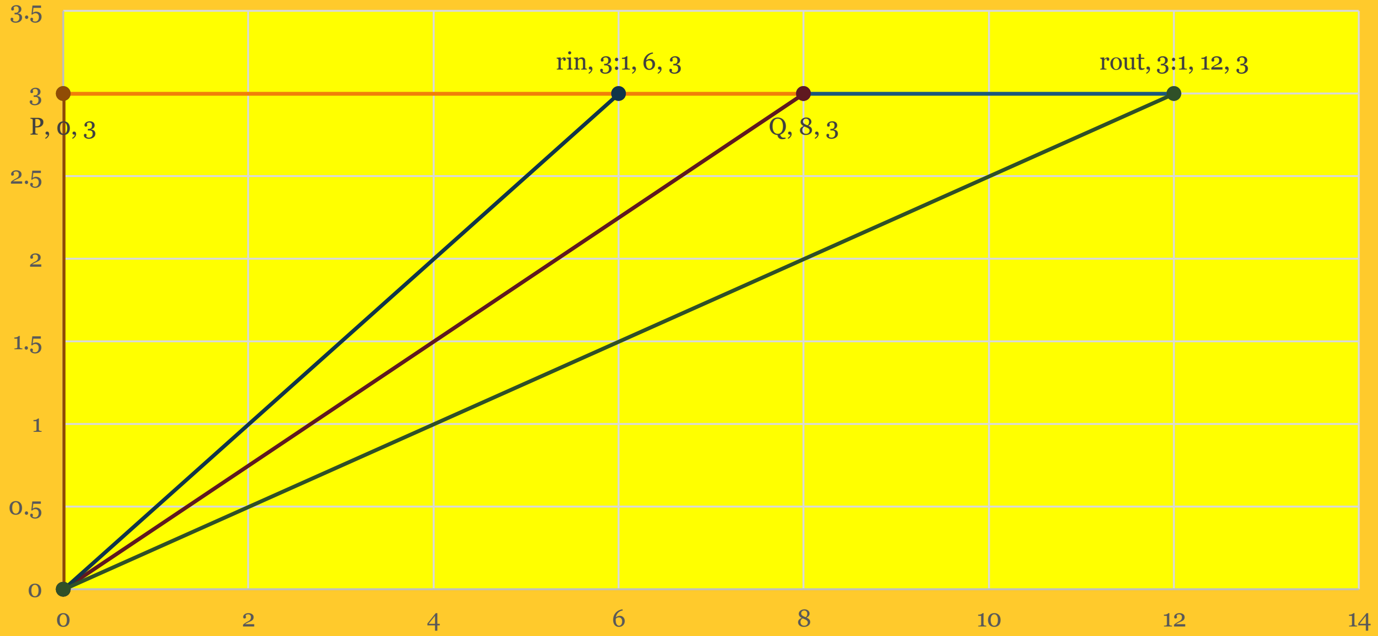
- $x = \frac{m\,x2+n\,x1}{m+n}$
- $y = \frac{m\,y2+n\,y1}{m+n}$



SECTION FORMULA

- P=[x1, y1 z1]
- Q=[x2, y2,z3]
- Rin=[x,y] divides PQ in the ratio m:n INTERNALLY
- x= (m*X2+n*X1)/(m+n), y= (m*Y2+n*Y1)/(m+n)
- Rout=[x,y] divides PQ in the ratio m:n EXTERNALLY
- x= (m*X2-n*X1)/(m-n), y= (m*Y2-n*Y1)/(m-n)

Section Formula



General Rule:

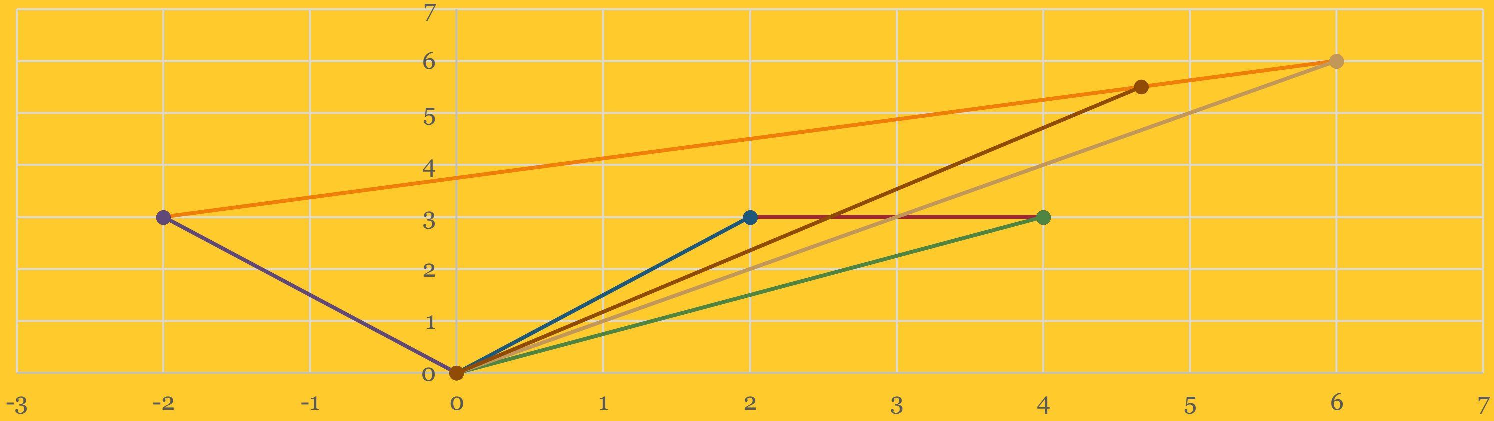
1) Internally (m:n) : Calculate the distance (L) between the points.
   x= m*L/(m+n), y=n*L/(m+n)
1) Externally (m:n): Calculate the distance (L) between the points.
   x= m*L/(m-n), y=n*L/(m-n)

Example-11

Consider two points P and Q with position vector OP=3a-2b and OQ=a+b. Find the position vector of a point R which divides the line segment joining P & Q in the ratio 2:1 (i) internally, and (2) externally

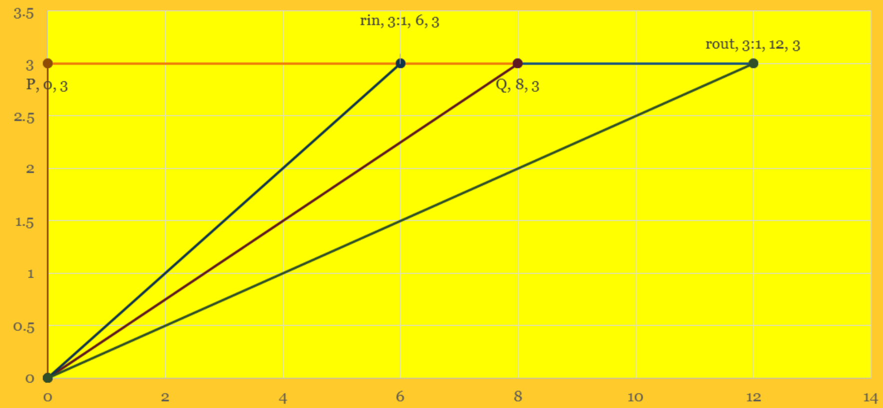
1) Internally, x=$\frac{2\,(a+b)+1\,(3a-2b)}{2+1}$ =$\frac{5a}{3}$
2) Externally, x=$\frac{2\,(a+b)-1\,(3a-2b)}{2-1}$ =4 b -a

Ex-11

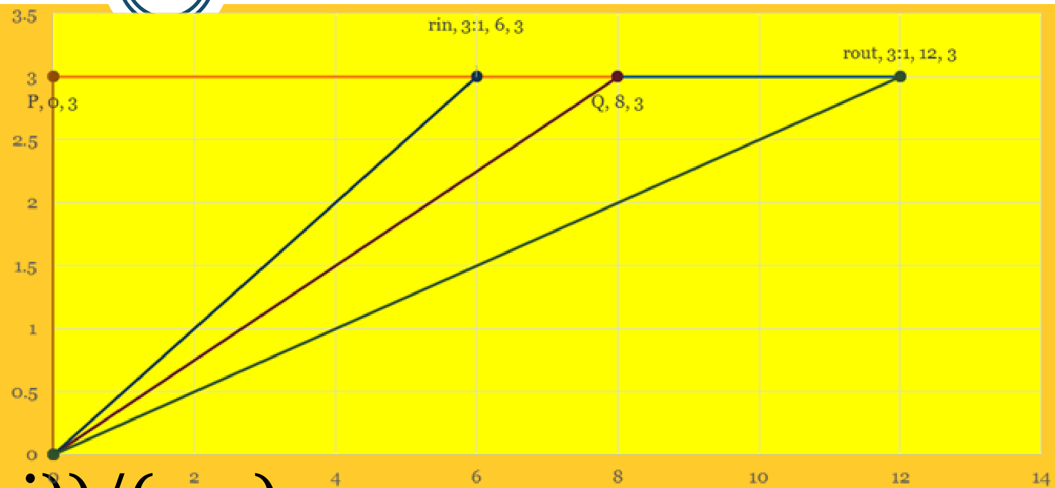


Section Formula for Vectors(Internally)

- $v1=(0i+3j)$
- $V2=(8i+3j$
- $m:n=3:1$(Internally)
- $r=m*v2+n*v1/(m+n)$
- $r=(3*(8i+3j)+1*(0i+3j))/(4+1)$
- $r=(24i+9j+0i+3j)/4$
- $r=(6i+3j)/5$
- $r=6i+3j$



Section Formula for Vector(Externally)



- v1=(0i+3j)
- V2=(8i+3j
- m:n=3:1(Internally)
- r=m*v2-n*v1/(m-n)
- r=(3*(8i+3j)-1*(0i+3j))/(3-1)
- r=(24i+9j-0i-3j)/2
- r=(24i+6j)/2
- r=12i+3j

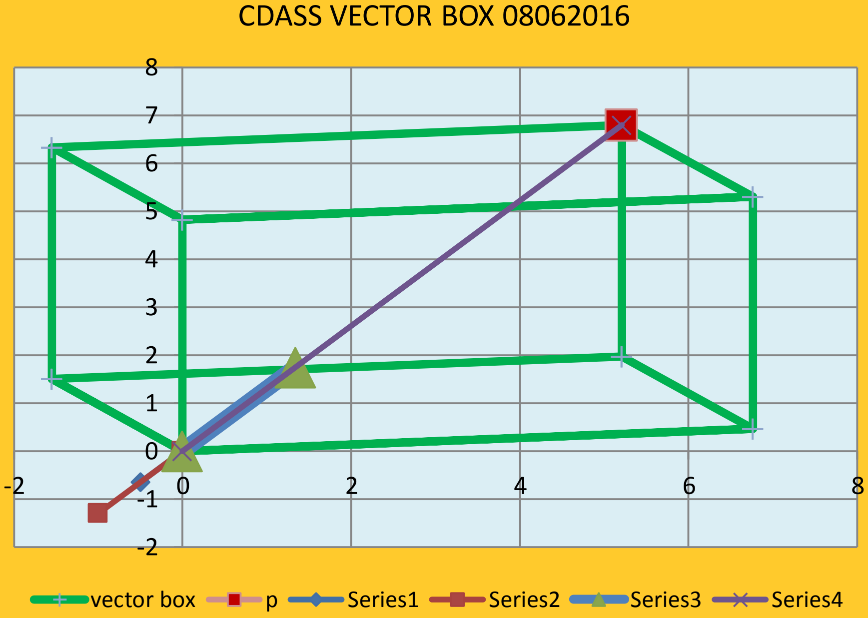
Direction Cosines

VECTOR OP=[7, 5, 6]
COMPONENT OF VECTOR OP:
X_COMPONENT=[7,0,0]
Y_COMPONENT=[0,5, 0]
Z_COMPONENT=[0,0,6]

r=|OP|=sqrt(7^2+5^2+6^2)=10.488

l=cos(α)=x/r = 7/10.488
m=cos(β) =y/r=5/10.488
n=cos(γ) = z/r =6/10.488

Direction ratios:
a=l r, b=m r, c=n r (r=-n to n)



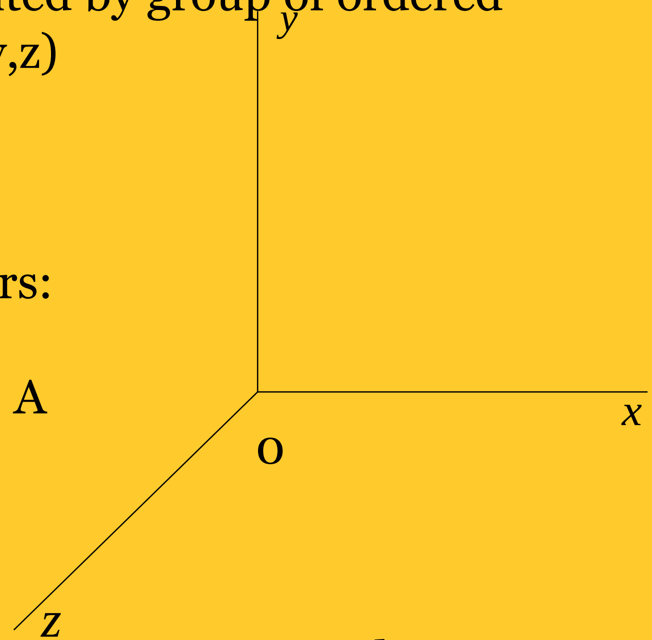
0.840052	1.073864	0.96176	Radian
48.13146	61.52787	55.10477	Degree

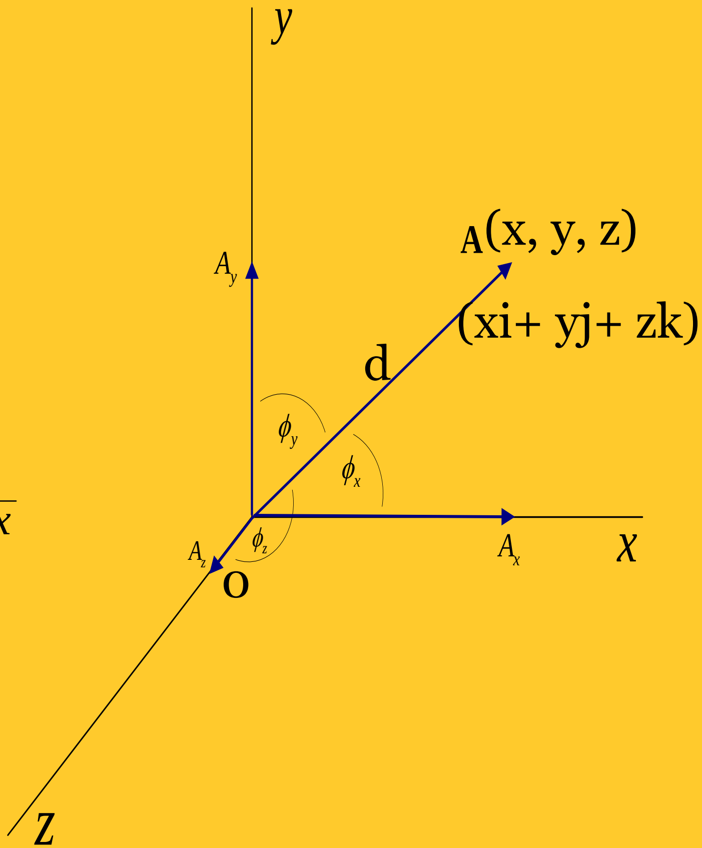
VECTORS

- Vector is a directed line segment.
- Vector has both Magnitude and Direction
- Vector is represented by group of ordered numbers (x,y), (x,y,z)

Properties of Vectors:

1. Initial Point : o
2. Terminal Point: A
3. Magnitude: d
4. Direction, t

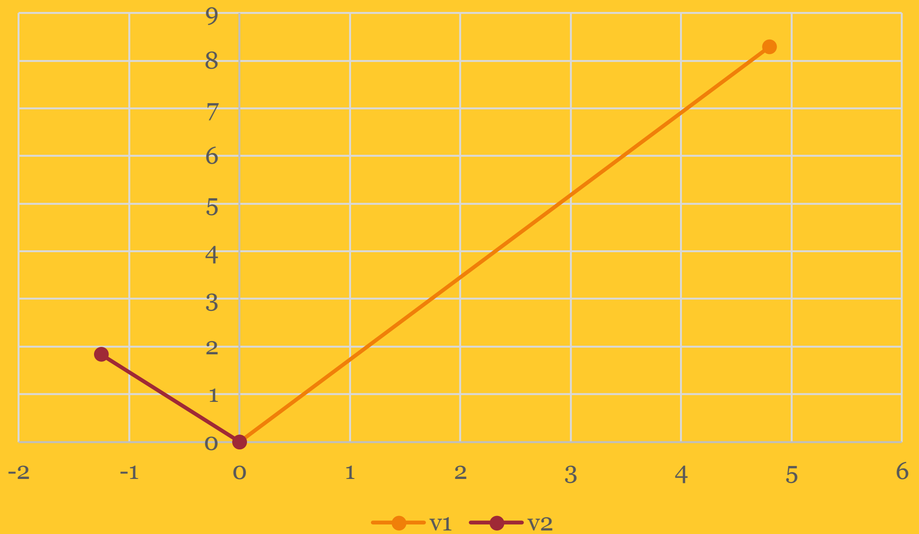




Represented as
Position Vector (x, y, z)

Vector vs Null Vector

- Vector, v1 = (1, 6)
- Vector, v2=(1,6,-13)
- Vector, v3=(0, 0, 0)
- Geometrically these vectors looks like:



VECTORS

Vector Representation:

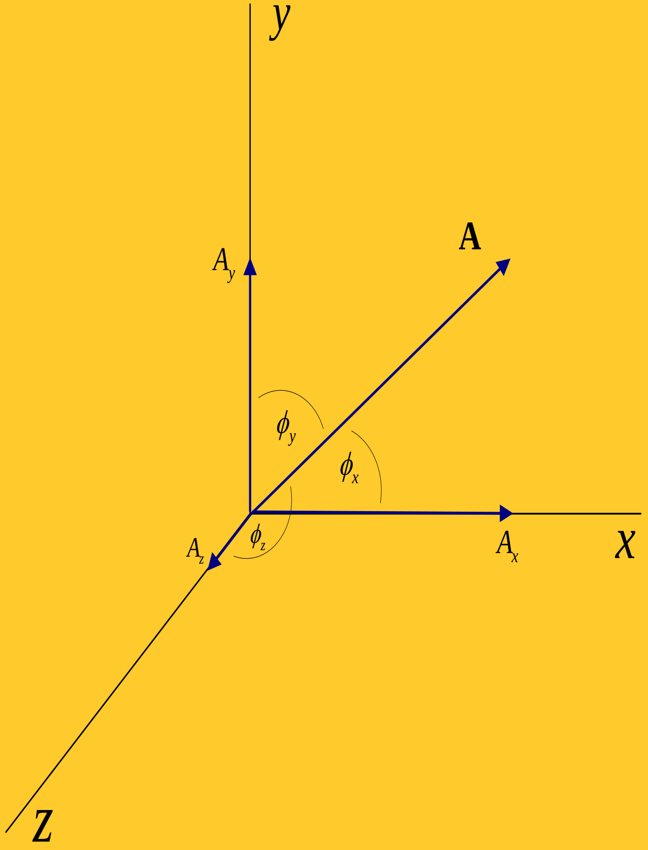
$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$A = (A_x, A_y, A_z)$$

(A_x, A_y, A_z are scalars)

Magnitude or Absolute Value:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Example:

$$\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$$

$$F = \sqrt{(3)^2 + (4)^2 + (-12)^2}$$

$$= 13 \text{ N}$$

Vector Representation

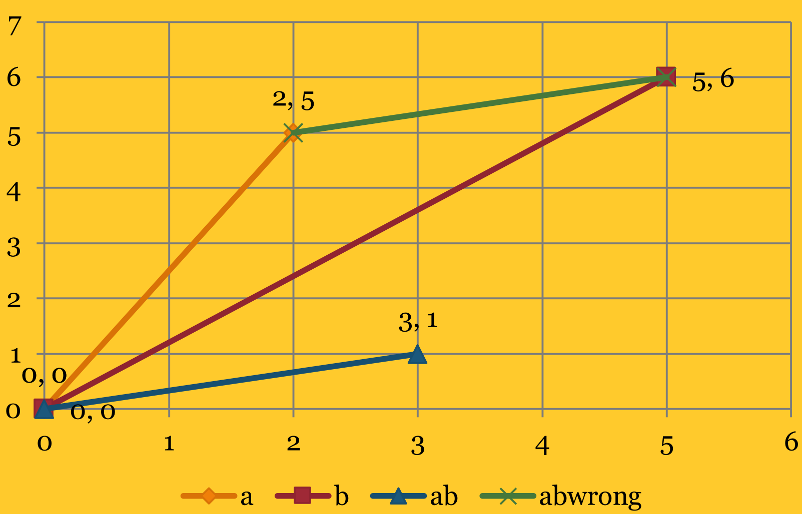
- Vectors are represented by (x, y) in 2d and (x, y, z) in 3d
- Vectors has an initial point represented by (x_1,y_1) or (x_1,y_1,z_1) and end point (x_2,y_2) or (x_2,y_2,z_2).

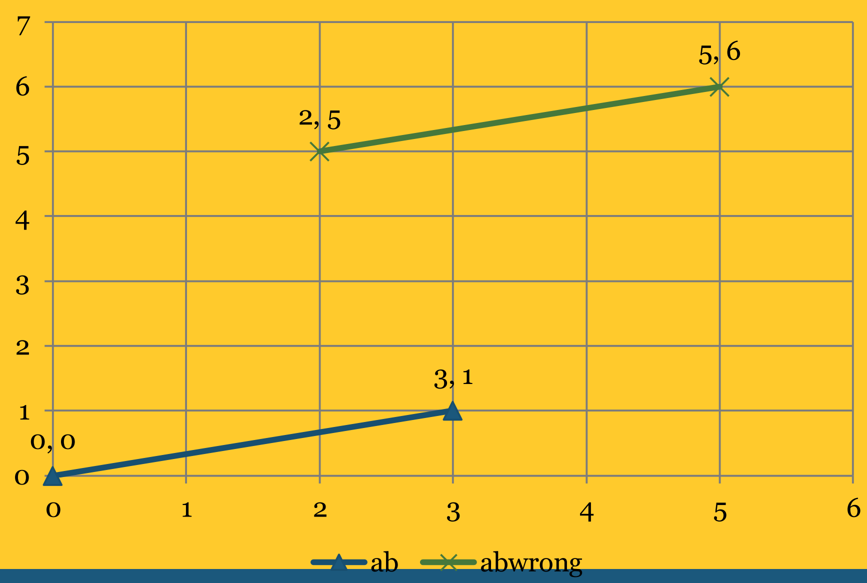
- Then how the vectors represented by (x,y) or (x,y,z)

- Example: Point A=(2, 5), B=(5,9).
- What is the vector AB?

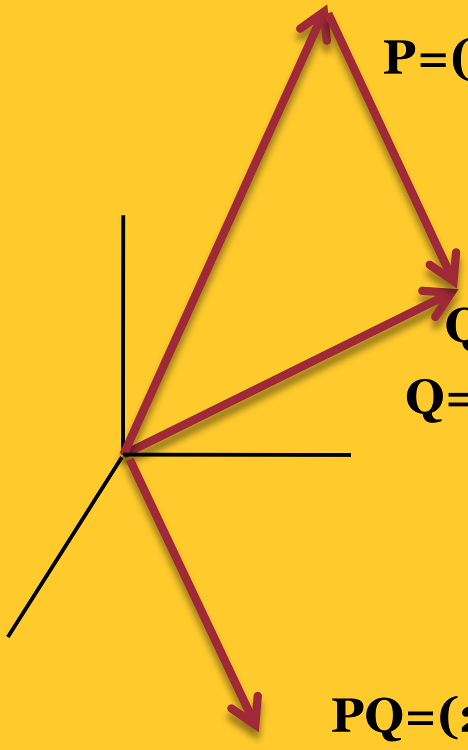
Vector Representation

- Example: Point a=(2, 5), b=(5,6).
- What is the vector ab?
- Answer: The given vector is (5-2),(6-5)) =(3, 1)
- The given vectors are shown below:





Vector in 3d



- Vector PQ

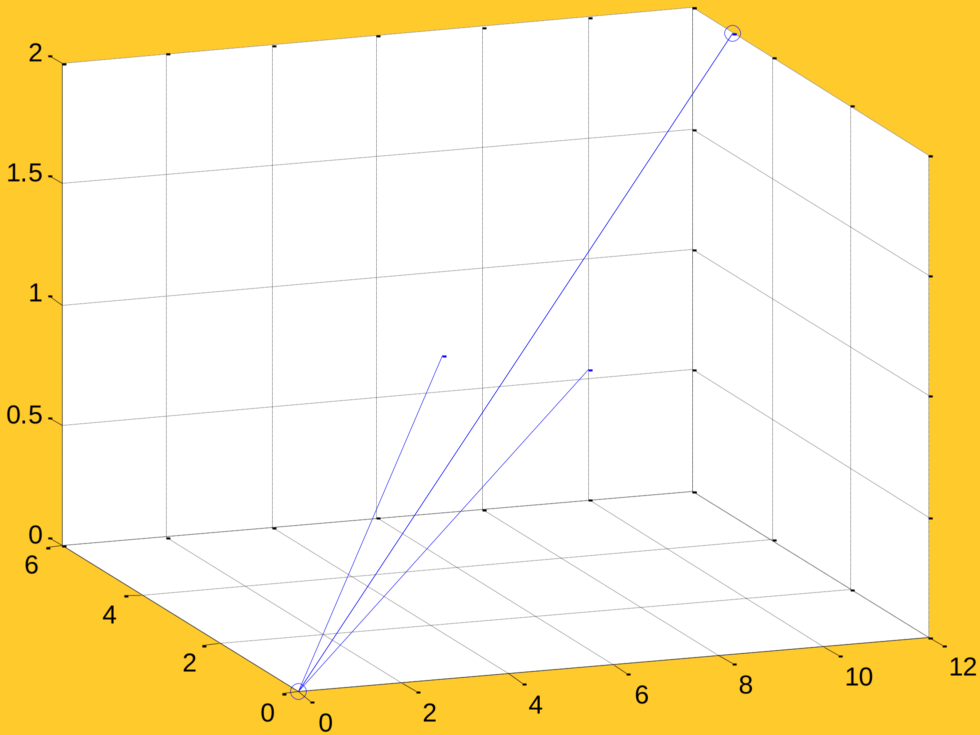
=(5-7, 3-1, 7-1)= (-2, 2, 6)

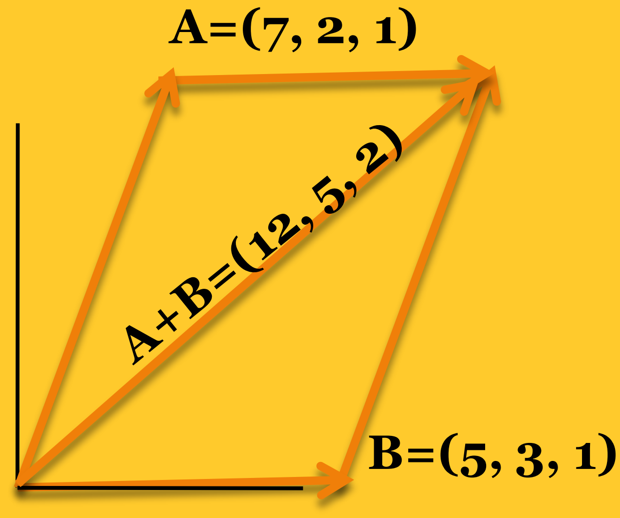
- PQ=(5-7)i+(3-1)j+(7-1)k= -2i+2j+6k

- What is the Initial Point of PQ??

Vector Operations-Addition

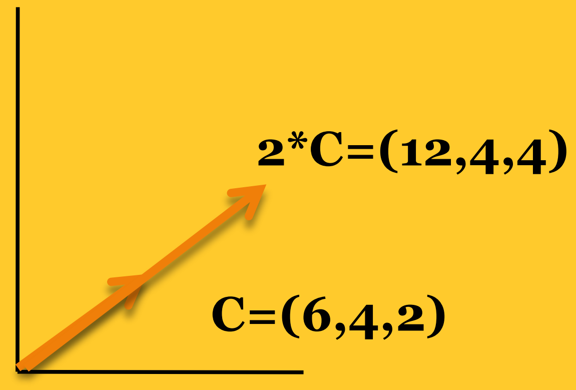
- Addition: **A+B**





Scalar Multiplication

- Multiplication of a vector by a scalar: 2*C
- =2*(6,4,2)
- =(12,8,4)
- Scalar multiplication of a vector, makes it larger and smaller.
- This is a major topic in eigen value problems.



Product of two vectors

- Points to remember:
- Product of two numbers is a number.
- Product of two matrices is a matrix
- Functions are multiplied in two ways-elementwise or compositionwise.
- Similarly, product of two vectors are done in two ways-scalar product and vector product.

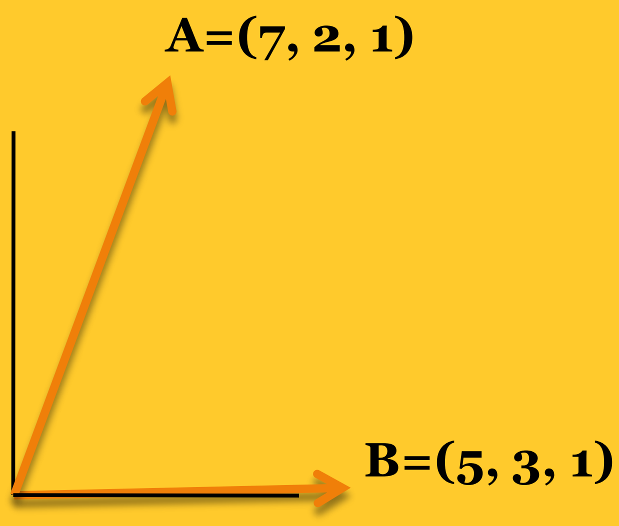
Vector Operations-Dot Product

- A=[xa, ya,za], B=[xb, yb, zb]
- Scalar or Dot Product: **A•B**

a.b=xa*xb+ya*yb+za*zb

$$\begin{pmatrix}7 & 2 & 1\end{pmatrix} * \begin{pmatrix}5\\3\\1\end{pmatrix} = 7*5+2*3+1*1 = 42$$

Result is a Scalar



Multiply Column-wise and then add
7 2 1
5 3 1
35 +6+1 = 42

Dot Product (Output is Scalar)

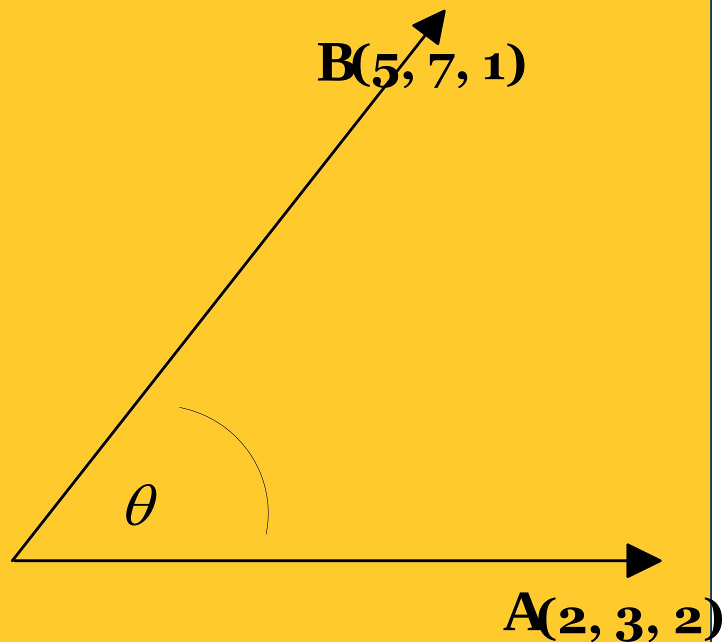
Definition:

$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$$

Computation:

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

$$A.B = \begin{bmatrix} 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 2\times 5 + 3\times 7 + 2\times 1 \end{bmatrix} = 33$$



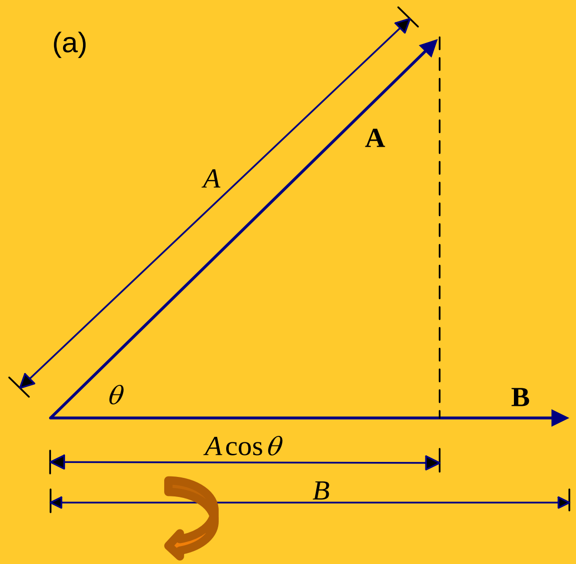
(2i+3j+2k).(5i+7j+1k)=10i^2+15 ij+10kj+14ij+21j^2+14kj+2 ik+3 jk+2 k^2
=10 i^2+21 j^2+2 k^2 =10+21+2=33
As ij=jk=ki=ik=kj=ji=0 and i^2=j^2=k^2=1

$$(2i+3j+2k).(5i+7j+1k) = (10i^2 + 21j^2 + 2k^2) = 33$$

$$(\because i^2 = j^2 = k^2 = 1)$$

Note: Because cos 0=1, cos 90 = 0

Geometrical Interpretation of Dot Product



Projection of **A** on **B**

Projection of A on B=|A| cos(t)

Projection of a vector on a line

Magnitude of projection of a = |a| cos (t)

Projection of a vector on a vector

Magnitude of projection of a = |a| cos (t)

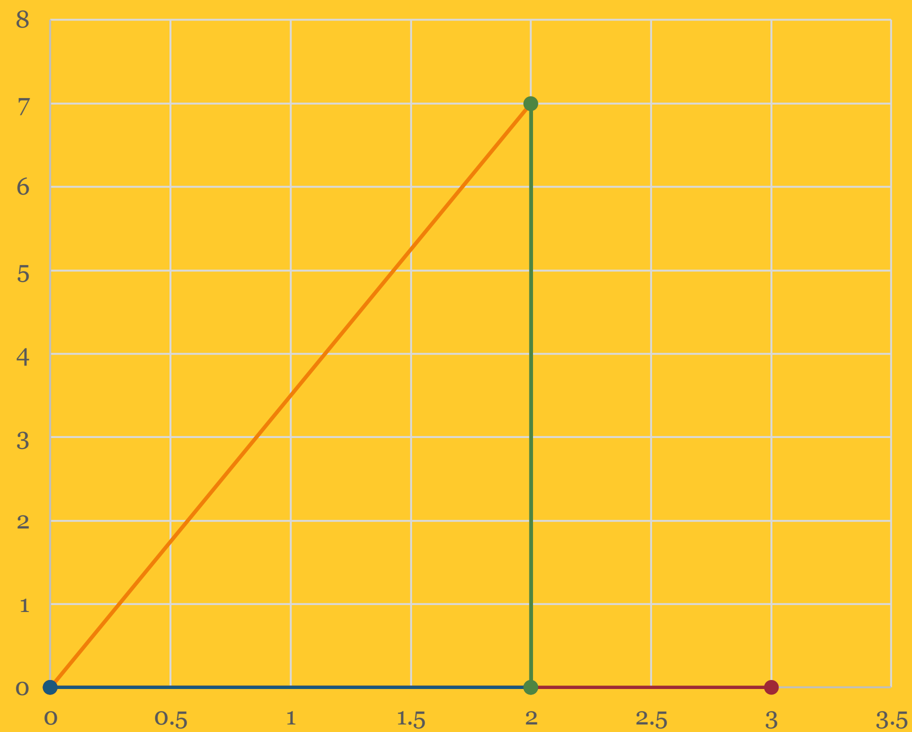
We know that a.b=|a|.|b| cos(t)
or Magnitude of Projection of a on b, |a| cos(t) = a.b/|b|

The projection vector of a on b is p=a.b/|b|*b/|b|

Basis: Unit vector in the direction of b and dot product of a and unit vector in direction of b

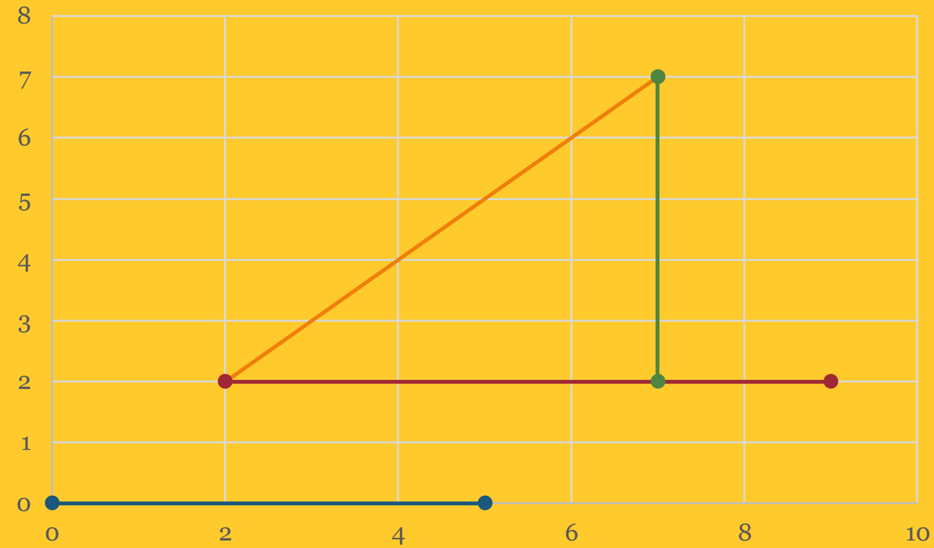
Projection of a vector on a vector

Projection of a on b					
	a	0	0		
	a	2	7	|a|=	7.28011
	b	0	0		
	b	3	0	|b|=	3
	unit b	1	0		
	a*b	6	0		
	a.b	6			
	cos t=	0.274721			
	a cos t=	2			
	p_vector	2	0		
		0	0		
		2	7		
		2	0		



Projection of a vector on a vector

Projection of a on b					
	a	5	5		
	b	7	0		
	a	2	2		
	a	7	7	|a|=	7.071068
	b	2	2		
	b	9	2	|b|=	7
	unit b	1	0		
	a*b	35	0		
	a.b	35			
	cos t=	0.707107			
	a cos t=	5			
	p_vector	5	0		
		0	0		
		7	2		
		7	7		



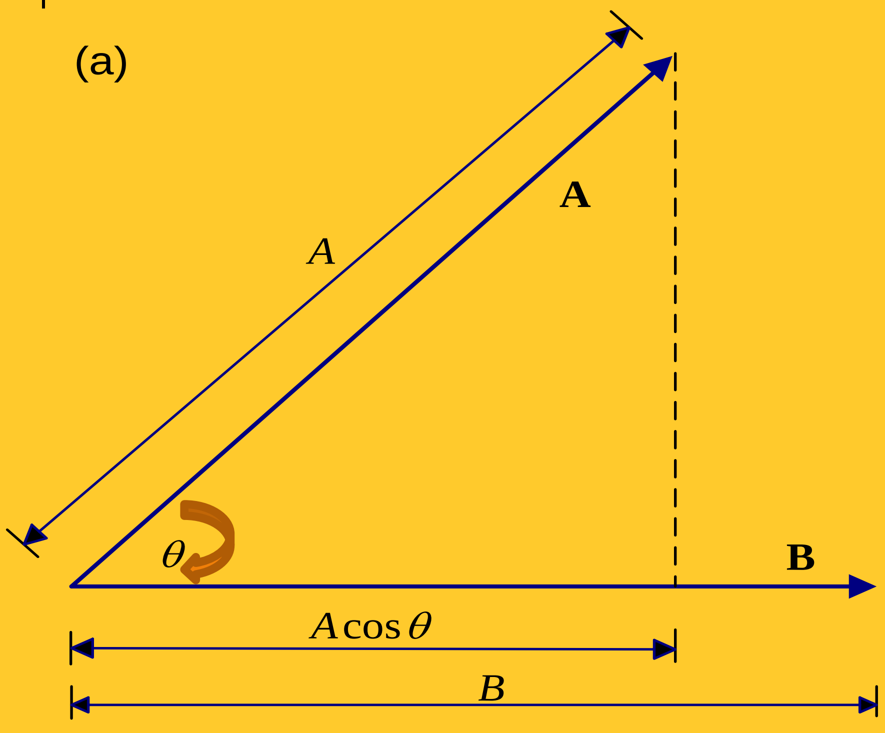
Finding Unit Vector

- Let v=xi+yj+zk
- Then |v|=sqrt(x^2+y^2+z^2)
- Unit Vector v = xi/|v|+yj/|v|+zk/|v|

- Unit vector is required for calculating Cross Product of Two Vector, finding projection vector, etc.

Finding Angle between two vectors from Dot Product

Step-1: Find the Magnitude of both the vectors.
Step-2: Find the dot product of the vectors
Step-3: Calculate cos(t)=A.B/|A|*|B| =x
Step-4: Calculate t=acos(x)



To calculate Projection of A on B, A cos (t) use the formula
cos(t)=A.B/|A|*|B|
Or |A| cos(t) = A.B/|B|

To find the coordinates of Projection Vector,

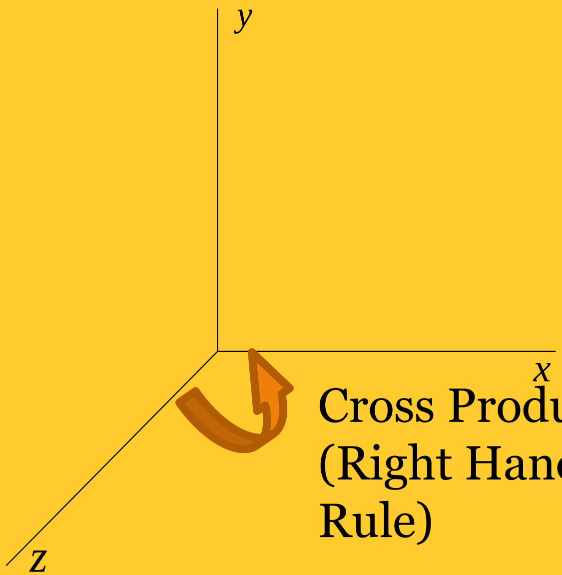
V=A.B/|B|*B/|B|

Cross Product (Vector Product)

Definition:

$$\mathbf{A} \times \mathbf{B} = \left(AB \sin\theta \right) \mathbf{u_n}$$

u is the vector perpendicular to plane of A and B vectors



Cross Product (Right Hand Rule)

Computation:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

={(ya*zb-za*yb),-(xa*zb-xb*za), (xa*yb-ya*xb)}

Computation of Cross Product

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & -2 & 1\\ 3 & 4 & 12\end{vmatrix}$$

A= (2, -2, 1), B=(3, 4, 12)

$$\mathbf{A}\times\mathbf{B}=\left[(-2)(12)-(1)(4)\right]\mathbf{i}-\left[(2)(12)-(1)(3)\right]\mathbf{j}$$

$$+\left[(2)(4)-(-2)(3)\right]\mathbf{k}$$

$$=-28\mathbf{i}-21\mathbf{j}+14\mathbf{k}$$

AXB=[-28, -21, 14]

Vector Cross Product

- AxB=|A| |B|sin (t) n (n=unit vector along normal)

Observations:

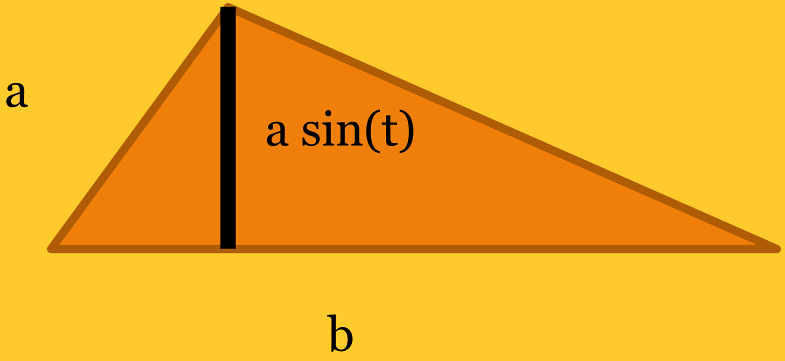
1. axb is a vector
2. axb is 0 iff the two vectors are parallel or collinear
3. If t=90 then axb=|a||b|n
4. ixi=jxj=kxk=0 and ixj=k, jxk=i, kxi=j
5. sin(t)=|axb|/|a||b|
6. ixj#jxi

Finding Area of a Triangle

- $A \times B = |A|\,|B| \sin(t)\, n$ (n=unit vector along normal)

Observations:

7. Area of the triangle formed by the vectors as adjacent sides is ½*|axb|=1/2*|a||b||sin(t)



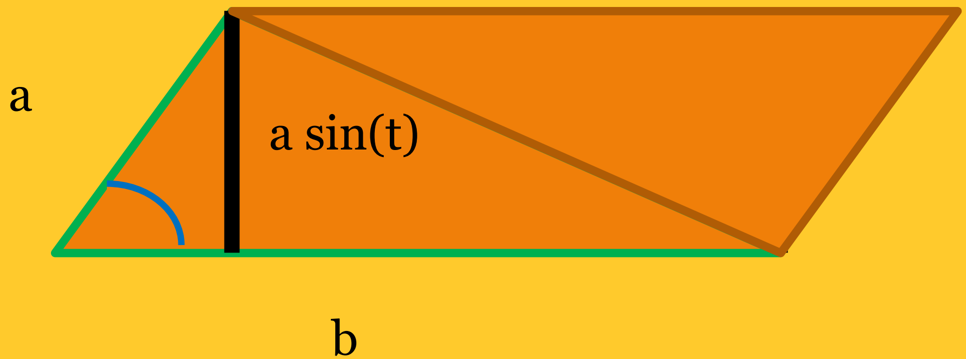
From the formula:
Area=1/2 base x Height

Vector Product

- AxB=|A| |B|sin (t) n (n=unit vector along normal)

Observations:

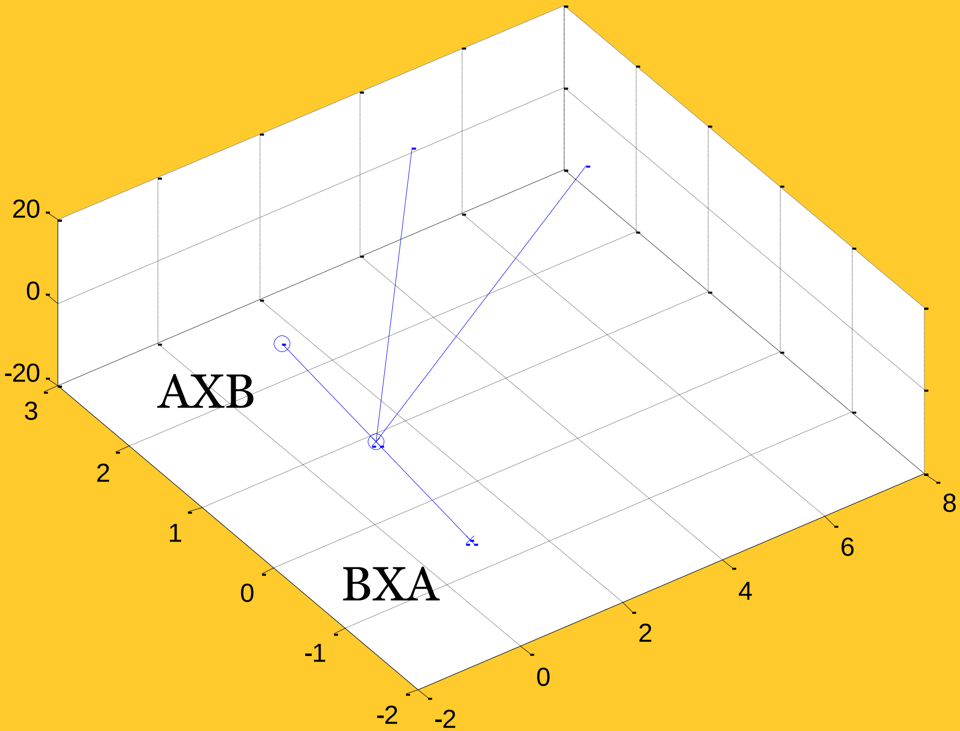
7. Area of the parallelogram formed by the vectors as adjacent sides is |axb|=|a||b||sin(t)

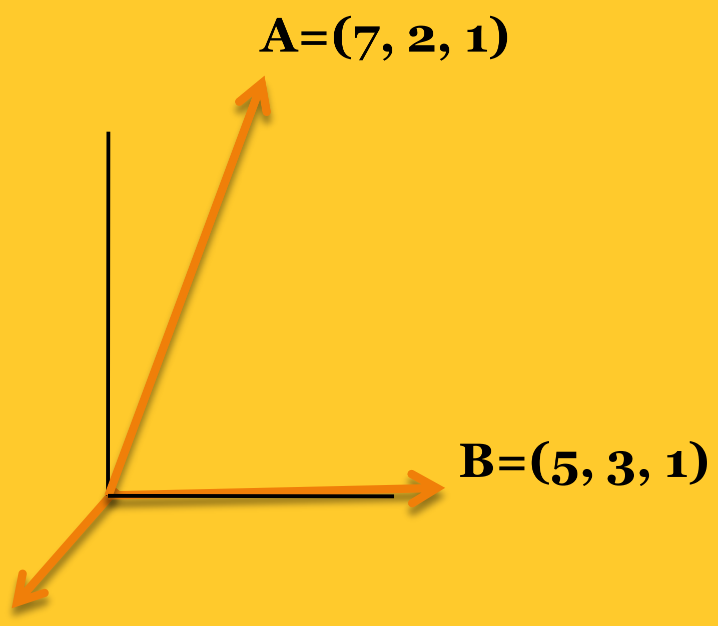


From the formula:
Area= Base x Height

Vector Operations-Cross Product

- Vector or Cross Product: **AxB**





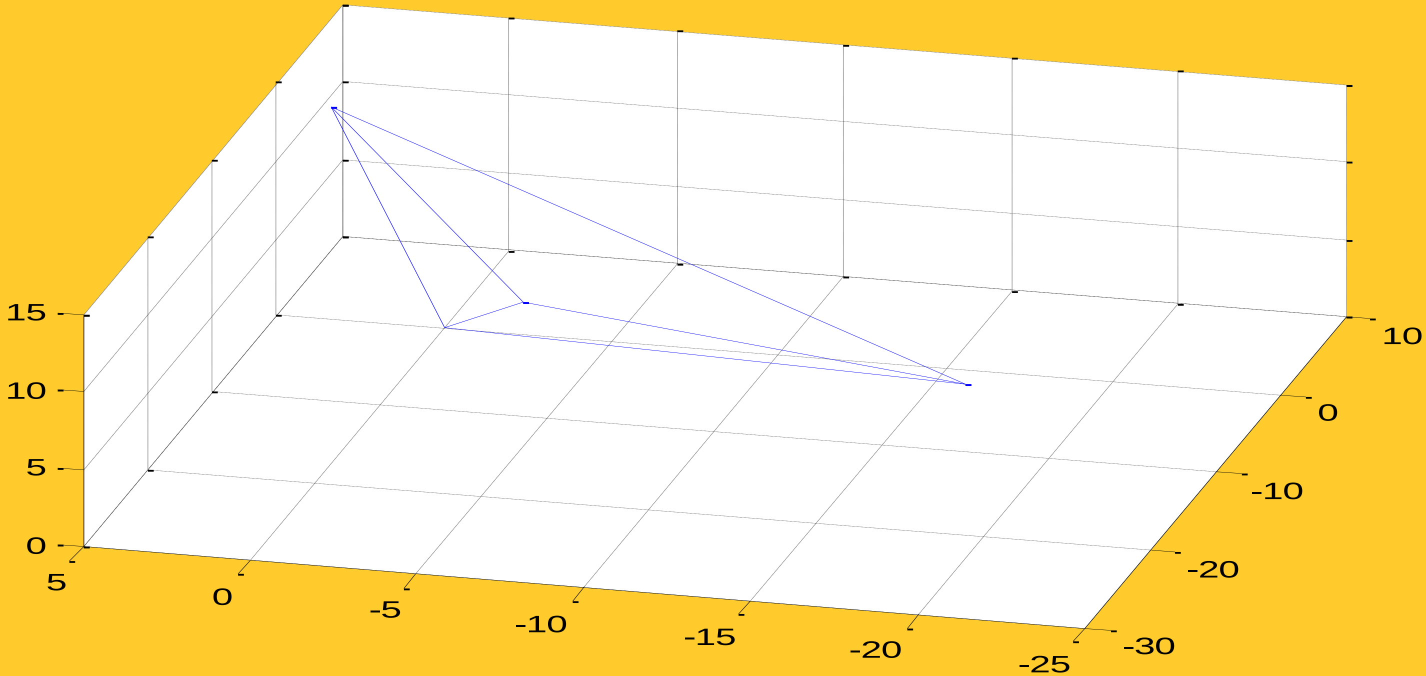
NON-COMMUTATIVE

Scalar Triple Product

- Triple Scalar Product: (**AxB)•C**
- **=(7, 2, 1)X(5,3,1).(6,4,2)**
- **=(-1,-2,11).(6,4,2)**
- **=8**
- **This triple product will generate a scalar equal to the volume of the parallelepiped formed by the vectors**

Cross Product of Previous Problem

2	**-2**	**1**
3	**4**	**12**
-28	**-21**	**13**



Magnitude of All Vector

2	**-2**	**1**
3	**4**	**12**
-28	**-21**	**13**

A=3
B=13
C=37.3363

Dot and Cross Product Formulas

xa	ya	za
2	3	1
xb	yb	zb
3	5	2

a.b=xa*xb+ya*yb=21

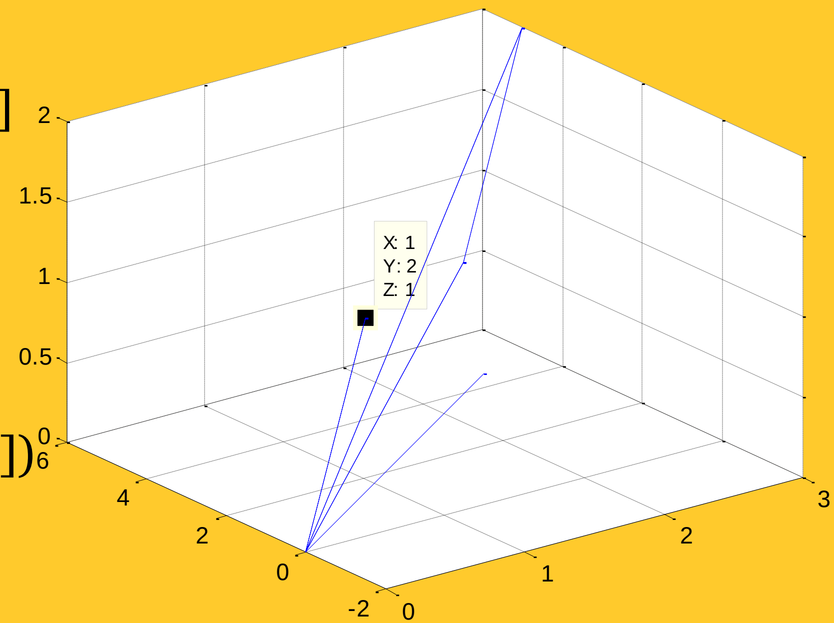
a.b=xa*xb+ya*yb+za*zb=23

Axb=(x,y,z)
={(ya*zb-za*yb),-((xa*zb-xb*za), (xa*yb-ya*xb)}=(1,-1,1)

Angle from Dot Product

Dot Product of two vectors A and B gives the angle formed by the vectors with MATLAB command

```
a=[2,3,1]
b=[3,5,2]
ab=[b(1)-a(1), b(2)-a(2), b(3)-a(3)]
adotb=dot(a,b)
axb=cross(a,b)
plot3([0,a(1)],[0,a(2)],[0,a(3)])
hold on
plot3([0,b(1)],[0,b(2)],[0,b(3)])
plot3([0,ab(1)],[0,ab(2)1],[0,ab(3)])
grid
absa=norm(a)
absb=norm(b)
angle=acos((adotb)/(absa*absb))
t=angle*180/pi
```



Angle from Dot Product

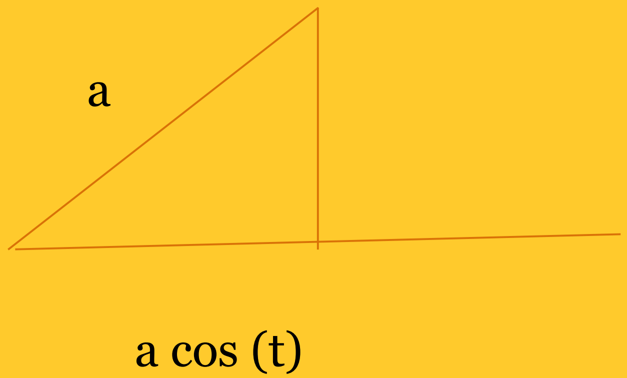
Dot Product of two vectors A and B gives the angle formed by the vectors with MATLAB command

```
a=[2,3,1]
b=[3,5,2]
ab=[b(1)-a(1), b(2)-a(2), b(3)-a(3)]
adotb=dot(a,b)
axb=cross(a,b)
plot3([0,a(1)],[0,a(2)],[0,a(3)])
hold on
plot3([0,b(1)],[0,b(2)],[0,b(3)])
plot3([0,ab(1)],[0,ab(2)1],[0,ab(3)])
grid
absa=norm(a)
absb=norm(b)
angle=acos((adotb)/(absa*absb))
t=angle*180/pi
```

```
a =      2    3    1
b =      3    5    2
ab =     1    2    1
adotb =    23
axb =  1   -1    1
absa =   3.7417
absb =   6.1644
angle =  0.0752
t =   4.3066
```

Projection of a vector on a line

If the vector **a** makes an angle t with the directed line in anticlockwise direction then the projection vector of a on l is a*cos(t)

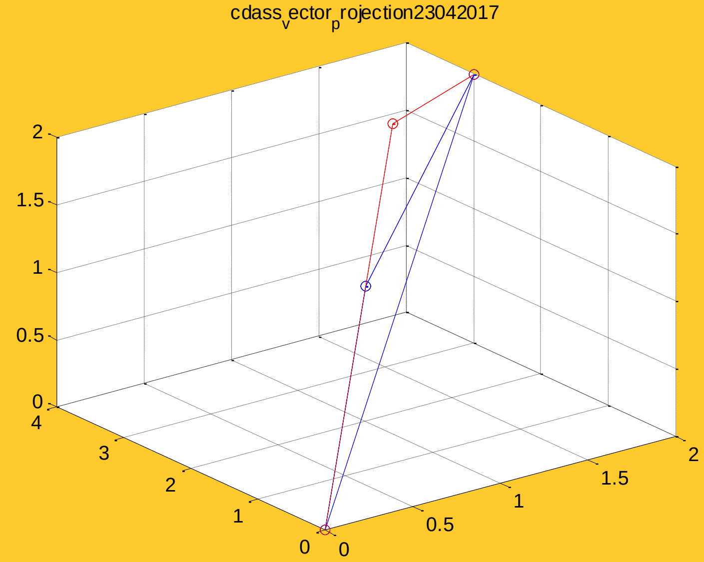


Observation: Projection of a vector a on vector b is |a| cos(t)=a.b/|b|

Now, if t =0, then va=a, if t=180, then va=-a, if t=pi/2, or t=3pi/2, va=0

Example-16

- Find the projection of a vector a = 2i+3j+2k on the vector b=i+2j+k
- The projection of vector a, |a|cos(t)=a.b/|b|



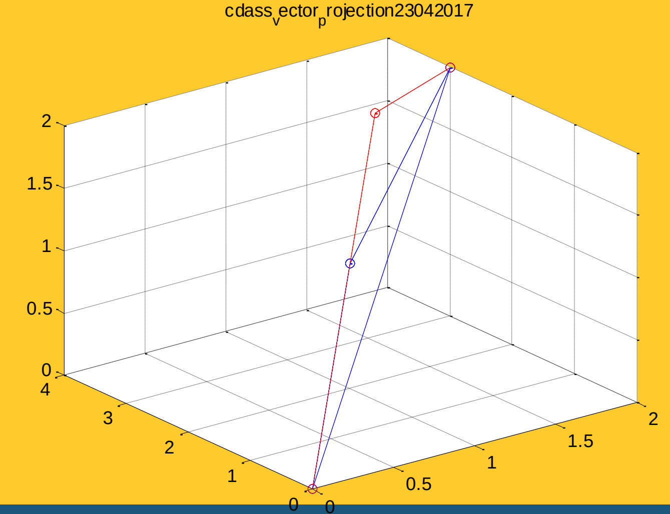
- Projection vector of a on b
- |a| cos(t)*[b/|b|]
- =(a.b/|b|)*[b/|b|]

Example-16

- Find the projection of a vector a = 2i+3j+2k on the vector b=i+2j+k
- The projection of vector a, |a|cos(t)=a.b/|b|

a = 2 3 0
b = 1 2 0
moda = 3.6056
modb = 2.2361
adotb = 8
acrossb = 0 0 1
projaonb = 3.5777
projanb = 1.6000 3.2000 0

- Projection vector of a on b
- |a| cos(t)*[b/|B|]
- =(a.b/|b|)*[b/|b|]

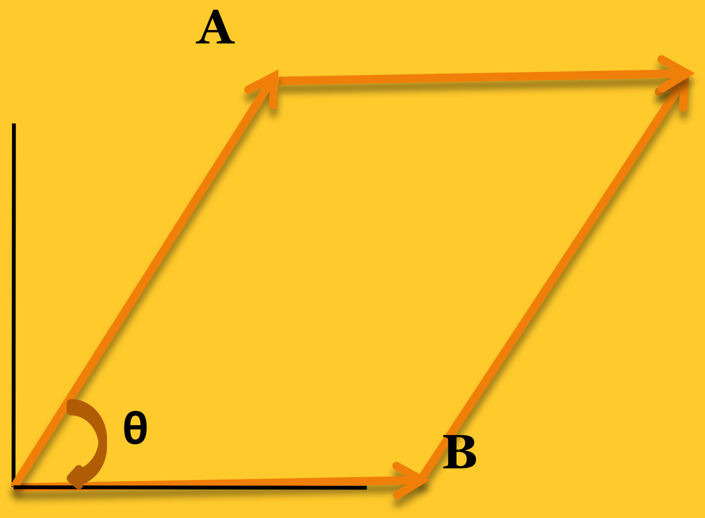


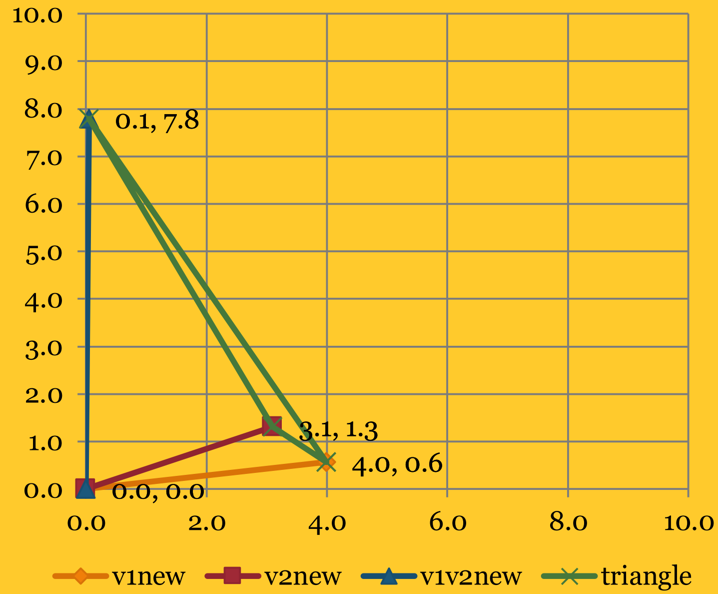
Magnitude of vectors

$|a.b| >= |a| * |b|$

a=[2,3,1]
b=[3,5,2]
|a|=3.741
|b|=6.165
a.b=23
|a|*|b|=23.06

Geometrical Interpretation of Cross Product





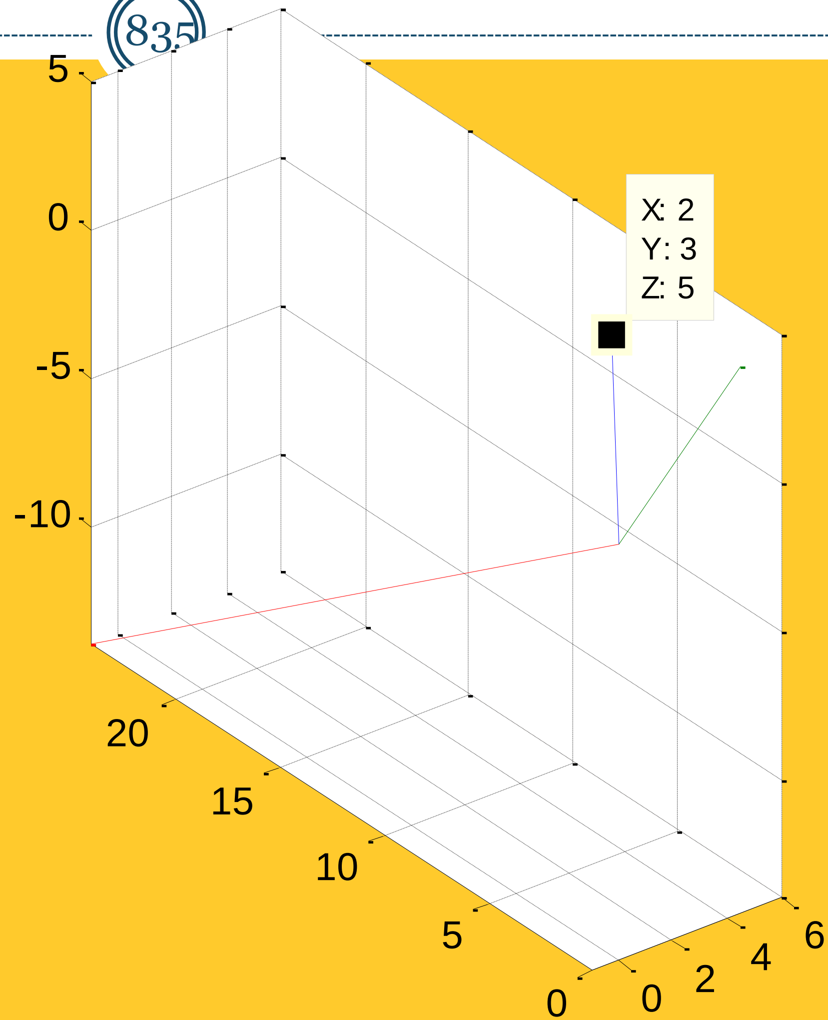
Cross Product of two vectors A and B gives the value of area of parallelogram formed by A and B as adjacent sides

DOT and CROSS product

- x=[2 3 5]
- y=[6 2 3]
- dotxy=dot(x,y)
- cxy=cross(x,y)

Result

- dotxy =33
- cxy = -1 24 -14



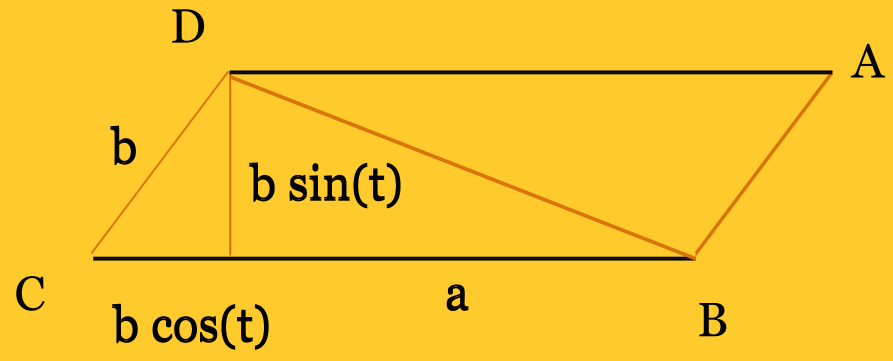
How Vector Product Finds Area?

- Important note:
- Vector product axb is only defined when a=(a1,a2,a3) and b = (b1,b2,b3) have three elements or three dimension vectors
- axb=(a2b3-a3b2, a3b1-a1b3, a1b2-a2b1)
- |axb|^2=(a2b3-a3b2)^2+(a3b1-a1b3)^2+ (a1b2-a2b1)^2
- |axb|^2=a2^2 b3^2 +a3^2 b2^2-2*a2a3b2b3+a3^2b1^2+a1^2b3^2-2a1a3b1b3+a1^2 b2^2 +a2^2 b1^2-2a1a2b1b2
- |axb|^2=(a1^2+a2^2+a3^2)(b1^2+b2^2+b3^2)-(a1b1+a2b2+a3b3)^2
- |axb|^2 =|a|^2||b|^2-(a.b)^2
- |axb|^2 =|a|^2||b|^2-|a|^2|b|^2 cos(t)^2
- |axb|^2 =|a|^2||b|^2 (1-cos(t)^2)
- |axb|^2 =|a|^2||b|^2 sin(t)^2)
- |axb|=|a||b|sin(t)
- axb is a vector perpendicular to a and b and its length is |axb|=|a||b|sin(t)

Finding the Area

- We can find the area of the quadrilateral formed by the vectors a and b.

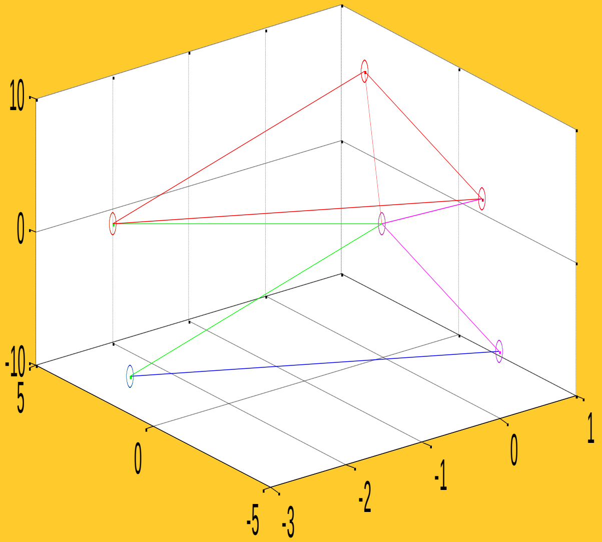
- Area=|axb|=|a||b|sin(t)



- Area of a rectangle=Length x Height
- In ABCD, Length=|a|, Height=|b| sin(t)

Finding area in 3d

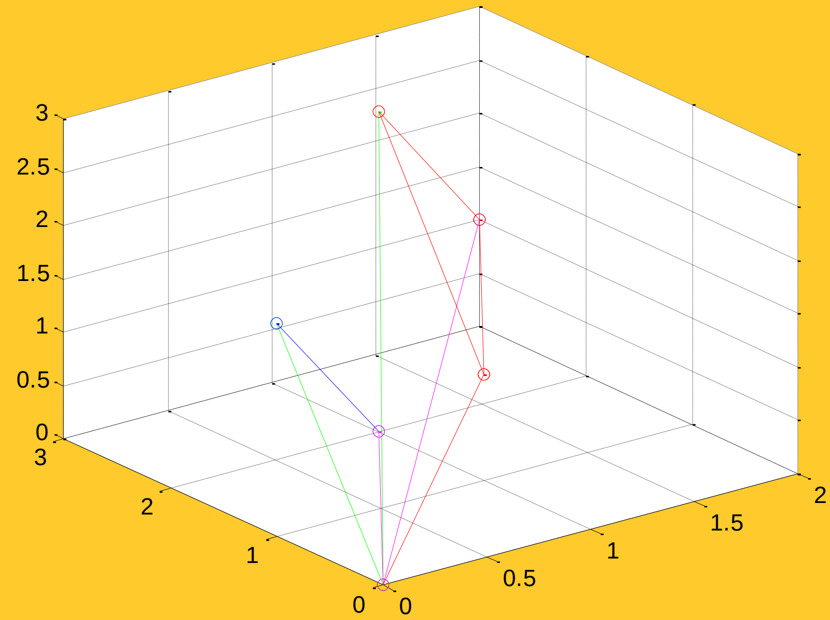
p	1	4	6
q	-2	5	-1
r	1	-1	1



p = 1 4 6
q = -2 5 -1
r = 1 -1 1
pq = -3 1 -7
pr = 0 -5 -5
qr = 3 -6 2
normpr = 7.0711
normpq = 7.6811
cost = 0.5523
t = 0.9856
st = 0.8336
area =1/2* 45.2769

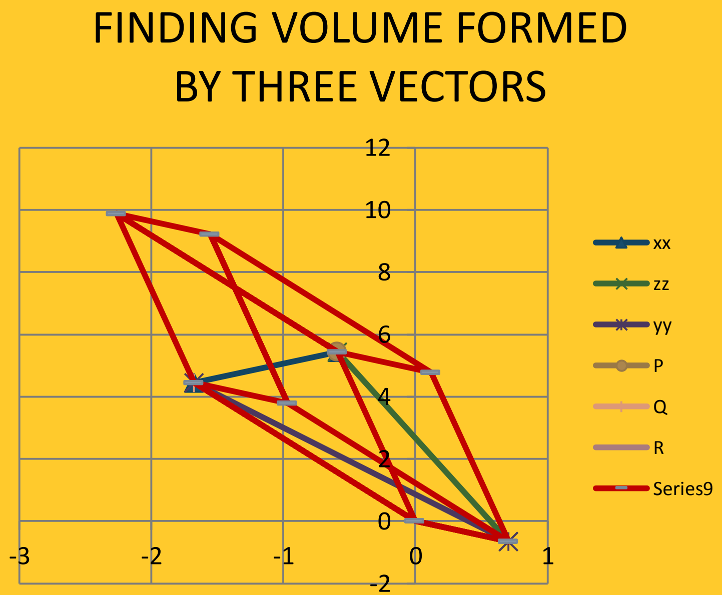
Example-22

p = 1 1 1
q = 1 2 3
r = 2 3 1
pq = 0 1 2
pr = 1 2 0
qr = 1 1 -2
normpr = 2.2361
normpq = 2.2361
cost = 0.4000
t = 1.1593
st = 0.9165
area = 4.5826
areab=norm(cross(pq,pr))

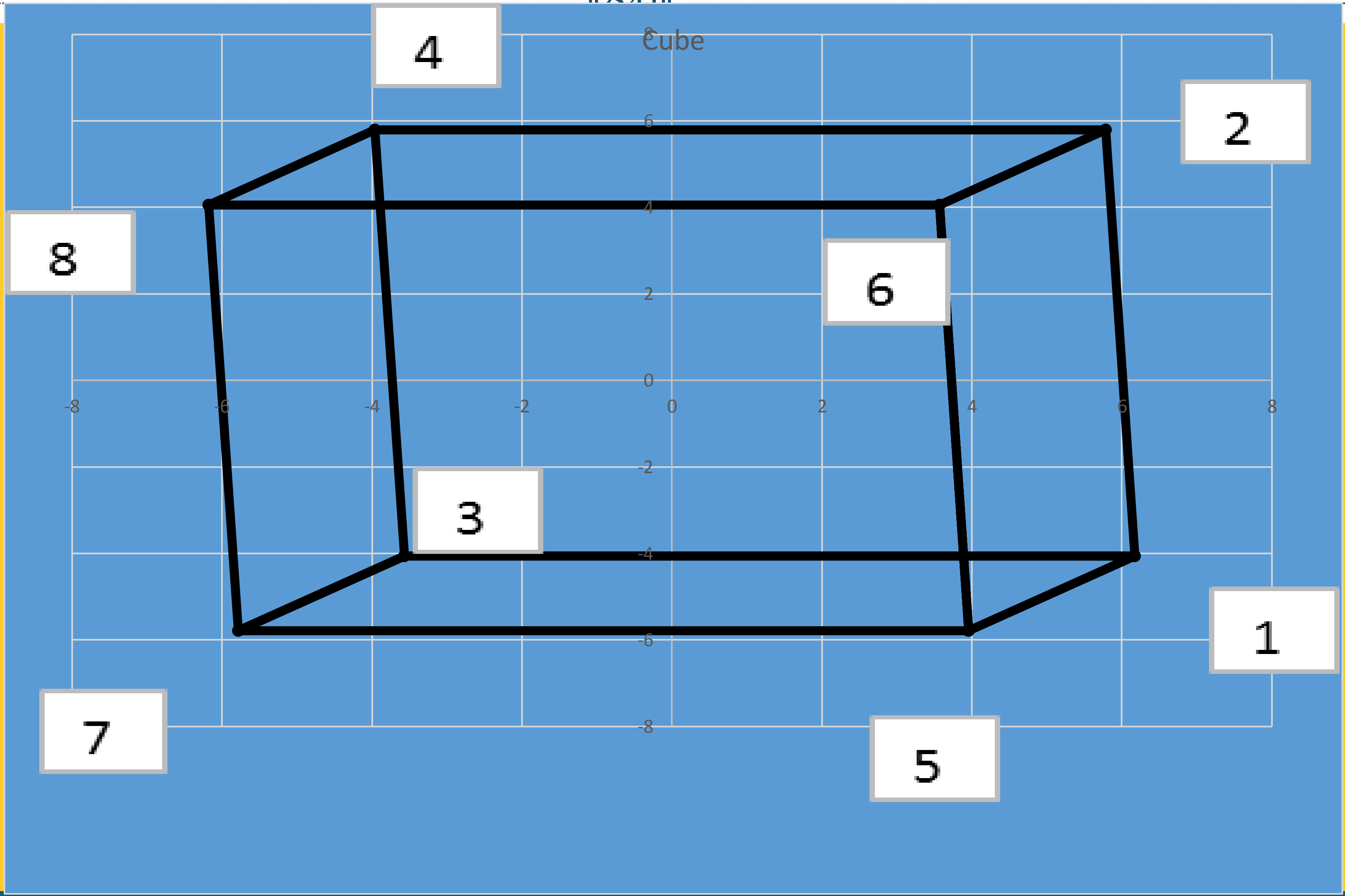


Finding Volume

p	1	4	6
q	-2	5	-1
r	1	-1	1



Volume Calculation



Volume Calculation

P=8	1	4	6
Q=5	-2	5	-1
R=3	1	-1	1
O=7	0	0	0
4=3+8	2	4	7
1=3+5	-1	4	0
2=1+8	0	8	8
6=5+8	-1	9	5

There are eight vertex in a cube. Four Vertex are given. We have to calculate the coordinates of remaining vertices

7	0	0	0	1
5	-2	5	-1	1
6	-1	9	5	1
8	1	4	6	1
7	0	0	0	1
3	1	-1	1	1
1	-1	4	0	1
2	0	8	6	1
4	2	3	7	1
8	1	4	6	1
6	-1	9	5	1
2	0	8	6	1
1	-1	4	0	1
5	-2	5	-1	1
7	0	0	0	1
3	1	-1	1	1
4	2	3	7	1

Torque

- Stewart Page 758: Ex-6:
- Force=40 NEWTON at 75 degree
- Distance= .25 m

- Here, magnitude of force and distance given.

- We know that magnitude of torque = |t|=|r||f|sin(t)
- |t|=40*.25*sin(75)=9.66 N.m (It is a scalar)
- Torque is |t|*unit vector in the direction of perpendicular to page.

Three Dimensional Geometry and Vectors

- In dealing with 3 dimensional geometry with Cartesian coordinate system, many time, it becomes difficult to analyse. Use of Vector makes the study simple and more effective.
- Topics covered:
  1. Direction cosines and direction ratios of a line
  2. Direction cosines and direction ratios of a line joining two points
  3. Equation of lines
  4. Equation of Planes
  5. Distance between lines
  6. Distance between a point and a plane

Direction Ratios

- Direction ratios provide a convenient way of specifying the direction of a line in three dimensional space.
- Direction cosines are the cosines of the angles between a line and the coordinate axes.
- Given a vector r = ai + bj + ck, its direction ratios are a : b : c.
- This means that to move in the direction of the vector we must move a units in the x direction and b units in the y direction for every c units in the z direction.

Direction cosines as l m n

- If a directed line passing through origin and makes angle $\propto, \beta, \gamma$ with x, y, z axis then these angles are called direction angles and cosine of these angles

$\cos(\propto)$, $\cos(\beta)$ *and* $\cos(\gamma)$ are called direction cosines.

- If we reverse the direction of the line, then the direction cosines will be

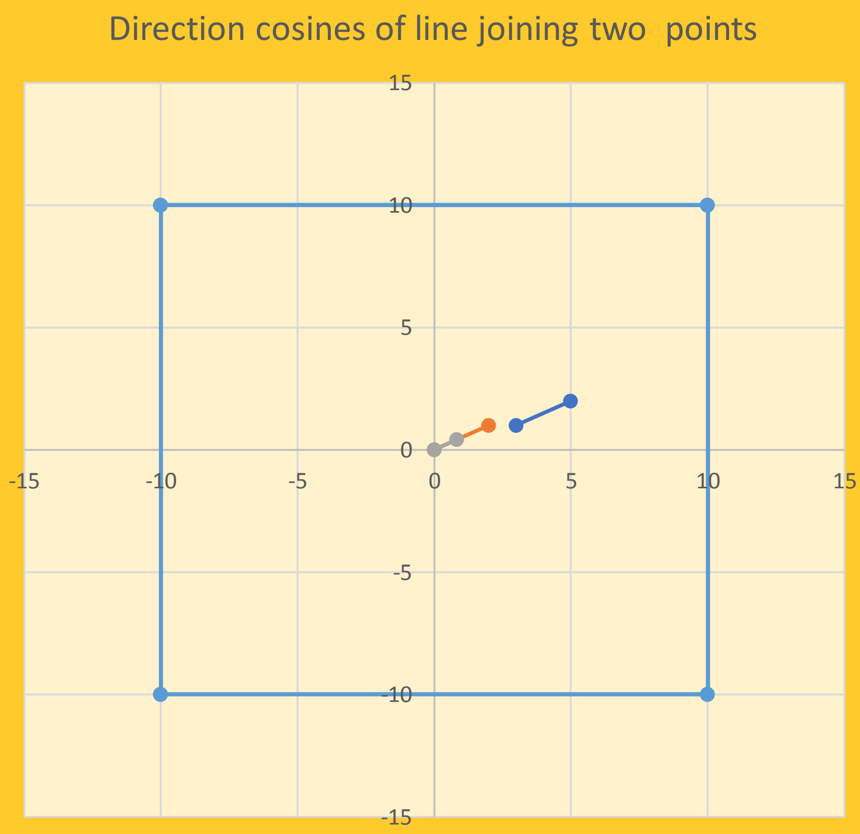
$\cos(\pi +\propto)$, $\cos(\pi + \beta)$ *and* $\cos(\pi + \gamma)$ and

$-\cos(\propto)$, $-\cos(\beta)$ *and* $-\cos(\gamma)$.

- As same line should not have two direction ratios, we take l, m, n as direction cosines.

Direction cosines of a line passing through 2 point

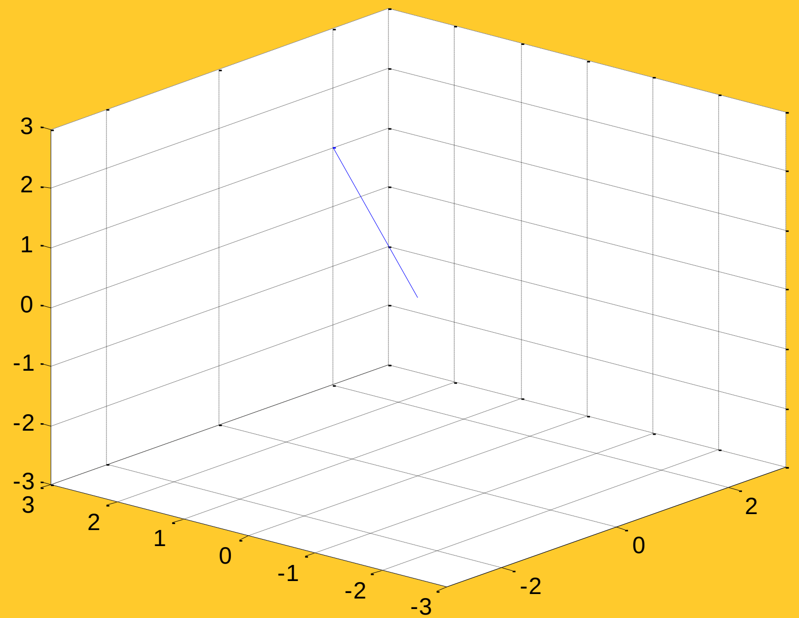
2	3	1
3	5	2
x	y	z
1	2	1
dist=	2.45	
0.41	0.82	0.41



Direction Cosines

a=[2,3,1]
|a|=3.741

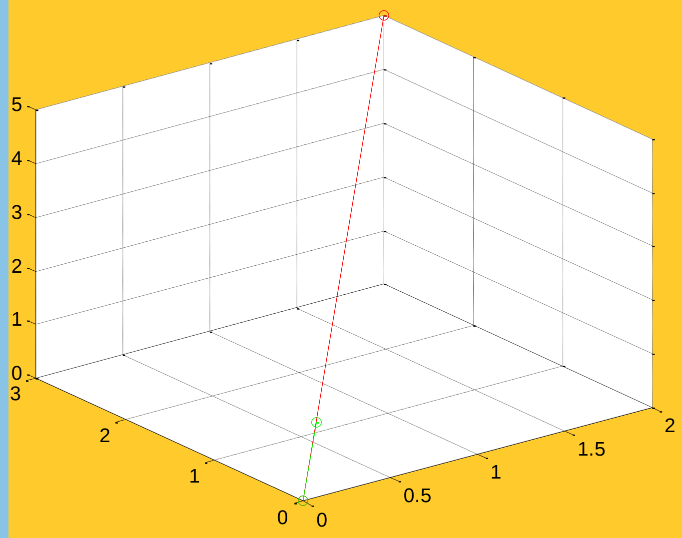
l=cos ∝=2/3.741
m=cos β=3/3.741
n=cos γ=1/3.741



Direction Ratios

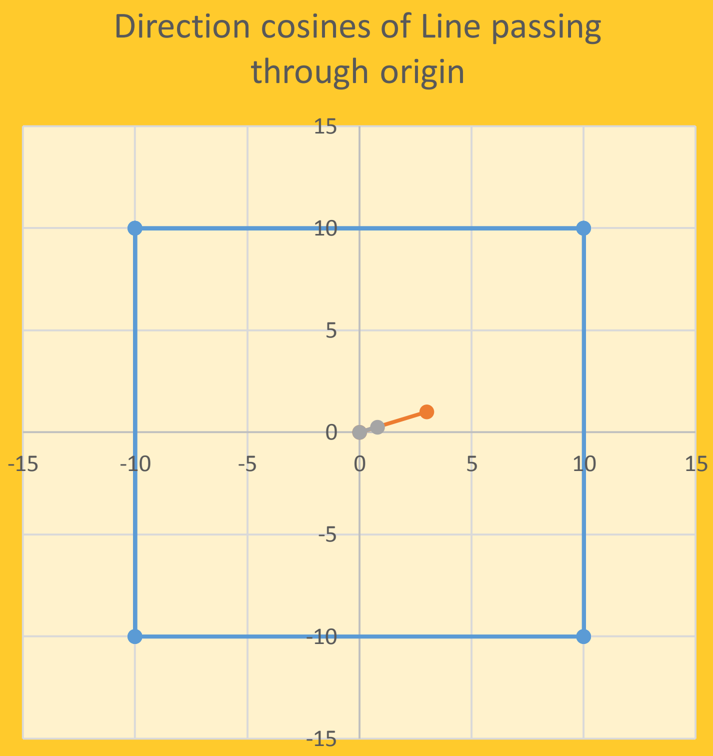
a=[2,3,5]
r=|a|=6.1644
l=cos ∝=2/6.1644
m=cos β=3/6.1644
n=cos γ=5/6.1644
dir_cosins = 0.3244 0.4867 0.8111
angles = 1.2404 1.0625 0.6248
angles_degree = 71.0682 60.8784 35.7958

Direction Angles= ∝, β, γ
Direction Cosines=l, m, n = x/|a|, y/|a|, z/|a|
l=x/r, m=y/r, n=z/r
Direction Ratios=x, y, z=2, 3, 5
x=l r, y=m r, z= n r



Direction cosines of a line

a	b	c
2	3	1
k=	3.741657	
l	m	n
0.534522	0.801784	0.267261



Equation of a line in space
(Vector representation of line)

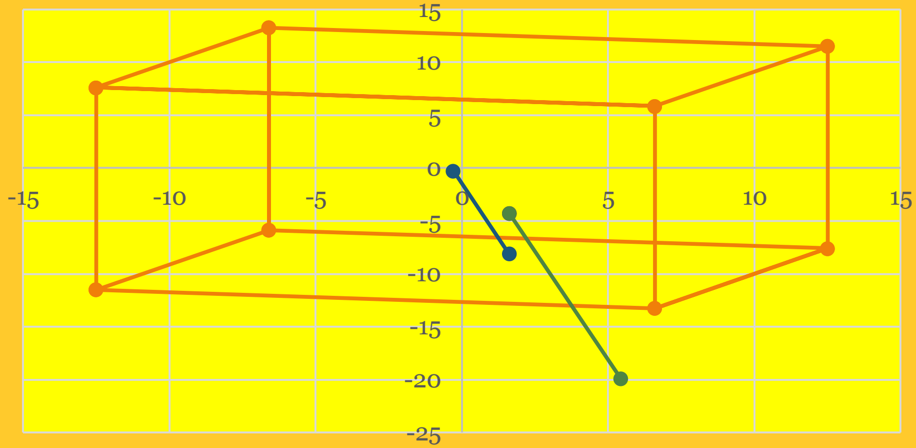
- A line in 3d is uniquely determined if

1. it passes through given point and has given direction
2. It passes through given points

Vector Equation of Line
Passing Through a point and parallel to a vector b

- A line passing through a given point and parallel to a given vector b:

a=given po	5	2	-4
b=given ve	3	2	-8
	0	0	0
r=a+λb	11	6	-20
	5	2	-4
	11	6	-20



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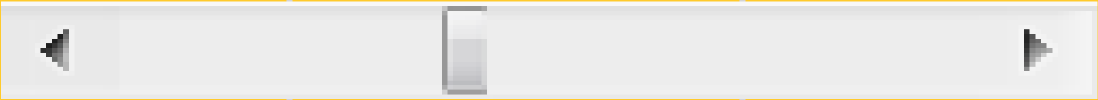


We are required to find a vector r which will represent a line
From Triangular law of vector addition,
λb=r-a or r=a+λb, λ is a parameter and can assume any arbitrary value.

Equation of Line
derivation of Cartesian form from vector form

Let the coordinate of given point A is (x0+y0+z0)
And Direction ratios of the line l are a, b, c

Then, a=x0i+y0j+z0k
b=ai+bj+zk
We have to find out, r=xi+yj+zk=(x0+ λ a)i+(y0+ λb)j+(z0+ λc)k

We know r=a+λb, λ is a parameter and can assume any arbitrary value.

Hence, x=x0+ λa
y=y0+ λb
z=z0+ λc
From this equations, we can write, (x-x0)/a=(y-y0)/b= (z-z0)/c= λ

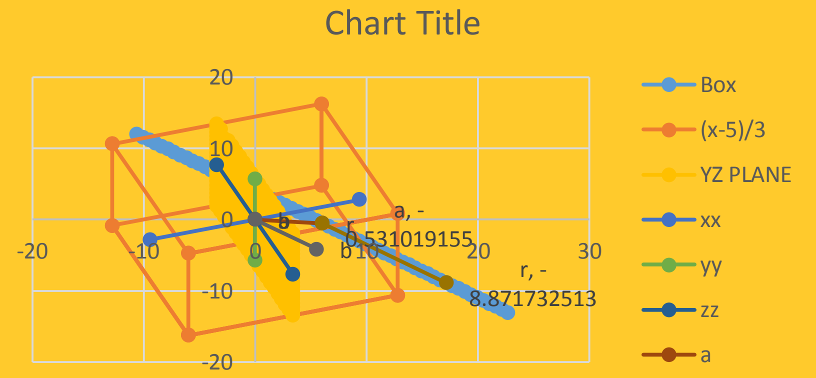
Example of the vector and Cartesian equation

- Find the vector and Cartesian equation of the line through the point a=(5, 2, -4) and which is parallel to the vector b=3i+2j-8k.

- Vector equation is $r=a+\lambda b$
- Hence, $r=5i+2j-4k+\lambda * (3i+2j-8k)$
- For Cartesian equation,
- $r=xi+yj+zk=(5+3\lambda)i+(2+2\lambda)j+(-4-8)k$
- $(x-5)/3 = (y-2)/2=(z+4)/-8=t$
- Parametric Equation: $x=3t+5$, $y=2+2t$. $z=-4+8t$

Example of the vector and Cartesian equation

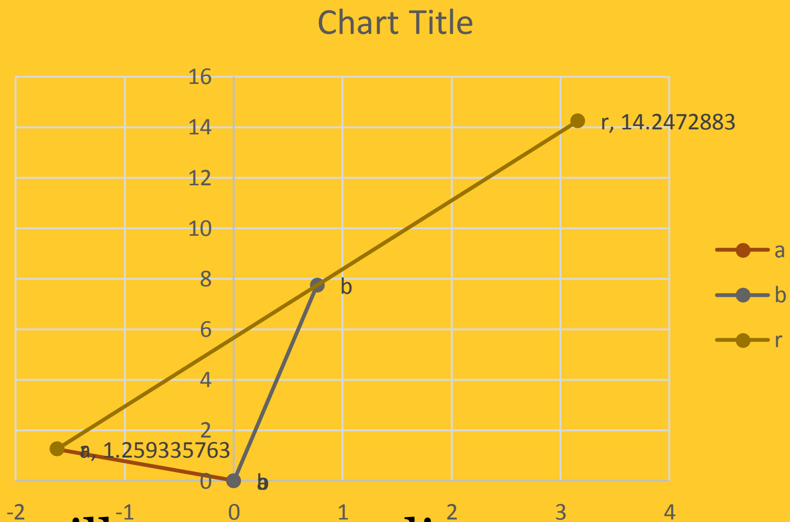
- Vector equation is $r = a + \lambda b$
- Hence, $r = 5i + 2j - 4k + \lambda * (3i + 2j - 8k)$
- For Cartesian equation,
- $r = xi + yj + zk = (5 + 3\lambda)i + (2 + 2\lambda)j + (-4 - 8)k$
- $(x-5)/3 = (y-2)/2 = (z+4)/-8$



Vector Equation of Line Passing Through Two points

- A line passing through a given point and parallel to a given vector b:

a=point1	-1	0	2
b=point2	3	4	6
	0	0	0
r=a+λ(b-a)			



We are required to find a vector r which will represent a line
From Triangular law of vector addition and laws of scalar multiplication, λ (b-a)= (r-a) or r=a+λ(b-a), λ is a parameter and can assume any arbitrary value.

Equation of Line
derivation of Cartesian form from vector form

Let the coordinate of given points are a=x1+y1+z1 and b=x2+y2+z2

Then, a=x1+y1+z1
b=x2+y2+z2
We know r=a+λ(b-a), λ is a parameter and can assume any arbitrary value.

We have to find out, r=xi+yj+zk= a+λ(b-a)
=(x1+ λ (x2-x1))i+(y1+ λ(y2-y1)j+(z1+ λ(z2-z1))k

Hence, x=x1+ λ(x2-x1)
y=y1+ λ(y2-y1)
z=z1+ λ(z2-z1)
From this equations, we can write, (x-x1)/(x2-x1)=(y-y1)/(x2-x1)= (z-z1)/(x2-x1)

Angle Between two Lines

Case-1: When lines pass from origin

Let l1=[a1, b1, c1] and l2=[a2, b2, c2] direction ratios

We know that the directed lines are vectors with components as a, b, c (a, b, c are direction ratios)

Now from dot product we can write, cos(t)=a.b/|a||b|

Or

cos(t)=|a1a2+b1b2+c1c2/((sqrt(a1^2+b1^2+c1^2)*sqrt(a2^2+b2^2+c2^2))|

Angle Between two Lines

Case-2: When lines do not pass from origin

Let l1=[a1, b1, c1] and l2=[a2, b2, c2] direction ratios

Here we will take two line pass through origin and parallel to given line.

We know that the directed lines are vectors with components as a, b, c (a, b, c are direction ratios)

Now from dot product we can write, cos(t)=a.b/|a||b|

Or

cos(t)=|a1a2+b1b2+c1c2/((sqrt(a1^2+b1^2+c1^2)*sqrt(a2^2+b2^2+c2^2))|

Angle Between two Lines

Case-3: When the direction cosines are given

Let l1=[l1,m1, n1] and l2=[l2, m2, n2] direction ratios

Here we will take two line pass through origin and parallel to given line.

We know that the directed lines are vectors with components as l,m,n

Now from dot product we can write, cos(t)=a.b/|a||b|

Or

cos(t)=|l1l2+m1m2+n1n2/((sqrt(l1^2+m1^2+n1^2)*sqrt(l2^2+m2^2+n2^2))|

Angle Between two Lines

Case-4: When the angle is 90 degree

$|l_1l_2+m_1m_2+n_1n_2|=0$

Case-4: When the angle is 0 degree, the

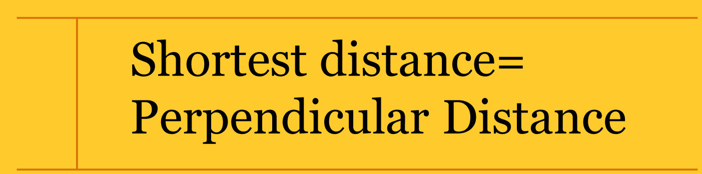
$a_1/a_2=b_1/b_2=c_1/c_2$

Shortest Distance Between Two Lines

- Case-1: When two lines intersect, then shortest distance is 0

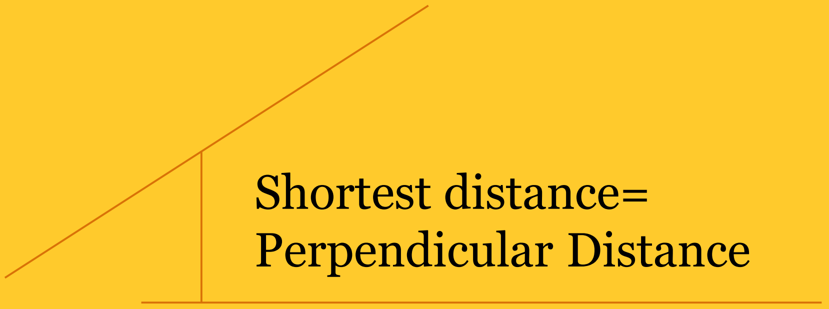
- Case-2: When two lines are parallel- Then the distance between them is the perpendicular distance. This the length of the perpendicular drawn from a point in one line on the other line.

Shortest distance=0



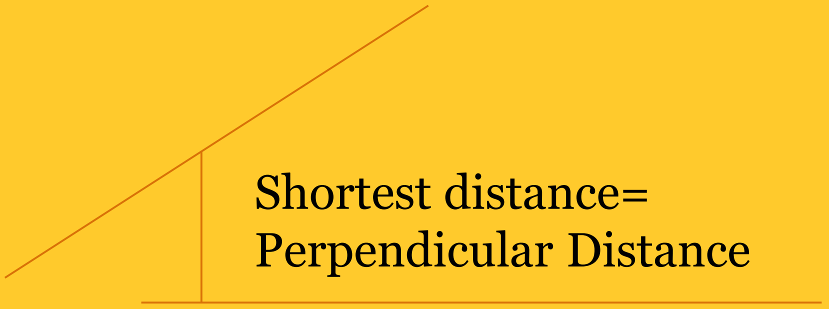
Shortest Distance Between Two Lines

- Case-3: In space or 3d, there may be lines that are neither intersect nor parallel. These lines are non coplanar and called skew lines.



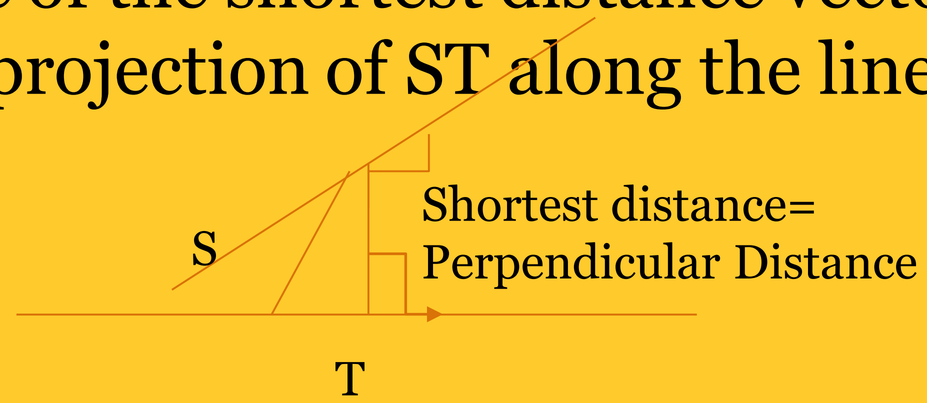
Shortest Distance Between Two Skew Lines

- For skew lines, the line of shortest distance is the line perpendicular to both the lines



Shortest Distance Between Two Skew Lines

- We know that the cross product gives us the perpendicular to both vectors
- Let, $l_1=a_1+\lambda b_1$ and $l_2=a_2+\mu b_2$, Hence a_1 and a_2 are two points on lines l_1 and l_2. Let these points are S and T.
- Then the magnitude of the shortest distance vector will be equal to the projection of ST along the line of shortest distance.

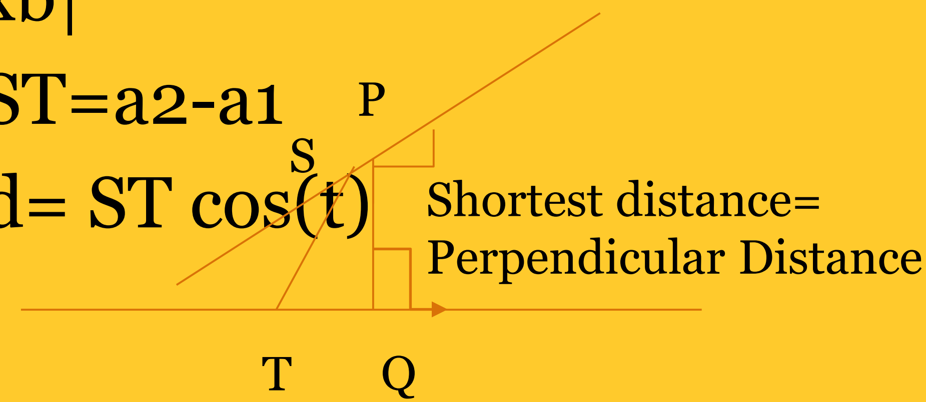


Shortest Distance Between Two Skew Lines

- For calculating the shortest, follow the following step

Given vectors are l1=a1+λb1 and l2=a2+μb2

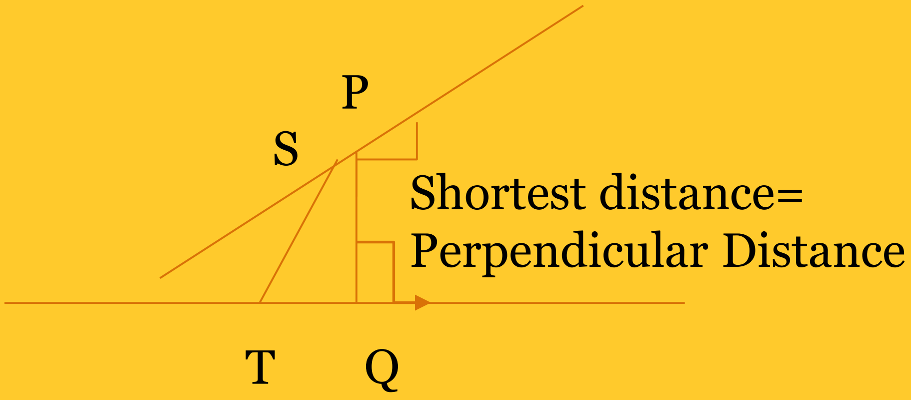
1. Step-1: Calculate the perpendicular Vector to b1 and b2=b1xb2
2. Step-2: Calculate unit vector along b1xb2=b1xb2/|b1xb|
3. Step-3: Calculate ST=a2-a1
4. Step-4: Calculate d= ST cos(t)



Shortest Distance Between Two Skew Lines

1. Step-4: Calculate cos(t) $=\left|\frac{PQ.ST}{|PQ||ST|}\right|$
2. Step-5: Calculate d = |ST| cos(t) = $|ST|\left|\frac{PQ.ST}{|PQ||ST|}\right|$

Or d= $\left|\frac{PQ.ST}{|PQ|}\right|=\left|\frac{b1xb2.(a2-a1)}{|b1xb2|}\right|$



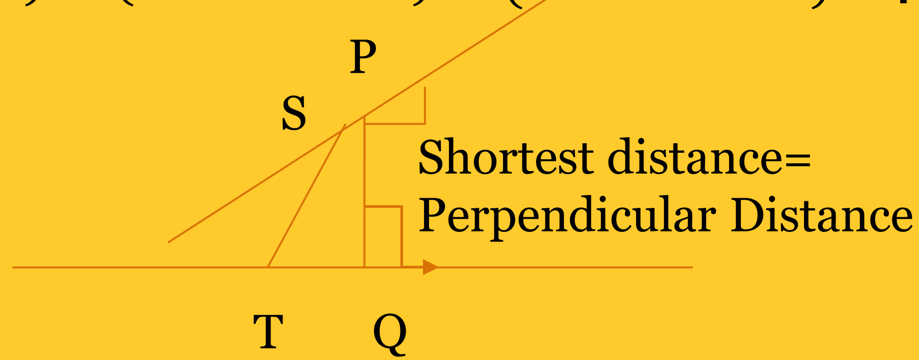
Shortest Distance Between Two Skew Lines

When the lines are in Cartesian form:

Then l1=x-x1/a1=y-y1/b1=z-z1/c1

And l2=x-x2/a2=y-y2/a2=z-z2/c2

$$\text{Then, } d=\left|\frac{\begin{vmatrix} x2-x1 & y2-y1 & z2-z1 \\ a1 & b1 & c1 \\ a2 & b2 & c2 \end{vmatrix}}{sqrt(b1c2-b2c1)^2+(c1a2-c2a1)^2+(a1b2-b1a2)\hat{}2}\right|$$



Shortest Distance Between Parallel Lines

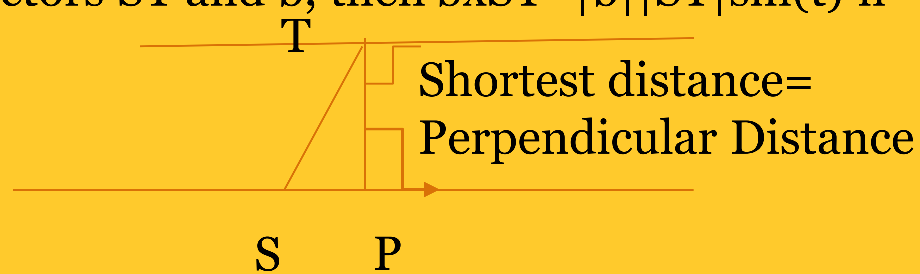
We know that the cross product gives us the perpendicular to both vectors
Let, $l_1=a_1+\lambda b$ and $l_2=a_2+\mu b$,

Hence a_1 and a_2 are two points on lines l_1 and l_2. Let these points are S and T.
As the lines are parallel then b is same for both the line

Then the magnitude of the shortest distance vector will be equal to the projection of ST along the line of shortest distance TP.

Let t be the angle between the vectors ST and b, then $b \times ST=|b||ST|\sin(t)\ n$

Now $ST=a_2-a_1$
$b \times (a_2-a_1)=|b|PT*1$
Hence, $d=TP=|b \times (a_2-a_1)/|b||$



Plane

- A plane is determined uniquely if any one of the following parameters is known:
  1. The normal of the plane and its distance from the origin. It is the equation of plane in normal form.
  2. It passes through a point and perpendicular of a given direction
  3. It passes through three given non collinear points

Equation of a plane in normal form

- The normal of the plane and its distance from the origin is given. We have to find the equation of plane in normal form.

- Let the normal vector , v=a i+b j+ c k
- Perpendicular Distance of the plane from the origin is d

- First calculate unit normal vector, n= = v/|v|
- Consider a plane whose perpendicular distance (ON) from the origin is d and n is unit normal vector. Then ON = d n

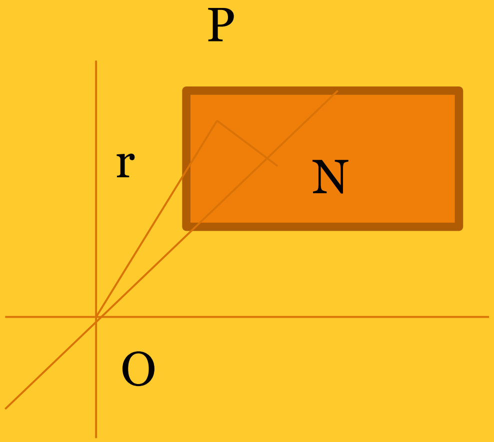
- Let r be the position vector of the point P and P(x, y, z) be any point in the plane. Hence NP is perpendicular to ON.
- Hence dot product of ON and PN is 0, i.e., ON.PN=0……….(i)
- Now, OP=ON+NP (Triangular Law of Vector Addition)
- Or r=d n+NP
- Or NP=r-d n……………………(ii)
- Now, from (i), d n . (r-d n)=0, Or n.(r-d n)=0(As d#0) Or n r- d n n =0
- Or n.r=d as (n.n=1)
- This is the vector form of the plane

  where n is the unit normal vector r is position vector OP

  or r = xi+yj+zk

The Cartesian form is

nx*x+ny*y+nz*z=d



Drawing a plane in normal form

- Given: The normal of the plane and its distance from the origin.
- r . n=d
- This is the vector form of the plane
- To draw the plane, we require to convert it to Cartesian form:

nx*x+ny*y+nz*z=d

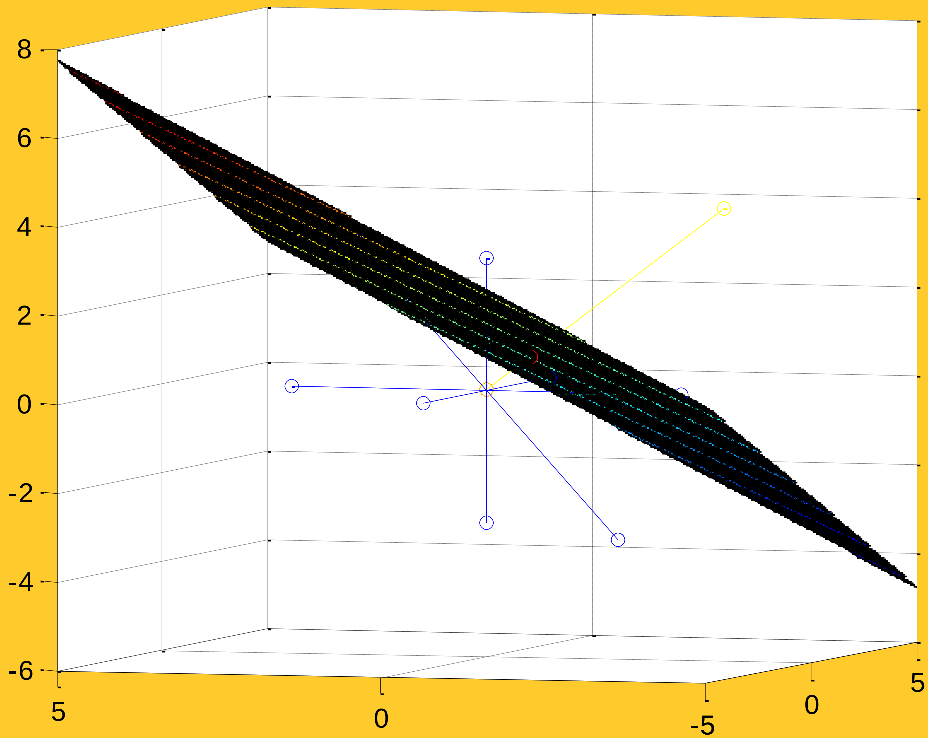
Drawing a plane in normal form

Given normal is [2 -3 4] and the perpendicular distance =6/sqrt(29)

```
Unit normal- [2/sqrt(29) i-3/sqrt(29) j + 4/sqrt(29)
r.n=d
[x i + y j + z k].[0.371390676i-0.557086015j+0.742781353k]=1.114172029
Or 0.371390676xi-0.557086015yj+ 0.742781353zk] =1.114172029
For Plotting:xx=-5:.1:5
yy=xx'
[x,y]=meshgrid(xx,yy)
z=(1.114172029-0.371390676*x+0.557086015*y)/0.742781353
surf(x,y,z)
hold on
plot3([0 2],[0 -3],[0 4],'oy-')
plot3([0 0.371390676],[0 -0.557086015],[0 0.742781353],'or')
```

Drawing a plane in normal form

- The normal of the plane and its distance from the origin is given. It is required to find the equation of plane in normal form.



Equation of a plane perpendicular to a given vector and passing through a given point

- There can be many planes that are perpendicular to the given vector. Through a given point P(x0, y0, z0), only one such plane exists.
- Let a plane pass through a point A with position vector a perpendicular to the vector N.
- Let r be the position vector of any point P(x, y, z) in the plane.
- Then the point P lies in the plane if and only if AP is perpendicular to N, i.e., AP.N=0.

Equation of a plane perpendicular to a given vector and passing through a given point

- But AP=r-a.

- Therefore, (r-a).N=0
- Cartesian Form:
- Given Point A be (x_0, y_0, z_0)
- Direction ratios of N=Ai+Bj+Ck
- r=xi+yj+zk
- (r-a).N=0
- So $[(x-x_0)i+(y-y_0)j+(z-z_0)k].[Ai+Bj+Ck]=0$
- $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

Example

- Example-17 Find the vector and Cartesian equation of the plane passing through the point (5, 2, -4) and perpendicular to the line with direction ratios 2,3, -1.
- Point is (5,2,-4), hence Position vector is P=5i+2j-4k
- Normal vector N=2i+3j-k
- Let the Vector representing the plane R=xi+yj+zk
- From dot product, we get, (R-P).N=0
- Cartesian form:
- (x-5)*2+(y-2)*3+(z+4)*-1=0
- 2x+3y-z=20
- z=(20-2x-3y)/-1

Example

- 2x+3y-z=20
- xx=-5:.1:5;
- yy=xx';
- [x,y]=meshgrid(xx,yy);
- z=(-20+2*x+3*y);
- surf(x,y,z)
- hold on
- plot3([0 2],[0 3],[0 -1],'^m-')

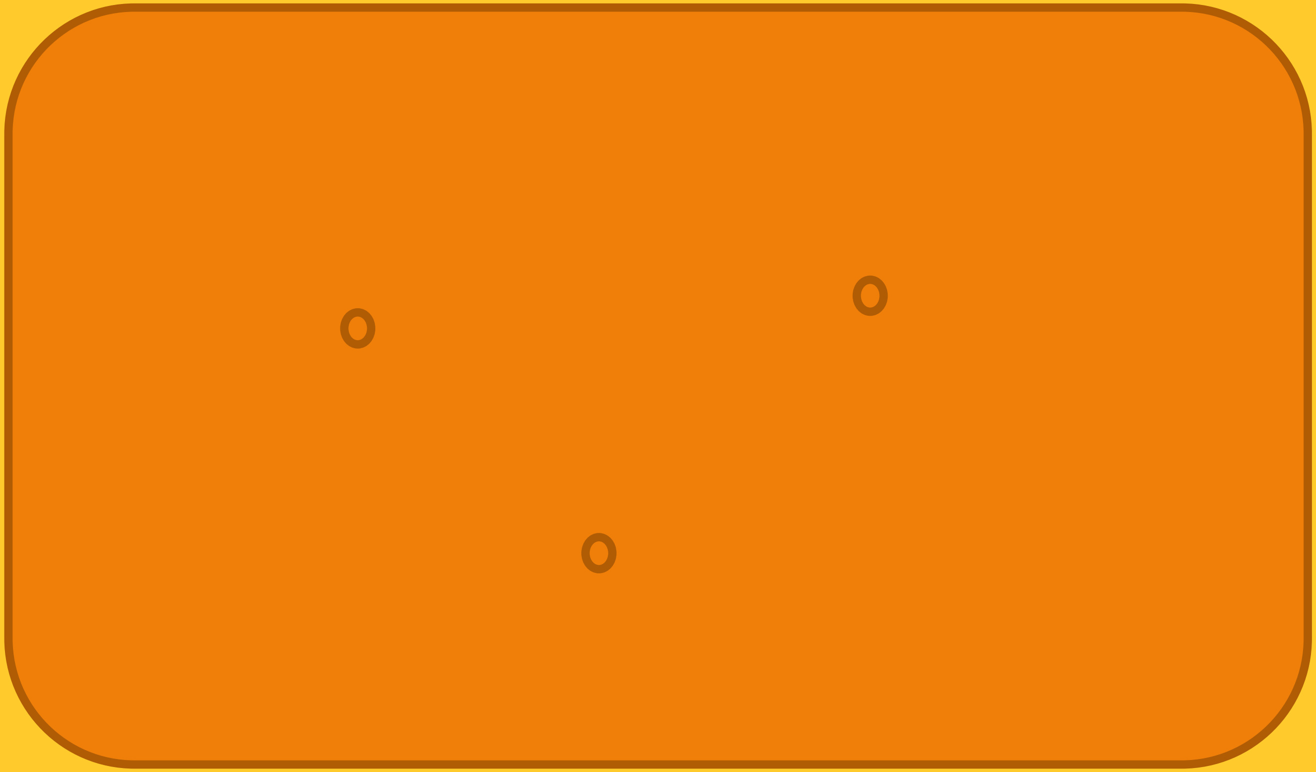
There can be many plane to a perpendicular line

- xx=-5:.1:5;
- yy=xx';
- [x,y]=meshgrid(xx,yy);
- z=(-20+2*x+3*y);
- surf(x,y,z)
- hold on
- plot3([0 2],[0 3],[0 -1],'^m-')
- z1=(10-2*x-3*y)/-1;
- z2=(30-2*x-3*y)/-1;
- surf(x,y,z1)
- surf(x,y,z2)

Example

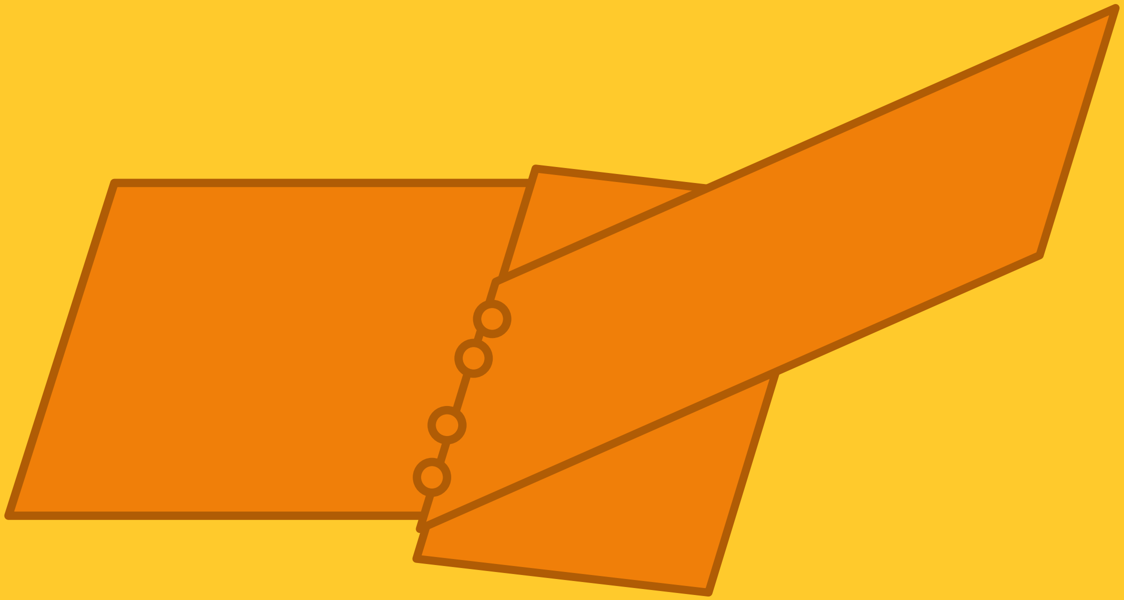
- Find the vector and Cartesian equations of the plane which passes through the point (5, 2, -4) and perpendicular to the line with direction ratios 2, 3, -1

Equation of a plane passing through three non collinear Points



Why was it necessary the three points should be collinear

- From collinear points, many planes can be drawn



Example - Books

Intercept form of the equation of a Plane

Plane Passing through intersection of two given plane

Coplanarity of Two Lines

Angle Between Two Planes

Distance of a point from a plane

Angle between a line and a plane

Analytical Functions

Definition:

The functions which describes a rule relating x and y that can be expressed by a definite formula are called analytical function.

Example: $y=ax^2+bx+c$

Different Types of Function

- Single Variable Functions
- Multi-variable Functions
- Explicit Functions
- Implicit Functions
- Monotonic Functions
- Algebraic Functions
- Transcendental Functions
- Polynomial Functions
- Constant Functions

Algebraic Functions

- 1. Polynomial Functions
- 2. Constant Functions
- 3. Linear Functions
- 4. Quadratic Functions
- 5. Cubic Functions
- 6. Rational Functions
- 7. Even and Odd functions

Transcendental Functions

- Exponential Functions
- Logarithmic Functions
- Trigonometric Functions

Observation:
There are hundreds of functions but geometrically they represent a line

How does these information help

Known vs Unknown

Breaking the problems at micro level

Points

Describing Math Through Points

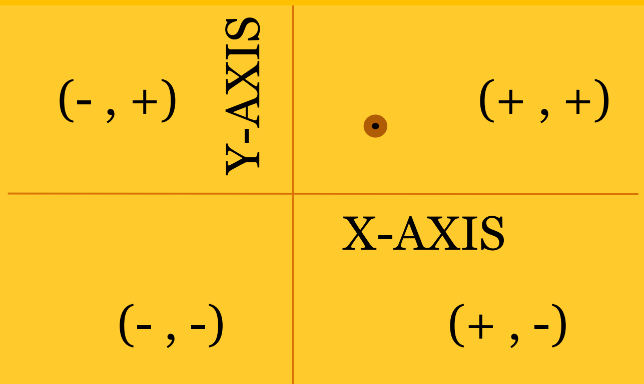
Define a Point?

What is the position of the point?

Points have dimension=0

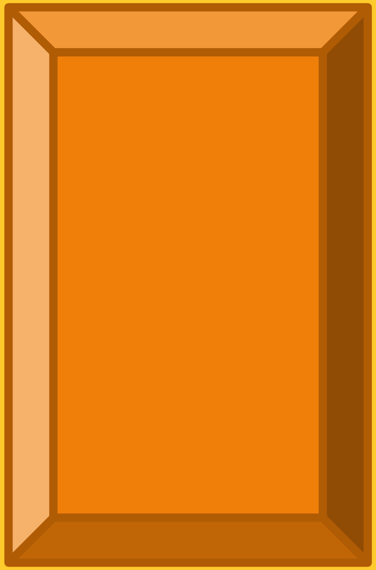
What is Point????

Note-Point described without reference is meaningless.



- ❖ Reference Lines
- ❖ Domain and Range
- ❖ Scale (Nano/Micro/Macro/Mega)
- ❖ Dimension
- ❖ Right Hand System

Formation of objects Dimensions



Drag and Create

1. 0 Dimension - Point
2. 1 Dimension – Line
3. 2 Dimension – Square
4. 3 Dimension – Cube
5. 4 Dimension – Hyper Cube
6. 5 Dimension – Hyper Hyper Cube
7. 0.2 , 0.75 Dimension - Fractals

Drag and Create: Dimensions

Object	Vertex	Edges	Faces	Solids	Hyper Solid	Dimension
Point	**1**	**0**	**0**	**0**	**0**	**0**
Line	**2**	**1**	**0**	**0**	**0**	**1**
Square	**4**	**4**	**1**	**0**	**0**	**2**
Cube	**8**	**12**	**6**	**1**	**0**	**3**
Hyper Cube	**16**	**32**	**24**	**8**	**1**	**4**
Hyper Hyper Cube	**32**	**80**	**80**	**40**	**10**	**5**

Relationship in 3d Objects: Vertex + Face=Edge+2

Platonic Solids

1. TETRAHEDRON
2. HEXAHEDRON
3. OCTAHREDRON
4. ICOSAHEDRON
5. DODECAHEDRON

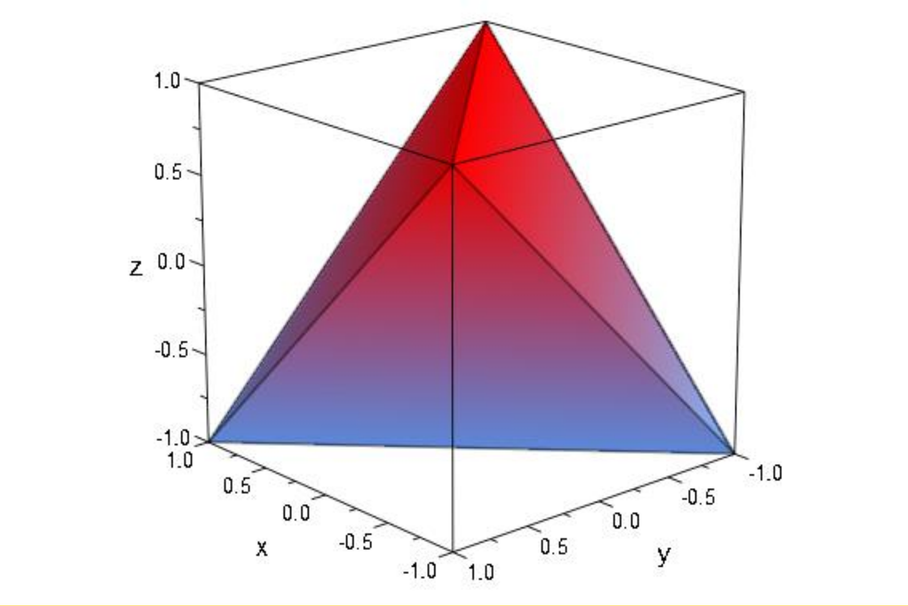
Characteristics:

In three-dimensional space, a **Platonic solid** is a regular, convex polyhedron. It is constructed by congruent regular polygonal faces with the same number of faces meeting at each vertex. Five solids meet those criteria:

Platonic Solids


```
tetra:=plot::Tetrahedron(Center=[0,0,0], Radius=1):
    plot(tetra)
```

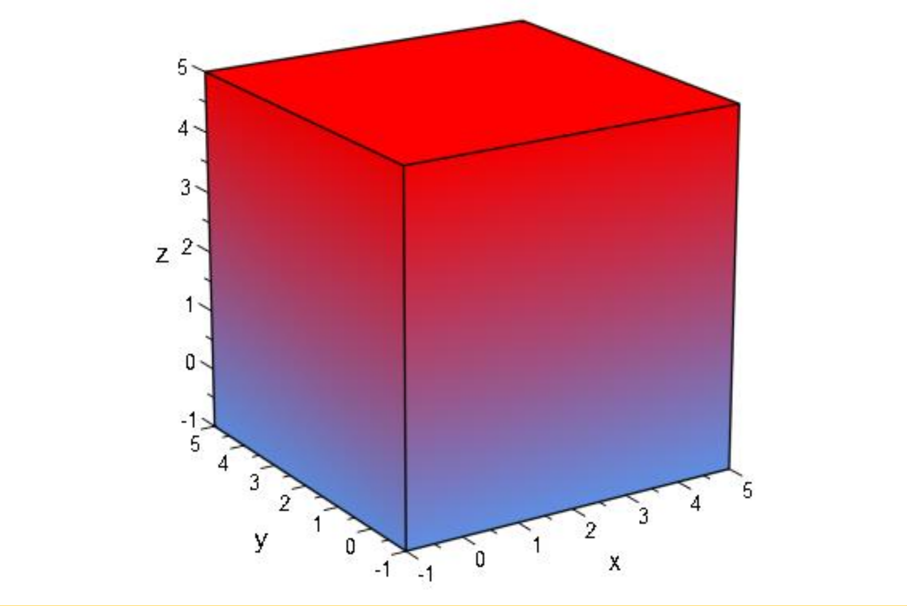
Faces - 4



Platonic Solids


```
hexahedron:=plot::Hexahedron(Center=[2,2,2],Radius=3):
    plot(hexahedron)
```

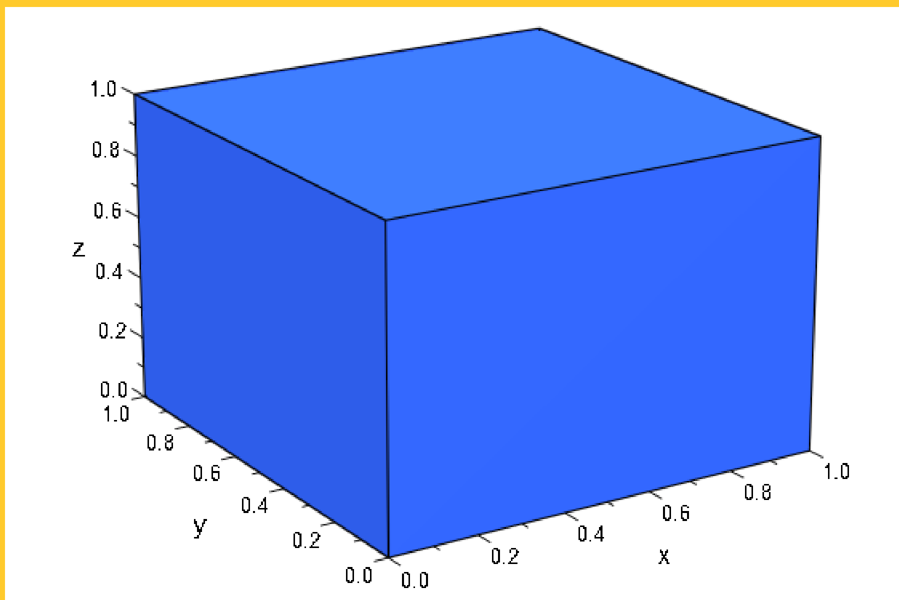
Faces : 6



Platonic Solids

Cube:

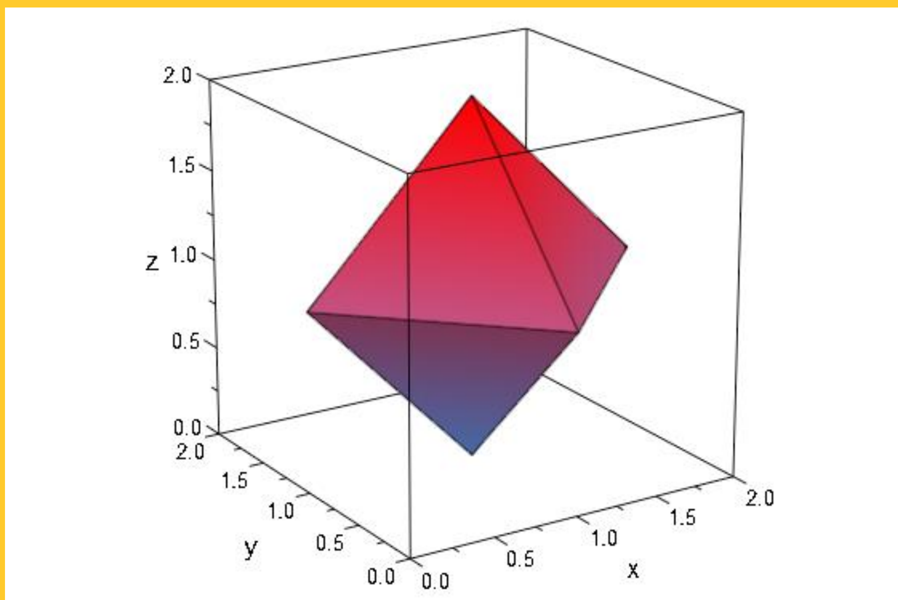
```
cube:=plot::Box([0,0,0],[1,1,1]):
    plot(cube)
```



Platonic Solids

octa:=plot::Octahedron(Center=[1,1,1],Radius=1):
plot(octa)

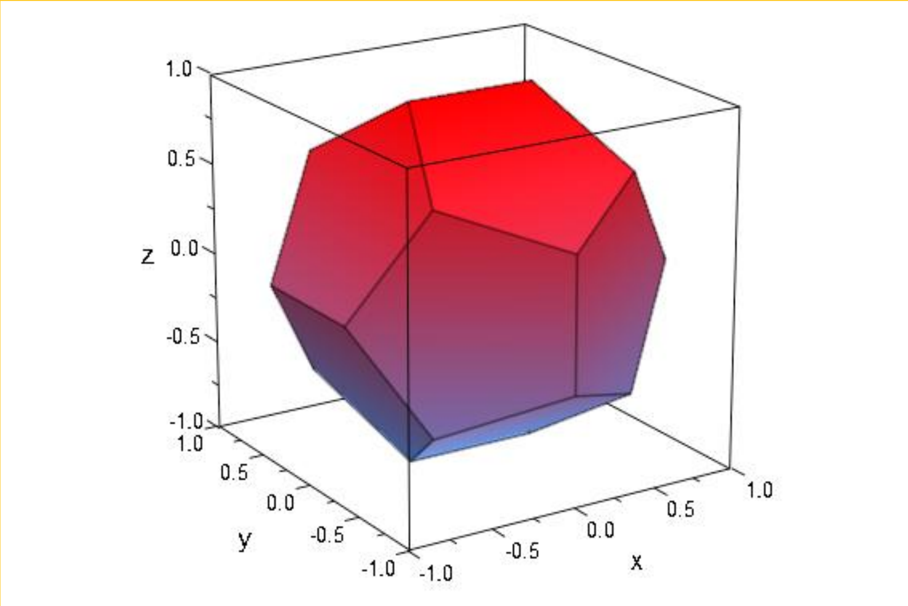
Faces : 8



Platonic Solids


```
dodeca:=plot::Dodecahedron(Center=[0,0,0],Radius=1):
    plot(dodeca)
```

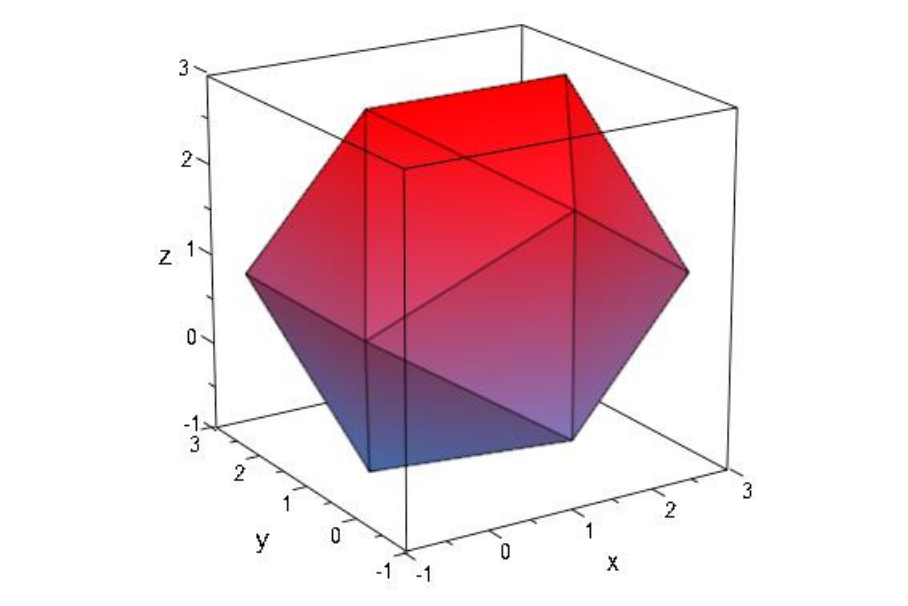
Faces: 12



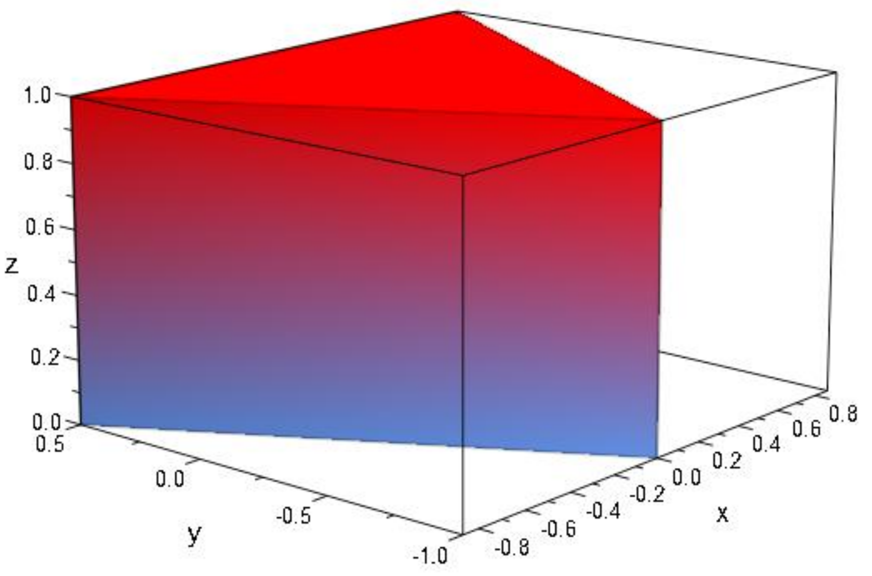
Platonic Solids

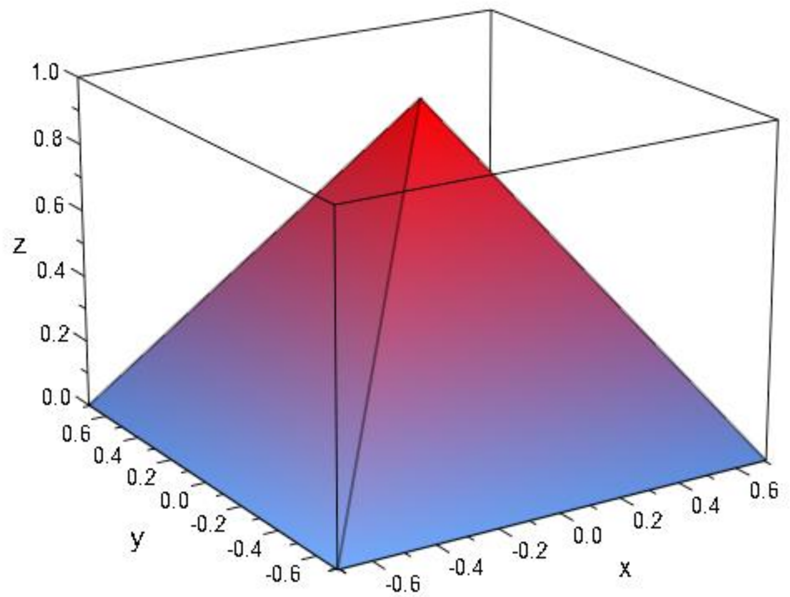

```
icosa:=plot::Icosahedron(Center=[1,1,1],Radius=2):
    plot(icosa)
```

Faces : 20



Solids-Prism, Pyramid





Representation of Points

Points have a definite position in a Specific Coordinate System

Points

Represented by coordinates

(x, y)

where x and y are the coordinates of a point

Expressed as Vector/Matrix

Matrix of (1x2) dimension: a=[x, y] which form elements of matrix

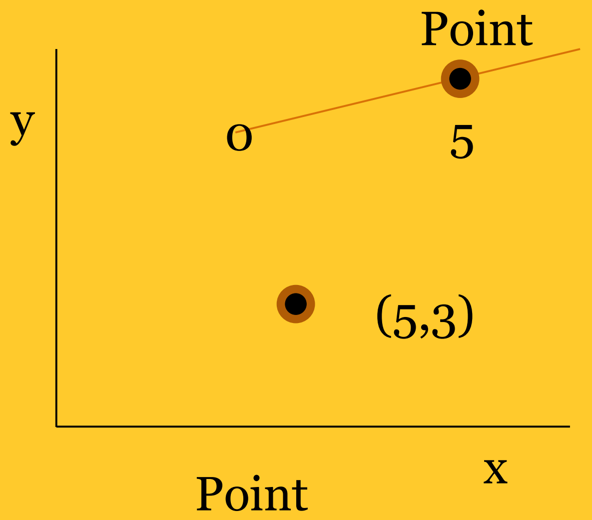
Representation of Vectors

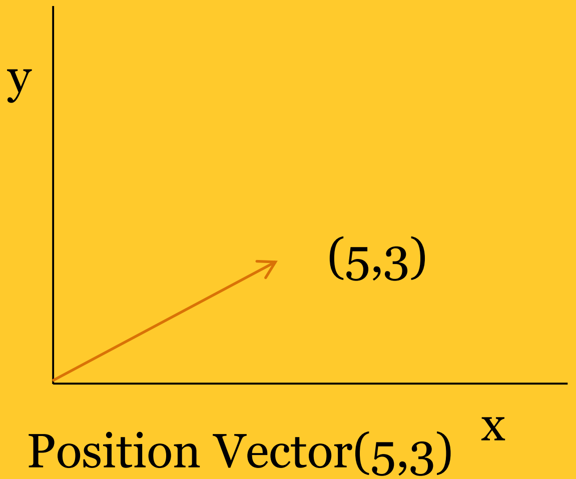
Vectors

- Represented by a directed line segment
  - Expressed as Coordinates (x, y), Have magnitude and direction
- They don't have any specific position (Location)
  - Matrix [x ,y] represents a vector

Points and Vectors

- Any Vector $(\vec{a})$ in the plane can be transformed i.e. scaled, reflected, rotated, translated or sheared by a Transformation Matrix, 't'.





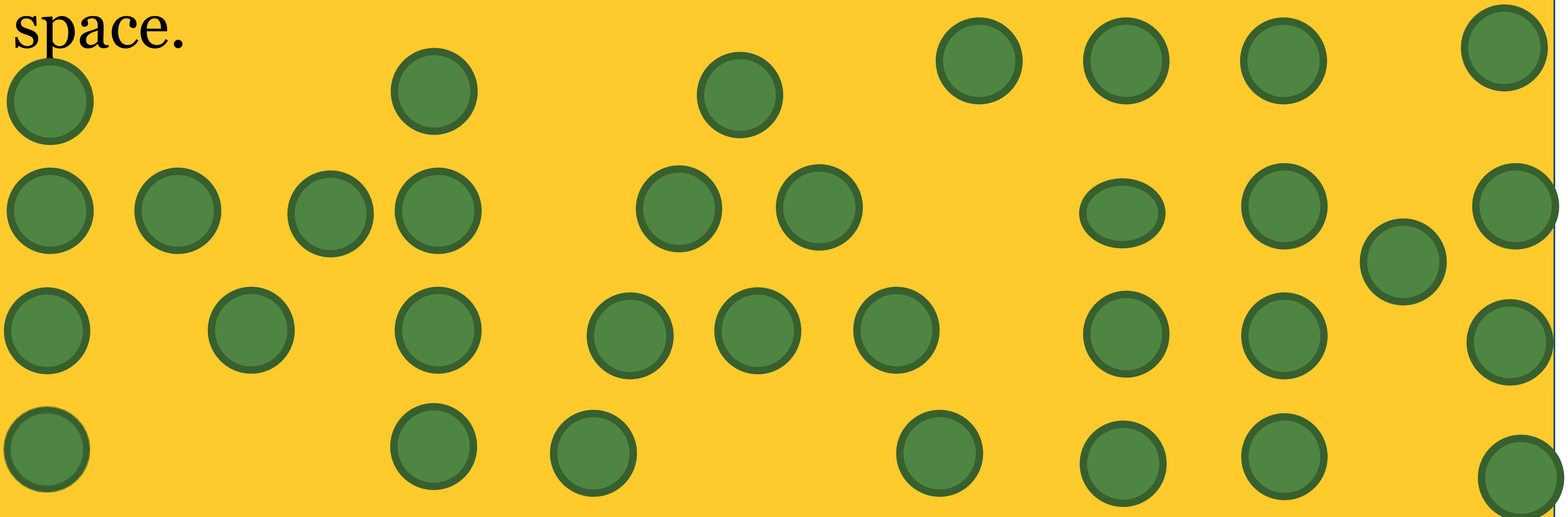
NOTE: Every point can be thought of a position vector
When a point is manipulated by a transformation matrix, the point do not move, the vector represented by the point moves.

Draw a Point

- $(5, 3)$
- $(5, 3, 2)$
- $(5, 3, 2, 1)$

Definition of Mathematics-Finding Point

- Mathematics is geometrically finding one component of a point from other component.
- Mathematics tracking the movement of the point in space.



Matrices – moving from static to dynamic world

- Representation of a point or a vector by its coordinates as [x, y], [x, y, z], can be viewed as well organized ordered numbers.
- This type of organized numbers is termed as MATRIX. It can have any number of rows/ columns.

$$\begin{bmatrix} 5 \end{bmatrix}$$

(1x1) matrix

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(2row x 1column) matrix

$$\begin{bmatrix} 2 & 3 \end{bmatrix}$$

(1x2) matrix

Column

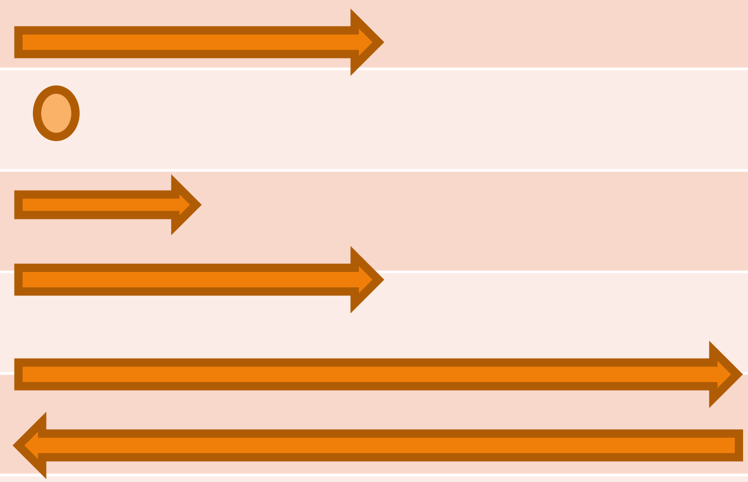
$$\begin{bmatrix} 3 & 2 & 5 \\ 7 & 1 & 0 \\ 9 & 4 & 6 \end{bmatrix}$$

Row

(3x3) matrix

<u>Scalar Multiplication of 2 numbers(a,t):</u> at= a x t

For a number 'a', depending upon the value of 't', the value of 'a*t' remains same, decreases, increases or changes sign.

a	t	a x t	Geometrical Representation
5			
5	0	5x0=0	
5	0.5	5x.5=2.5	
5	1	5x1=5	
5	2	5x2=10	
5	-2	5x-2=-10	Plot these results in excel

Matrix Multiplication

- Input (x, y)
- Output(x*, y*)
- Easy way to move a point is matrix Multiplication

Matrix Multiplication

Matrix Multiplication:

- Rule: Number of columns of 1st matrix should be equal to number of rows of 2nd matrix.

$$[m \times a] * [a \times n] = [m \times n]$$

- A 1x2 matrix forms when a 1x2 matrix is multiplied by a 2x2 matrix

$$[x \quad y] * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = [ax + cy \quad bx + yd]$$

- [x,y] is a row matrix represents a point as well as a vector. The (2x2) matrix is a transformation matrix.
- We are interested to know the effects of changes in individual elements of a transformation matrix on the resultant vector/point.

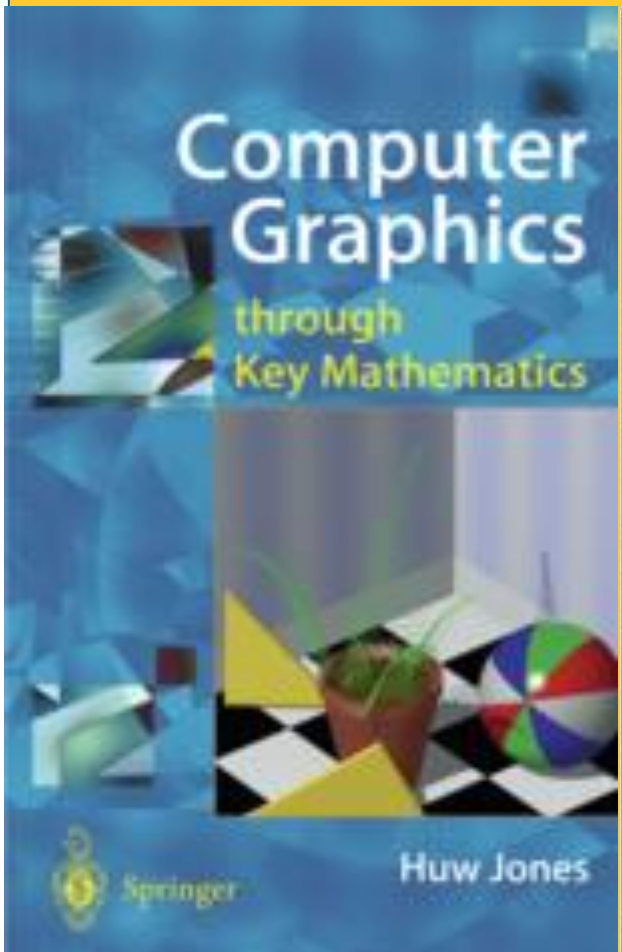
Matrix Multiplication in Excel

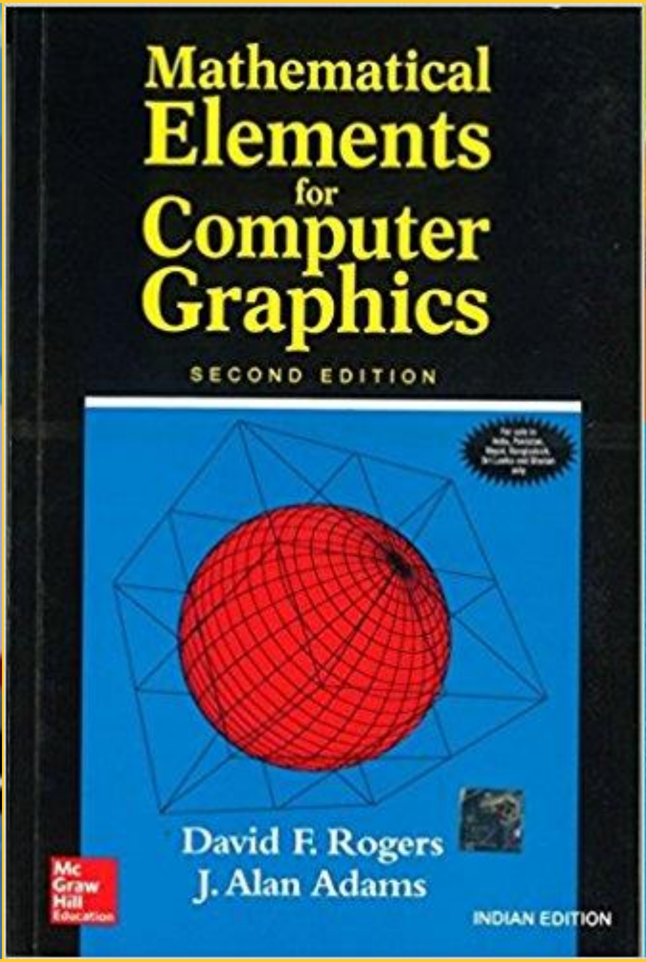
- Matrix multiplication of a 1xn matrix by a nxn matrix results in a 1xn matrix
- Importance of Matrix Multiplications – It helps in transformations
- Through these transformations, we can capture the dynamism around the world.

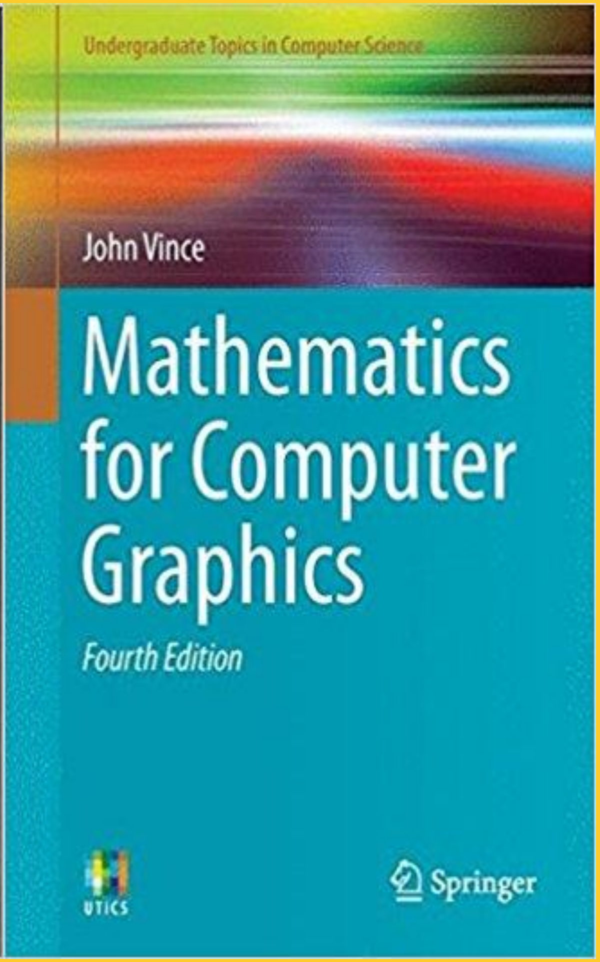
Dynamic World
Type of Transformations

- Scaling
- Reflection
- Shearing
- Rotation
- Translation
- Projection

Reference Books







Transformation and Matrices

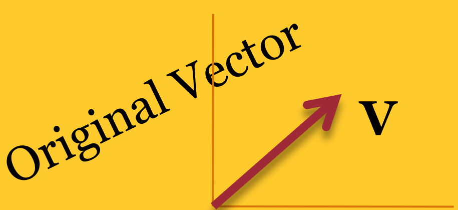
- Let a position vector (v) =[5,3]
- The Transformation Matrix (t)= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- Then Resultant Vector (vt) = [Xt Yt]

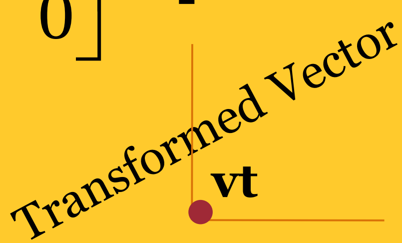
CASE 1:

a	b	c	d
0	0	0	0

$$t= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$v*t= vt= \begin{bmatrix} 5 & 3 \end{bmatrix} * \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5*0+3*0 & 5*0+3*0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$





Result: Multiplication of a vector by a zero matrix produces a Zero Vector.

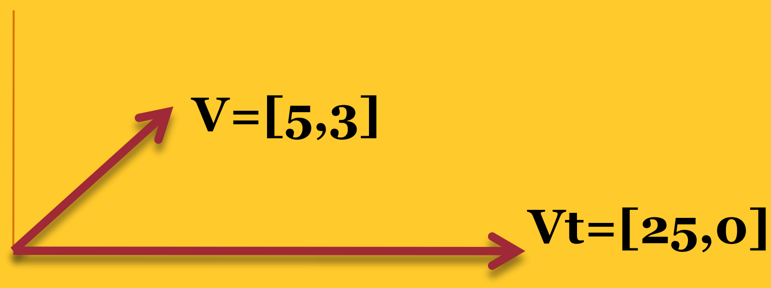
SCALING

CASE 2:

a	b	c	d
5	0	0	0

$$t=\begin{bmatrix}5 & 0\\ 0 & 0\end{bmatrix}$$

$$vt=\begin{bmatrix}5 & 3\end{bmatrix}*\begin{bmatrix}5 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}5*5+3*0 & 5*0+3*0\end{bmatrix}=\begin{bmatrix}25 & 0\end{bmatrix}$$



RESULT: Scaling in X- Direction

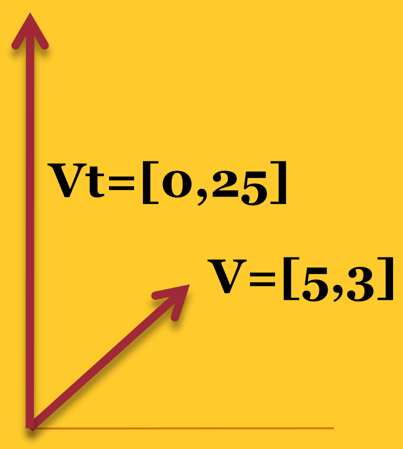
SHEARING

CASE 3:

a	b	c	d
0	5	0	0

$$t=\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$$

$$vt=\begin{bmatrix} 5 & 3 \end{bmatrix}*\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 5*0+3*0 & 5*5+3*0 \end{bmatrix}=\begin{bmatrix} 0 & 25 \end{bmatrix}$$



RESULT: Shearing in Y- Direction

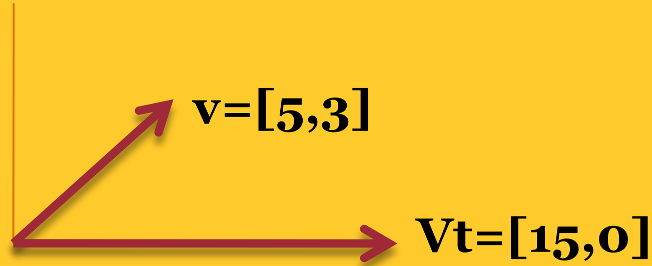
SHEARING

CASE 4:

a	b	c	d
0	0	5	0

$$t= \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix}$$

$$vt= \begin{bmatrix} 5 & 3 \end{bmatrix} * \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 5*0+3*5 & 5*0+3*0 \end{bmatrix} = \begin{bmatrix} 15 & 0 \end{bmatrix}$$



RESULT: Shearing in X- Direction

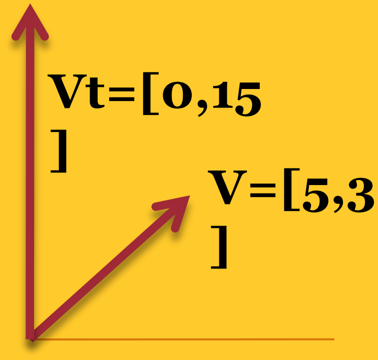
SCALING

CASE 5:

a	b	c	d
0	0	0	5

$$t=\begin{bmatrix}0 & 0\\ 0 & 5\end{bmatrix}$$

$$vt=\begin{bmatrix}5 & 3\end{bmatrix}*\begin{bmatrix}0 & 0\\ 0 & 5\end{bmatrix}=\begin{bmatrix}5*0+3*0 & 5*0+3*5\end{bmatrix}=\begin{bmatrix}0 & 15\end{bmatrix}$$



RESULT: Scaling in Y- Direction

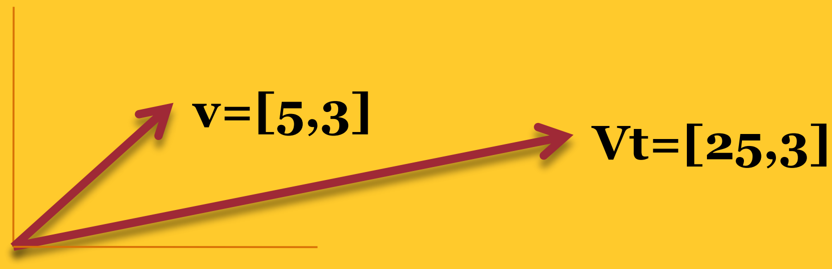
SCALING

CASE 6:

a	b	c	d
5	0	0	1

$$t=\begin{bmatrix}5 & 0\\0 & 1\end{bmatrix}$$

$$vt=\begin{bmatrix}5 & 3\end{bmatrix}*\begin{bmatrix}5 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}5*5+3*0 & 5*0+3*1\end{bmatrix}=\begin{bmatrix}25 & 3\end{bmatrix}$$



RESULT: Scaling in X Component

SCALING

CASE 6:

a	b	c	d
1	0	0	5

t= $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

$$vt = \begin{bmatrix} 5 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5*1+3*0 & 5*0+3*5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \end{bmatrix}$$

RESULT: Scaling in Y Component

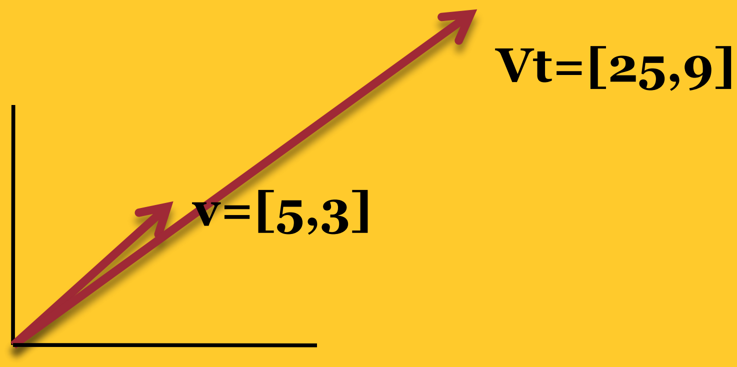
SCALING

CASE 7:

a	b	c	d
5	0	0	3

$$t = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$$

$$vt = \begin{bmatrix} 5 & 3 \end{bmatrix} * \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5*5+3*0 & 5*0+3*3 \end{bmatrix} = \begin{bmatrix} 25 & 9 \end{bmatrix}$$



RESULT: Scaling in both the coordinates

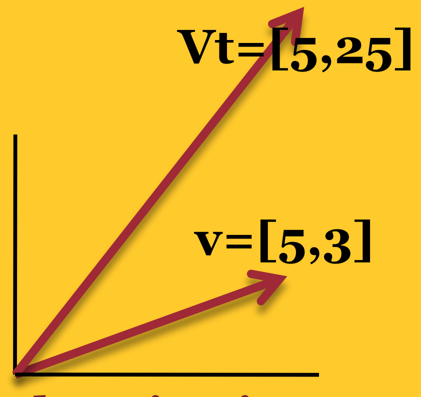
SHEARING

CASE 8:

a	b	c	d
1	5	0	0

$$t=\begin{bmatrix}1 & 5\\ 0 & 0\end{bmatrix}$$

$$vt=\begin{bmatrix}5 & 3\end{bmatrix}*\begin{bmatrix}1 & 5\\ 0 & 0\end{bmatrix}=\begin{bmatrix}5*1+3*0 & 5*5+3*0\end{bmatrix}=\begin{bmatrix}5 & 25\end{bmatrix}$$



RESULT: Shearing in Y- Component

SCALING

Observation from Cases:

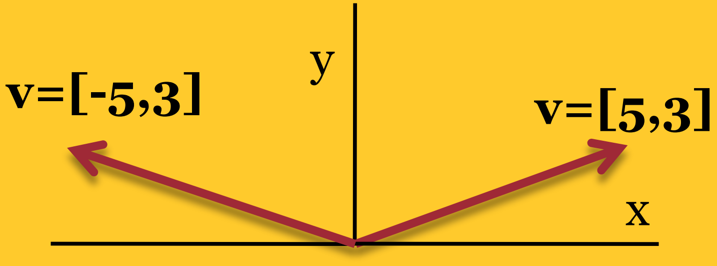
$$[x \quad y] * \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} = [ax \quad dy]$$

- A transformation matrix whose primary diagonal elements are non-zero, results scaling in both axis.
- If a=d, then scaling are equal.
- When a=d>1, b=0, c=0 then pure enlargement occurs.
- If b=0, c=0 , 0<a<1, 0<d<1, then a compression of coordinates of vectors occurs.

REFLECTION

- If ‘a’ and/or ‘d’ are negative, then reflection through a plane or axis occurs.

$$[5 \quad 3] * \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [-5 \quad 3]$$



1. If a=-1, reflection through Y axis occurs.
2. If d=-1, then reflection through X axis occurs.
3. If a=-1, d=-1, then reflection through origin occurs.

Note=Reflection Occurs In 3d

Shear

Keep track on notes

NOTE 1:

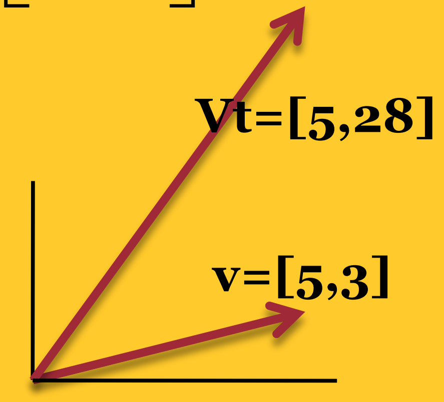
Reflection and Scaling involve main diagonal elements of Transformation Matrix.

Effect of Off-diagonal elements in the resultant vector

a	b	c	d
1	5	0	1

$$\begin{bmatrix}5 & 3\end{bmatrix}*\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix}=\begin{bmatrix}5*1+3*0 & 5*5+3*1\end{bmatrix}=\begin{bmatrix}5 & 28\end{bmatrix}$$

<u>Results= Shear</u>



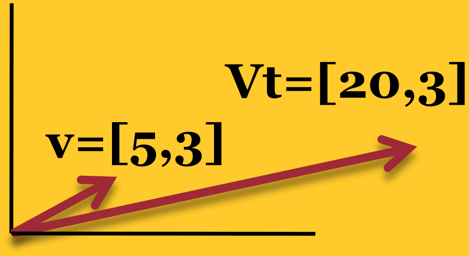
Result: X coordinate unchanged while Y coordinate changes linearly on the coordinates.

Shear…

- Similarly, as in the above cases,

a	b	c	d
1	0	5	1

$$t=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$$



Produces a shear proportional to X coordinate.

$$\begin{bmatrix}5 & 3\end{bmatrix}*\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}=\begin{bmatrix}5*1+3*5 & 5*0+3*1\end{bmatrix}=\begin{bmatrix}20 & 3\end{bmatrix}$$

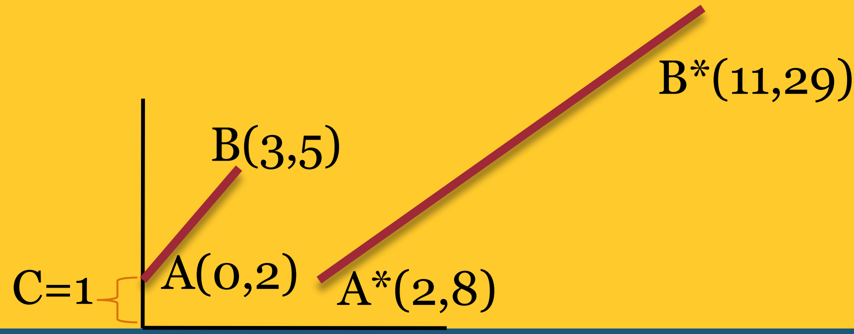
- **<u>NOTE 2:</u>**
  OFF DIAGONAL TERMS produces Shear.

Transformation of Straight Lines

- A straight line can be defined by two position vectors which specify the coordinates of its end points.
- Let A=[0 2] and B=[3,5] are two position vectors joining the endpoints of a line AB. Now, let t= $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ be the transformation matrix.

$$\begin{bmatrix} 0 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0*2+2*1 & 0*3+2*4 \\ 3*2+5*1 & 3*3+5*4 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 11 & 29 \end{bmatrix}$$

A*B* are the endpoints of the Transformed Line



NOTE 4: Transformation of a line changes the length and Orientation

Midpoint Transformation:

- Line (AB) transformed Into A*B*, Points on the second line have a one to one correspondence with the points on the first line. This is true for end points and mid points also.
- A=[0, 2] and B=[3, 5]. Midpoints between A and B is

m1= $\begin{bmatrix} \frac{0+3}{2} & \frac{2+5}{2} \end{bmatrix} = [1.5 \quad 3.5]$

- Midpoint of the Transformed Line A*[2, 8] and B*[11,29] is

m2= $\begin{bmatrix} \frac{2+11}{2} & \frac{8+29}{2} \end{bmatrix} = [6.5 \quad 18.5]$

Note-3: one to One correspondence: A=[0, 2] and B=[3, 5] , A*=[2, 8] and B*=[11, 29]

Midpoint Transformation…

- Let us see whether the transformation matrix transforms m1 to m2 or not-

$$[1.5 \quad 3.5]*\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = [1.5*2+3.5*1 \quad 1.5*3+3.5*4] = [6.5 \quad 18.5]$$

- Hence midpoint of the original line transformed the mid point of the transformed line.
- NOTE 4:

  Any Straight line can be transformed into any other straight line in any position by simply transforming its end points and redrawing the line between the end points.

Note-5: Slope of parallel line (m) is same

- A 2x2 transformation matrix transforms a pair of parallel lines into another pair of parallel lines.
- Let AB// EF (slope=m)
- If AB is transformed by a transformation matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then,

$$\begin{bmatrix} xt_1 & yt_1 \\ xt_2 & yt_2 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix}$$

Contd..

$$m^* = \left[\frac{(bx_2 + dy_2) - (bx_1 + dy_1)}{(ax_2 + cy_2) - (ax_1 + cy_1)}\right] = \left[\frac{b(x_2 - x_1) + d(y_2 - y_1)}{a(x_2 - x_1) + c(y_2 - y_1)}\right]$$

$$m^* = \left[\frac{b + dm}{a + cm}\right]$$

$$t = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

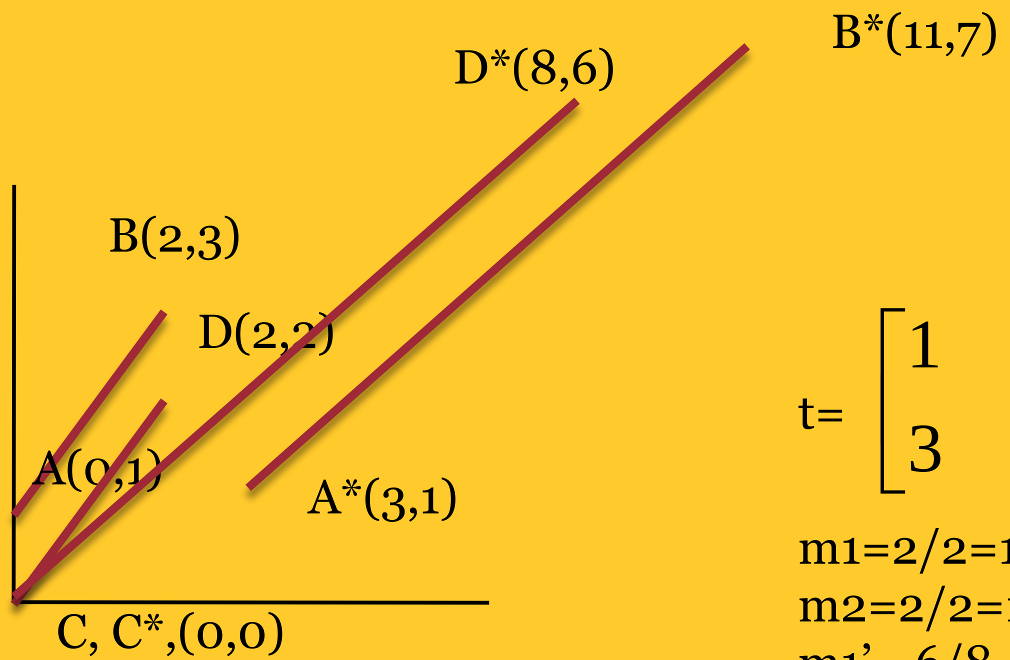
- m* is independent of coordinates
- m* is same for both A*B* and E*F*.
- Note 6:

Parallel Lines Transforms into parallel lines when operated by a general 2x2 transformation matrix (Affine Transformation).

Parallel Lines Remain Parallel After Transformation

0 1
2 3

- 0 0
  2 2

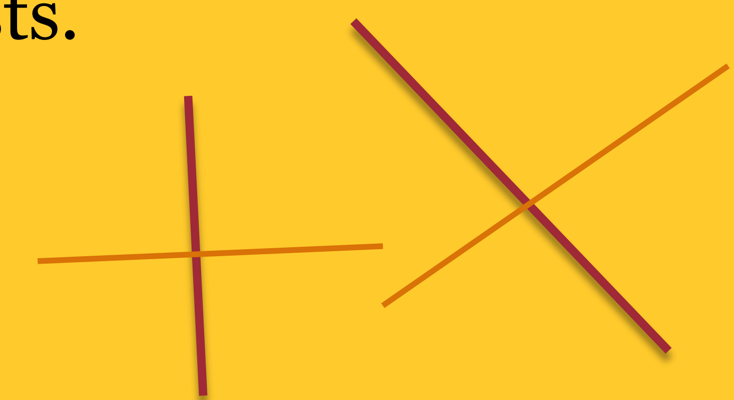


$$t = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

m1=2/2=1
m2=2/2=1
m1'= 6/8=0.75
m2'= 6/8=0.75

Transformation of Intersecting Lines

- Two intersecting lines → have a common point which means that a solution to the pair of equations representing the lines exists.
- Let the two equations be-



$$y = m_1 x + c_1$$
$$y = m_2 x + c_2$$
$$or$$
$$-m_1 x + y = c_1$$
$$-m_2 x + y = c_2$$
$$or$$
$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} -m_1 & -m_2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

Matrix calc

$$M^{-1} = \begin{bmatrix} \dfrac{1}{m_2 - m_1} & \dfrac{m_2}{m_2 - m_1} \\ \dfrac{-1}{m_2 - m_1} & \dfrac{-m_1}{m_2 - m_1} \end{bmatrix}$$

Transformation of Intersecting Lines

- **Note 7:**

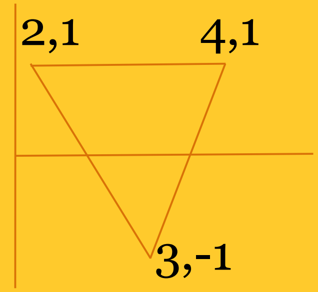
  Two non- perpendicular intersecting lines when multiplied by a 2x2 transformation matrix produces two intersecting perpendicular lines and vice-versa.

- Transformation of intersecting lines involves a rotation, a reflection and a scaling. Let us consider these effects individually on a plane object .....

Transformation of Plane Rotation

- Let ABC be the triangle formed by A(3, -1), B(4, 1) and C(2, 1) and the triangle is rotated 90˚ clockwise. Then,

$$t= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



Hence the Transformed triangle is

$$A^*B^*C^* = \begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -1 & 4 \\ -1 & 2 \end{bmatrix}$$

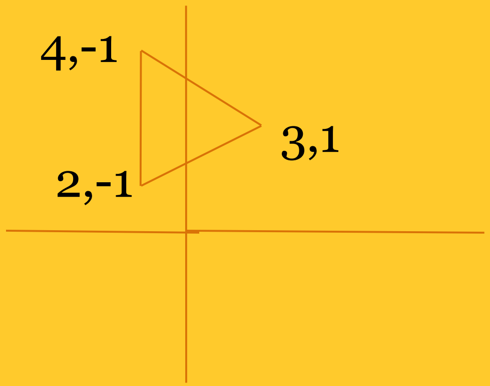
- The general rotation about the origin is governed by, t=

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

- The rotation is positive counter clock wise

Rotation in Excel

Rotation Matrix = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

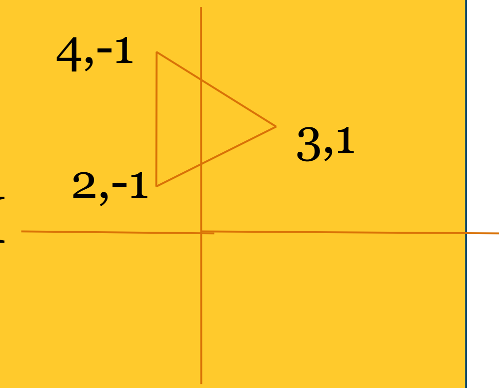


- Draw a rotation matrix in Excel and insert slider to show the dynamic condition of rotation

Rotation

- The determinant of rotation transformation matrix is $(\cos\theta \times \cos\theta)-(\sin\theta \times -\sin\theta)= \cos^2\theta+ \sin^2\theta=1$
- The transpose of rotation transformation matrix

$$t^T= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Since $t^* \, t^t= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Which indicates that $t^t=t^{-1}$

- **NOTE 8:**

  The determinant of a pure rotational matrix is +1. Such matrices are called orthogonal matrix (Find it in Excel).
- **Inverse Matrix= Transpose Matrix**

Reflection

- The 2D rotation in the xy plane occurs entirely in the two dimensional plane about an axis normal to the xy plane, a reflection is a 180° rotation out into 3d space and back into 2d space about an axis in the xy plane.
- A reflection about y=0, i.e., x axis is obtained by transformation matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Reflection about x=0, i.e, y axis is obtained by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
- Reflection about y=x is obtained by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- Reflection about y=-x is obtained by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Reflection

- The transformation matrices having determinant=-1 produces reflection.
- **NOTE 9:**

  Two successive reflection about two lines passing through origin, results pure rotation.

- **NOTE 10:**

  The reflection matrices are orthogonal meaning that its transpose is its inverse.

$$t^T = t^i$$

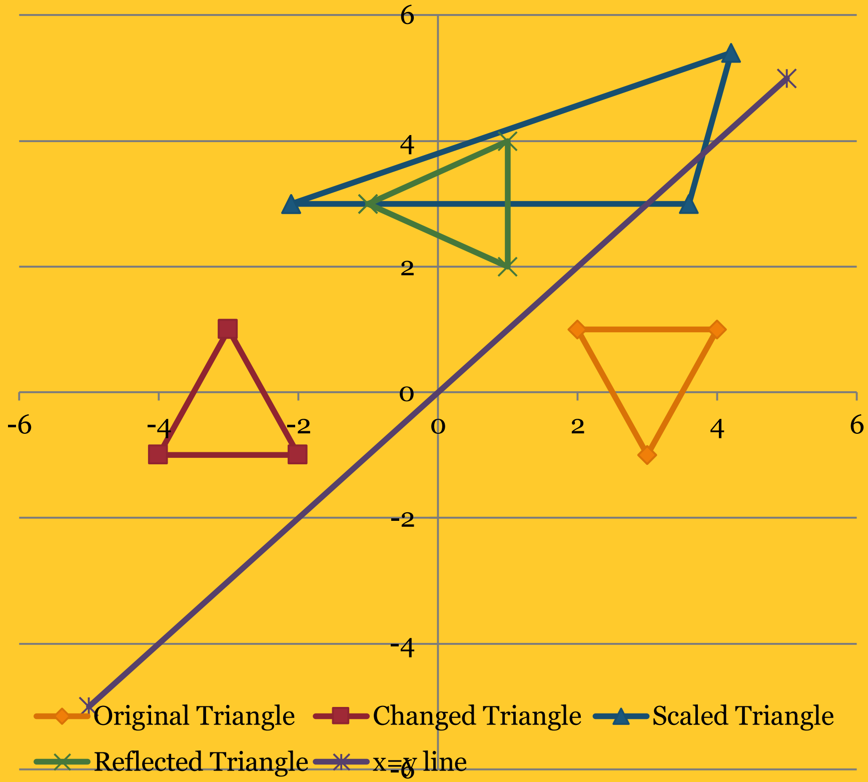
Recap: Scaling

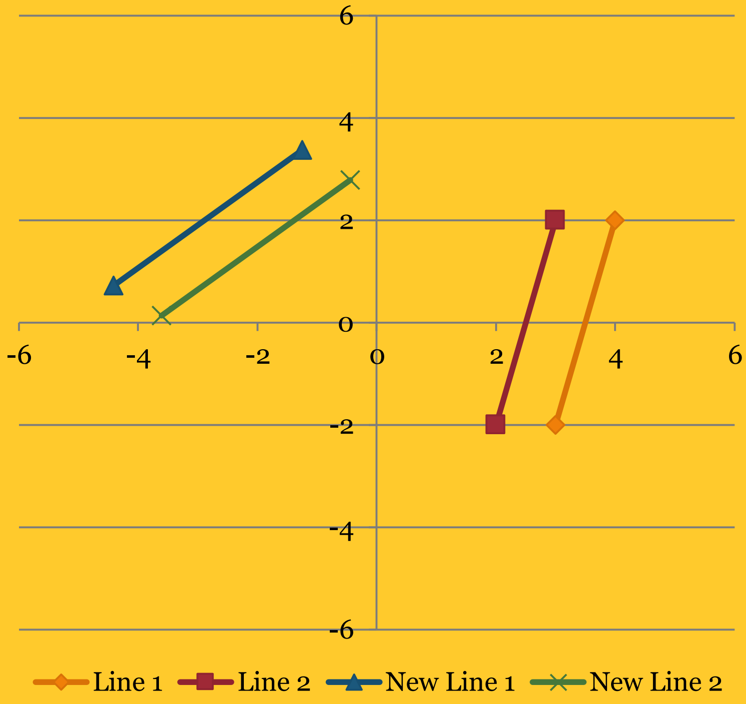
- The main diagonal of transformation matrix governs scaling.
- If t= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with a=d, b=0, c=0, a uniform scaling occurs.
- With a=d>1, a uniform expansion occurs.
- With 0<a=d<1, then a uniform compression occurs.
- Non uniform scaling occurs when a≠d.
- **NOTE 11:**

  Main diagonal element of the transformation matrix governs scaling.

Transformation of Intersecting Line

Rotation and Reflection do not change intersecting angle:
Scaling may change intersecting angles





Transformation of Lines

Observation:

1. During transformation, Parallel Lines will always remain parallel
2. During Transformation, angles of intersecting lines may change

Note: This observation has tremendous effect when applying transformation matrix.

Combined Transformation

- Position vectors defines the vertices, shapes and position of the object.
- By performing matrix operations, the transformations can be controlled.
- Since matrix multiplication is non-commutative, the order of the transformation is important.

- **<u>NOTE 12:</u>**
  Order of the transformation is important as matrix multiplication is non commutative.

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

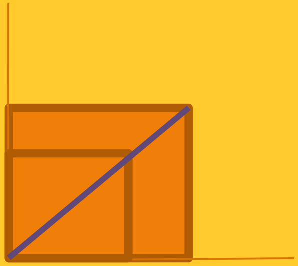
$$\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix}$$

Transformation of the Square

- Transformation matrix operates in every point in the plane.
- Under 2x2 transformation, origin remains invariant. This transformation may be interpreted as stretching of original object into a new shape.
- Let ABCD is a unit rectangle with ABCD= $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$
- The 2x2 transformation matrix is t= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- A*B*C*D*= $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ a & b \\ a+c & b+d \\ c & d \end{bmatrix}$

0	0
2	3
6	5
4	2
0	0

2 3
4 2



Transformation of the Square

- Results: A*=origin is not affected, A=A*
- Coordinates of B* is changed to the first row of matrix.
- Coordinates of D* is changed to second row of transformation matrix.
- Coordinates of C* is a+c and b+d
- The determinant of the transformation matrix determines the scaling factors.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ a & b \\ a+c & b+d \\ c & d \end{bmatrix}$$

2 3
4 2

Transformation of a Square

- If A_i is the initial area of the object and A_t be the transformed area of the object, then

$$A_t = A_i * \det(A)$$

- **NOTE 13:**

Determinant of transformation matrix governs the scaling factors of the transformed objects.

- **NOTE 14:**

Orthogonal matrices whose determinants are +1 or -1 do not change the area of transformed object.

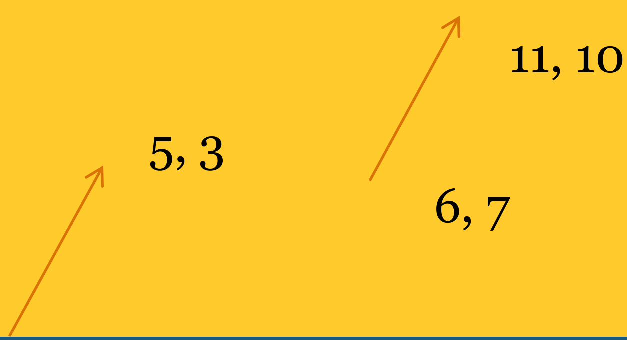
Issues

- Determining the condition when perpendicular lines transforms into perpendicular lines.

- **NOTE 15:**
  - **Rotation and reflections preserve angle and magnitude of the intersecting position vectors.**
  - **Uniform scaling preserves the angle between intersecting lines but not the magnitude of transformed vectors.**
  - **Non uniform scaling changes angle and magnitude of intersecting lines**

Translation & Homogeneous Coordinates

- Scaling, shearing, reflection, rotation can be achieved when the origin is invariant.
- One of the most important form of Transformation is Translation. In one way, translation is the most simple transformation.
- The addition of a vector with the original vector translates the original vector to the desired position.
- v=[5,3], t=[6,7]
- Then vt=[11,10]

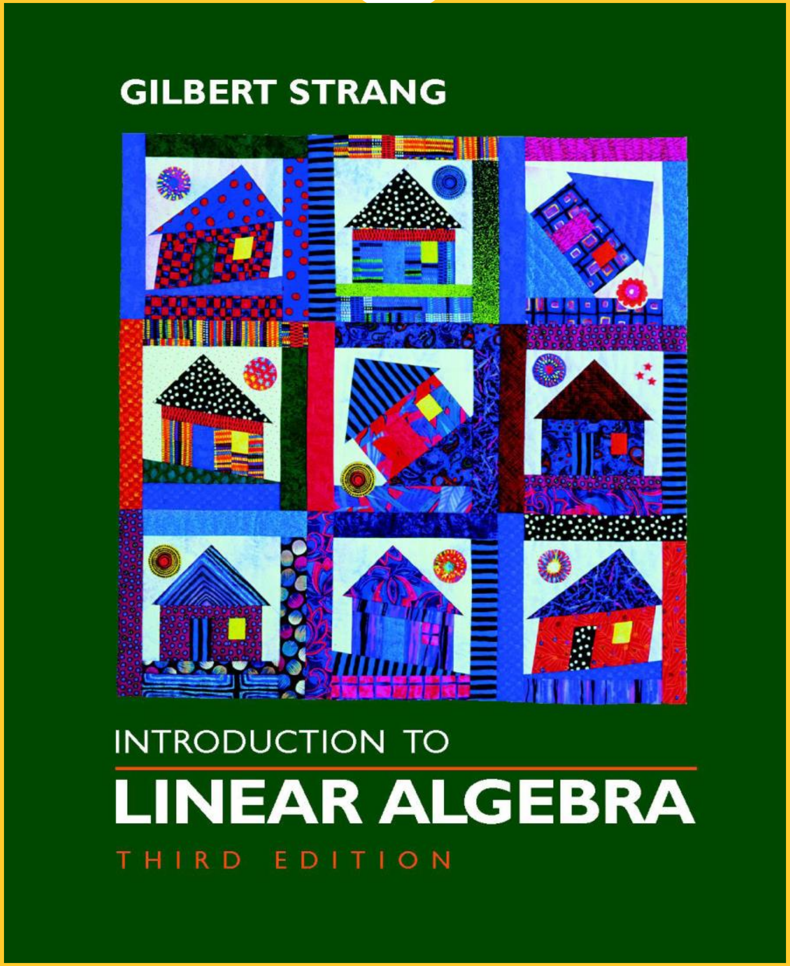


Introduction of Homogeneous Coordinates

- By Matrix addition all points are TRANSLATED, but cannot achieve rotation, reflection, scaling and shearing
- By Matrix multiplication all points Transformed except origin and translation can not be achieved.

- Homogeneous Coordinates help for complete transformation .. Rotation, Scaling, Shear, Reflection, translations…
- Homogeneous Coordinates helps in shifting of origin also.

Linear Algebra

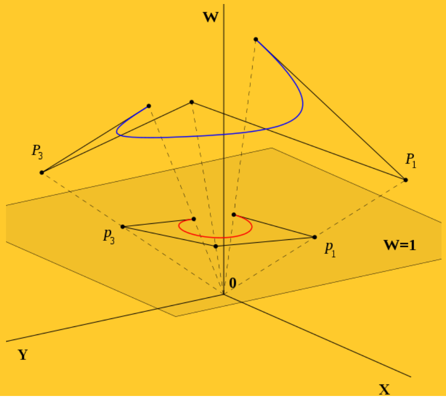


Homogeneous Coordinates

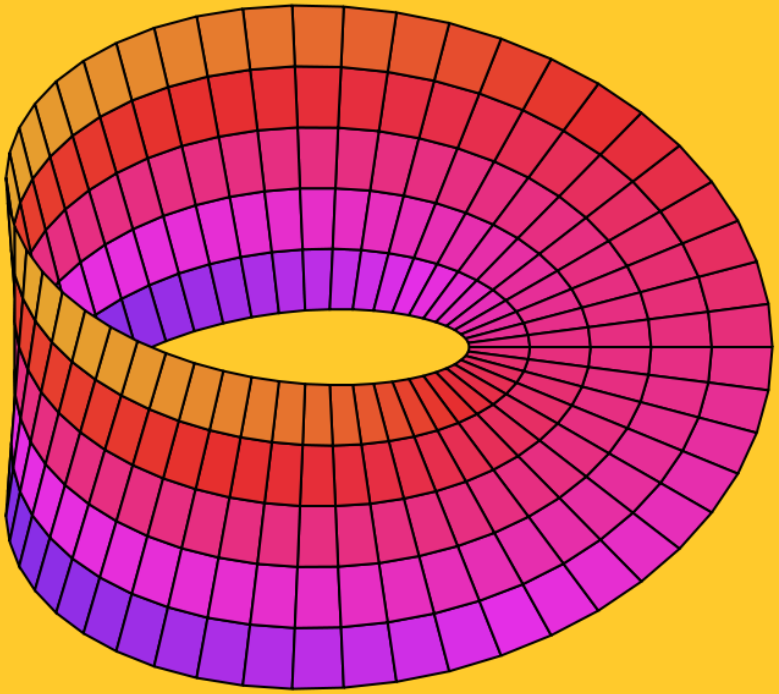
- The number of coordinates required is, in general, +1 than the dimension of the projective space being considered.
- For example, 3 homogeneous coordinates required to specify a point on a projective plane. 4 homogeneous coordinates are required to specify a point on a projective space and so on.
- The origin in homogeneous coordinate system in 2D is (0,0,1) and not (0,0) or (0,0,0).
- If 2D Cartesian coordinate is (x, y), corresponding homogeneous coordinate is (x,y,1).

German mathematicians

August Ferdinand Möbius 1827



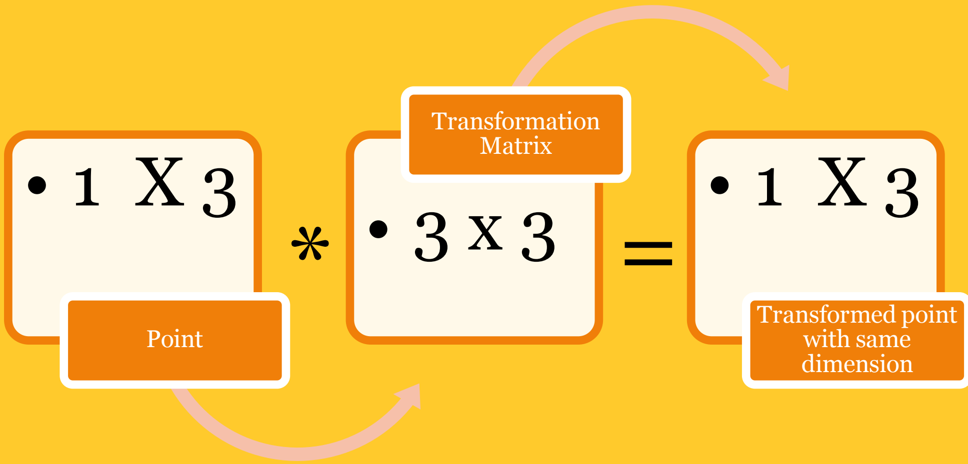
From WIKI



From Wiki- Homogeneous coordinates

- In mathematics, **homogeneous coordinates** or **projective coordinates**, introduced by August Ferdinand Möbius in his 1827 work *Der barycentrische Calcül*,[1][2] are a system of coordinates used in projective geometry, as Cartesian coordinates are used in Euclidean geometry. They have the advantage that the coordinates of points, including points at infinity, can be represented using finite coordinates. Formulas involving homogeneous coordinates are often simpler and more symmetric than their Cartesian counterparts. Homogeneous coordinates have a range of applications, including computer graphics and 3D computer vision, where they allow affine transformations and, in general, projective transformations to be easily represented by a matrix.

Homogeneous Coordinates



- So for a point (x,y) in 2D system is represented by a point (x,y,1) in homogeneous coordinate system.
- As the point is (1x3) matrix, the transformation matrix , t, is given by t=

$$\begin{bmatrix} a & b & p \\ c & d & q \\ l & m & s \end{bmatrix}$$

Translation

- [5, 3]+[5, 4]=[10, 7]
- Let v=[5,3,1] and t= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$ =[10, 7,1]

  Then vt=[10,7,1]

- [0, 0]+[5,4]=[5,4]
- For origin, v=[0,0,1], t= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$ =[5, 4, 1]

  Vt=[5,4,1] indicating that origin has shifted.

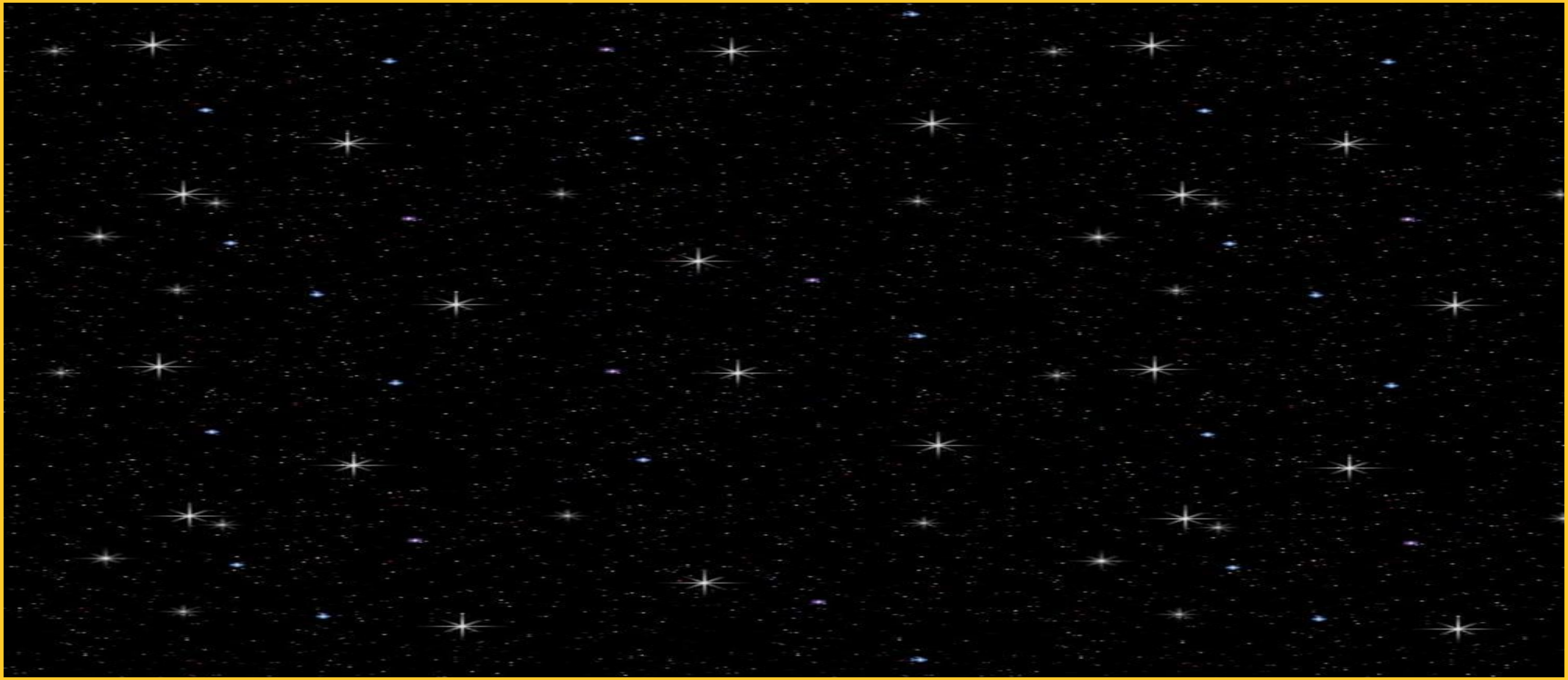
Note-Try it in excel.

- **NOTE 16:**

  Homogeneous coordinates help in translation and shifting of all points.

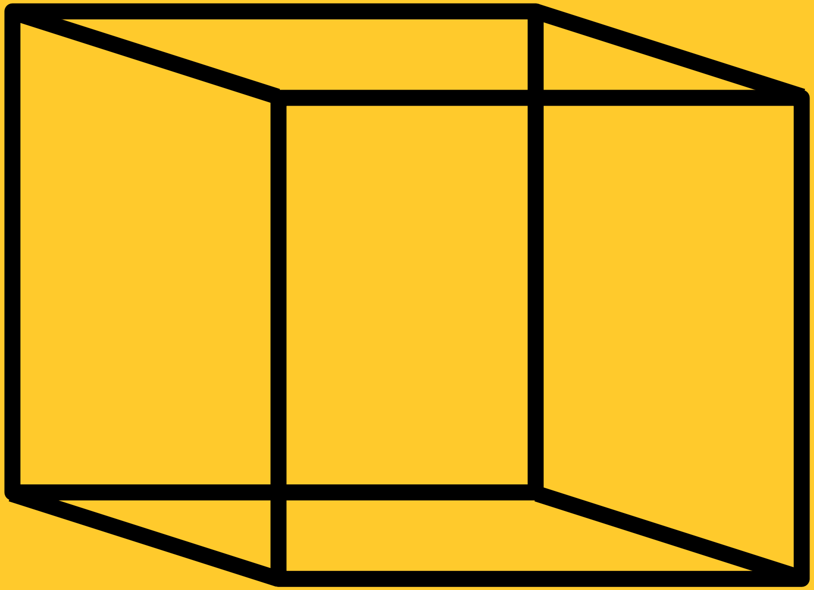
Physical World vs Visual World

If I ask , what is this photograph?
You will answer Night sky, Star. I can not differ.
God has given us an incredible gift – our eyes. It can be tiny but it is so powerful that we can see these stars at an infinite distance.



Physical World vs Visual World

Similarly if I ask you what is this? You will answer cube. I will say, no it is not cube. You will argue. But I will stick in my word.



And the problem lies here.

Physical World vs Visual World

- Similarly if I ask you what is this? You will answer, it is railway track. I will say, no it is not a rail track. You will argue. But I will stick in my word.

- And the problem lies here.
- What we see is different from the real world.

Technique of projection..

- v=[x y 1] t= $\begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix}$

  vt= [x, y, px+qy+1]
- Here h=px+qy+1. We can divide the coordinates of original vector by h=px+qy+1 to bring the points back to the homogeneous plane whose h value is 1.

Projection in Homogeneous Coordinates

$$\begin{bmatrix}1 & 3 & 1\\4 & 1 & 1\end{bmatrix} * \begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 3 & 5\\4 & 1 & 6\end{bmatrix} \Rightarrow h = \begin{bmatrix}5\\6\end{bmatrix}$$

- Now if we change the transformed coordinates to h=1 plane, then we get $\begin{bmatrix}1/5 & 3/5 & 1\\4/6 & 1/6 & 1\end{bmatrix}$
- This action of converting h=1 can be termed as PROJECTION.
- Try in Excel for a triangle

- The general 3x3 transformation matrix for 2D homogeneous coordinates can be subdivided into four parts-
- t= $\left[\begin{array}{cc|c} a & b & p \\ c & d & q \\ \hline m & n & s \end{array}\right]$
- The a, b, c, d elements produce scaling, rotation, reflection and shearing,
- m, n produces translation,
- p, q of the third column produces projection.
- s produces scaling

Overall Scaling

- The term 'S' here causes uniform Scaling. It changes the coordinates in a homogeneous plane other than h=1.
- If 0<S<1, then enlargement occurs and if S>1 then compression occurs.

$$t= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

- If we bring the transformed points to h=1, then

$$vt= \begin{bmatrix} 1/s & 3/s & 1 \\ 4/s & 1/s & 1 \end{bmatrix}$$

Three Dimensional Transformation

- Transformation matrix for Three Dimensional transformation is
- Point= $\alpha = [x, y, z]$

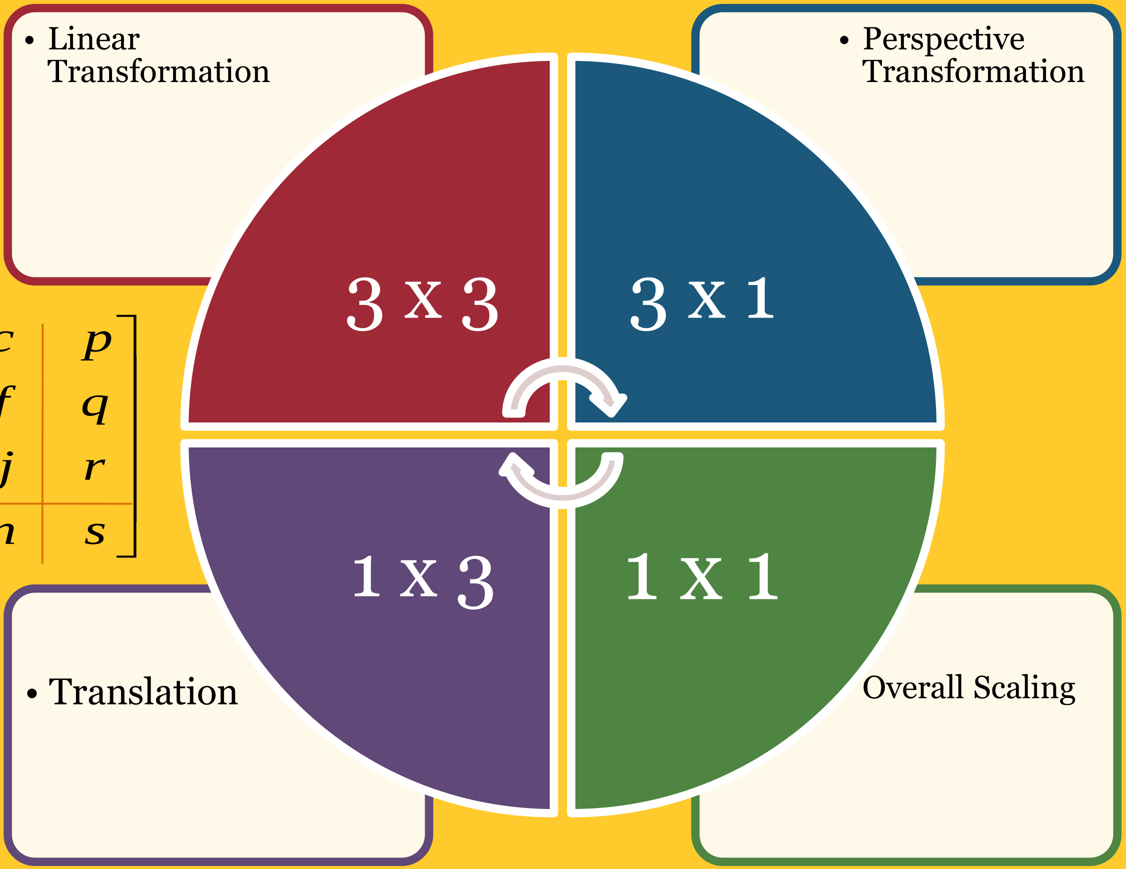
t= $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

- The equivalent homogeneous coordinates in 3d is
- Point = $\alpha = [x, y, z, 1]$

[T]= $\begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ l & m & n & s \end{bmatrix}$

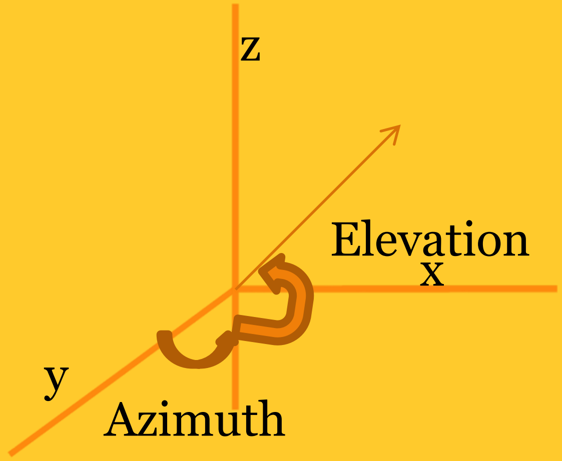
Three Dimensional Transformation

$$[T]= \left[\begin{array}{ccc|c} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ \hline l & m & n & s \end{array}\right]$$



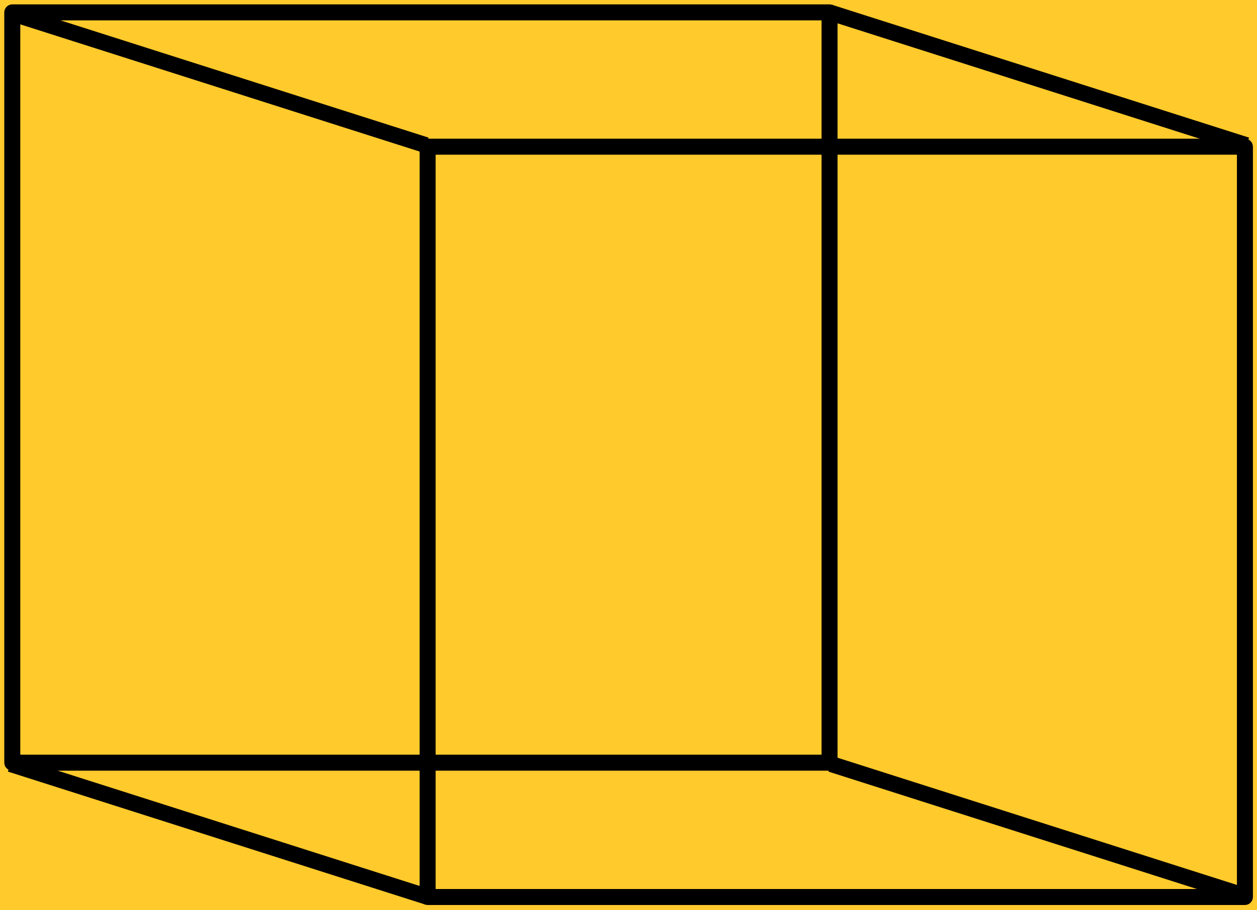
View Angle

- Viewing angles ⇨ Azimuth, Elevation



- Azimuth is the angle from –ve y-axis
- Elevation is the angle above xy plane to observer
- Orthogonal View (Front View) (90˚,0)

A 3d cube in 2d



Creating a cuboid in 3d

x	y	z	h
0	0	1	1
2	0	1	1
2	3	1	1
0	3	1	1
0	0	1	1
0	0	0	1
2	0	0	1
2	3	0	1
0	3	0	1
0	3	1	1
2	3	1	1
2	3	0	1
2	0	0	1
2	0	1	1
0	0	1	1
0	0	0	1
0	3	0	1

py	py	py	py
=Cos(x)	0	=Sin(x)	0
0	1	0	0
=-sin(x)	0	=Cos(x)	0
0	0	0	1

1	0	0	0
0	=Cos(y)	=sin(y)	0
0	=-sin(y)	=cos(y)	0
0	0	0	1

pz	pz	pz	pz
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	1

Three Dimensional Scaling (Local)

- ts= $\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ produces local scaling about x, y, z coordinate axis.

- ts= $\begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Reduces x coordinate by ½ and y coordinate by 1/3 and z unchanged.

Three Dimensional Scaling (Overall)

- ts= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$
- When s<1, h reduced to less than 1, converting h=1 produces enlargement.
- When s>1, h>1, when h is made equal to 1, produces compression.
- Main diagonal produces scaling
- The overall scaling can also be achieved by means of uniform local scaling factor 1/s.

Three Dimensional Shearing

- The off diagonal terms of 3x3 upper left sub matrix of the generalized 4x4 transformation matrix produces shearing.
- $t_{sh} = \begin{bmatrix} 1 & b & e & 0 \\ c & 1 & f & 0 \\ g & i & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is a shear matrix.
- The origin remains unaffected.

3 Dimensional Rotation

- There is no formula for 3d rotation
- We play a trick to achieve 3d rotation
- 2D rotation formula for rotation will be used

Three Dimensional Rotation

- Before considering three dimensional rotation about an arbitrary axis, let us examine rotation about x axis. For rotation about x-axis, the x coordinates of the position vectors do not change and the transformation matrix is given by

$$t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Contd..

- The rotation about Z-axis is:

$$t = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The rotation about Y-axis is:

$$t = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The requirement of pure rotation is determinant=+1

Three Dimensional Reflection

- The reflection occurs through a plane. During the reflection through XY plane, the Z coordinate value reversed in sign.
- $tz = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for XY Plane
- $tx = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for YZ Plane
- $ty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for XZ Plane

Three Dimensional Translation

$$[Tr] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix}$$

Multiple Transformations

- Successive transformations can be combined or concatenated into a single 4x4 transformation that yields the same results. Since matrix methods are non-commutative, the order of multiplication is important ([A] [B] ≠ [B] [A]).

What is Plane Geometric Projection

- The process of converting a three dimensional object into a two dimensional object for viewing purpose is called projection.

- The Projection Matrix from 3d to 2d always contains a column of zero.

- The projections of objects are formed by the intersection of lines called projectors with a plane called plane of projection.

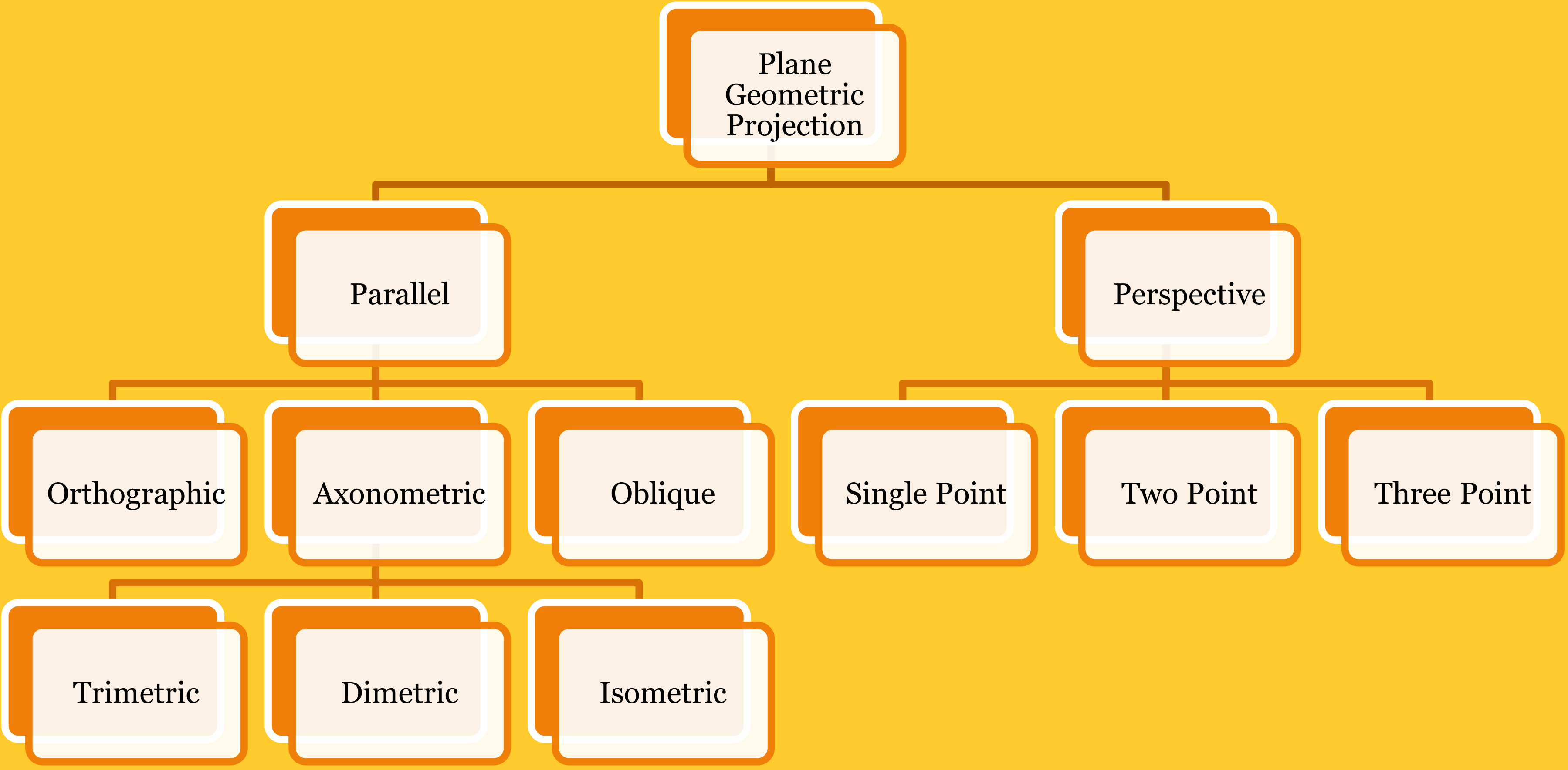
Plane Geometric Projection

What is Plane Geometric Projection

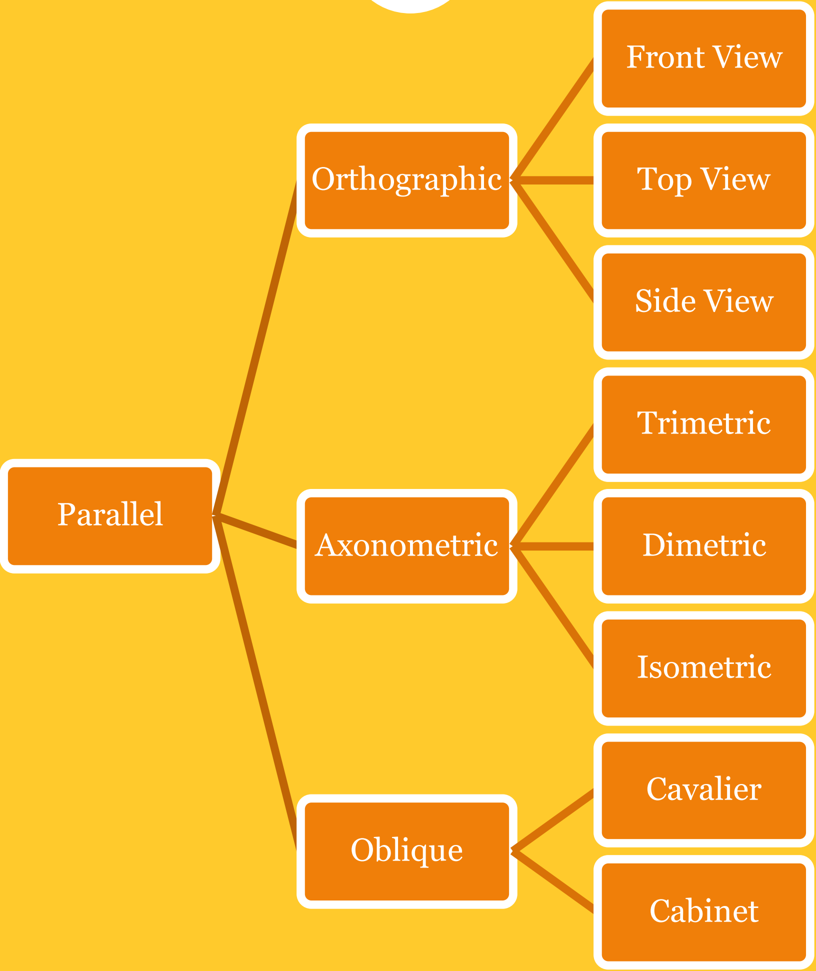
- Projectors: These are the lines from an arbitrary point called center of projection(CP), through each point in object.
- How we see objects?
- Approach:

1) Object Fixed, Center of projection is free to move (Used for large object)
2) Center of projection is fixed, object is manipulated to obtain any required view (Used to describe small object)

Type of Projections



Type of Projections



Orthographic Projections

- It is the projection where one of the coordinate plane is zero.
- The matrix for projection into x=0, y=0, z=0 are given below:
- $$P_X = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Parallel Projection-Orthographic-Front View

- In orthographic Projection the center of projection is at infinity.
- Front View (Elevation): The center of projection at the infinity in the positive z-axis
- Projection is formed in x-y plane where z=0

- The Projection matrix is:

$$Pz=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Parallel Projection-Orthographic-Right View

- In orthographic Projection the center of projection is at infinity.
- Right View: The center of projection at the infinity in the positive x-axis
- Projection is formed in y-z plane where x=0

- The Projection matrix is:

$$Px=\begin{bmatrix}0 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Parallel Projection-Orthographic-Top View

- In orthographic Projection the center of projection is at infinity.
- Top View: The center of projection at the infinity in the positive y-axis
- Projection is formed in x-z plane where y=0

- The Projection matrix is:

$$Py=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Parallel Projection-Orthographic-Bottom View

- In orthographic Projection the center of projection is at infinity.
- Bottom View: First the object is rotated at 90 degree about x axis and the project it at z=0 plane.
- The center of projection at the infinity in the positive z-axis
- Projection is formed in x-y plane where z=0

- The Projection matrix is:

$$\text{Protx90}=\begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & -1 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix} \qquad \text{Pz}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Parallel Projection-Orthographic-Left View

- In orthographic Projection the center of projection is at infinity.
- Left View: First the object is rotated at 90 degree about y axis and then project it at z=0 plane.
- The center of projection at the infinity in the positive z-axis
- Projection is formed in x-y plane where z=0

- The Projection matrix is:

$$\text{Proty90}=\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{Pz}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Parallel Projection-Orthographic-Rear View

- In orthographic Projection the center of projection is at infinity.
- Rear View: First the object is reflected about z=0 plane and then project it at z=0 plane.
- The center of projection at the infinity in the positive z-axis
- Projection is formed in x-y plane where z=0

- The Projection matrix is:

$$\text{Prefz}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \qquad \text{Pz}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Orthographic Projection

- Orthographic projection matrices

$$[T_z] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; [T_y] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; [T_x] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic Views

View	C.O.Projection	Proj. Plane
Front	On +ve z axis	Z=0 (xy)
Right Side	On +ve x axis	X=0 (yz)
Top	On +ve y axis	Y=0 (xz)
Rear	On -ve z axis	Z=0 (xy)
Left Side	On -ve x axis	X=0 (yz)
Bottom	On -ve y axis	Y=0 (xz)

Parallel Projection-Auxiliary View

- An object with planes not parallel to one of the coordinate planes, orthographic view do not show the correct shape of the plane.
- In auxiliary view, the normal of the auxiliary plane is rotated and translated to make it coincident with one of the coordinate system.
- The result is then projected onto the coordinate plane perpendicular to that axis.
- The Projection matrix is:

$$\text{Prot}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & 1\end{bmatrix},\ \text{Ptran}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix},\quad \text{Pz}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Parallel Projection-Axonometric

- In orthographic projection we can view only one face. In axonometric projection at least three adjacent faces are shown.
- This is achieved by rotation, translation and then projection in one plane, generally z=0 plane.
- The center of projection is at infinity. As the faces are not parallel to the plane of projection, the projection does not show true shape.
- However, parallel lines equally fore shortened.
- The fore shortening factor is the ratio of the projected length to the true length.
- There are three types of Axonometric projection:

  (1) Trimetric (2) Dimetric and (3) Isometric

Axonometric Projections

- An axonometric projection is used to show three adjacent faces of an object.

- After a rotation with few degree, translations to a desired distance, the result is then projected from a centre of projection at infinity onto one of the coordinate planes, usually z=0 plane or xy plane.

Parallel Projection-Axonometric-Trimetric

- Axonometric projection is formed by first rotating the object in y direction, then by x-direction followed by projection in z=0 plane.
- In Trimetric Projection, the foreshortening factor in x , y and z direction is different.
- The Projection matrix is:

$$\text{Proty}=\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Protx}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\text{Pz}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{T=Proty*Protx*Pz},$$

Parallel Projection-Axonometric-Trimetric

- In Trimetric Projection, the foreshortening factor in x , y and z direction is calculated by applying the concatenated transformation matrix to the unit vectors along the principal axes.
- The Projection matrix is:

$$Proty=\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Protx=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Pz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T=Proty*Protx*Pz,\ U=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Parallel Projection-Axonometric-Trimetric

- In Trimetric Projection, the foreshortening factor in x , y and z direction is calculated by applying the concatenated transformation matrix to the unit vectors along the principal axes.
- Let, the angle of rotation in y is p, and the angle of rotation in x is t.
- The Projection matrix is:

$$\text{Proty}=\begin{bmatrix} cp & 0 & sp & 0 \\ 0 & 1 & 0 & 0 \\ -sp & 0 & cp & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Protx}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & ct & st & 0 \\ 0 & -st & ct & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Pz}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\text{T=Proty*Protx*Pz, T}=\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{U}=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Parallel Projection-Axonometric-Trimetric

- The calculation of foreshortening factor:

Proty= $\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Protx= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Pz= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

T=Proty*Protx*Pz, U= $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T= $\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

U*=U*T

Fx= $\sqrt{xx^2 + xy^2}$, Fy= $\sqrt{xy^2 + yy^2}$, Fz= $\sqrt{xz^2 + yz^2}$

Fx= $\sqrt{cp^2 + spst^2}$, Fy= $\sqrt{0^2 + ct^2}$, Fz= $\sqrt{sp^2 + -cpst^2}$

Parallel Projection-Axonometric-Dimetric

- In diametric projection, two the three foreshortening factor is equal.
- The third one is arbitrary.

- calculation of foreshortening factor:

Proty= $\begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Protx= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Pz= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

T=Proty*Protx*Pz, U= $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T= $\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Original length of the unit vector = 1.

Fx= $\sqrt{xx^2 + xy^2}$, Fy= $\sqrt{xy^2 + yy^2}$, Fz= $\sqrt{xz^2 + yz^2}$

Fx= $\sqrt{cp^2 + spst^2}$, Fy= $\sqrt{0^2 + ct^2}$, Fz= $\sqrt{sp^2 + -cpst^2}$

Parallel Projection-Axonometric-Dimetric

- In diametric projection, two the three foreshortening factor is equal.
- The third one is arbitrary.

- calculation of foreshortening factor:

T=Proty*Protx*Pz, U= $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T= $\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

U*=U*T

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Fx= $\sqrt{xx^2 + xy^2}$, Fy= $\sqrt{xy^2 + yy^2}$, Fz= $\sqrt{xz^2 + yz^2}$

Fx= $\sqrt{cp^2 + spst^2}$, Fy= $\sqrt{0^2 + ct^2}$, Fz= $\sqrt{sp^2 + -cpst^2}$

For two foreshortening factors to be equal, p=asin(fz/ $\sqrt{2 - fz^2}$)

t=asin(+-fz/ $\sqrt{2}$)

Parallel Projection-Axonometric-Dimetric

- Calculation of foreshortening factor:

U*=U*T

- Original length of the unit vector = 1.

Fx=$\sqrt{xx^2 + xy^2}$, Fy=$\sqrt{xy^2 + yy^2}$, Fz=$\sqrt{xz^2 + yz^2}$

Fx=$\sqrt{cp^2 + spst^2}$, Fy=$\sqrt{0^2 + ct^2}$, Fz=$\sqrt{sp^2 + -cpst^2}$

For two foreshortening factors to be equal,

p=asin(+-fz/$\sqrt{2 - fz^2}$)

t=asin(+-fz/$\sqrt{2}$)

- It is mentioned that FF is between 0 to 1.
- For each fz between 0 to 1, there are four possible diametric projections depending on angles p and t.

Parallel Projection-Axonometric-Isometric

- In isometric projection, all three foreshortening factors are equal.
- calculation of foreshortening factor:

T=Proty*Protx*Pz, U= $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, T= $\begin{bmatrix} cp & spst & 0 & 0 \\ 0 & ct & 0 & 0 \\ sp & -cpst & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Original length of the unit vector = 1.

Fx= $\sqrt{xx^2 + xy^2}$, Fy= $\sqrt{xy^2 + yy^2}$, Fz= $\sqrt{xz^2 + yz^2}$

Fx= $\sqrt{cp^2 + spst^2}$, Fy= $\sqrt{0^2 + ct^2}$, Fz= $\sqrt{sp^2 + -cpst^2}$

- For isometric view, Fx=Fy=Fz. This can be achieved when FF=0.8165
- There are four isometric view with p=+_45 degree and t=+-35.26 degree.

Parallel Projection-Oblique Projection

- The center of projection is at infinity.
- The projectors intersect the projection plane at an oblique angle.
- Faces parallel to the plane of projection shows true shape and size.
- Faces that are not parallel to the plane of projection are distorted.

- The Projection matrix is:

$$P\_oblique=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Parallel Projection-Oblique Projection

- In two-dimension the projector P'O can be obtained from PO, where PO is the unit vector by translating P to the point P' at (-a –b 1)

$$P\_oblique=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

- Where a=f*cos(t) and b=f*sin(t) and f is the projected length of the z-axis unit vector, i.e., the foreshortening factor and t is the angle between projected z-axis and x-axis.

Parallel Projection-Oblique Projection

- P_oblique= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,
- Where a=f*cos(t) and b=f*sin(t)
- Let b=angle of projector and the plane of projection. Then b=acot(f)
- When b=90 degree, f=0. When f=1, b=acot(1)=45 degree. This condition is called Cavalier Projection.
- When foreshortening factor is ½, the angle t=acot(1/2)=63.435 degree.

Perspective Transformations

- When any of the first three elements of the fourth column of the homogeneous coordinate is non-zero, a perspective transformation results.
- It is a transformation in one 3d space to another 3d space.
- In perspective transformation parallel lines converge, object size is reduced with increasing distance from the centre of projection and non- uniform foreshortening of the lines in the object as a function of orientation and distance of the object from the centre of projection occurs.
- Center of projection at a finite distance from the Projection Plane.

Perspective Transformations

- When any of the first three elements of the fourth column of the homogeneous coordinate is non-zero, a perspective transformation results.
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- Center of projection at a finite distance from the Projection Plane.

Perspective Transformations

- Perspective Transformation Matrix

$$P=[x,y,z,1],\ Tp=\begin{bmatrix}1 & 0 & 0 & p\\ 0 & 1 & 0 & q\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix},\ Tz=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

P*=P*Tp*Tz=[x y 0 px+qy+rz+1]

- The point is to be brought back to the h=1 plane.
- This is done by dividing all elements of

[x y z px+qy+rz+1] by px+qy+rz+1.

Perspective Transformations-Single Point

- Perspective Transformation Matrix

$$P=[x,y,z,1],\ Tp=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & r\\0 & 0 & 0 & 1\end{bmatrix},\ Tz=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

P*=P*Tp*Tz=[x y 0 rz+1]

- The point is to be brought back to the h=1 plane.
- This is done by dividing all elements of

[x y z rz+1] by rz+1.

Perspective Transformations-Single Point

- The projection of P(x, y, z) in the projection plane is given by P'(x*, y*, z*)
- The relation can be established by,
  - x*/zc=x/(zc-z), x*=x/(1-z/zc), Let, r=-1/zc, gives, x*=x/(1+rz)
  - y*/zc=y/(zc-z), y*=y/(1-z/zc), let r=-1/zc, gives, y*=y/(1+rz)
  - The result is same.
  - The perspective projection matrix is T=Tp*Tz.
  - $T=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Perspective Transformations-Single Point

- The perspective projection matrix is T=Tp*Tz.
- T= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- This matrix produces perspective projection on to z=0 plane from a center of projection, zc=-1/r on the z-axis.
- Perspective projection occurs in two steps-first is the perspective transformation and then projection.
- The perspective transformation image intersects the z-axis at z=+1/r.
- This intersection point represents the intersection point of the line parallel to z-axis and z-axis at infinity into the finite point at z=+1/r on the z-axis. This point is called vanishing point.
- Vanishing point lies on the opposite side of the plane of projection at an equal distance of 1/r.
- All lines parallel to z axis passes through [0 0 1/r 1] point, the vanishing point.

Perspective Transformations-Single Point

- The perspective projection in x-axis is T=Tp*Tz.
- T= $\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and xc=[-1/p 0 0 1], vanishing point = [1/p 0 0 1]

- The perspective projection in y-axis is T=Tp*Tz.
- T= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- and yc=[0 -1/q 0 1], vanishing point = [0 1/p 0 1]

Perspective Transformations-Two Point

- Single point perspective projection, does not provide adequate perception of the three dimensional shape of the object. Two point perspective projection helps in this direction.

- The perspective projection matrix is T=Tp*Tz.
- Tp= $\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Tz= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, **T=Tp*Tz**
- **This matrix produces perspective projection on to z=0 plane from two center of projection, xc=-1/p on the x-axis and yc=-1/q on y axis..**
- Vanishing point lies on the opposite side of the plane of projection at an equal distance of 1/p and 1/q in x and y axis.
- All lines parallel to x and y axis passes through [1/p 0 0 1] and [0 1/q 0 1] point in respective axes, the vanishing points.

Perspective Transformations-Three Point

- For adequate perception of the three dimensional shape of the object, three point perspective projection is used.

- The perspective projection matrix is T=Tp*Tz.
- Tp= $\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$, Tz= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, **T=Tp*Tz**
- **This matrix produces perspective projection on to z=0 plane from three center of projection, xc=-1/p on the x-axis and yc=-1/q on y axis and zc=-1/r in z-axis.**
- Vanishing point lies on the opposite side of the plane of projection at an equal distance of 1/p, 1/q and 1/r in x, y and z axis.
- All lines parallel to x, y and z axis passes through [1/p 0 0 1], [0 1/q 0 1] and [0 0 1/r 1] point in respective axes, the vanishing points.

Size of Transformation Matrix

- 2D Plane – The transformation matrix for a 2d object is a 3x3 matrix

- 3D Space – The transformation matrix of a 3d object is a 4x4 matrix

- The number of points representing the object can be infinite but the transformation matrix is only 3x3 or 4x4 matrix.

Depth Perception

Q&A…

How this knowledge help in learning Mathematics

- Now we can create any object – in 2-dimension as well as 3 – dimensions.
- We can transform these objects
- We can create graphs of one variable, two variable functions.
- We can demonstrate Geometry, Trigonometry, Coordinate Geometry, Vectors, Complex Functions easily.
- We can demonstrate all mathematical concepts related to any branch of mathematics and remove the abstractness of mathematics.

Coordinate Systems

- Quadrants: The axes of a two-dimensional Cartesian system divide the plane into four infinite regions, called quadrants, each bounded by two half-axes.

2nd Quadrants,
--+

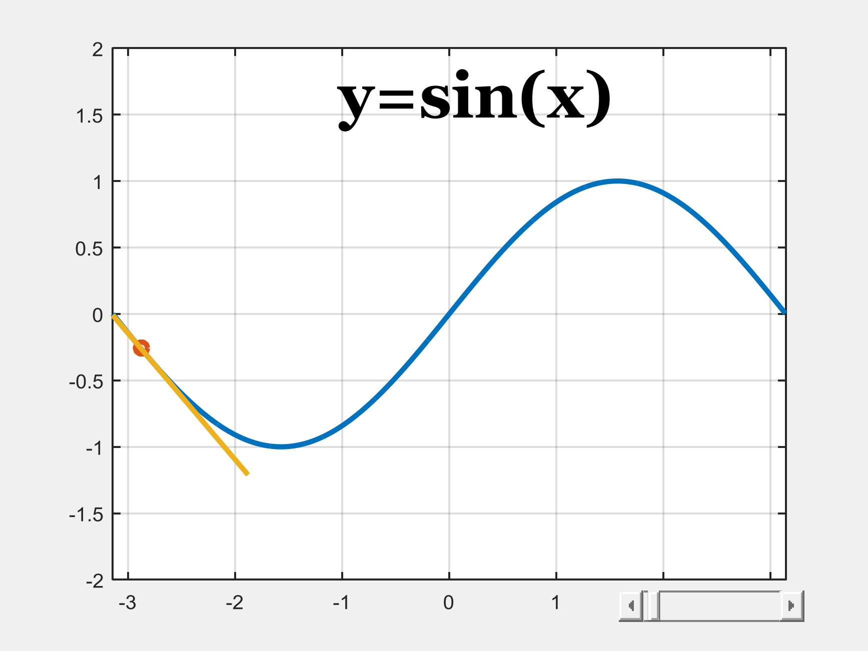
1st Quadrants,
++

3rd Quadrants, -
--

4th Quadrants,
+-

Construction Methods and Animations

- Function of Single Variable, y=sin(x)
- t=-pi:.01:pi;
- x=sin(t)
-
- plot(t,x,'LineWidth',2.5)
- axis([-pi pi -2 2])
- grid

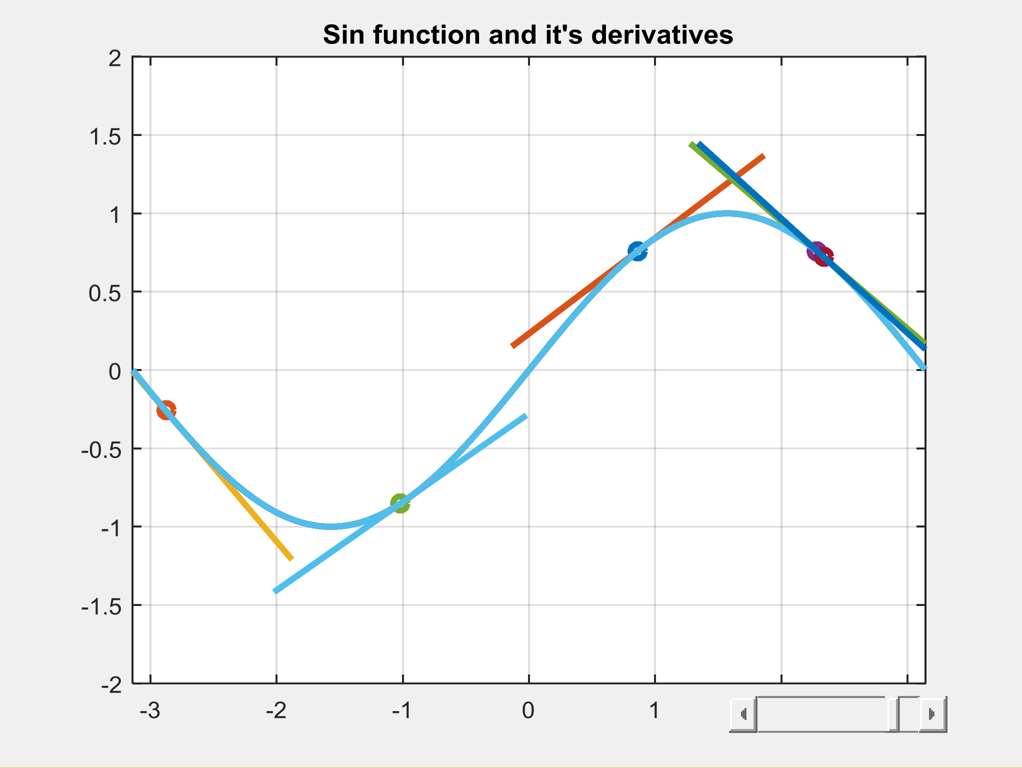


Putting a Slider in Script file

- Animating derivative of a Function of Single Variable
- % 1. Create a figure and axes
-         f = figure()
-         ax=axis()
- % 2. Create slider
-         sld = uicontrol('Style', 'slider','Min',-pi,'Max',pi,'Value',-pi+.2,...
-         'Position', [400 20 120 20],'Callback', @sldcall);
-
- % 3. Create a call back
-                 function sldcall(source,event)
-                 val = get(source,'Value')

Animating Derivative of Sin function

- plot(t,x,'LineWidth',2.5)
- axis([-pi pi -2 2])
- grid
- hold on
- x1=val
- y1=sin(x1)
- plot(x1,y1,'o','LineWidth',2.5)
- x2=x1+.1
- y2=sin(x2)
- m=(y2-y1)/(x2-x1)
- c=y1-m*x1
-
- x3=x1-1
- x4=x1+1
- y3=x3*m+c
- y4=x4*m+c
- plot([x1,x3,x4],[y1,y3,y4],'LineWidth',2.5)



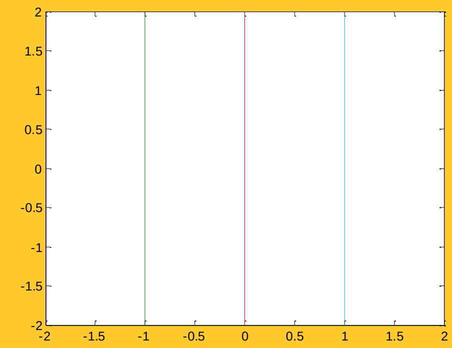
Functions of 2 variable: Plotting xy Grid

- x y grid is the domain of a two variable function z=f(x,y)

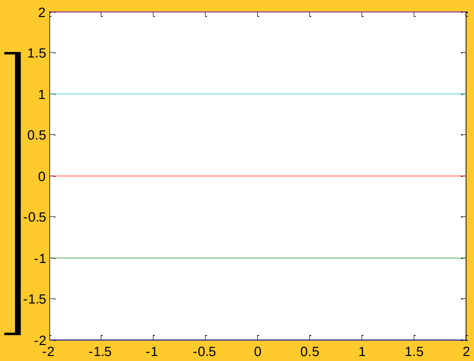
- It is very important to learn how to make a grid
- Matlab Commands:
- X=-2:1:2
- Y=X'
- [x,y]=meshgrid(X,Y)
- plot(x,y)
- figure
- plot(y,x)
- figure
- plot(x,y,y,x)

xy Grid Data

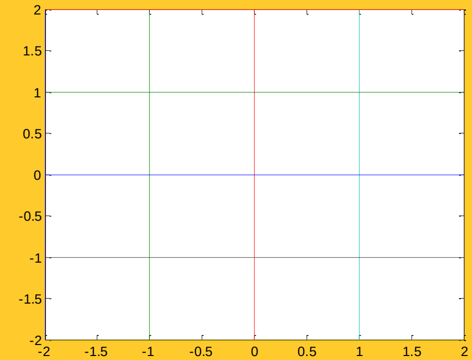
- X = [-2 -1 0 1 2]
- Y = [-2 -1 0 1 2]'
- x = $\begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}$
- y =
- $\begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix}$



y-grid



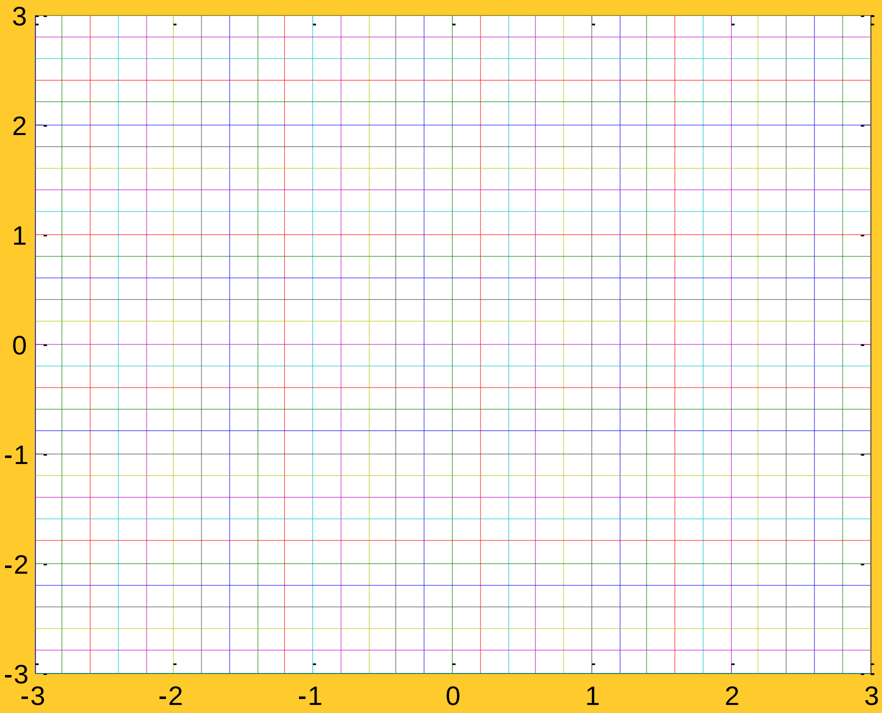
x-grid



x-y grid

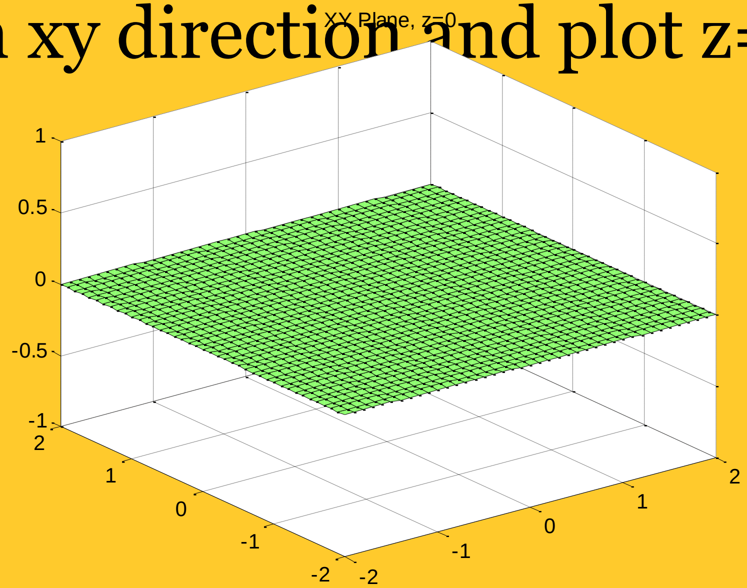
Mesh Grid

- xx=-3:.2:3
- yy=xx'
- [x,y]=meshgrid(xx,yy)
- plot(x,y,y,x)



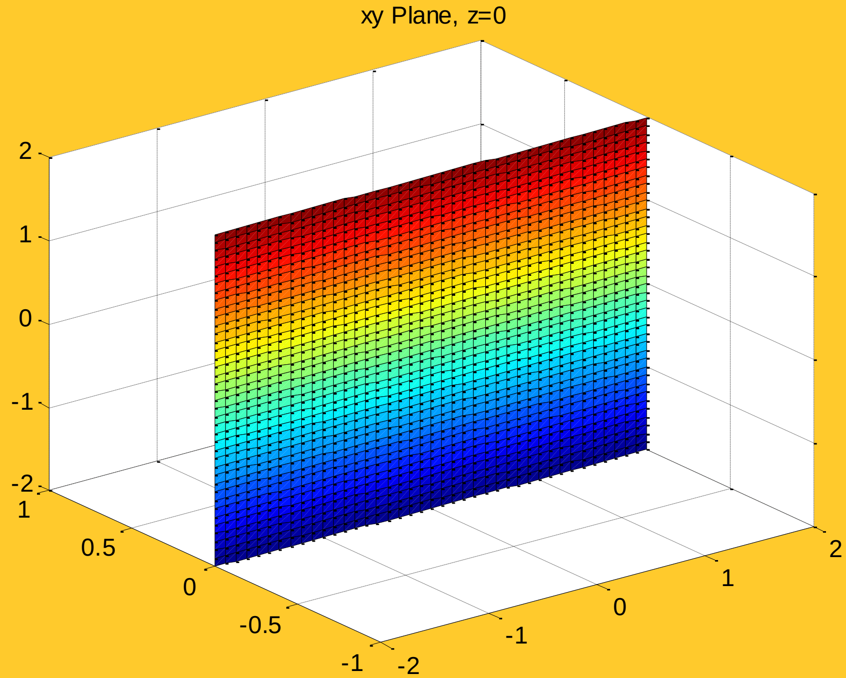
Creating XY Plane, Z=0

- A xy plane is that whose Z value is 0
- We have already to know how to create a grid. Now we will create a grid in xy direction and plot z=0 value in this grid.
- X=-2:.1:2;
- Y=X';
- [x,y]=meshgrid(X,Y);
- z=x.*y*0-0;
- surf(x,y,z)



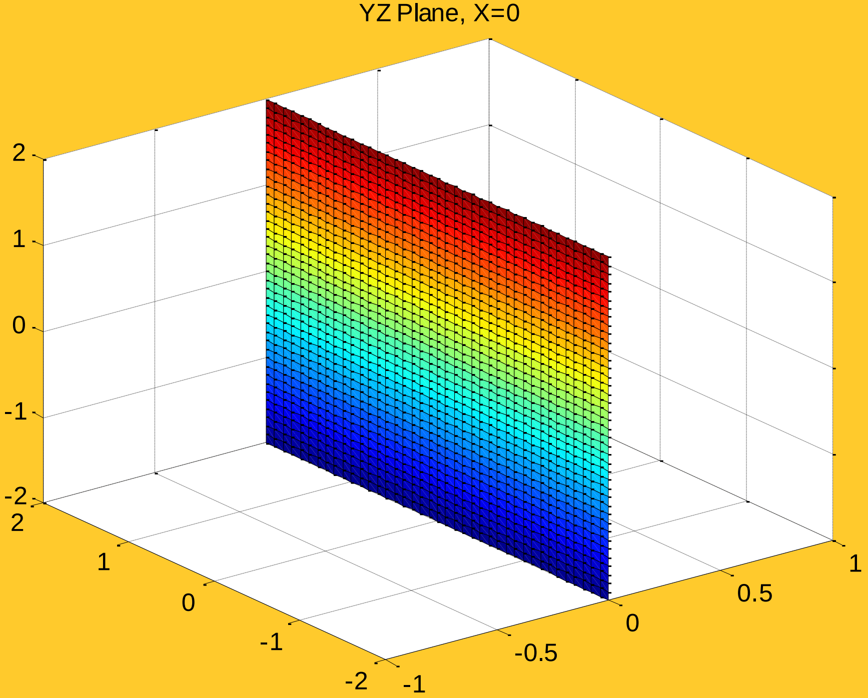
Creating Y=0 plane or xz plane

- A xz plane is that whose y value is 0
- We have already to know how to create a grid. Now we will create a grid in xz direction and plot y=0 value in this grid.
- %XZ PLANE
- X=-2:.1:2;
- Z=X';
- [x,z]=meshgrid(X,Y);
- y=x.*z.*0+0;
- figure
- surf(x,y,z)



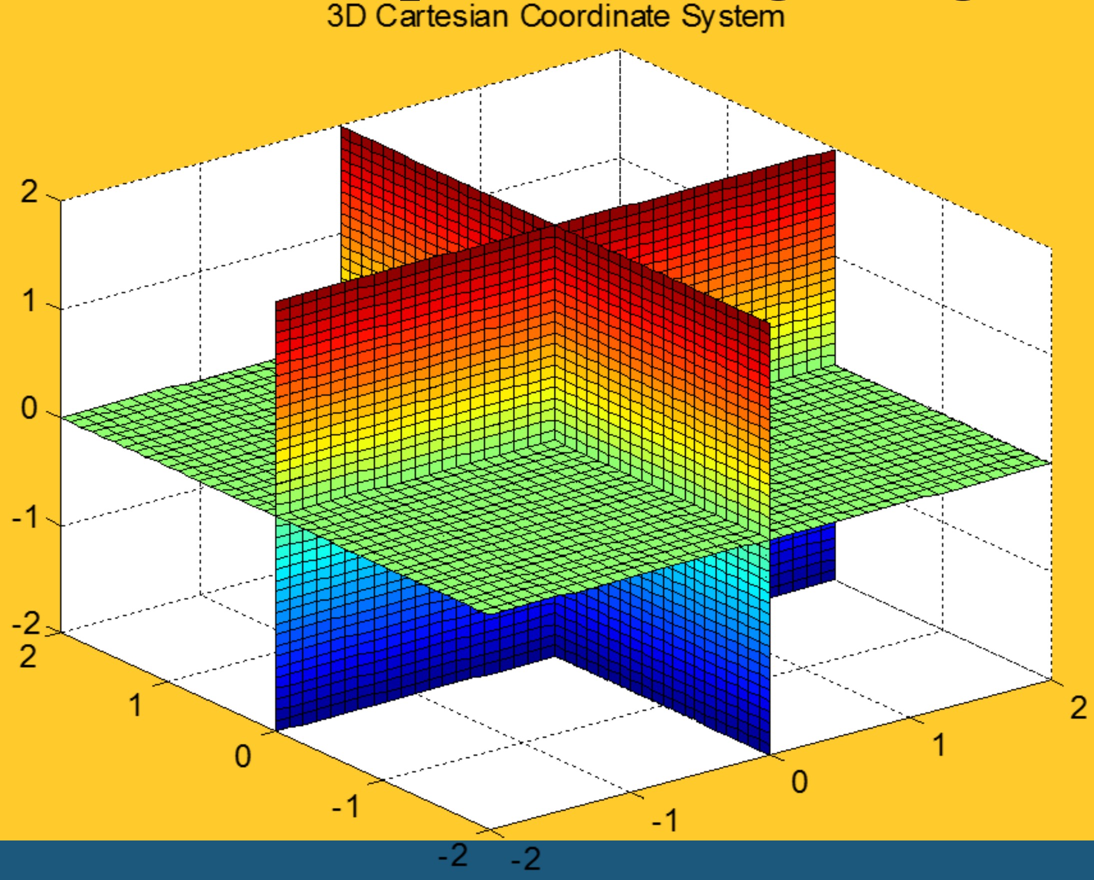
Creating YZ Plane, x=0

- A yz plane is that whose y value is 0
- We have already to know how to create a grid. Now we will create a grid in xz direction and plot y=0 value in this grid.
- %yz PLANE
- Y=-2:.1:2;
- Z=Y';
- [y,z]=meshgrid(Y,Z);
- X=Y.*z.*0+0;
- figure
- surf(x,y,z)



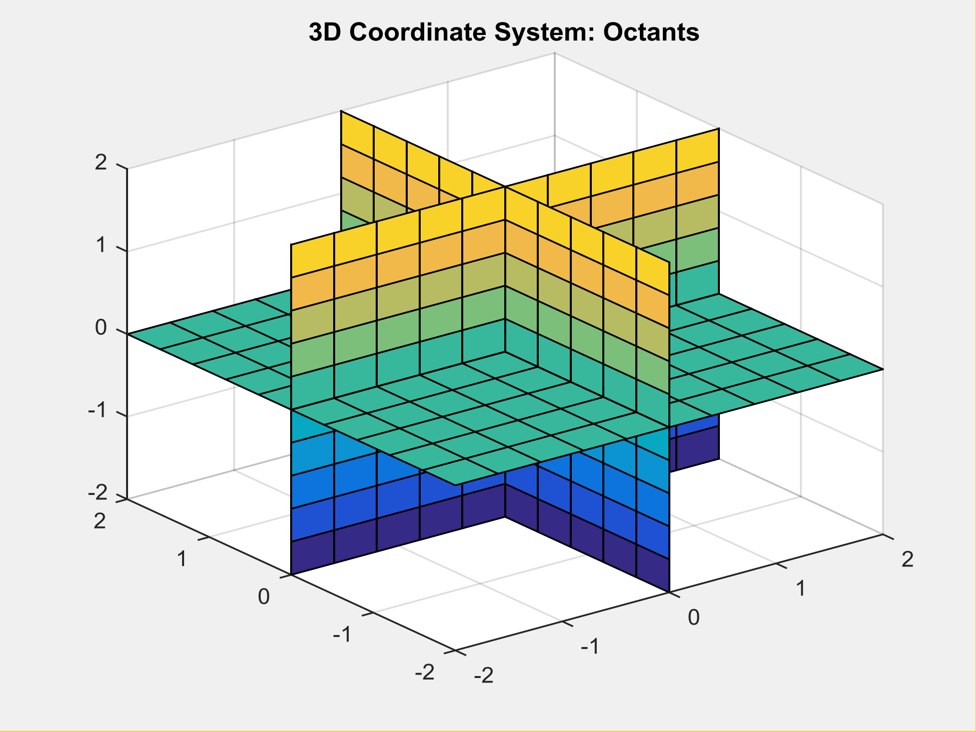
Coordinate Systems

- Octants: A three-dimensional Cartesian system defines a division of space into eight regions or octants.



3D Coordinate System

- %1. xz plane, y=0
- X=-2:.4:2;
- Z=X';
- [x,z]=meshgrid(X,Y);
- y=x.*z.*0+0;
- surf(x,y,z)
- hold on
- %2. xy plane, y=0
- X=-2:.4:2;
- Y=X';
- [x,y]=meshgrid(X,Y);
- z=x.*z.*0+0;
- surf(x,y,z)
- %3. yz plane, x=0
- Y=-2:.4:2;
- Z=Y';
- [y,z]=meshgrid(X,Y);
- x=y.*z.*0+0;
- surf(x,y,z)

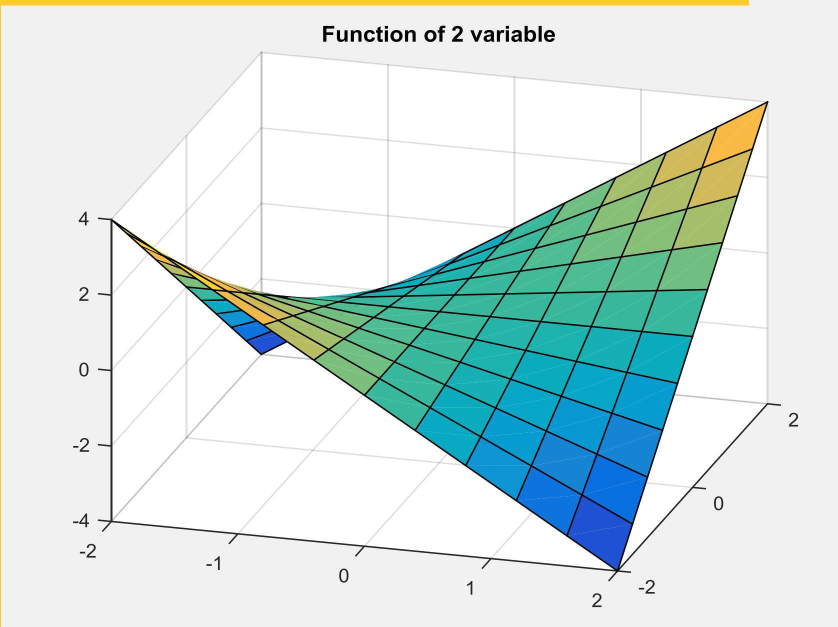


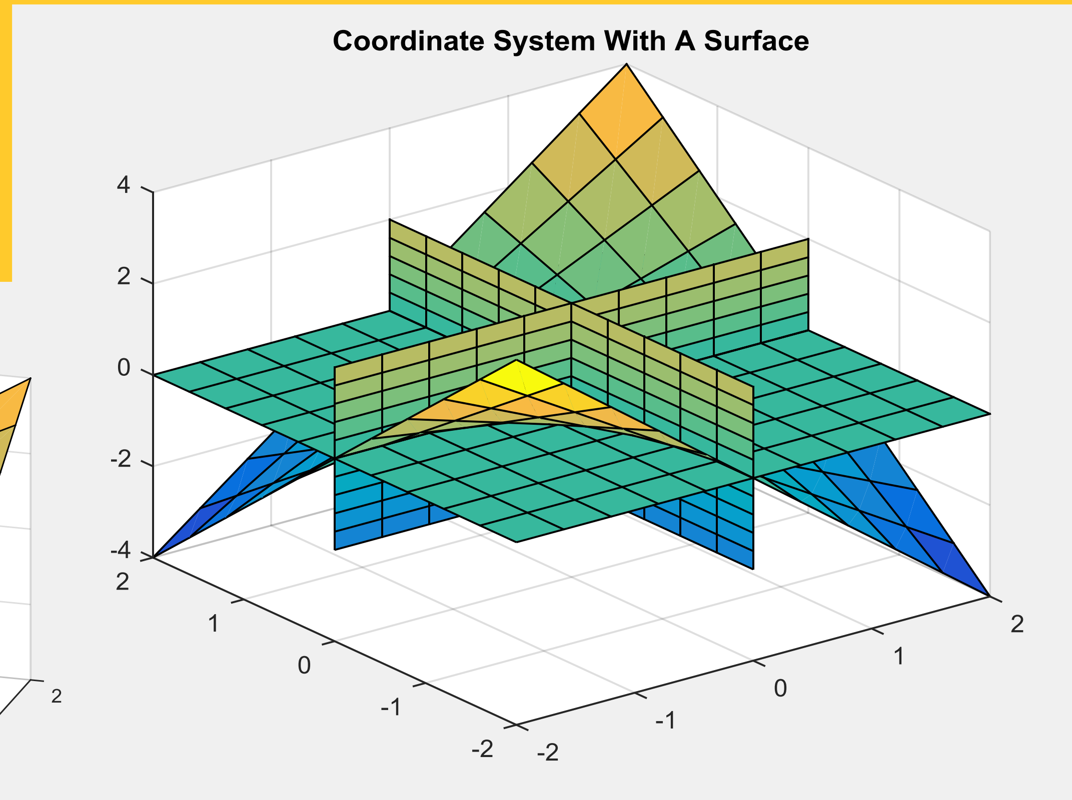
3D Coordinate System with a surface

- X=-2:.4:2;
- Y=X';

[x,y]=meshgrid(X,Y);

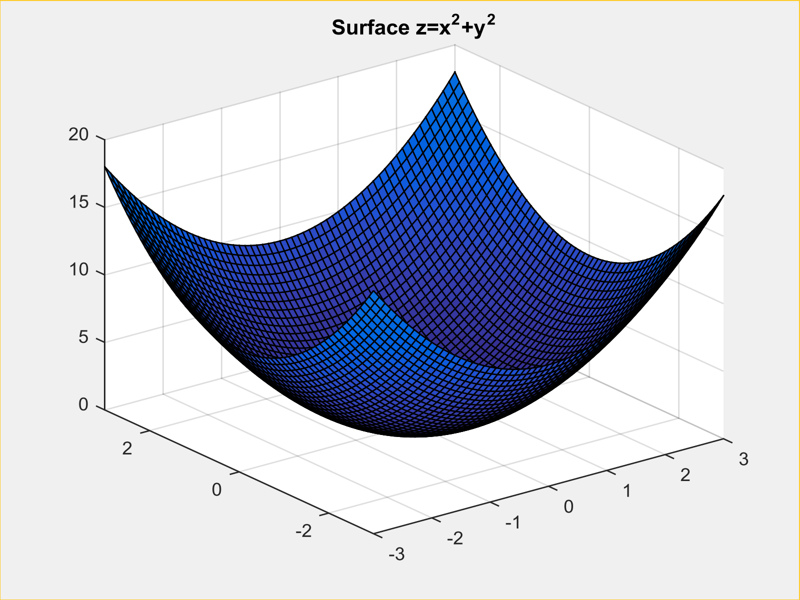
- z=x.*y;
- surf(x,y,z)





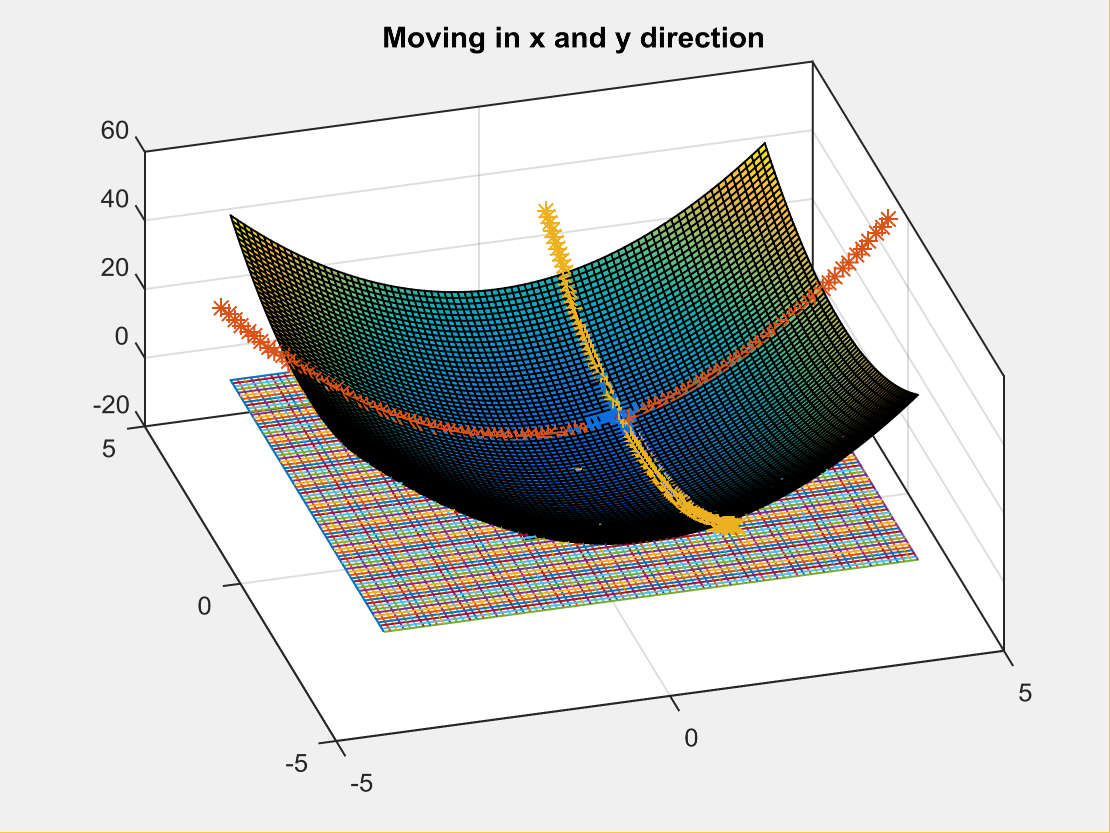
Drawing a surface

- z=x^2+y^2
- X=-7:.1:7
- Y=X'
- [x,y]=meshgrid(X,Y)
- z=x.^2+y.^2
- surf(x,y,z)
- axis([-3 3 -3 3 0 20])



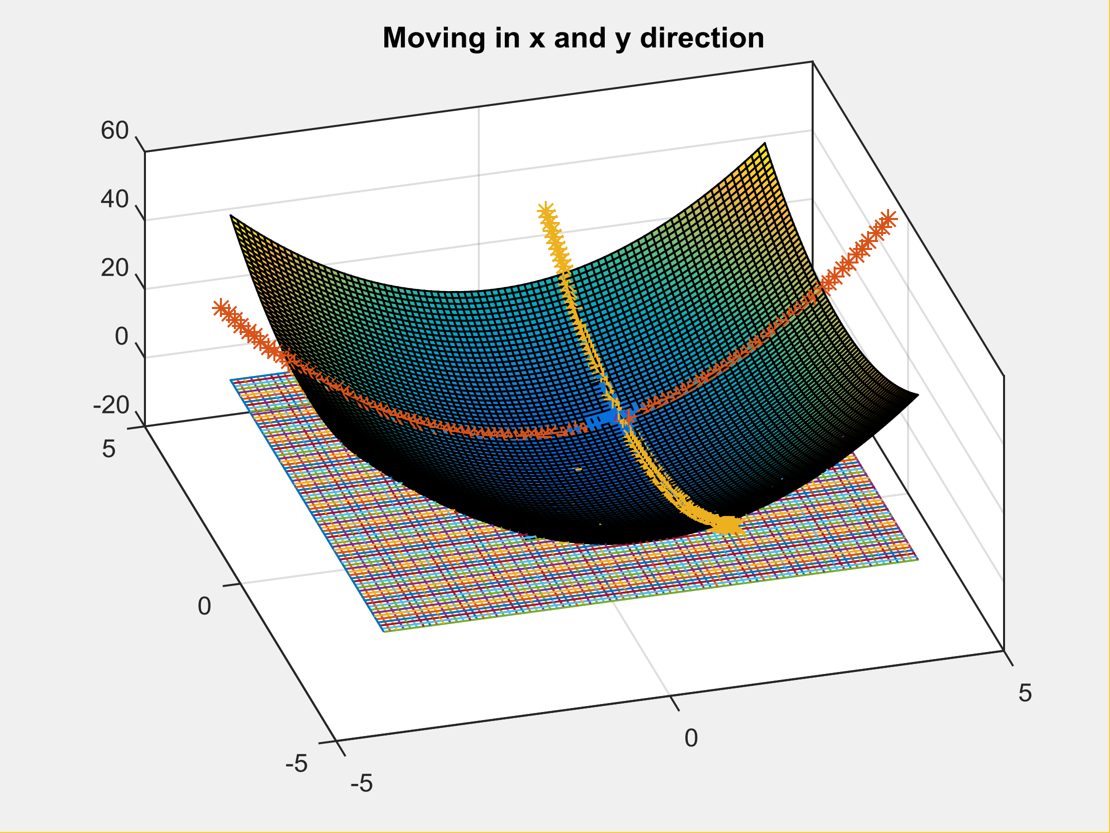
Moving in x-direction of a surface

- X=-4:.1:4;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- z=2.*x.^2+y.^2
- hold on
- surf(x,y,z)
- grid
- %Drawing a function
- %along x axis at y=1
- x=-5:.1:5;
- y=x*0+1;
- z=2.*x.^2+y.^2
- plot3(x,y,z,'*')



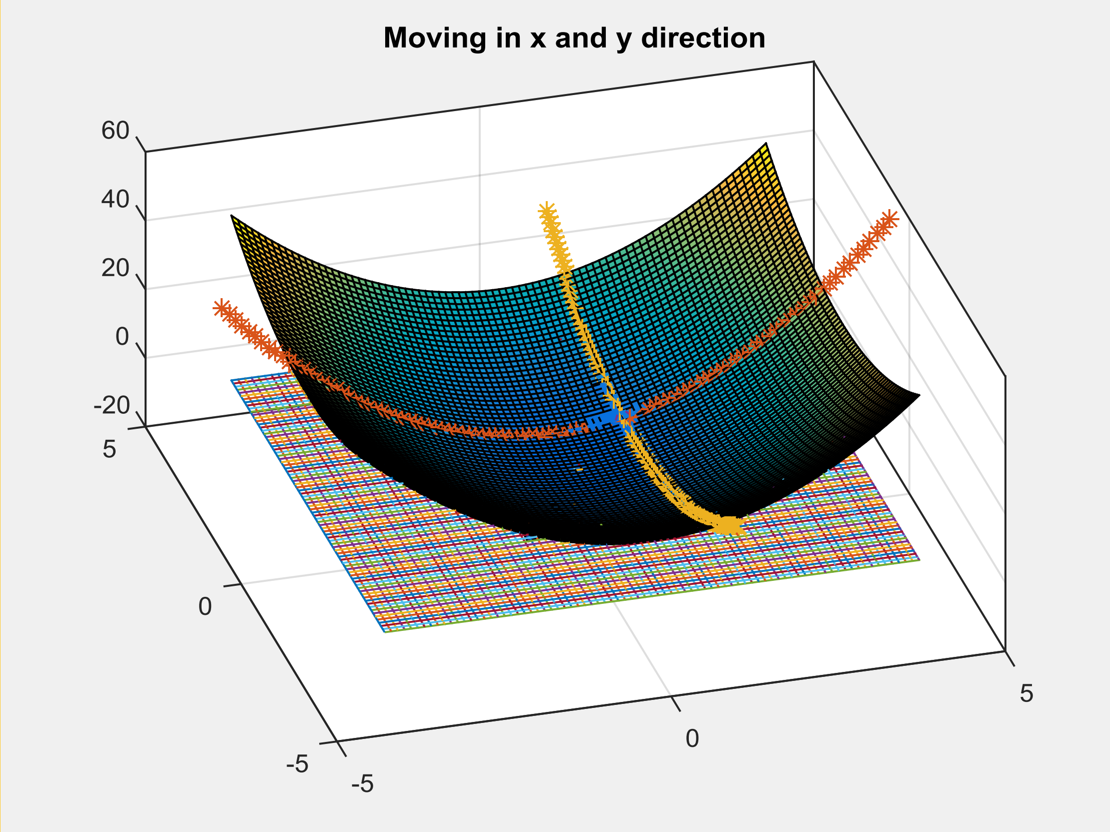
Moving in y-direction of a surface

- X=-4:.1:4;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- z=2.*x.^2+y.^2
- hold on
- surf(x,y,z)
- grid
- %Drawing a function
- %along y axis at x=1
- %y=-5:.1:5;
- x=y*0+1;
- z=2.*x.^2+y.^2
- plot3(x,y,z,'*')



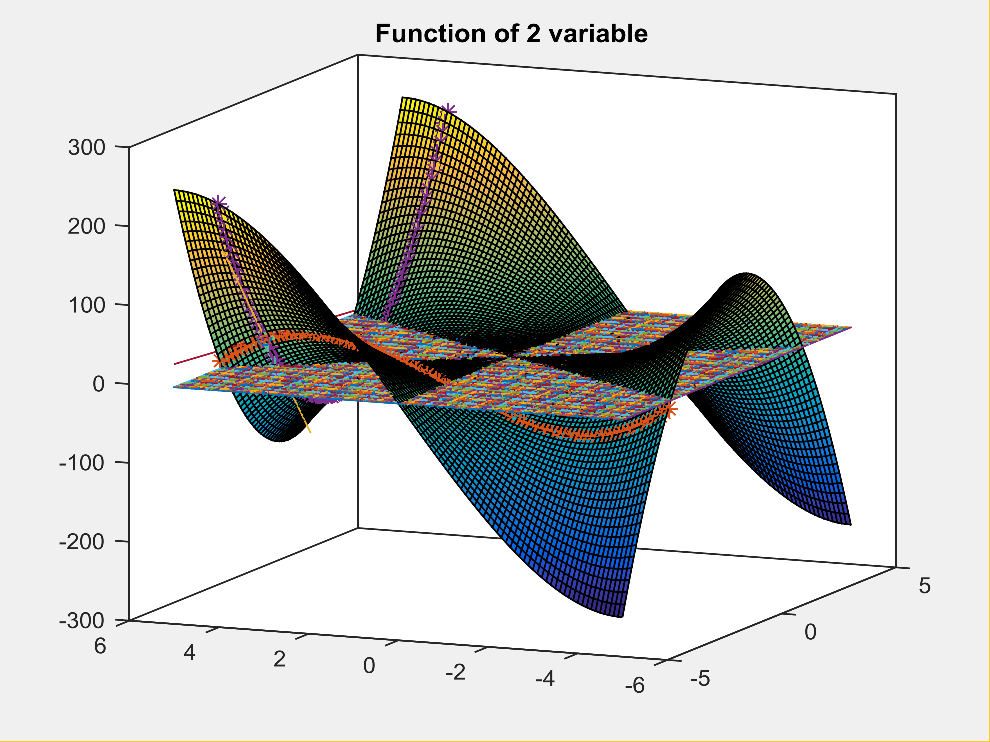
Drawing a Point in a surface

- X=-4:.1:4;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- z=2.*x.^2+y.^2
- hold on
- surf(x,y,z)
- Grid
- % Plotting the point
- xo=1
- yo=1
- zo=2.*xo.^2+yo.^2
- plot3(xo,yo,zo,'o')



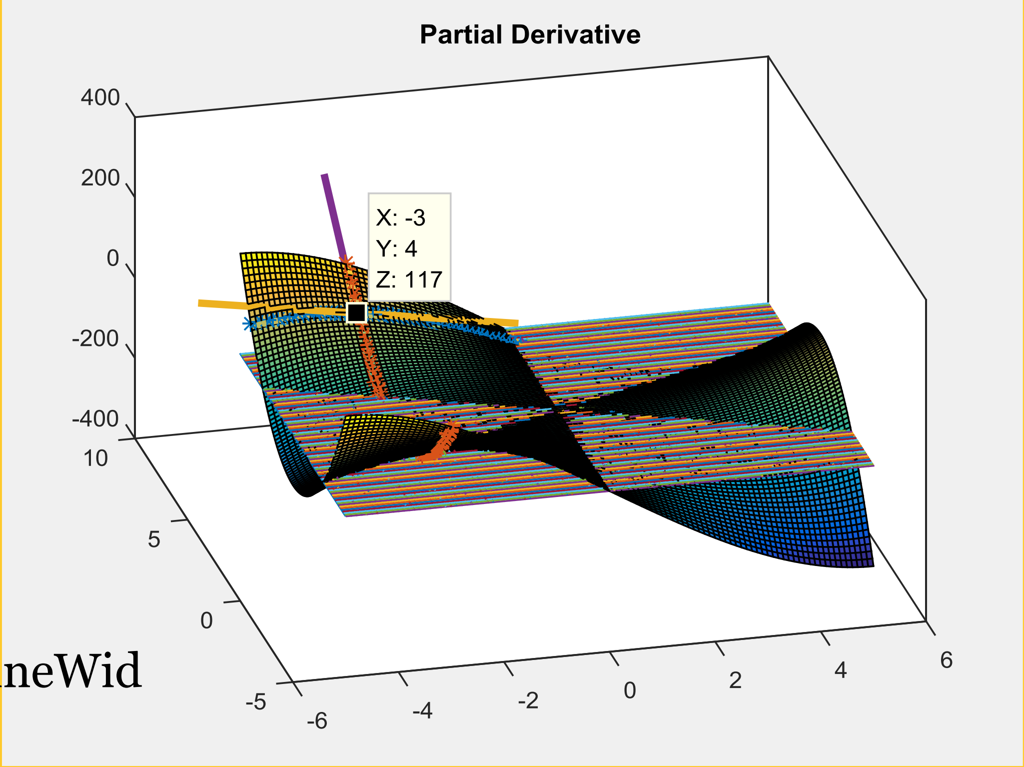
Drawing a surface with Grid

- X=-5:.1:5;
- Y=X';
- [x,y]=meshgrid(X,Y);
- plot(x,y,y,x)
- u = x.^3 - 3.*x.*y.^2;
- hold on
- surf(x,y,u)



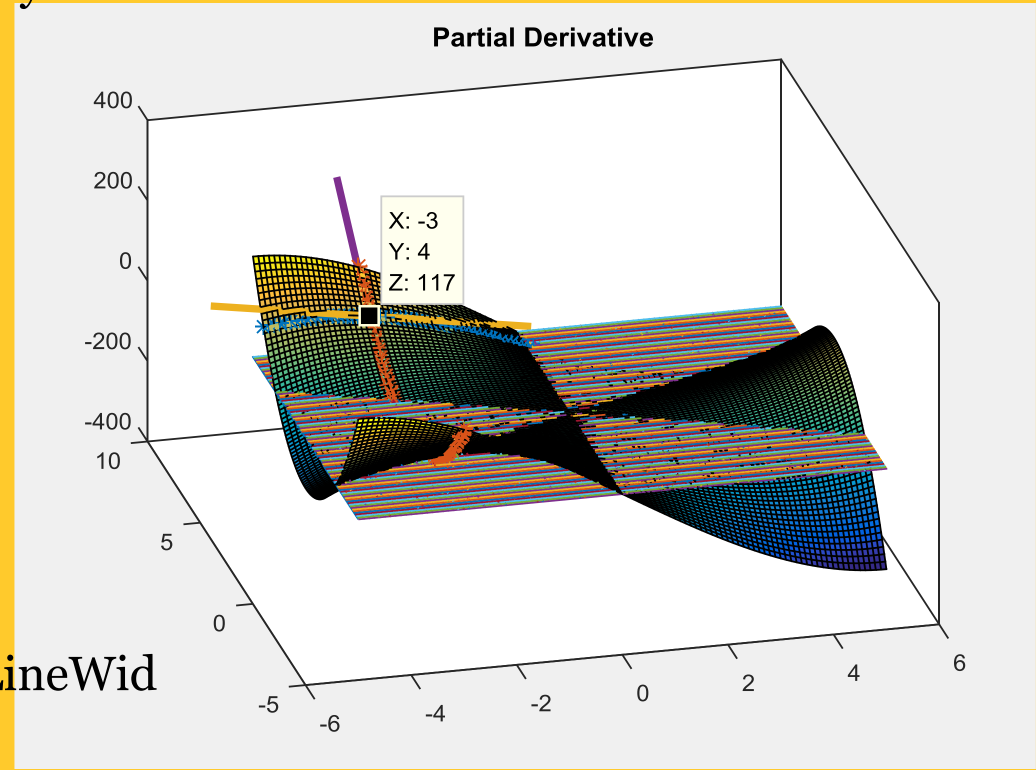
Drawing Partial Derivative at (3,4)


```
%Plotting the partial derivatives dudx
x=-3
y=4
u = x.^3 - 3.*x.*y.^2
m=3*x^2 - 3*y^2
c=u-m*x
x1=x-3
x2=x+3
u1=m*x1+c
u2=m*x2+c
y1=4
y2=4
plot3([x1,x,x2],[y1,y,y2],[u1,u,u2],'LineWid
th',3)
grid
```



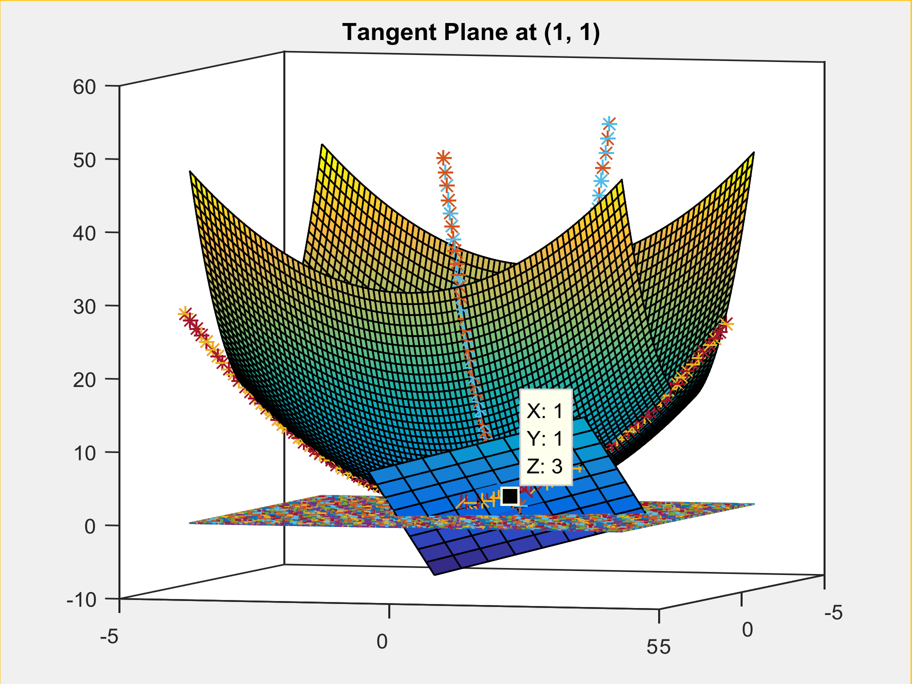
Drawing Partial Derivative at (3,4)


```
%Plotting the partial derivatives dudy
x=-3
y=4
u = x.^3 - 3.*x.*y.^2
m=-6*x*y
c=u-m*y
y1=y-3
y2=y+3
u1=m*y1+c
u2=m*y2+c
x1=-3
x2=-3
plot3([x1,x,x2],[y1,y,y2],[u1,u,u2],'LineWidth',3)
```



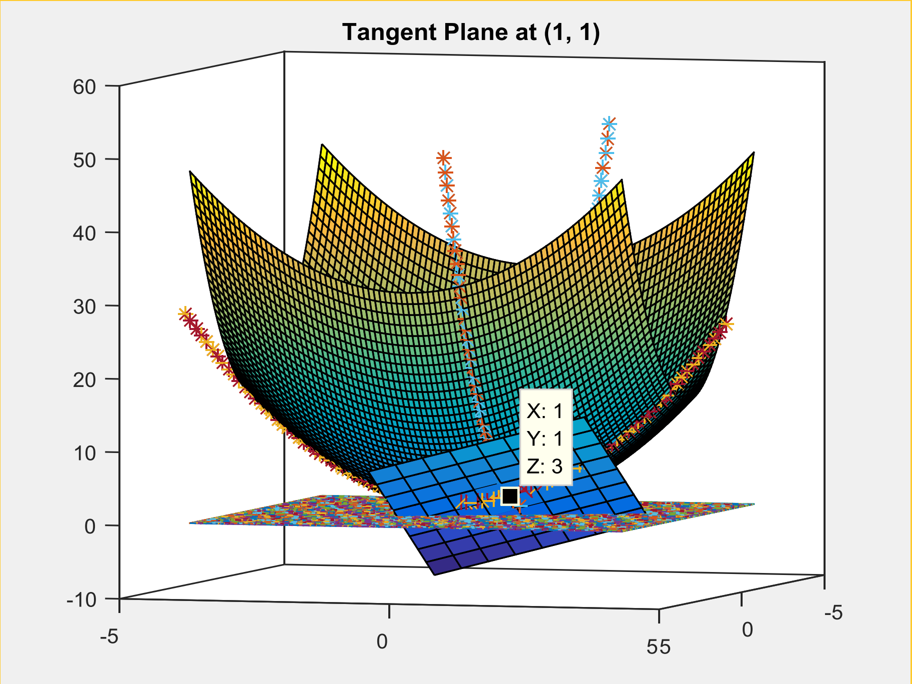
Drawing Tangent Plane


```
X=-4:.1:4;
Y=X';
[x,y]=meshgrid(X,Y);
plot(x,y,y,x)
z=2.*x.^2+y.^2
hold on
surf(x,y,z)
grid
```



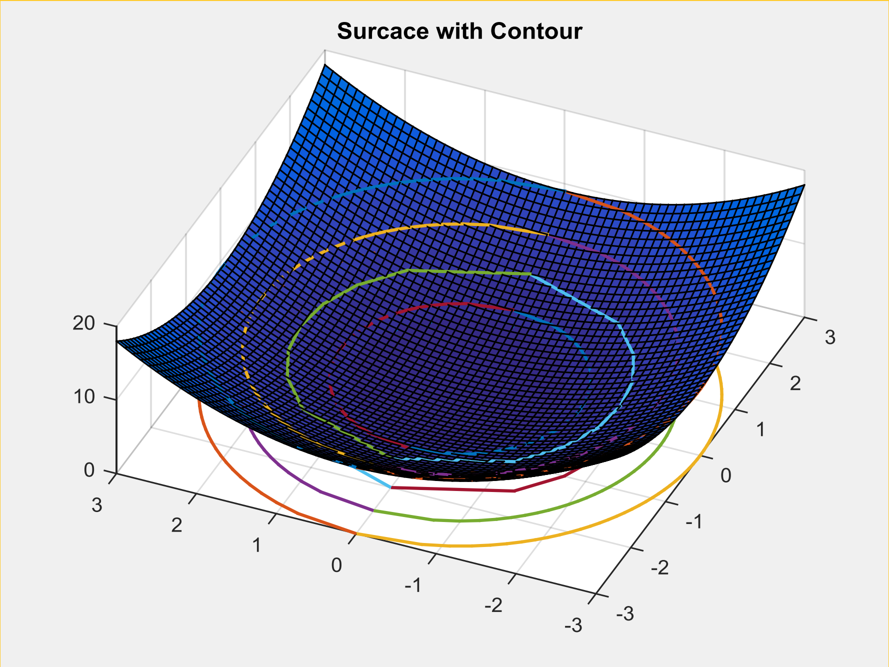
Drawing Tangent Plane


```
% Plotting the point
xo=1
yo=1
zo=2.*xo.^2+yo.^2
dzdx = 4.*x
dzdy=2.*y
plot3(xo,yo,zo,'o')
 %Tangent Plane
X=[xo-2,xo-1.5, xo-1,xo-
.5,xo,xo+.5,xo+1,xo+1.5,xo+2]
Y=[yo-2,yo-1.5, yo-1,yo-
.5,yo,yo+.5,yo+1,yo+1.5,yo+2]'
[x,y]=meshgrid(X,Y)
dzdx = 4.*x
dzdy=2.*y
z=zo+4.*(x-xo)+2.*(y-yo)
surf(x,y,z)
```



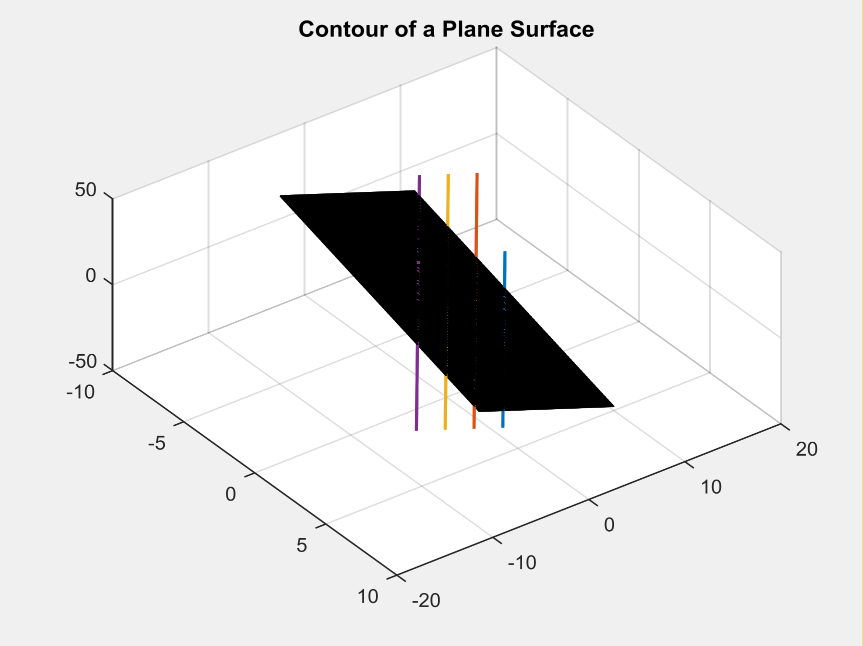
Drawing Contour of a surface

- %1
- x=-3:.1:3
- y=sqrt(9-x.^2)
- z=y.*0+9
- plot3(x,y,z,'LineWidth',1
- %2
- x=-3:.1:3
- y=-sqrt(9-x.^2)
- z=y.*0+9;
- plot3(x,y,z,'LineWidth',1.5)



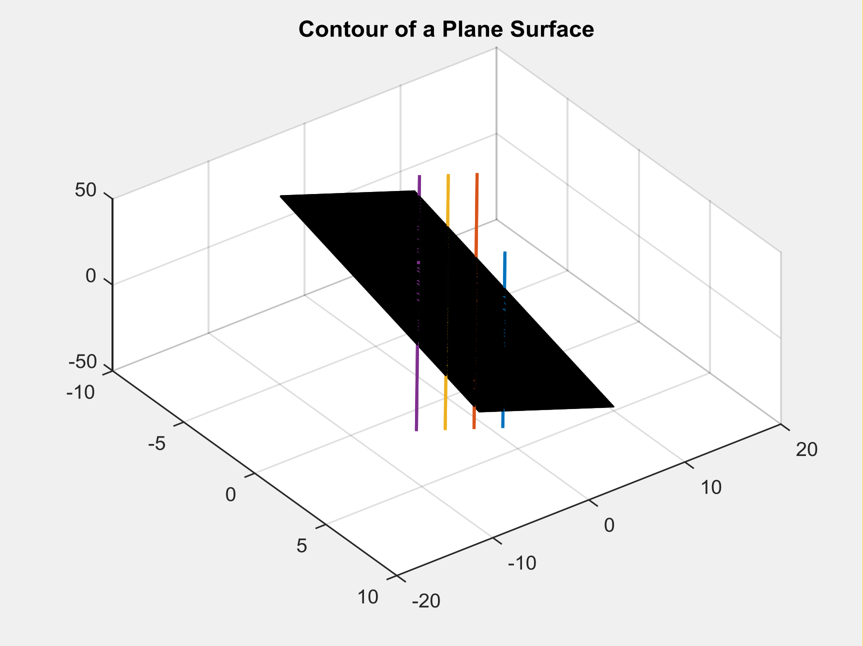
Drawing Contour of a Plane Surface

- %Contour of plane Surface
- X=-7:.1:7
- Y=X'
- [x,y]=meshgrid(X,Y)
- z=6-3*x-2*y
- surf(x,y,z)
-
- hold on
-
- x=-3:.1:8
- y=6-(1.5).*x
- z=y.*0+-6
- plot3(x,y,z,'LineWidth',1.5)
-
- x=-8:.1:8
- y=3-(1.5).*x
- z=y.*0+0
- plot3(x,y,z,'LineWidth',1.5)



Drawing Contour of a Plane Surface

- x=-8:.1:8
- y=-(1.5).*x
- z=y.*0+6
- plot3(x,y,z,'LineWidth',1.5)
-
- x=-8:.1:8
- y=-3-(1.5).*x
- z=y.*0+12
- plot3(x,y,z,'LineWidth',1.5)



Drawing a Cube

Sl no	Vertex	x	y	z	h
1	7	-10	-10	-10	1
2	5	10	-10	-10	1
3	6	10	10	-10	1
4	8	-10	10	-10	1
5	7	-10	-10	-10	1
6	3	-10	-10	10	1
7	1	10	-10	10	1
8	2	10	10	10	1
9	4	-10	10	10	1
10	8	-10	10	-10	1
11	6	10	10	-10	1
12	2	10	10	10	1
13	1	10	-10	10	1
14	5	10	-10	-10	1
15	7	-10	-10	-10	1
16	3	-10	-10	10	1
17	4	-10	10	10	1

Creating a cube in 3d

x	y	z	h
0	0	1	1
2	0	1	1
2	3	1	1
0	3	1	1
0	0	1	1
0	0	0	1
2	0	0	1
2	3	0	1
0	3	0	1
0	3	1	1
2	3	1	1
2	3	0	1
2	0	0	1
2	0	1	1
0	0	1	1
0	0	0	1
0	3	0	1

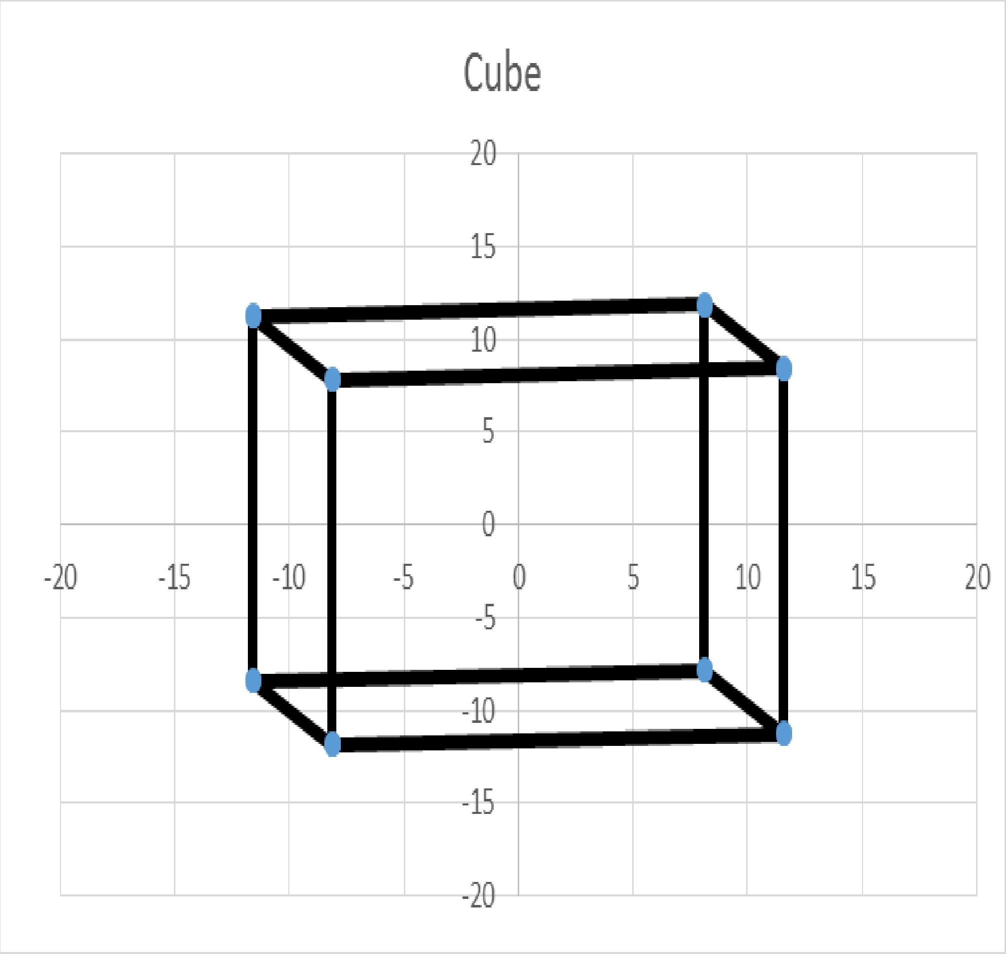
py	py	py	py
=Cos(x)	0	=Sin(x)	0
0	1	0	0
=-sin(x)	0	=Cos(x)	0
0	0	0	1

1	0	0	0
0	=Cos(y)	=sin(y)	0
0	=-sin(y)	=cos(y)	0
0	0	0	1

pz	pz	pz	pz
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	1

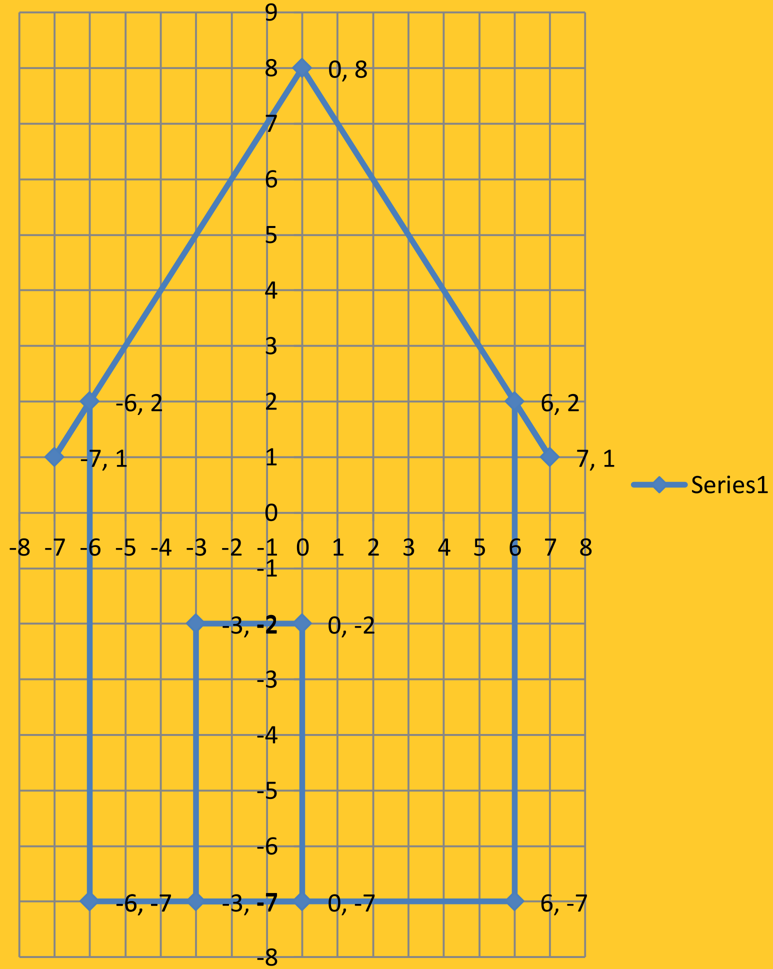
Creating a cube in 3d

x	y	z	h
0	0	1	1
2	0	1	1
2	3	1	1
0	3	1	1
0	0	1	1
0	0	0	1
2	0	0	1
2	3	0	1
0	3	0	1
0	3	1	1
2	3	1	1
2	3	0	1
2	0	0	1
2	0	1	1
0	0	1	1
0	0	0	1
0	3	0	1



Creating a House

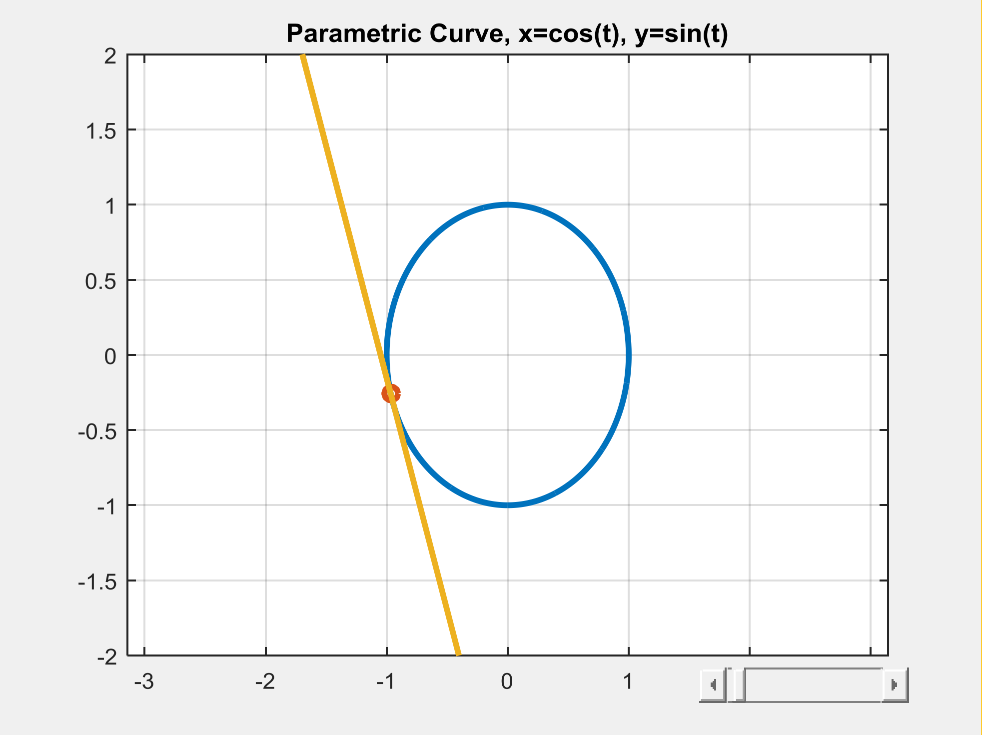
Vertex	x	y
1	-6	-7
2	-6	2
3	-7	1
4	0	8
5	7	1
6	6	2
7	6	-7
8	-3	-7
9	-3	-2
10	0	-2
11	0	-7
12	-6	-7



Parametric Curve with Derivative

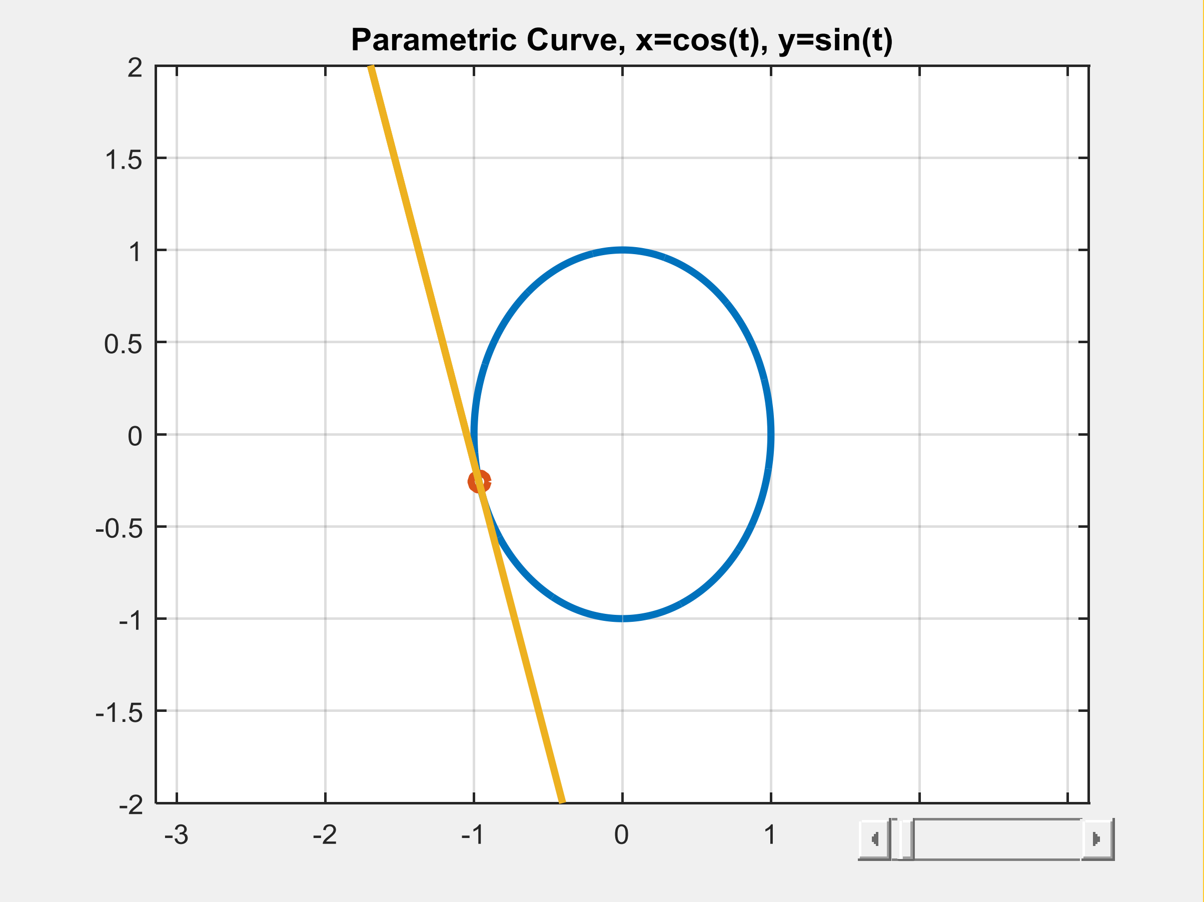
% 4. Write the code

```
t=-pi:.01:pi;
x=sin(t)
y=cos(t)
 plot(x,y,'LineWidth',2.5)
 axis([-pi pi -2 2])
 grid
 hold on
 t=val
 x1=cos(t)
 y1=sin(t)
plot(x1,y1,'o','LineWidth',2.5)
plot([x1,x3,x4],[y1,y3,y4],'LineWidth',2.5)
```

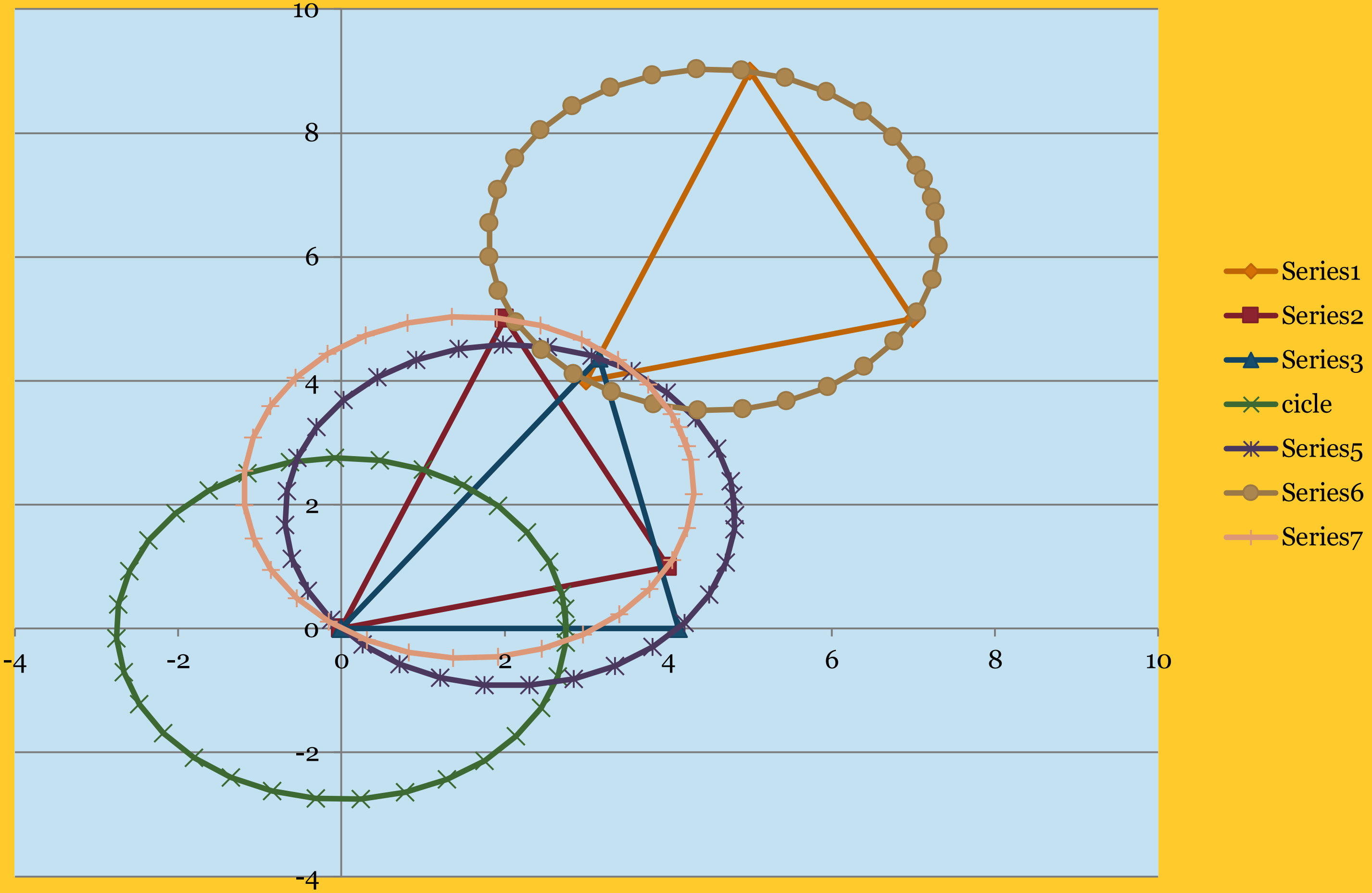


Parametric Curve with Derivative

x2=cos(t+.1)
y2=sin(t+.1)
m=(y2-y1)/(x2-x1)
c=y1-m*x1
x3=x1-1
x4=x1+1
y3=x3*m+c
y4=x4*m+c



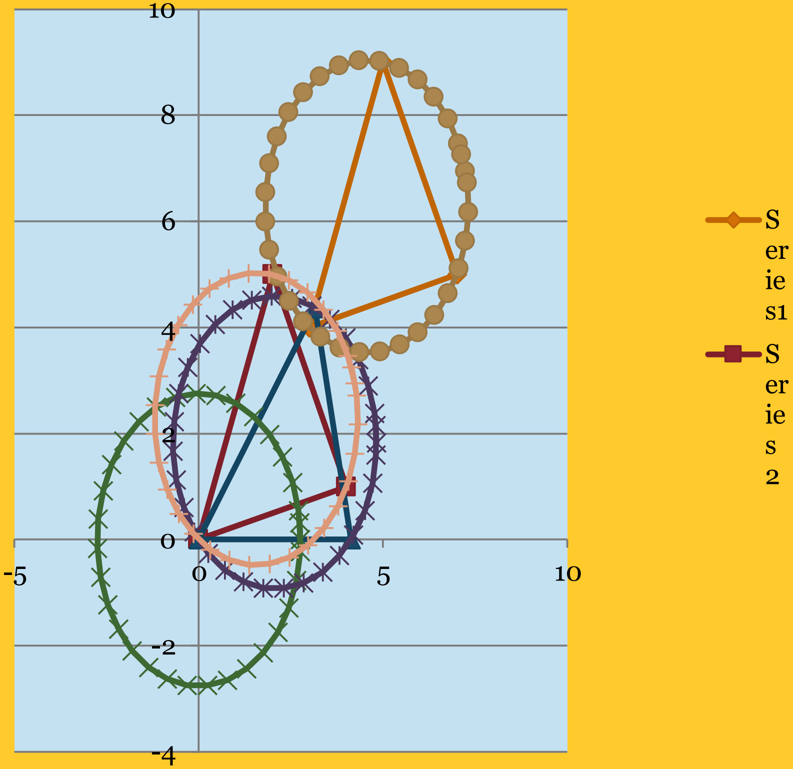
Creating a Circle from Three Point



Creating a Circle from Three Point

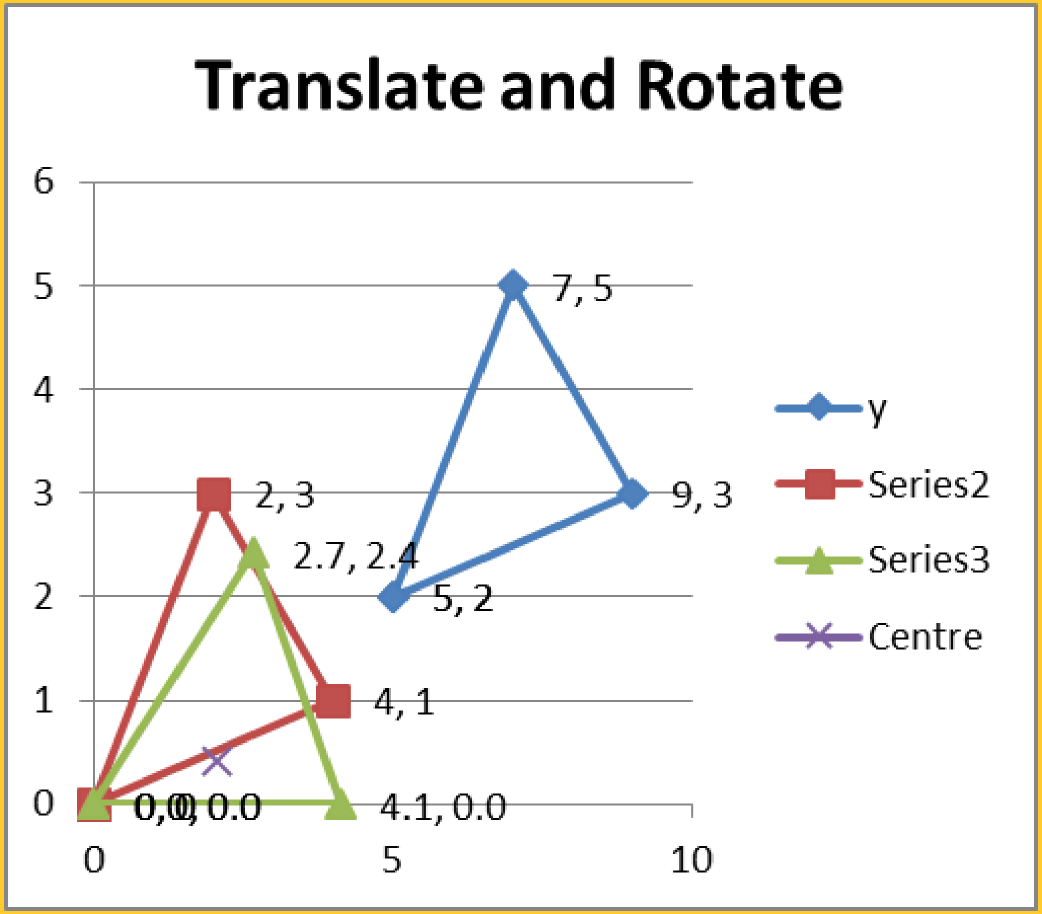
- Step-1: Find the center (h, k) of the circle from three given points.
- If we directly go for solving this, it will be very complex task.
- We try it graphically.

1. Translate all point so that one point coincide with origin
2. Rotate so that other point coincide with x-axis.
3. Calculate h=x2/2,
4. Calculate k=x3/2y3(x3-x2)+y3/2
5. Calculate r =sqrt(h^2+k^2)
6. Draw circle



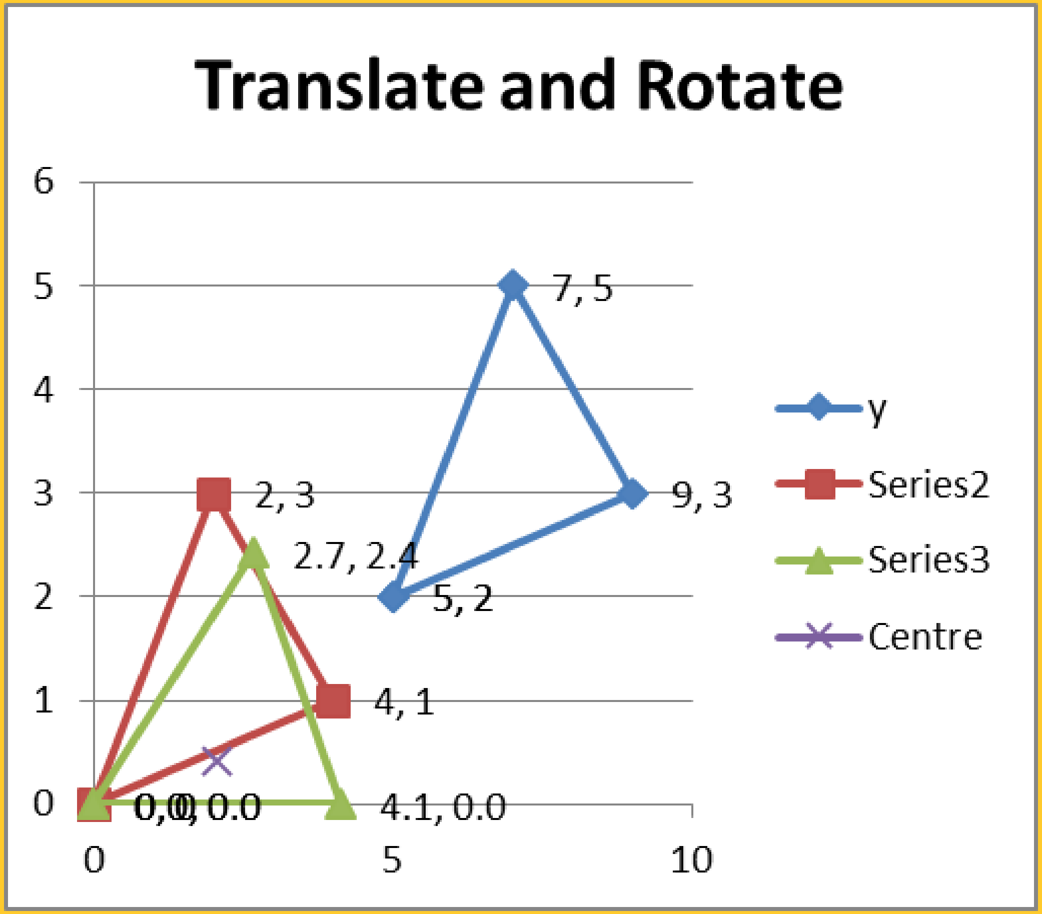
Creating a Circle from Three Point


```
%1. Draw The Points
p1=[5,2]
p2=[9,3]
p3=[7,5]
plot([p1(1)],[ p1(2)],'o')
hold on
plot([p2(1)],[ p2(2)],'o')
plot([p3(1)],[ p3(2)],'o')
grid

%2 Create the Triangle
x=[p1(1), p2(1), p3(1),p1(1)]

y=[p1(2), p2(2) p3(2),p1(2)]
o=[1 1 1 1]
plot(x,y)
axis([-2 10 -2 10 ])
```



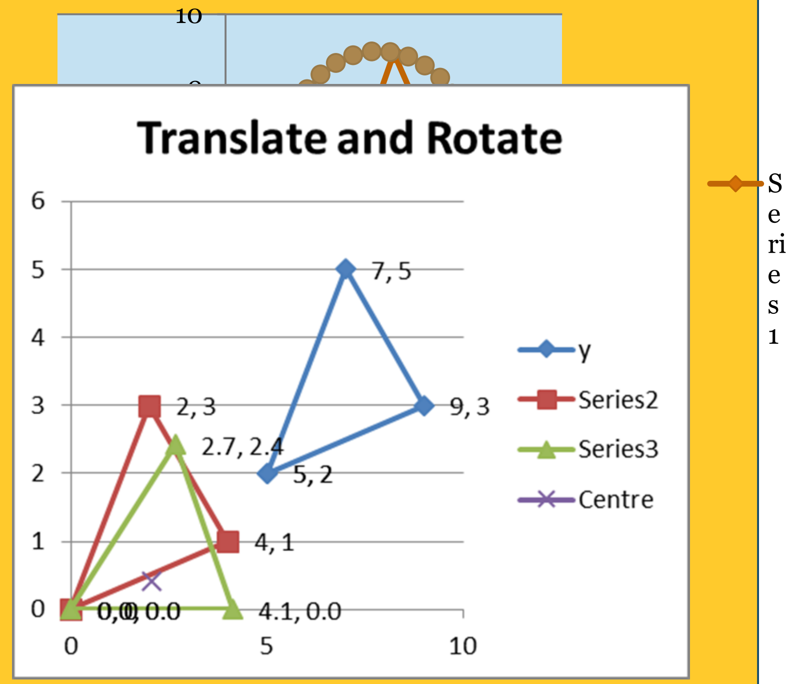
Creating a Circle from Three Point


```
%1. Draw The Points
p1=[5,2]
p2=[9,3]
p3=[7,5]
plot([p1(1)],[ p1(2)],'o')
hold on
plot([p2(1)],[ p2(2)],'o')
plot([p3(1)],[ p3(2)],'o')
grid

%2 Create the Triangle
x=[p1(1), p2(1), p3(1),p1(1)]

y=[p1(2), p2(2) p3(2),p1(2)]
o=[1 1 1 1]
plot(x,y)
axis([-2 10 -2 10 ])
```



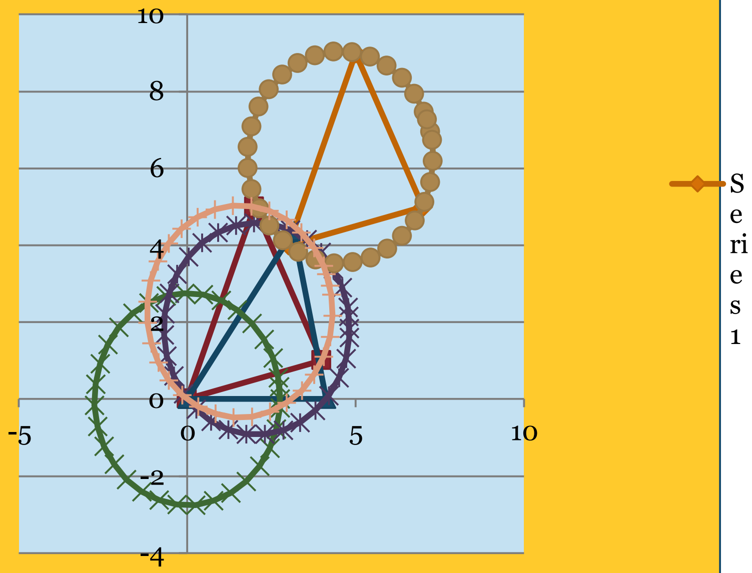
Creating a Circle from Three Point


```
%3Translate to Origin
m=[x;y;o]'
tt=[1 0 0; 0 1 0; -5 -2 1]
itt=inv(tt)
mt=m*tt
   x=mt( : , 1 )'
   y=mt( : , 2)'
plot(x,y)
tant=(mt(2,2)-mt(1,2))/(mt(2,1)-mt(1,1))
t=atan(tant)
```



Creating a Circle from Three Point

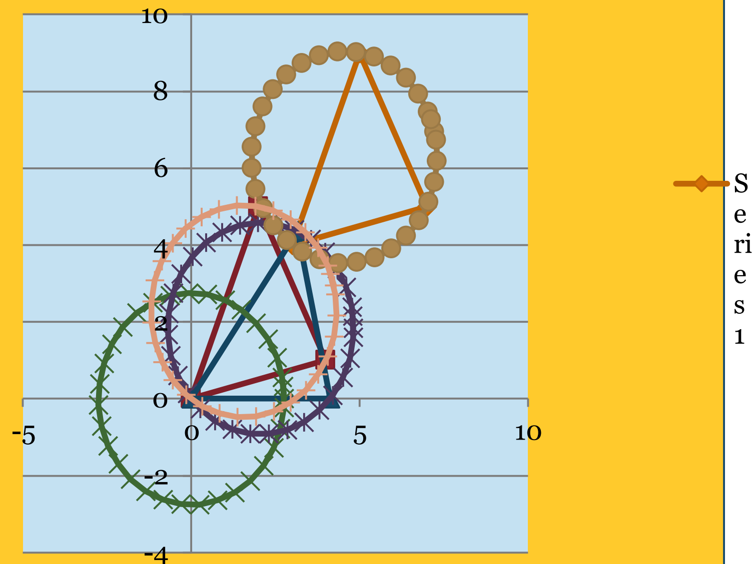

```
%4Rotation
tr=[cos(t) -sin(t) 0;sin(t)
cos(t) 0;0 0 1]
mr=mt*tr
itr=inv(tr)
x=mr( :, 1)'
y=mr( : , 2)'
plot(x,y)
```



Creating a Circle from Three Point

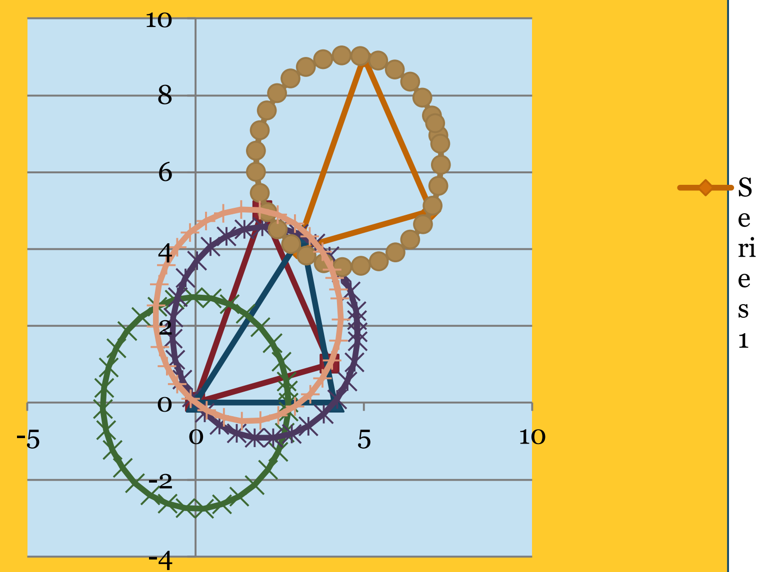
%5Calculate the centre, h and k and draw the circle

```
h=mr(2,1)/2
k=(mr(3,1)/(2*mr(3,2)))*(mr(3,1)-mr(2,1))+mr(3,2)/2
r=sqrt(h^2+k^2)
t=0:.1:2*pi
x=r*cos(t)
y=r*sin(t)
h=y*0+1
plot(x,y)
```



Creating a Circle from Three Point


```
%6Placing the circle at h and k
m=[x;y;h1]'
m=m*[1 0 0; 0 1 0; 2.0616 .4123 1]
%7Placing the circle to original
position
newm=m*itr*itt
x=newm( : ,1)
y=newm( : ,2)
plot(x,y)
p=[1 0 0 ; 0 1 0 ; h k 1]
newp=p*itr*itt
plot(newp(3,1), newp(3,2),'o')
```



Creating a Circle from Three Point

m =

5	2	1
9	3	1
7	5	1
5	2	1

tt =

1	0	0
0	1	0
-5	-2	1

mt =

0	0	1
4	1	1
2	3	1
0	0	1

tant = 0.2500; t = 0.2450

tr =

0.9701	-0.2425	0
0.2425	0.9701	0
0	0	1.0000

mr =

0	0	1.0000
4.1231	0	1.0000
2.6679	2.4254	1.0000
0	0	1.0000

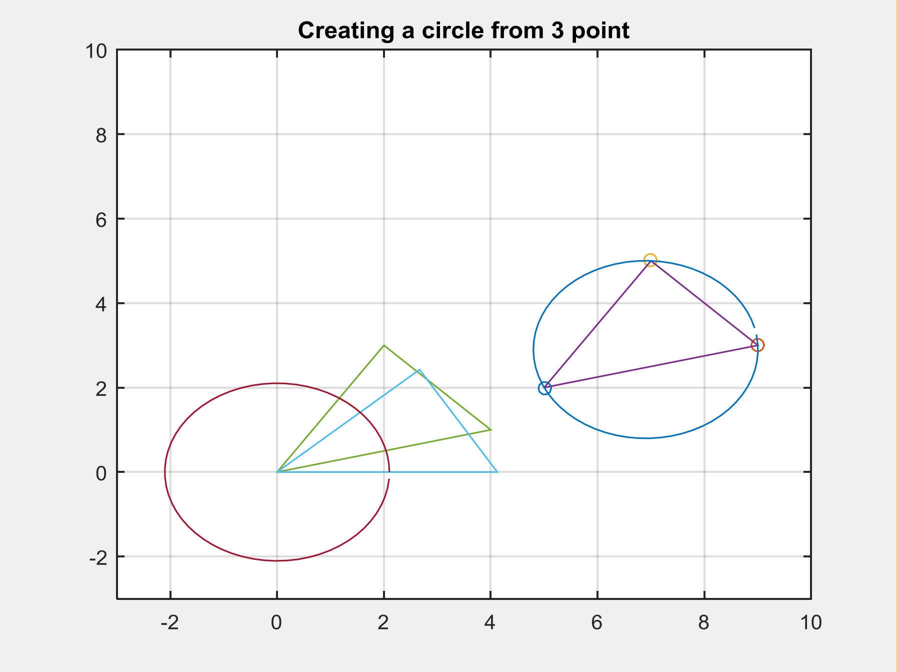
Creating a Circle from Three Point

mr =

0	0	1.0000
4.1231	0	1.0000
2.6679	2.4254	1.0000
0	0	1.0000

h = 2.0616

k = 0.4123



Creating a Circle from Three Point

mr =

0	0	1.0000
4.1231	0	1.0000
2.6679	2.4254	1.0000
0	0	1.0000

h = 2.0616

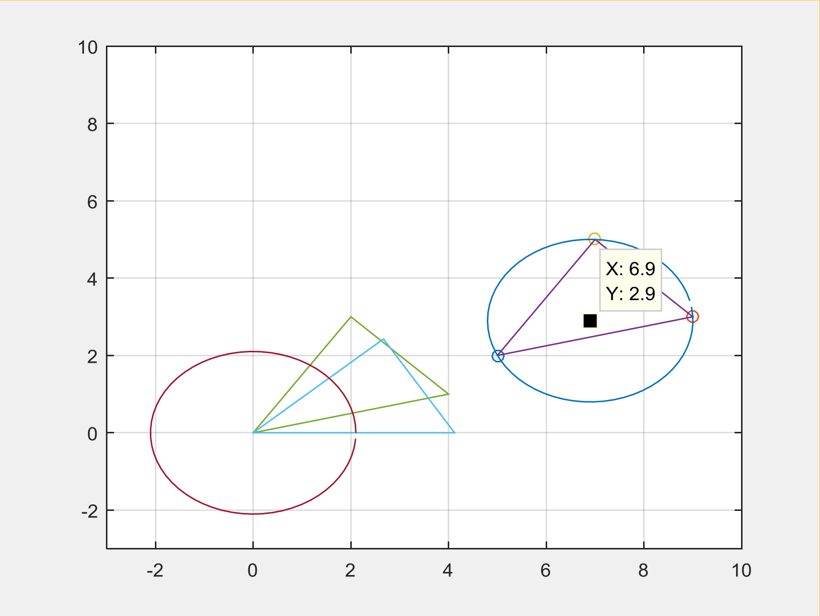
k = 0.4123

p =

1.0000	0	0
0	1.0000	0
2.0616	0.4123	1.0000

newp =

0.9701	0.2425	0
-0.2425	0.9701	0
6.9000	2.9000	1.0000



Creating Sphere


```
k =3
n = 2^k-1
theta = pi*(-n:2:n)/n
phi = (pi/2)*(-n:2:n)'/n
X = cos(phi)*cos(theta)
Y = cos(phi)*sin(theta)
Z = sin(phi)*ones(size(theta))
surf(X,Y,Z)
```

Creating Sphere

- k = 3
- n = 7

- theta = -3.1416 -2.2440 -1.3464 -0.4488 0.4488 1.3464 2.2440 3.1416

- phi =
- -1.5708
- -1.1220
- -0.6732
- -0.2244
- 0.2244
- 0.6732
- 1.1220
- 1.5708

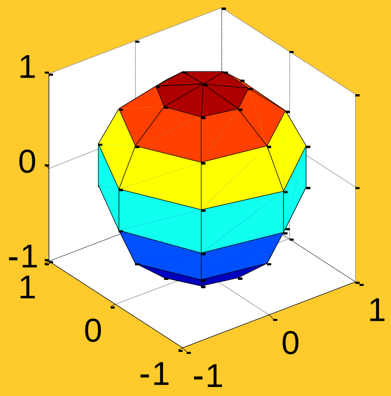
Creating Sphere

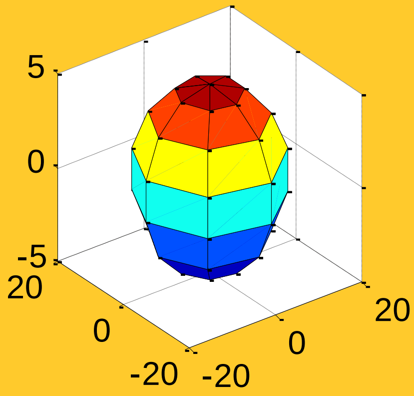
- k = 3
- n = 7

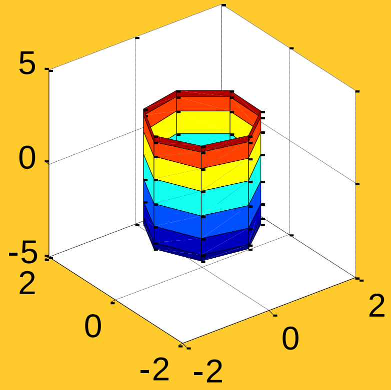
- theta = -3.1416 -2.2440 -1.3464 -0.4488 0.4488 1.3464 2.2440 3.1416

- phi =
- -1.5708
- -1.1220
- -0.6732
- -0.2244
- 0.2244
- 0.6732
- 1.1220
- 1.5708

Sphere, Ellipse, Cylinder







Creating Ellipse


```
%Ellipse
subplot(2,2,2)
k =3
n = 2^k-1
r1=3
r2=5
theta = pi*(-n:2:n)/n
phi = (pi/2)*(-n:2:n)'/n
X = r1*cos(phi)*r2*cos(theta)
Y = r1*cos(phi)*r2*sin(theta)
Z = r2*sin(phi)*ones(size(theta))

surf(X,Y,Z)
```

Creating Ellipse

- k = 3
- n = 7
- r1 = 3
- r2 = 5
- theta = -3.1416 -2.2440 -1.3464 -0.4488 0.4488 1.3464 2.2440 3.1416
- phi =
- -1.5708
- -1.1220
- -0.6732
- -0.2244
- 0.2244
- 0.6732
- 1.1220
- 1.5708

Creating Cylinder from Super formula


```
%Superformula
 subplot(2,2,3)
k =3
n = 2^k-1

theta = pi*(-n:2:n)/n
phi = (pi/2)*(-n:2:n)/n
a=2
b=2
m=5
n1=2
n2=2
n3=2
```

```
phi1 = (pi/2)*(-n:2:n)'/n
rx1=abs(1/a)*abs(cos(m*theta/4).^n2)+abs(1/b)*abs(sin(m*theta/4).^n3)
rx2=(cos(m*phi/4).^n2)/a+(sin(m*phi/4).^n3)/b
X = rx1*cos(phi1)*rx2'*cos(theta)
Y = rx1*cos(phi1)*rx2'*sin(theta)
Z = 3*sin(phi1)*ones(size(theta))
 surf(X,Y,Z)
axis square
```

Creating Ellipse with Compression Method

Creating All Conic Sections From One Formula

Interpolation

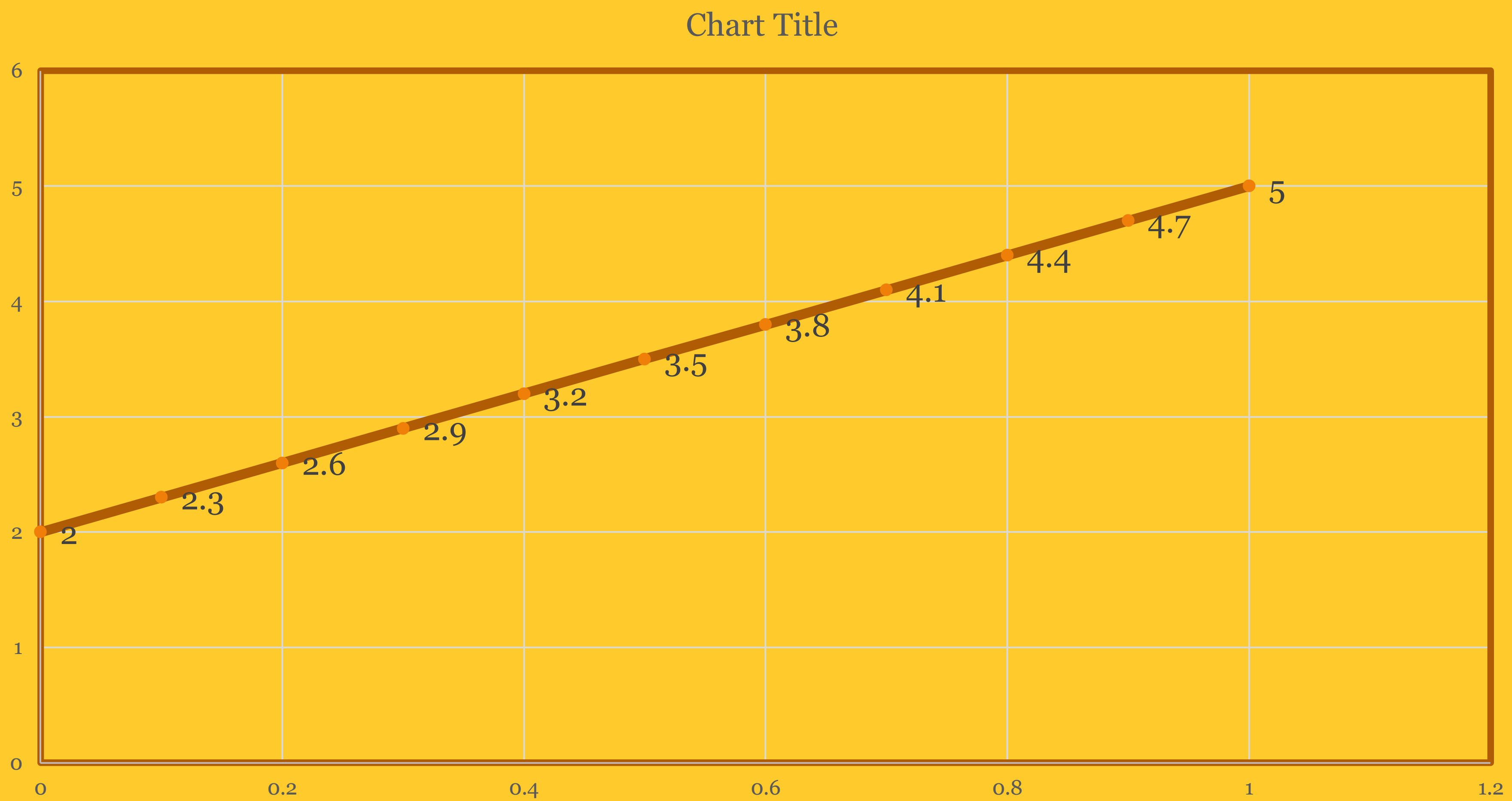
Interpolation:

Interpolation is the process of estimating values between data points

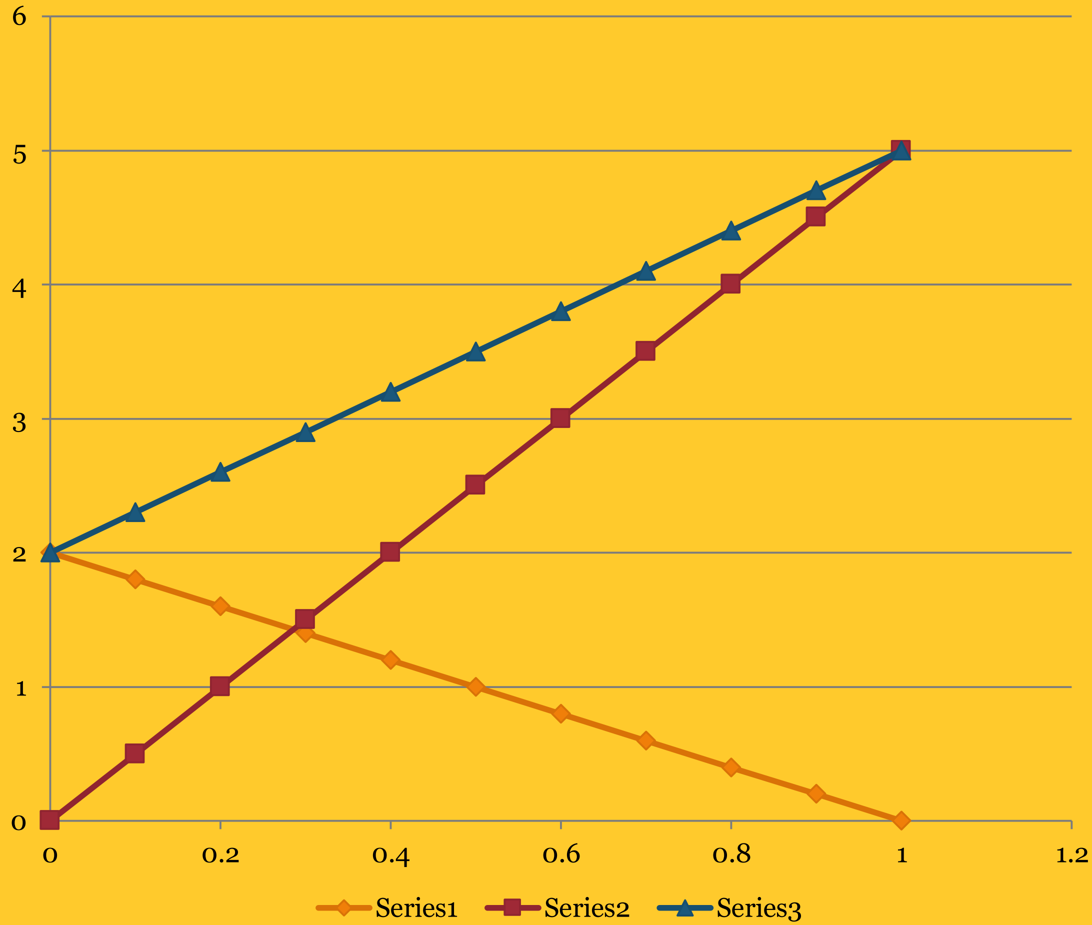
Note: There are many confusion about the objective of interpolation.

It can be a process of finding intermediate points or it can be moving from one point to other points.

Interpolation between two points



Two New Tool
Linear Interpolation and Slider



Interpolation from 2 to 5

t	N11=2*(1-t)	N22=5*t	N=N11+N22
0	2	0	2
0.1	1.8	0.5	2.3
0.2	1.6	1	2.6
0.3	1.4	1.5	2.9
0.4	1.2	2	3.2
0.5	1	2.5	3.5
0.6	0.8	3	3.8
0.7	0.6	3.5	4.1
0.8	0.4	4	4.4
0.9	0.2	4.5	4.7
1	0	5	5

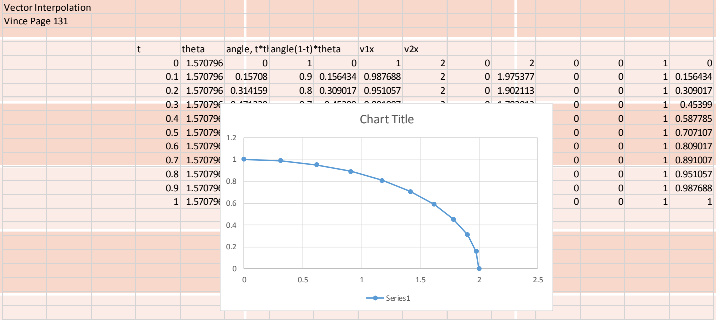
Derivation of interpolation matrix

- Interpolate from n1=2 to n2=5
- We require that at t=0, n=2 and t=1, n=5.
- This can be achieved if t=0, n1=2 and n2=0
- And at t=1, n1=0 and n2=5
- This can be achieved from the formula,
- n=n1+t(n2-n1)
- This can be written as: n=n1+t*n2-t*n1
- or n=n1*(1-t)+n2*t or [1-t t] $\begin{bmatrix} n1 \\ n2 \end{bmatrix}$ or [t 1] $\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} n1 \\ n2 \end{bmatrix}$

Interpolation from 2 to 5

t varies from 0 to 1
Point varies from 2 to 5

t	h	interpolant		From to	n
0	1	-1	1	2	2
0.1	1	1	0	5	2.3
0.2	1				2.6
0.3	1				2.9
0.4	1				3.2
0.5	1				3.5
0.6	1				3.8
0.7	1				4.1
0.8	1				4.4
0.9	1				4.7
1	1				5



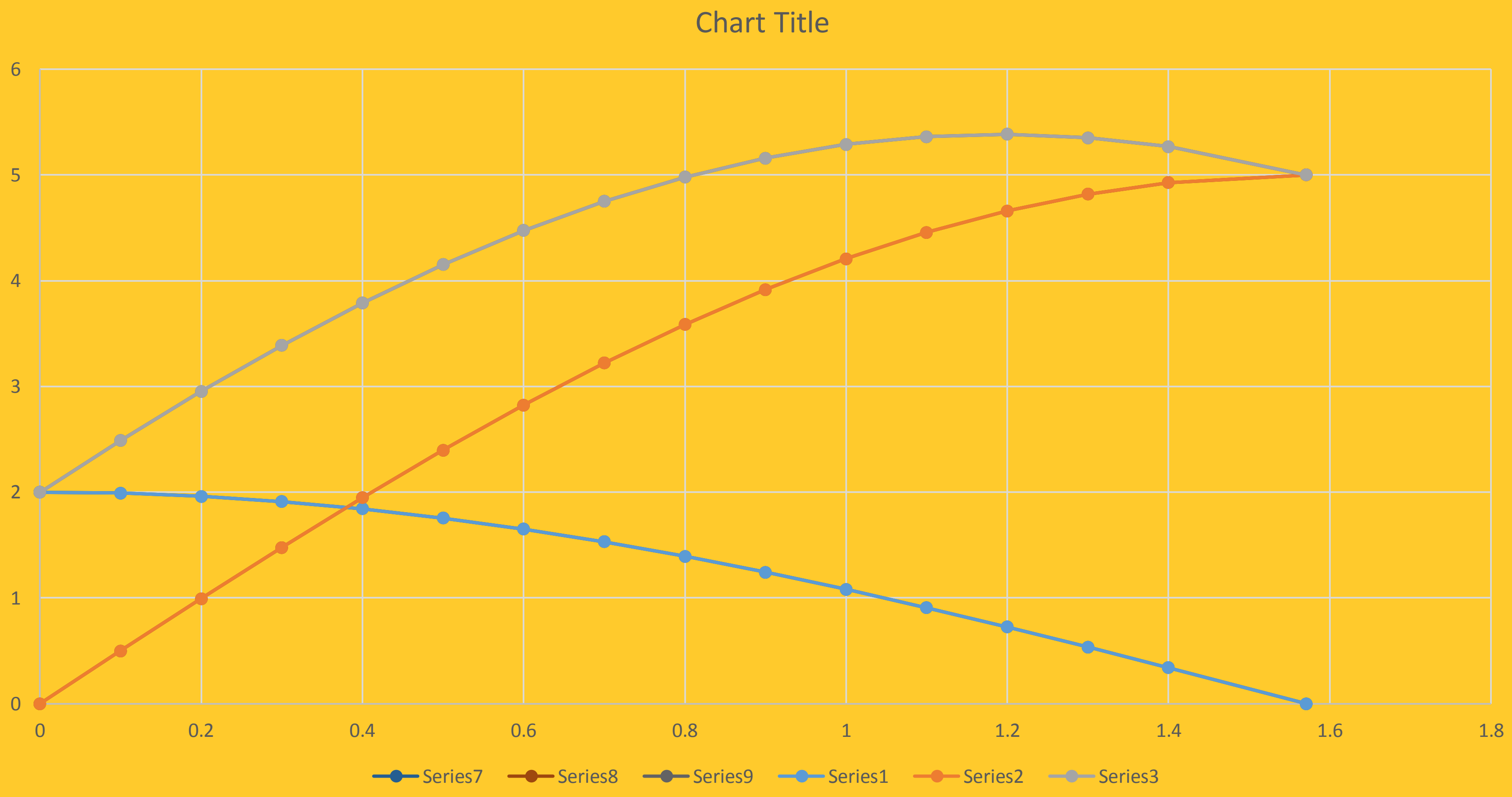
MMULT(MMULT(C4:D14,E4:F5),G4:G5)

Non linear interpolation

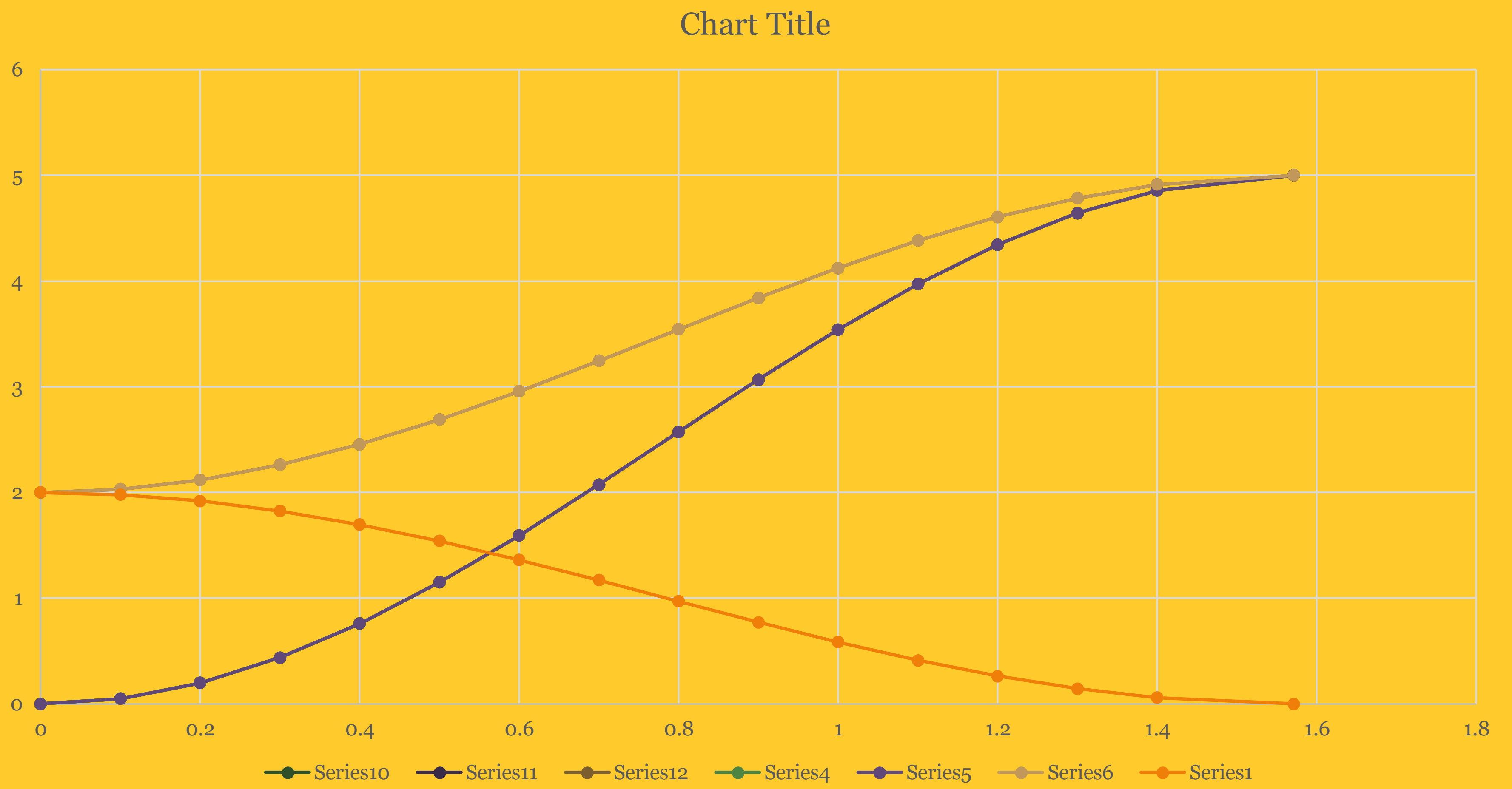
- Linear interpolant ensures equal steps in parameter t gives rise equal steps in the interpolated values.

- Many times it is required that equal steps in t gives unequal steps in the interpolated values.
- This can be achieved by :
  1. trigonometric functions ($\sin^2 x+\cos^2 x=1$) as x varies from 0 to pi/2
  2. Polynomial equations: $[(1-t)+t]^n=1$

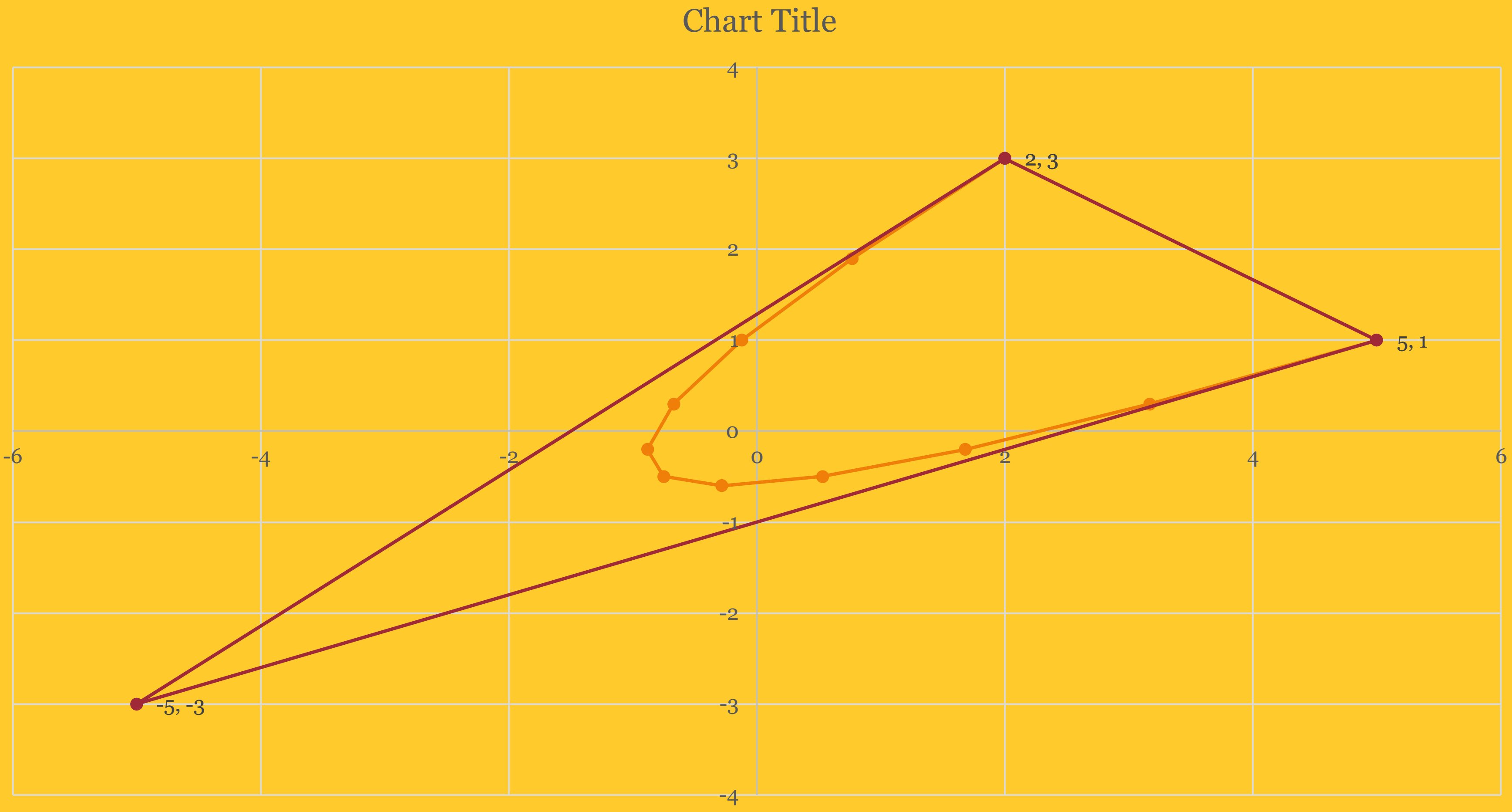
Trigonometric Interpolation



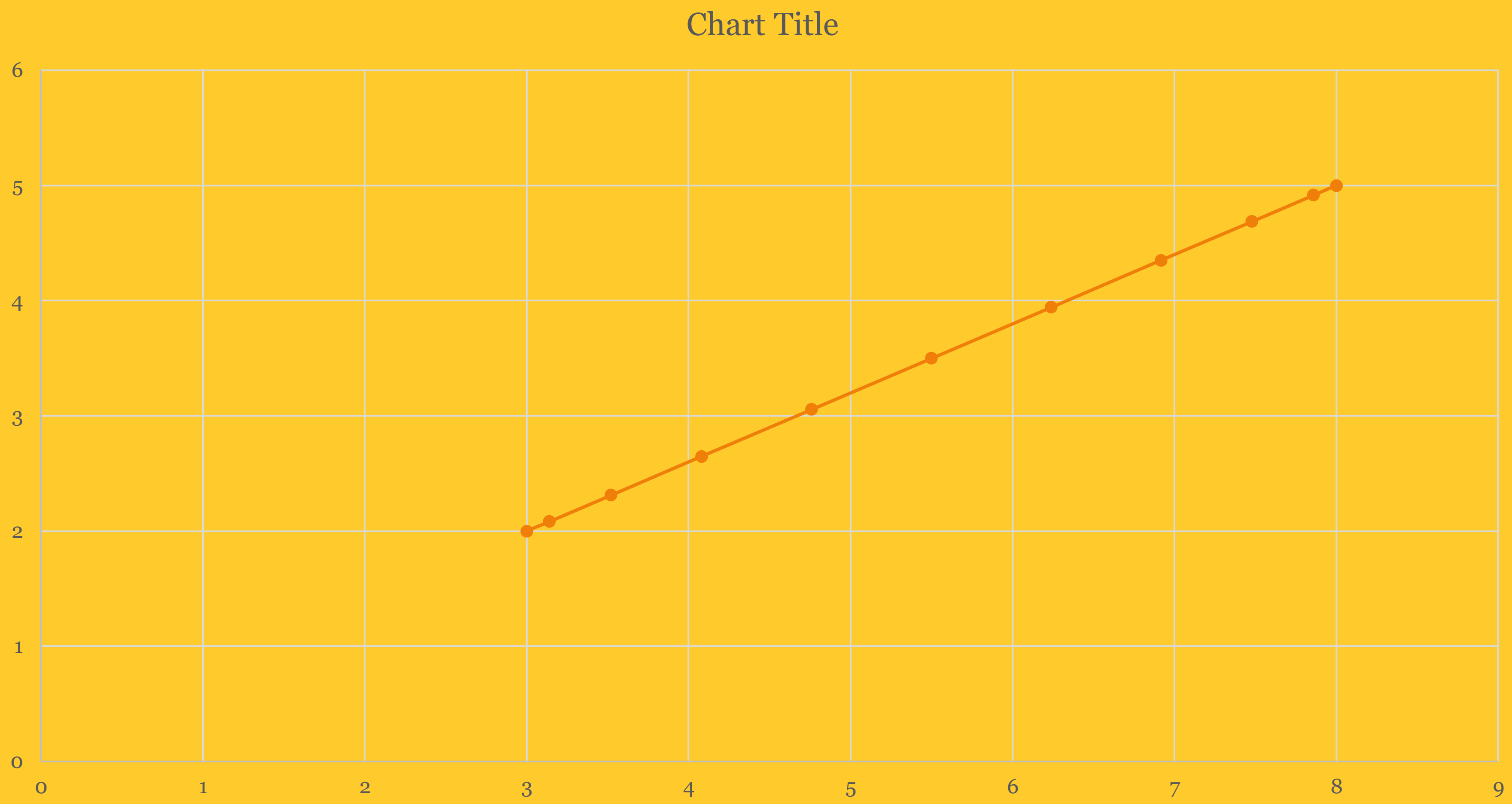
Trigonometric Interpolation



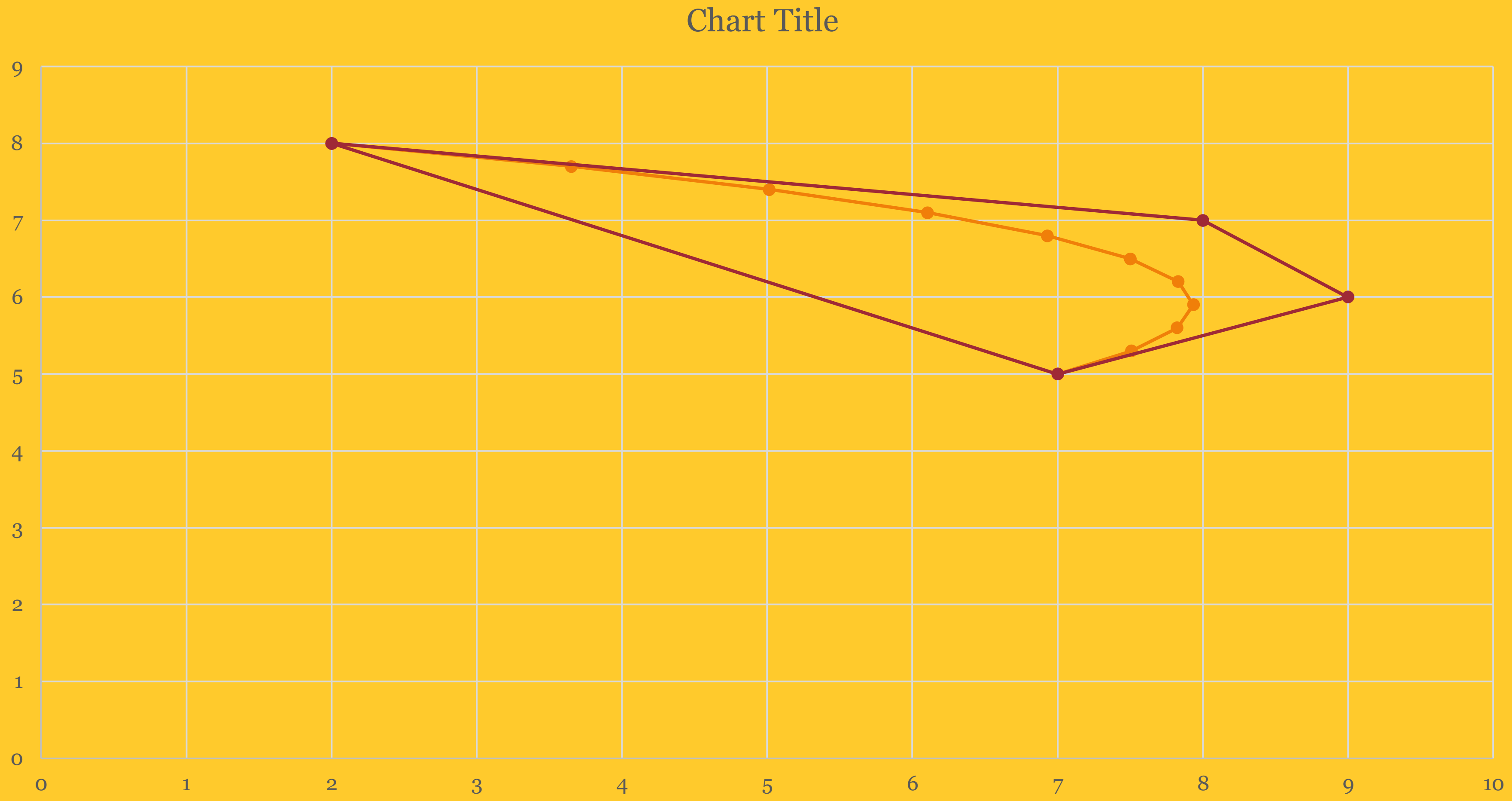
Quadratic Interpolation



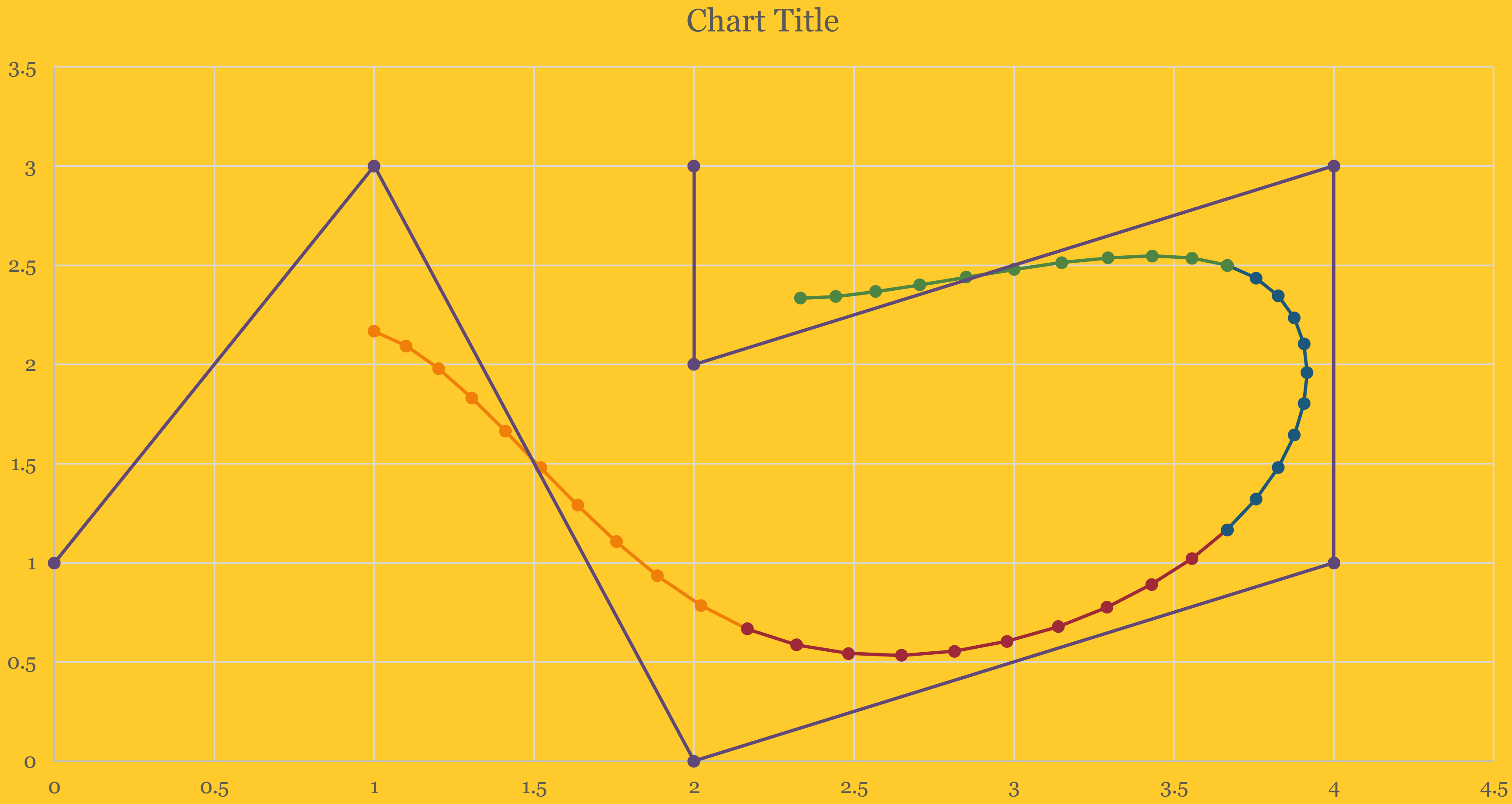
Cubic Interpolation



Bezier Curves



Uniform b-spline



Animation

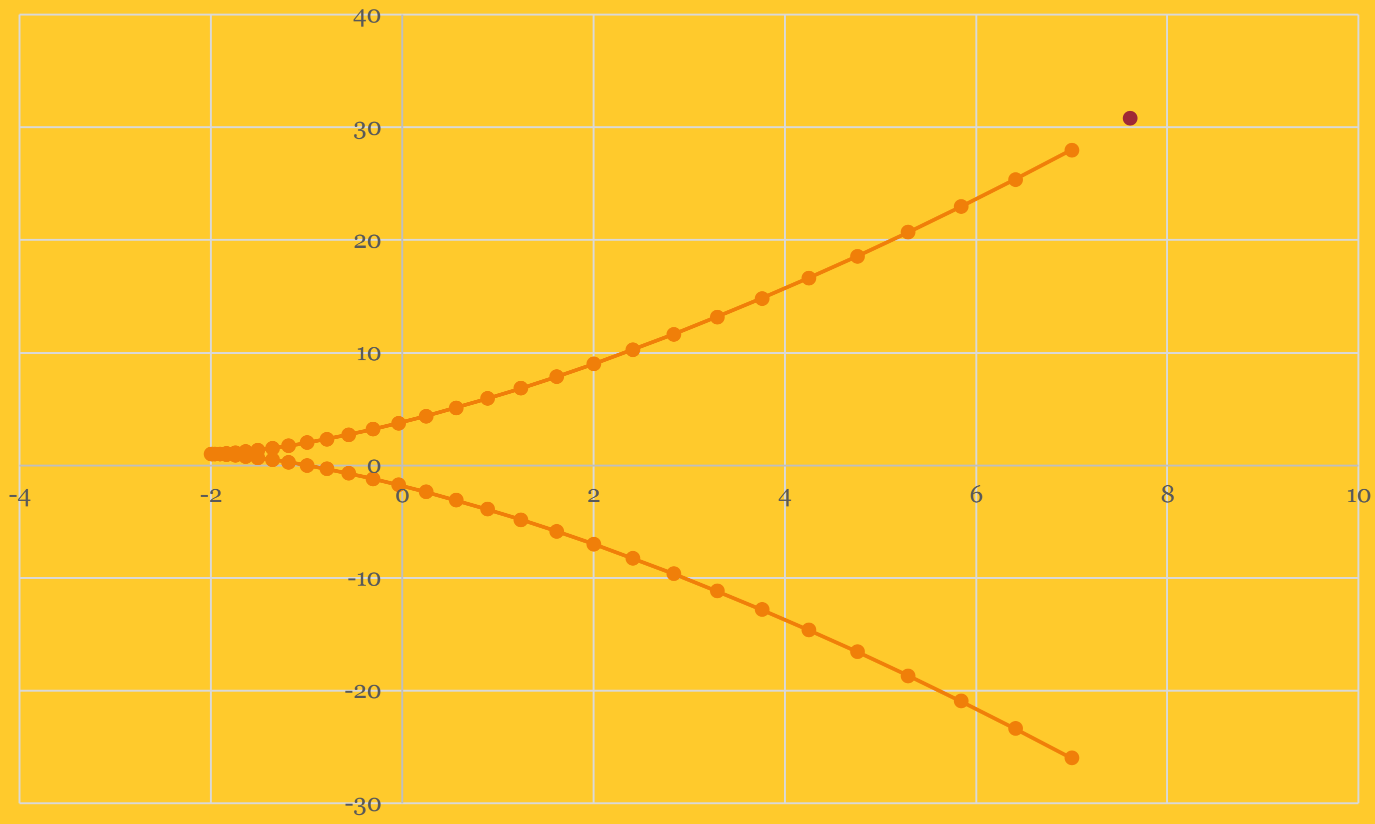
- There are many topics particularly the topics related to dynamic world can be well illustrated with the help of animation.

1) Animating an object movement in a plane: Let an object is moving in a plane whose coordinates are given by the formula – $x=t^2-2$ and $y=t^3+1$. We want to trace the position of the object from $t=-3$ to $t=3$ for increments of 0.1.

Steps for animation

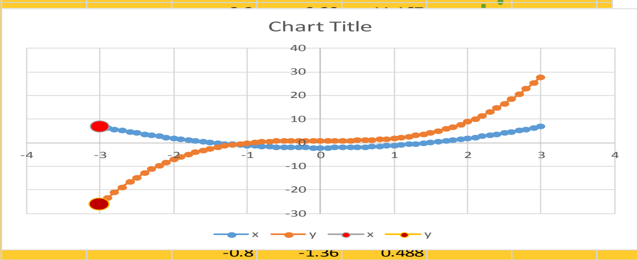
- Step-1: Calculate the values of x and y for different values of t from -3 to 3 with an increment of 0.1.
- Step-2: Plot x and y to create the path of the object.
- Step-3: Insert a slider.
- Step-4: Link the slider value to t
- Step-5: Calculate the corresponding x and y value
- Step-6: Plot the point as object position at time t

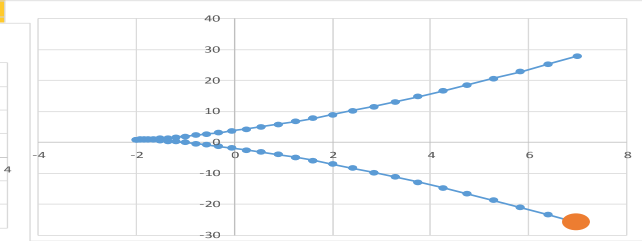
Animating an object movement



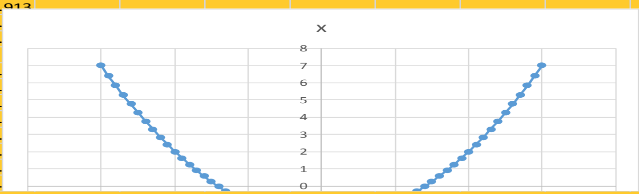
Animation of a moving object

Animation in excel							
25-05-2016							
DOUBLE DECKER	t	x	y		0		
DELHI TO JAIPUR	-3	7	-26	..\cdas	t	x	y
	-2.9	6.41	-23.389	sanim	-3	7	-26
	-2.8	5.84	-20.952				
	-2.7	5.29	-18.683				
	-2.6	4.76	-16.576				
	-2.5	4.25	-14.625				
	-2.4	3.76	-12.824				





	-0.8	-1.36	0.488
	-0.7	-1.51	0.657
	-0.6	-1.64	0.784
	-0.5	-1.75	0.875
	-0.4	-1.84	0.936
	-0.3	-1.91	0.973
	-0.2	-1.96	0.992
	-0.1	-1.99	0.999
	1.53E-15	-2	1
	0.1	-1.99	1.001
	0.2	-1.96	1.008
	0.3	-1.91	1.027
	0.4	-1.84	1.064
	0.5	-1.75	1.125
	0.6	-1.64	1.216
	0.7	-1.51	1.343
	0.8	-1.36	1.512
	0.9	-1.19	1.729
	1	-1	2
	1.1	-0.79	2.331
	1.2	-0.56	2.728
	1.3	-0.31	3.197
	1.4	-0.04	3.744
	1.5	0.25	4.375
	1.6	0.56	5.096
	1.7	0.89	5.913
	1.8	1.24	6.
	1.9	1.61	7.
	2	2	
	2.1	2.41	10.
	2.2	2.84	11.
	2.3	3.29	13.
	2.4	3.76	14.
	2.5	4.25	16.
	2.6	4.76	18.
	2.7	5.29	20.
	2.8	5.84	22.



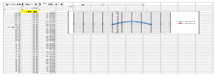
2. Animation of the shape of a graph

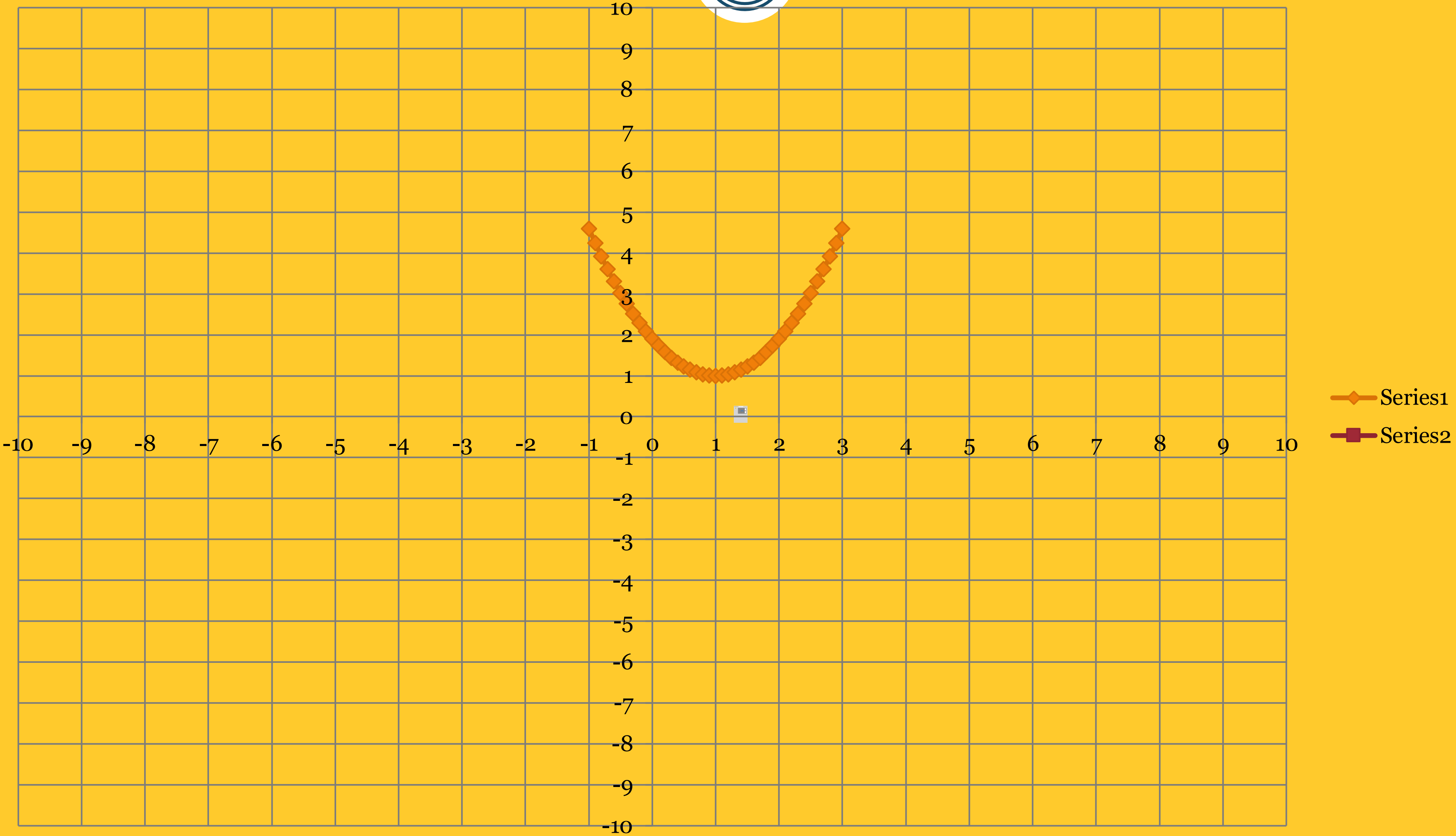
- Here the shape of a graph of a given function is animated. Let the function is given by y=a(x-1)^2+1.

- We are required to find the shape of the graph of this function for different values of a.

- For animating this graph we insert a slider whose value is to be linked to the values of a. There is a trick. All the a values are linked to the first value of a.

Animation of a Function





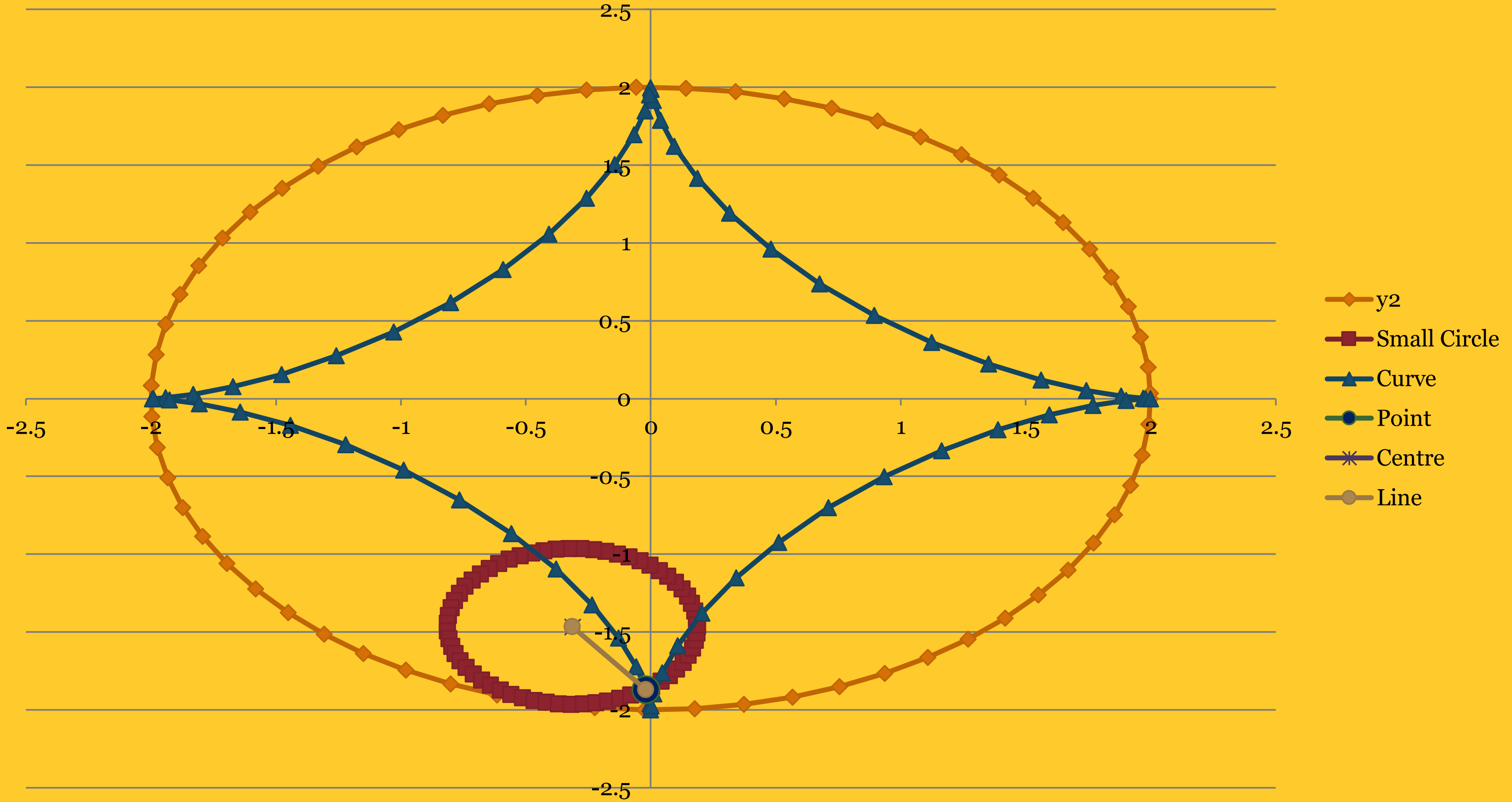
3. Animation of Hypocycloid

- This is a more complex animation. We want to animate a point on the edge of a small circle which rolls without slipping inside a large circle.

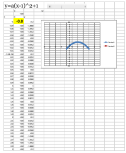
- The trace of the path of the particle is a closed plane curve known as hypocycloid.

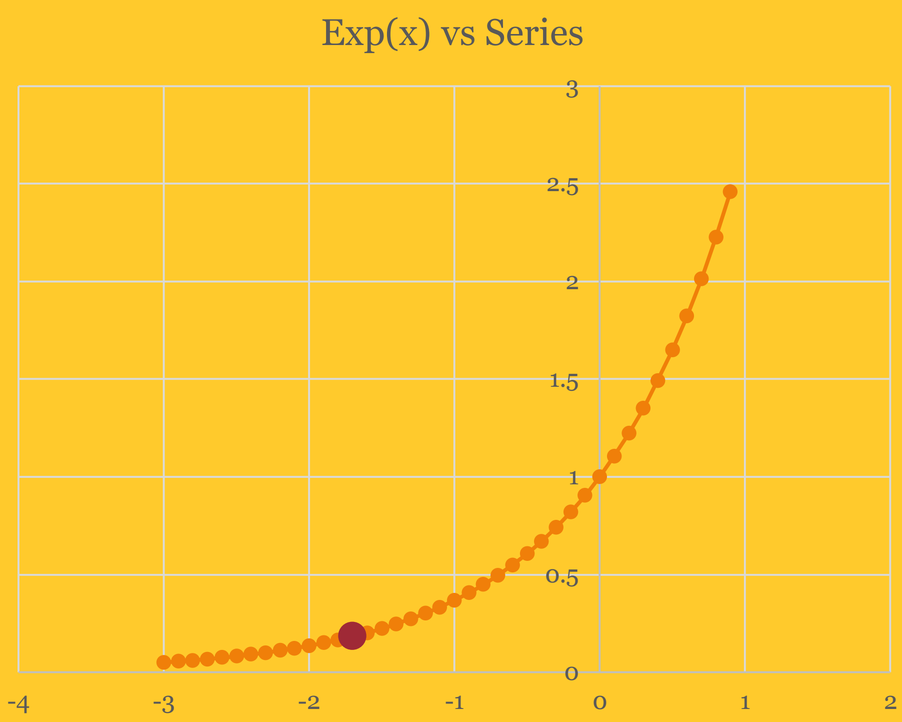
- We are required to draw a Large circle, a rotating small circle, the hypocycloid, centre of smaller circle and the point on the edge of the small circle.

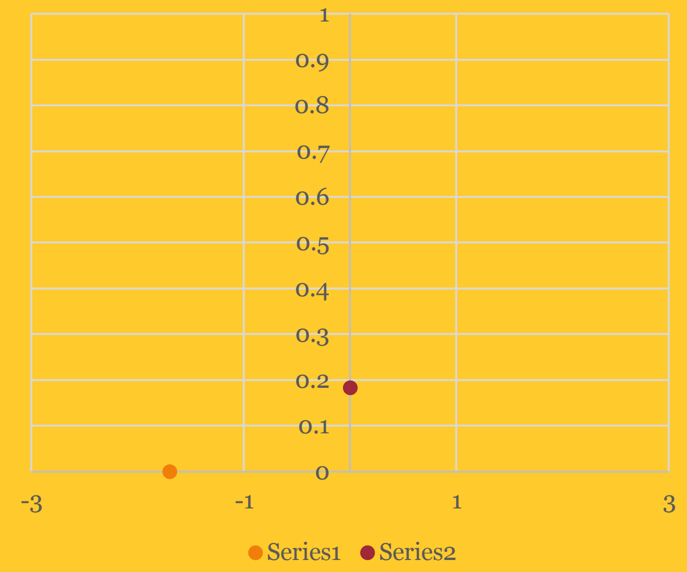
Animation of Hypocycloid



Animation of Exponential Series







Most Important and highest used Formulas

1. $$x^2 + y^2 = z^2$$

2. x=r*cos(t) and y=r*sin(t)

Conic Sections

ANALYTICAL VS MATRIX METHODS

Polar Equations

- Any point in the Cartesian coordinates are expressed as (x, y) and the same point in polar coordinate system is expressed as (r, t).
- Hence, x in Cartesian coordinate system can be expressed by

$$x=r*\cos(t)$$

and y can be expressed by

$$y=r*\sin(t).$$

Circle

- The circle is defined by the points whose distances from a fixed point (called origin) is same.
- The circle can be drawn by two methods-

1. By rotating the point with equal spacing
2. By incrementing the point with equal intervals.

- Ex: Draw a circle with radius r=1.
- Then x=1.cos(t)
- y=1.sin(t)

Draw Circle in Excel
Non Uniform Spacing

- Draw Circle with Radius r=1

- x=1.cost

- y=1.sint

- t=0 to 2Π
- Dt=.1

Draw Circle in Excel
Uniform Spacing of Points

- In this, points are equally spaced along the curve. First an initial point is calculated and subsequent points are calculated by adding a small incremental value.

Steps for Drawing a Circle with Uniformly spaced points

Step No.	Xi	Yi
1.	r*cos(ti)	r*sin(ti)
2.	x(i+1)=r*cos(ti+δt)	y(i+1)=r*sin(ti+δt)
Using the sum Angle Formula		
3.	x(i+1)=r(costi.cosδt-sinti.sinδt)	y(i+1)=r(costi.sinδt-cosδt.sinti)
4.	X(i+1)=(xi.cosδt-yi.sinδt)	Y(i+1)=(xi.sinδt-yi.cosδt)

Parabola

- In Cartesian coordinate system, the parabola is represented by $y = \pm\sqrt{4ax}$
- In this equation, y is having two values for each value of x. Hence this equation cannot be represented graphically easily.
- We can draw two curves to draw a parabola- one for (-y) values for each x and one for (+y) values for same x.

Parabola

- Parametric Representation of parabola is given by-

$$x = \tan^2 \phi$$
$$y = \pm\sqrt{a + a\tan\phi}$$

- In many cases, we get parabola as $y=x^2$
- But this is not the standard form of parabola as it is not aligned with the x-axis.

Hyperbola

- The standard form of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- The hyperbola has two separate branches that approach two asymptotes.
- Note: Asymptote is some boundary beyond which a curve will not pass. Beyond origin, x and y are very large compared to 1 in the right hand side of the equation, so it can be approximated as :

- or $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ or $$y = \frac{b}{a}x$$

$$y = -\frac{b}{a}x$$

Hyperbola

- These equations produces two straight line that pass through origin (c=0) and slope=b/a or –b/a and angle between the lines are arctan(b/a) and –arctan(b/a) to the x-axis.
- The parametric representation is:

  $x=\pm a\sec t$, [h/b]

  $y=\pm b\tan t$, [b/a]
- Another alternative parametric representation-

  $x=a \cosh t$

  $y=b \sinh t$

  $\cosh t=(e^{t}+e^{-t})/2$

  $\sinh t=(e^{t}-e^{-t})/2$

as t varies from 0 to ∞ hyperbola is traced out.

Hyperbola

- $x_i = a \cosh t_i$
  $y_i = b \sinh t_i$

- $x_{i+1} = a (\cosh (t_i + \delta_t))$
  $y_{i+1} = b (\sinh (t_i + \delta_t))$
  or
- $x_{i+1} = a (\cosh t_i . \cosh \delta_t - \sinh t_i . \sinh \delta_t)$
  $y_{i+1} = b (\sinh t_i . \cosh \delta_t + \cosh t_i . \sinh \delta_t)$

- $tmin = \cosh^{-1}(xmin/a)$
  $tmax = \cosh^{-1}(xmax/a)$

- $\cos^{-1} x = \ln(x + \sqrt{x^2 - 1})$

Equation of a Conic Section

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Can we determine the type graph - whether it is a Line, Circle, Ellipse, Parabola or Hyperbola, by looking at this equation?

Determinant Role in Identification

- If the equation: $ax^2 + bxy + cy^2 + dx + ey + f = 0$
- Then, the major determinant is-

$$\begin{vmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{vmatrix}$$

- The major determinant formed by the coefficients plays a major role in determining when an equation will be a line, an ellipse, a hyperbola or parabola.

Identifying the Type of Conic Section

$$ax^2 + bxy + cy^2 + dx + ey + f = 0........(1)$$

1. If a, b, c =0, then it is a straight line

   $$dx+ey+f=0$$

2. If equation 1 can be factorized then it is the equation of two lines

   $$(x-1)(y-2)=0 \qquad \text{i.e. } x=1,\ y=2 \text{ lines}$$

3. In other cases either it will be an ellipse or a parabola or a hyperbola
4. So the problem is to ascertain when an equation will be a line or circle or ellipse or parabola or hyperbola

Determinant Role in Identification

$$\begin{vmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{vmatrix}$$

- If $\Delta=0$, then the equation is a line (or pair of lines)
- When major determinant is not 0, then the equation is a Conic Section.
- But is it an Circle, Ellipse, Hyperbola or parabola?

Determinant Role in Identification

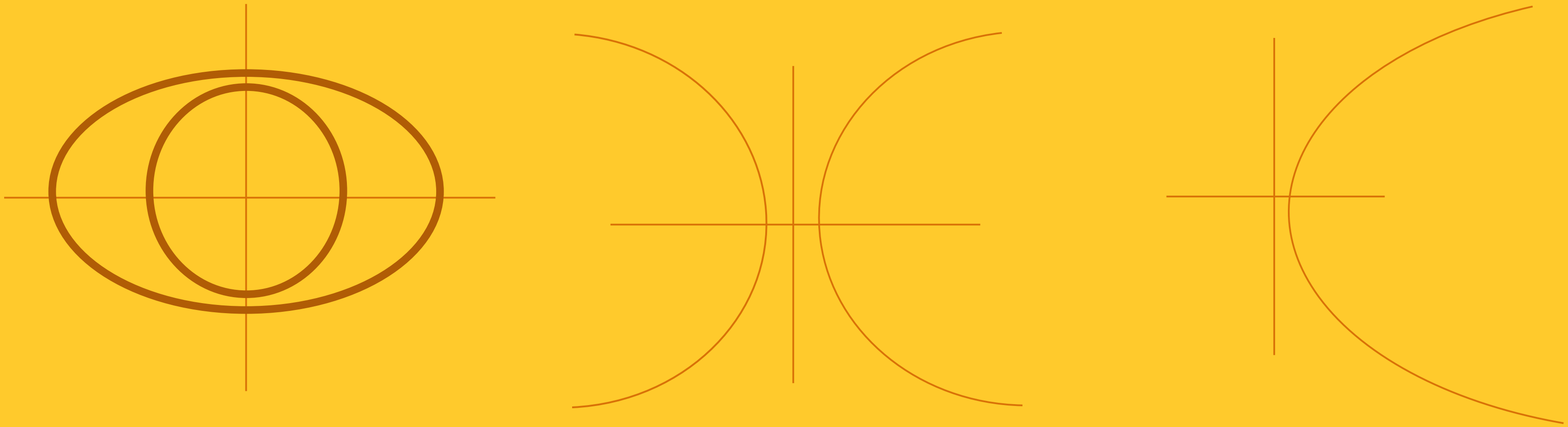
- Minor Determinant determines the type of conic section.
- Minor Determinant= $\begin{vmatrix} a & b/2 \\ b/2 & c \end{vmatrix} = \Delta$
- If Δ>0, Ellipse
- If Δ<0, Hyperbola
- If Δ=0, Parabola

- The conic section is ascertained here but how do we get the standard form of the conic section?

Two types of Conic Sections

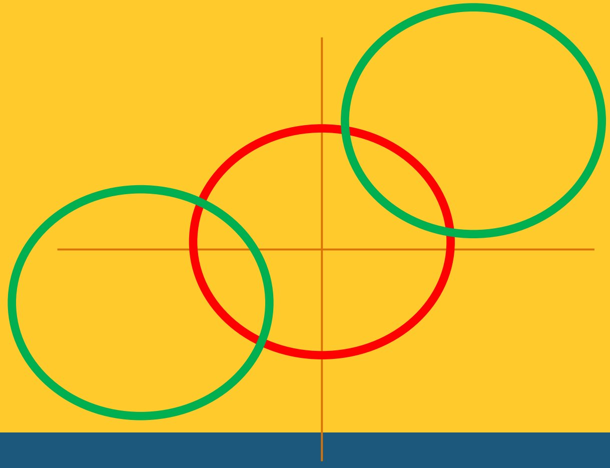
- Central
- Non- Central
- ELLIPSE, CIRCLE, HYPERBOLA- CENTRAL CONIC
- PARABOLA- NON CENTRAL

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$



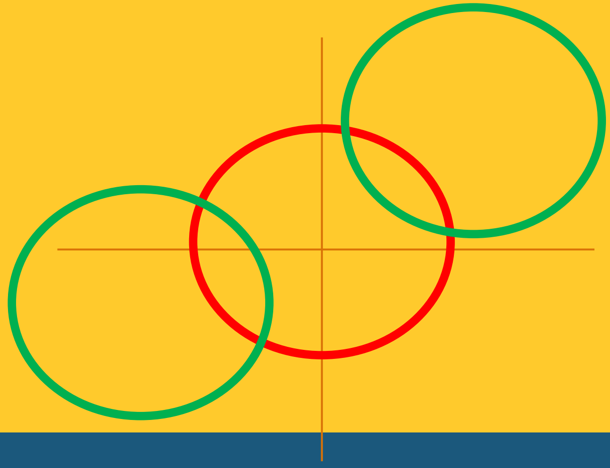
Standard form

- How to transform a Non-Standard equation into a Standard Form?
- In a non standard equation, the x*y term rotates the section in a certain angle and x and y shifts the conic section from origin.



Standard form

- Standard form of a circle= $x^2 + y^2 = r^2$
- Standard form of Ellipse= $\frac{x\hat{}2}{a\hat{}2} + \frac{y\hat{}2}{b\hat{}2} = 1$



Bringing Back to Standard Form

- In order to transform an equation to its standard form, it is to be rotated back to the standard position and translate the center to the origin.
- The rotation angle can be determined by:

$$t = \frac{1}{2} a\tan\left(\frac{b}{a-c}\right)$$

Bringing Back to Standard Form

- To determine the value of m and n which is to be translated to x and y direction are calculated from minor determinants-

$$m = \frac{\begin{vmatrix} d & b \\ e & c \end{vmatrix}}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}} \qquad n = \frac{\begin{vmatrix} a & d \\ b & e \end{vmatrix}}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}}$$

Bringing Back to Standard Form

- Rotate back the given equation
- Translate back to origin
- This will remove the x*y, x and y terms from the given equation to get the standard form
- Translation and Rotation Matrix in 2D Homogeneous Plane

$$t_r = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad t_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix}$$

Bringing Back to Standard Form

- If we apply the transformations shown below, we will get the coefficient matrix of the standard equation.

$$t_{ct} = t_t \times t_r \times t_c \times t_r^{-1} \times t_c^{-1}$$

$$t_r = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad t_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix} \quad t_c = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

Ensure whether Standard form corresponds to Original Equation?

- First the standard equation is to be formed
- Then the invariants of standard form and given equation is to be matched.
- 3 invariants do not change when we transform the equations. The invariants are-

1. a+c=a'+c'
2. $\begin{vmatrix} a & b \\ b & c \end{vmatrix} = \begin{vmatrix} a' & b' \\ b' & c' \end{vmatrix}$
3. ΔM=ΔM'

Now how do we draw such conic sections?

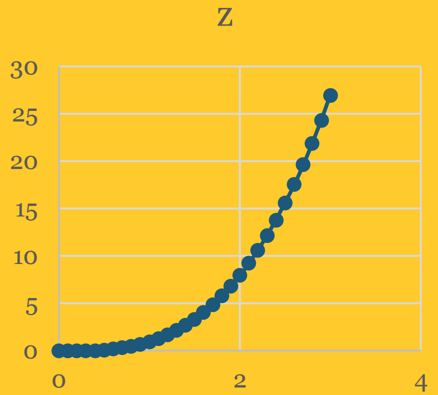
Vector Function

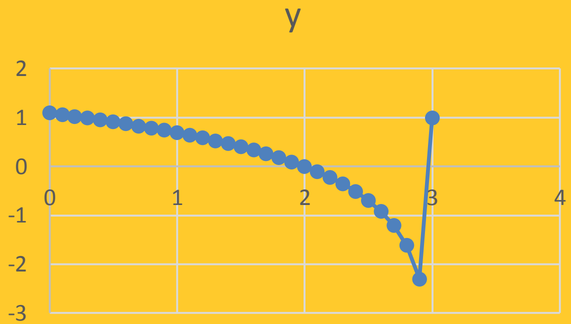
- A vector valued function is a rule that assigns to each element in domain (Reals) an element in range (vectors)
- It is expressed as $r(t)=[f(t),g(t),h(t)]$ or

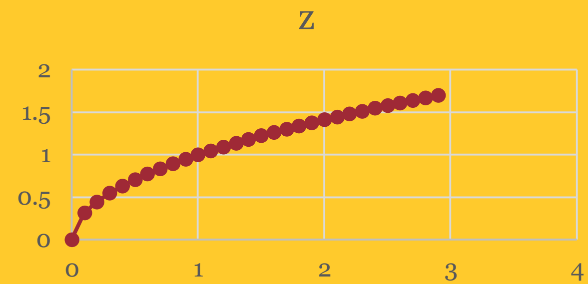
$$r(t)=f(t)i+g(t)j+h(t)k$$

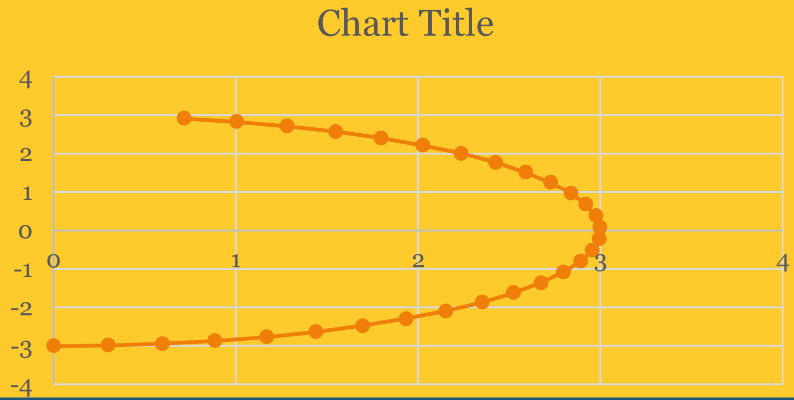
Example of Vector Function

- r=[t^3, ln(3-t), sqrt(t)]



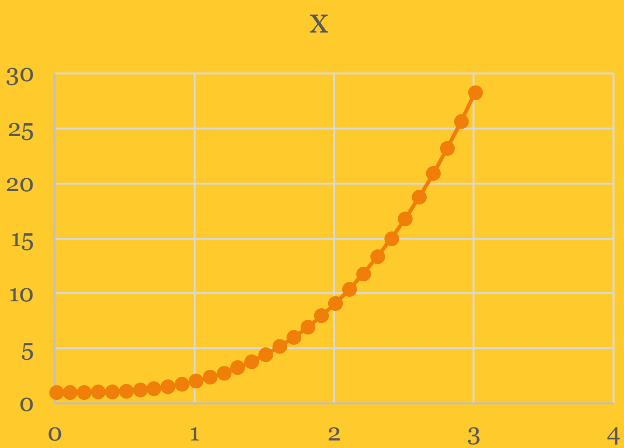


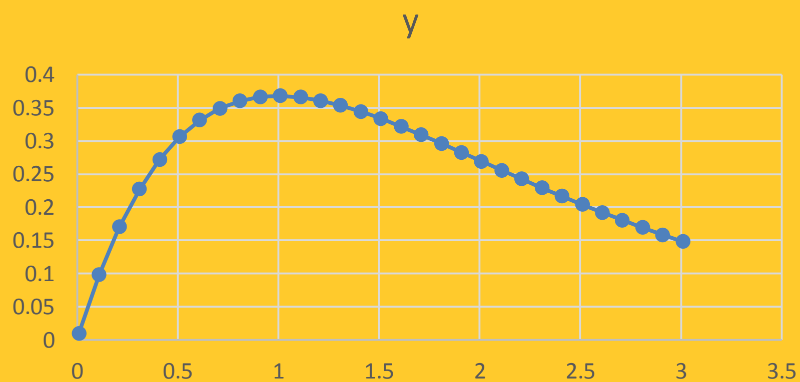


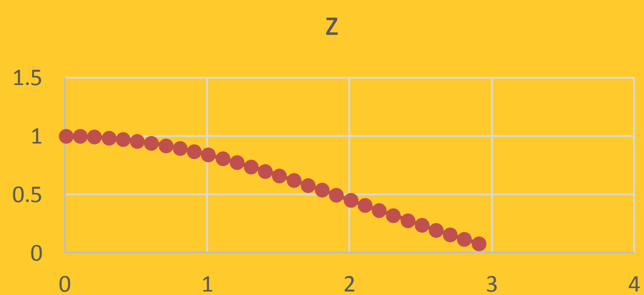


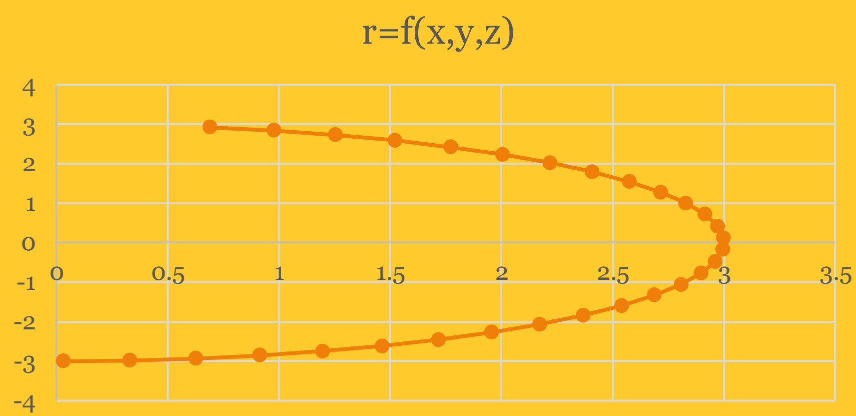
Example of Vector Function

- R=(1+t^3)i+t*exp(-t)j+sin(t)/t k



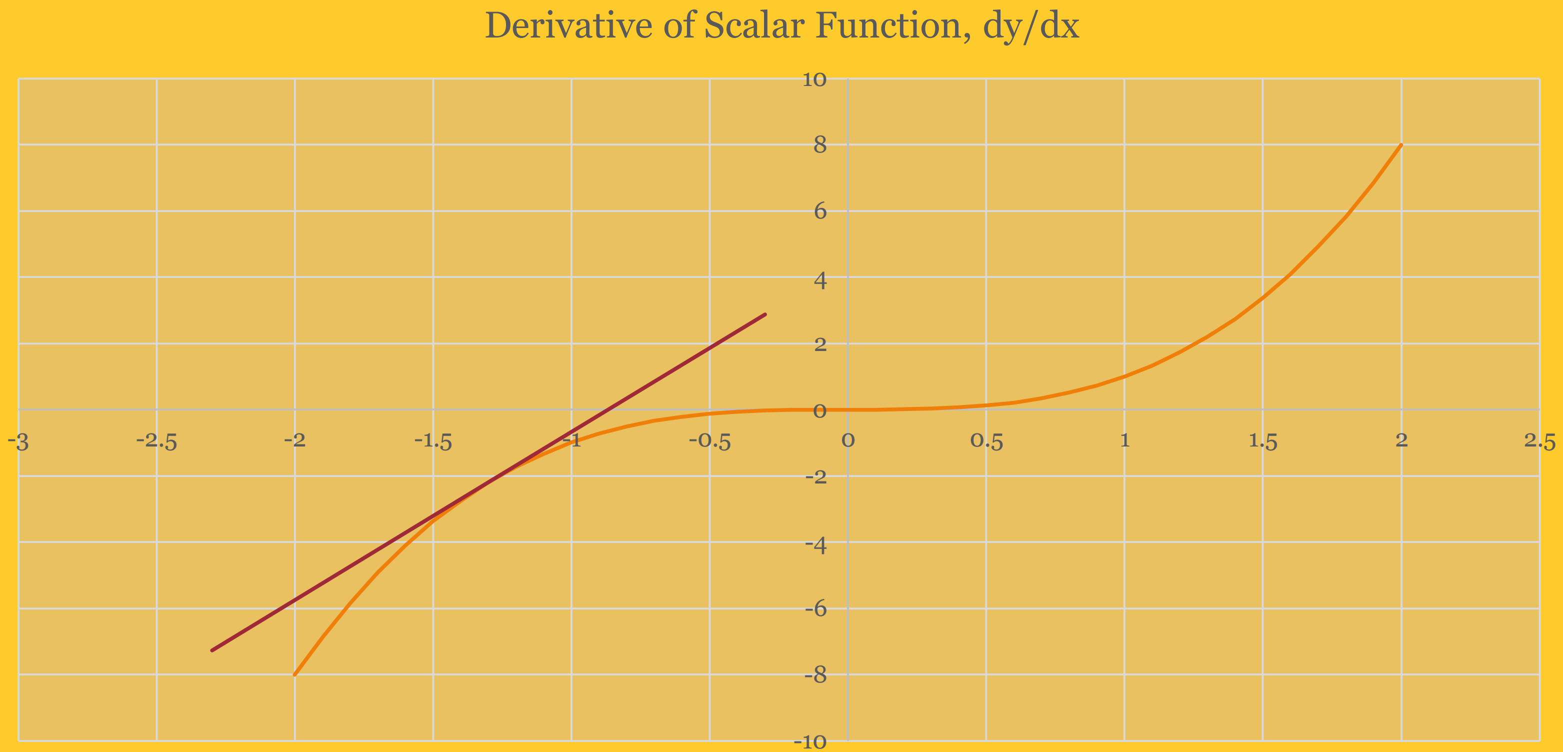






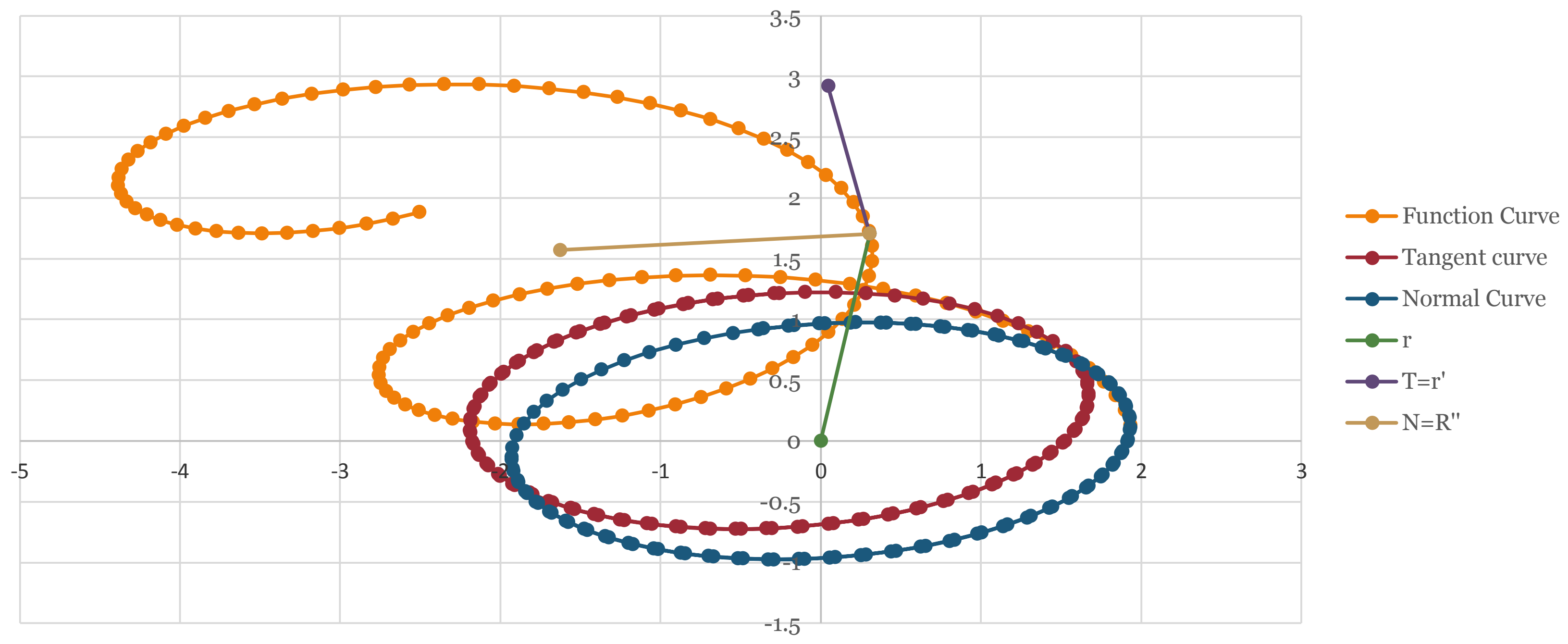
Scalar Function

y=x^3



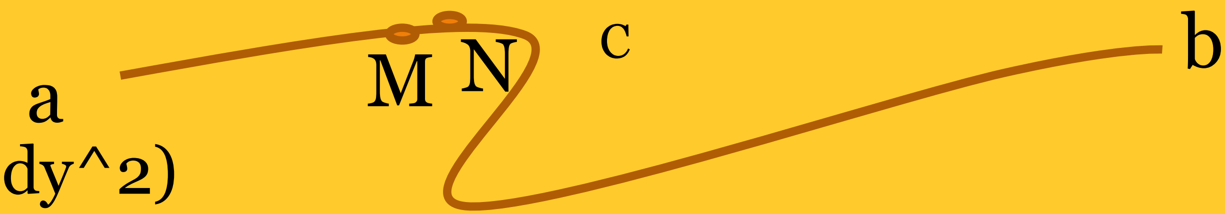
Vector Function

r	2 cos(t)	Sin(t)	t
r'	-2 sin(t)	Cos(t)	1
R’’	-2 cos(t)	-sin(t)	0



Arc Length

- If we take a infinitesimal arc MN, which is equivalent to the cord MN. Let, dx and dy is the increment of x and y for small increment of t, dt,



- $MN=\sqrt{(dx^2+dy^2)}$
- $MN=\sqrt{(((dx^2+dy^2)/dt^2)*dt^2)}$
- $MN=\sqrt{((dx^2/dt^2+dy^2/dt^2)*dt^2)}$
- $MN=\sqrt{((dx/dt)^2+(dy/dt^2))}\ dt$
- $s=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\quad dt$

Arc Length Parametrization of curve

- A parametric representation of a curve with arc length as parameter is called an arc length parametrization of the curve.
- Page-31:Example-4. Find the arc length parametrization of the line x=3t+2, y=2t-1 that has reference point (2,-1) and the same orientation as the original line.
- $s=\int_0^t \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ dt$
- $s=\int_0^t \sqrt{3^2+2^2}\ dt=\sqrt{13}\ t$
- $t=s/\sqrt{13}$ Hence, $x=3s/\sqrt{13}+2$ and $y=2s/\sqrt{13}-1$

Curvature-Use of Arc Length Parametrization

Q&A…

Q&A…

Pattern

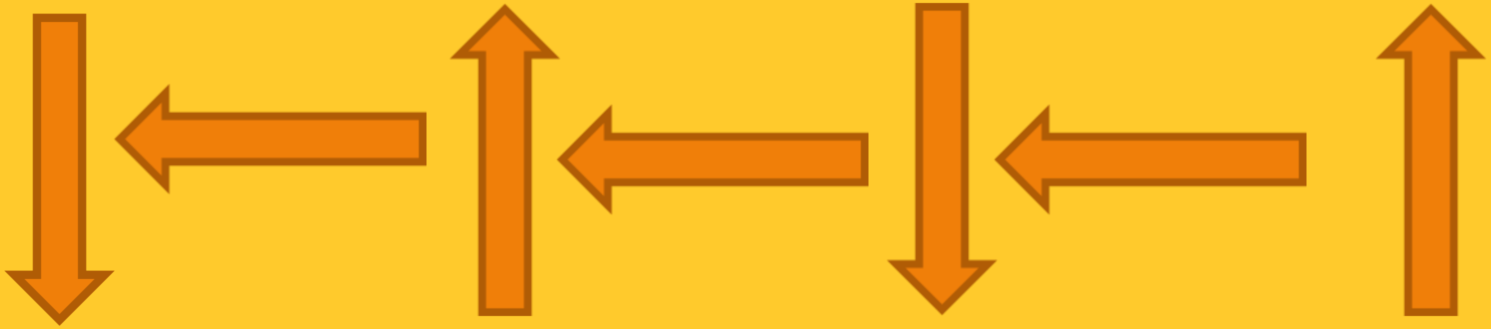
- Noun:
- A repeated decorative design
- A regular and intelligible form or sequence discernible in the way in which something happens or is done
- Verb:
- Decorate with a recurring design
- Give a regular or intelligible form to
-
- Cambridge Dictionary: A particular **way in which something is done, is organized, or happens**

Pattern

sl	Class	Chapter	Title
1	I	10	Patterns
2	II	5	Patterns
3	III	1	Where to look from (Visualization and Pattern)
4	III	10	Play with Patterns
4	IV	10	Play with Patterns
5	V	5	Does it look the same?
6	V	7	Can you see the Pattern?
7	VIII	10	Visualization of Solid Shapes

Pattern in Figure: What Comes next

What Comes next



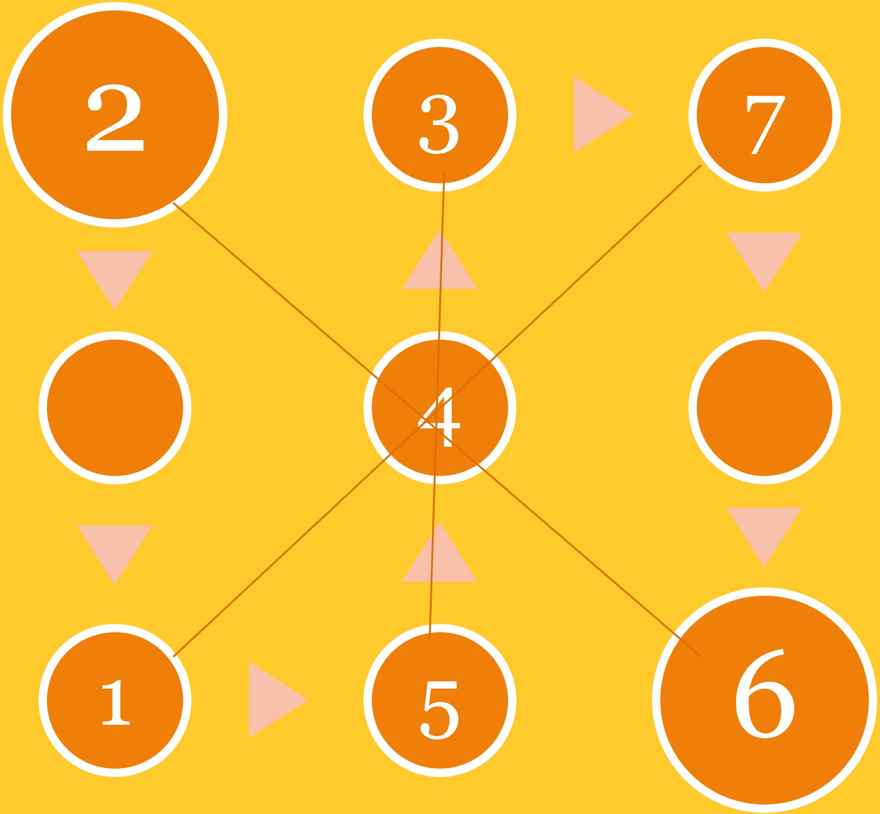
Pattern is Every Where

Number Box: No Number Appears twice in a line

1	**2**	**3**
3	1	2
2	3	1

Magic Pattern

Magic Pattern: Add to 12



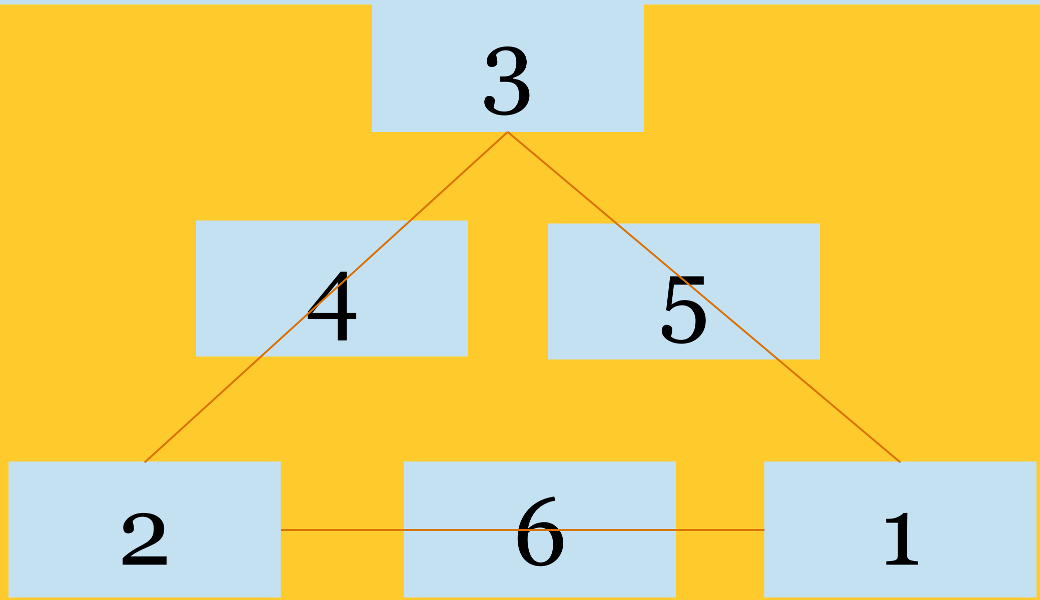
Pattern is Every Where

Fill cells with 1 to 9 so that each line add up to 15

2	**9**	**4**
7	5	3
6	1	8

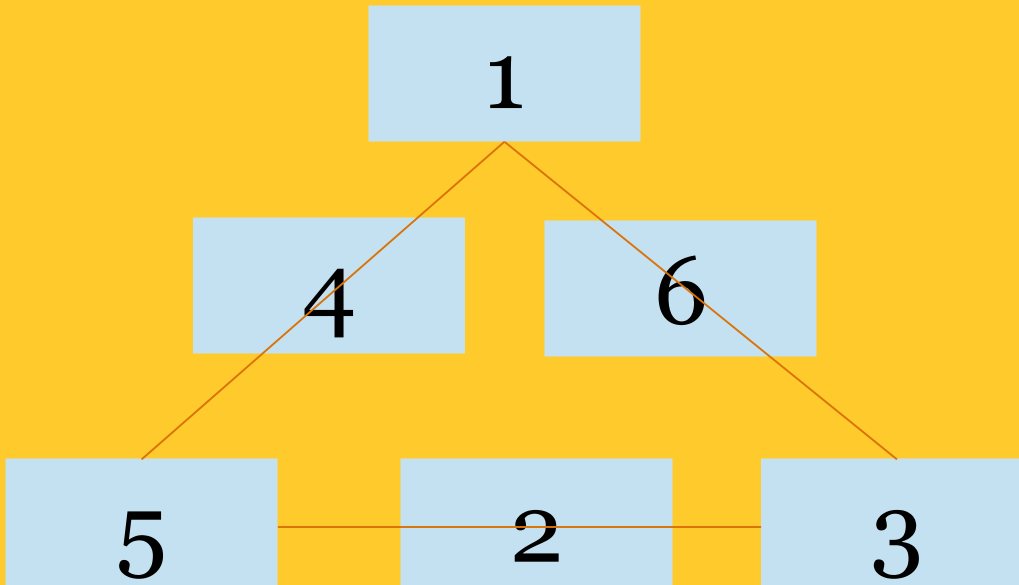
Magic Triangle

Fill cells with 1 to 6 so that each line add up to 9



Magic Triangle

Fill cells with 1 to 6 so that each line add up to 10



Number Tower

Rule: 2 cell bellow add up to above cell

		80		
	30		50	
10		20		30

Pattern with Addition

1	**+**	**2**	**+**	**3**	**=**	**6**
2	+	3	+	4	=	9
3	+	4	+	5	=	12
4	+	5	+	6	=	15
5	+	6	+	7	=	18

Observation: Sum grows by 3 each time

Pattern in Addition

1	+	2	+	3	+	4	=	10
2	+	3	+	4	+	5	=	14
3	+	4	+	5	+	6	=	18
4	+	5	+	6	+	7	=	22
5	+	6	+	7	+	8	=	26
6	+	7	+	8	+	9	=	30

Observation: Sum grows by 4 each time

Formation of complex objects

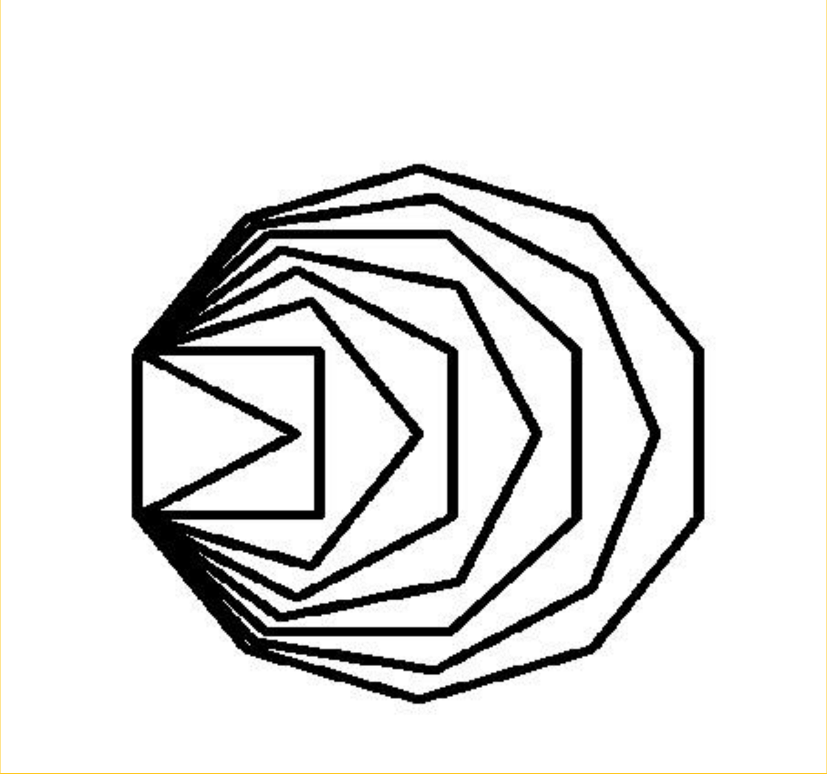
Many complex objects are formed by repeated drawing of a simple object repeatedly by following certain pattern.

Formation of Objects

- Majority of objects are formed following some pattern:

- If you have a straight line, draw this line repeatedly following certain pattern, you get all the regular polygons.

Formation of Polygons following a Pattern



Creating Regular Polygons

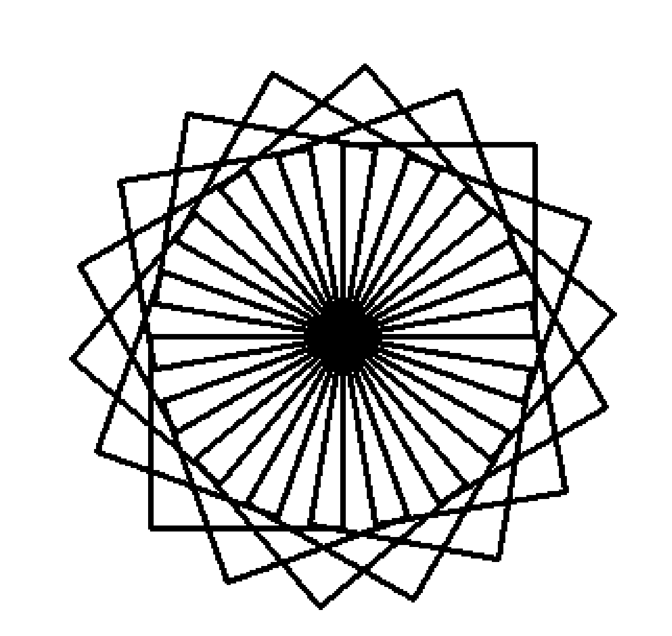
- Regular polygon follows a pattern
- There is a relationship between the no of sides, Angle and Length of the sides of the polygon
- Turtle graphic provides an elegant way of creating polygon
- The procedure to create a polygon in LOGO is

```
to polygon :sides :length
repeat :sides  [forward :length rt 360/:sides]
end
```

Procedure calls Procedure

Objective-To create pattern, we have to draw same object with different orientation. This is easily can be accomplished by calling a procedure within another procedure

The flower was created from the square created earlier

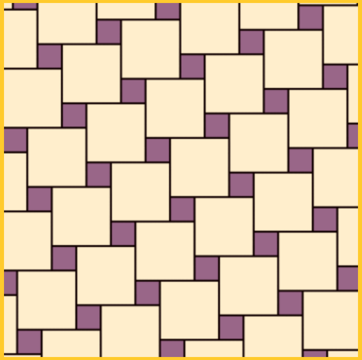


```
To square
Repeat 4 [fd 100 rt 90
End
To flower
Repeat 18 [ square rt 20]
end
```

Different Examples of Pattern

Nature provides examples of many kind of patterns, including symmetries, trees and other structures with a fractal dimension, spirals, meanders, waves, foams, tilings, cracks and stripes.

Examples of Pattern



Different Examples of Pattern

Symmetry

Snowflake sixfold symmetry
Animals that move usually have bilateral or mirror symmetry as this favours movement.
Plants often have radial or rotational symmetry.
Fivefold symmetry is found in the starfish, sea urchins, and sea lilies.
Crystals have a highly specific set of possible crystal symmetries; they can be cubic or octahedral.



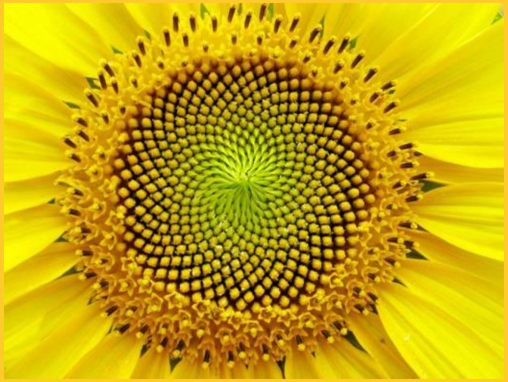


Different Examples of Pattern

Spirals

Spiral patterns are found in the body plans of animals, multiple spirals found in flower heads such as the sunflower and fruit structures like the pineapple.





Different Examples of Pattern

Chaos, flow, meanders

Vortex, street, turbulence

Chaos theory predicts that while the laws of physics are deterministic, events and patterns in nature never exactly repeat because extremely small differences in starting conditions can lead to widely differing outcomes. Many natural patterns are shaped by this apparent randomness, including vortex streets and other effects of turbulent flow such as meanders in rivers.

Different Examples of Pattern

Waves, dunes

Waves are disturbances that carry energy as they move. Mechanical waves propagate through a medium – air or water, making it oscillate as they pass by.

Wind waves are surface waves that create the chaotic patterns of the sea. As they pass over sand, such waves create patterns of ripples; similarly, as the wind passes over sand, it creates patterns of dunes.

Different Examples of Pattern

Bubbles, foam

Foams obey Plateau's laws, which require films to be smooth and continuous, and to have a constant average curvature. Foam and bubble patterns occur widely in nature, for example in radiolarians, sponge spicules, and the skeletons of silicoflagellates and sea urchins.

Different Examples of Pattern

Cracks

Cracks form in materials to relieve stress: with 120 degree joints in elastic materials, but at 90 degrees in inelastic materials. Thus the pattern of cracks indicates whether the material is elastic or not. Cracking patterns are widespread in nature, for example in rocks, mud, tree bark and the glazes of old paintings and ceramics.

Different Examples of Pattern

Spots, stripes

Different spots and stripes in animals, skins etc.

Different Examples of Pattern

Tilings/ *Tessellation and Tile*

In visual art, pattern consists in regularity which in some way "organizes surfaces or structures in a consistent, regular manner." At its simplest, a pattern in art may be a geometric or other repeating shape in a painting, drawing, tapestry, ceramic tiling or carpet, but a pattern need not necessarily repeat exactly as long as it provides some form or organizing "skeleton" in the artwork. In mathematics, a tessellation is the tiling of a plane using one or more geometric shapes (which mathematicians call tiles), with no overlaps and no gaps

Different Examples of Pattern

Architecture:

Virupaksha temple at Hampi has a fractal-like structure where the parts resemble the whole.
In architecture, motifs are repeated in various ways to form patterns. Most simply, structures such as windows can be repeated horizontally and vertically
Architects can use and repeat decorative and structural elements such as columns, pediments, and lintels.
Repetitions need not be identical.
Temples in South India have a roughly pyramidal form, where elements of the pattern repeat in a fractal-like way at different sizes.

Different Examples of Pattern

Fractals

Fractals are mathematical patterns that are scale invariant. This means that the shape of the pattern does not depend on how closely you look at it. Self-similarity is found in fractals. Examples of natural fractals are coast lines and tree shapes, which repeat their shape regardless of what magnification you view at. While self-similar patterns can appear indefinitely complex, the rules needed to describe or produce their formation can be simple (e.g. Lindenmayer systems describing tree shapes).

Symmetry

Symmetry:

1. The quality of being made up of exactly similar parts facing each other or around an axis
2. Correct or pleasing proportion of the parts of a thing

Symmetry

SL	CLASS	CHAPTER	TITLE
1	VI	13	Symmetry
2	VII	14	Symmetry

Symmetrical Objects

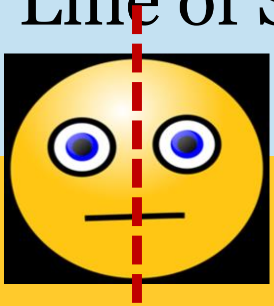
- Definition: Objects with evenly balanced proportions are called Symmetrical Objects.





Line of Symmetry

- In all the above objects, left half and right half matched.
- The objects whose left and right halves match exactly then the object is said to have line symmetry.
- If we put a mirror in the middle, then the image of one side of the object will fall exactly on the other side of the object.
- The mirror line is called the "Line of Symmetry" or "Axis of Symmetry".



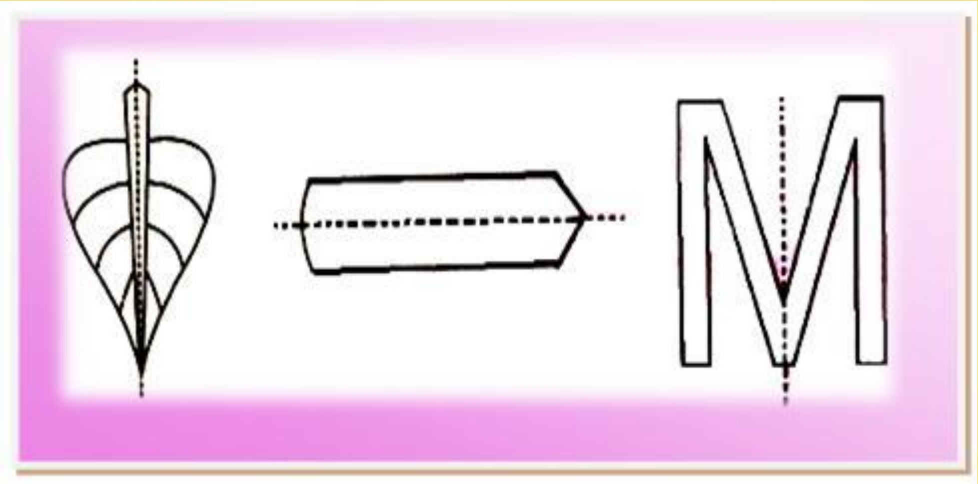
Line of Symmetry

Different types of Line of Symmetry:

1. One line symmetry
2. Two line symmetry
3. Three Line Symmetry
4. Four Line Symmetry
5. Five Line Symmetry
6. Multi Line Symmetry

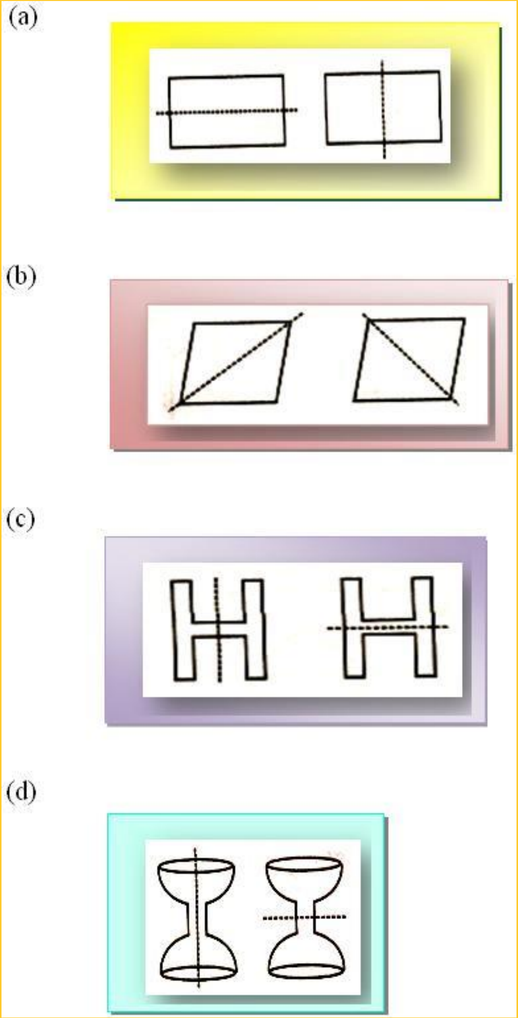
Symmetrical Shapes

- One Line Symmetry:



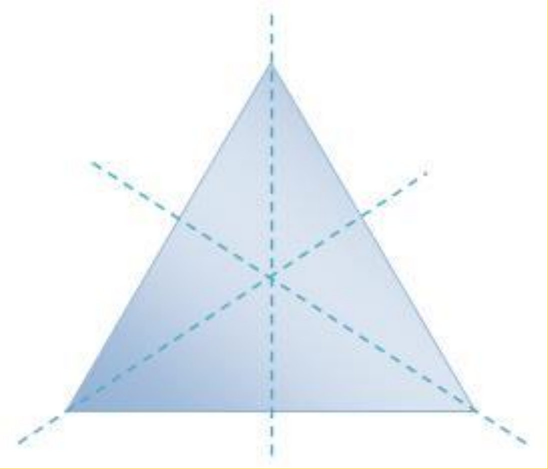
Symmetrical Shapes

- Two Line Symmetry:



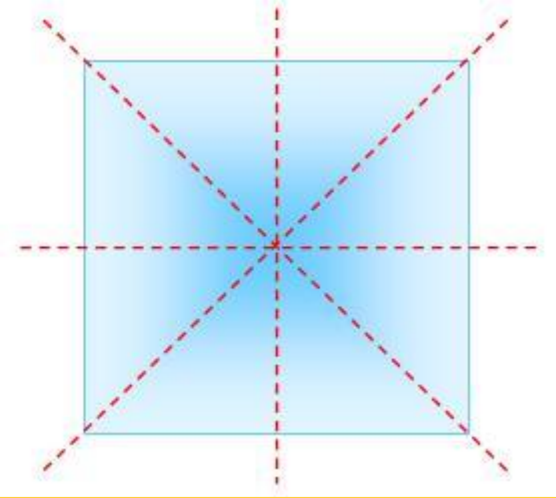
Symmetrical Shapes

- Three Line Symmetry:



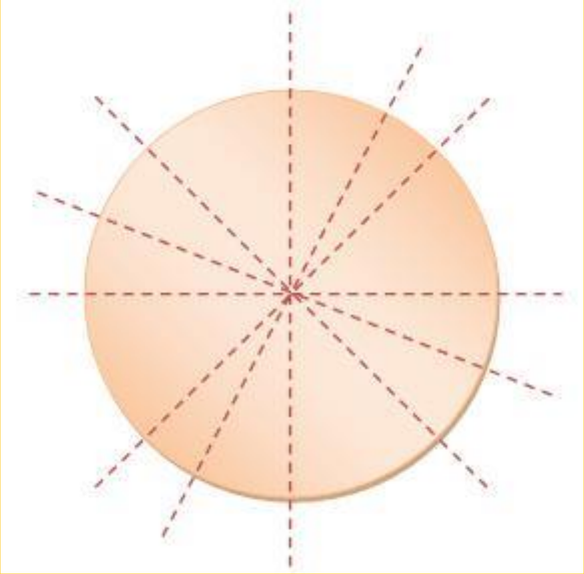
Symmetrical Shapes

- Four Line Symmetry:



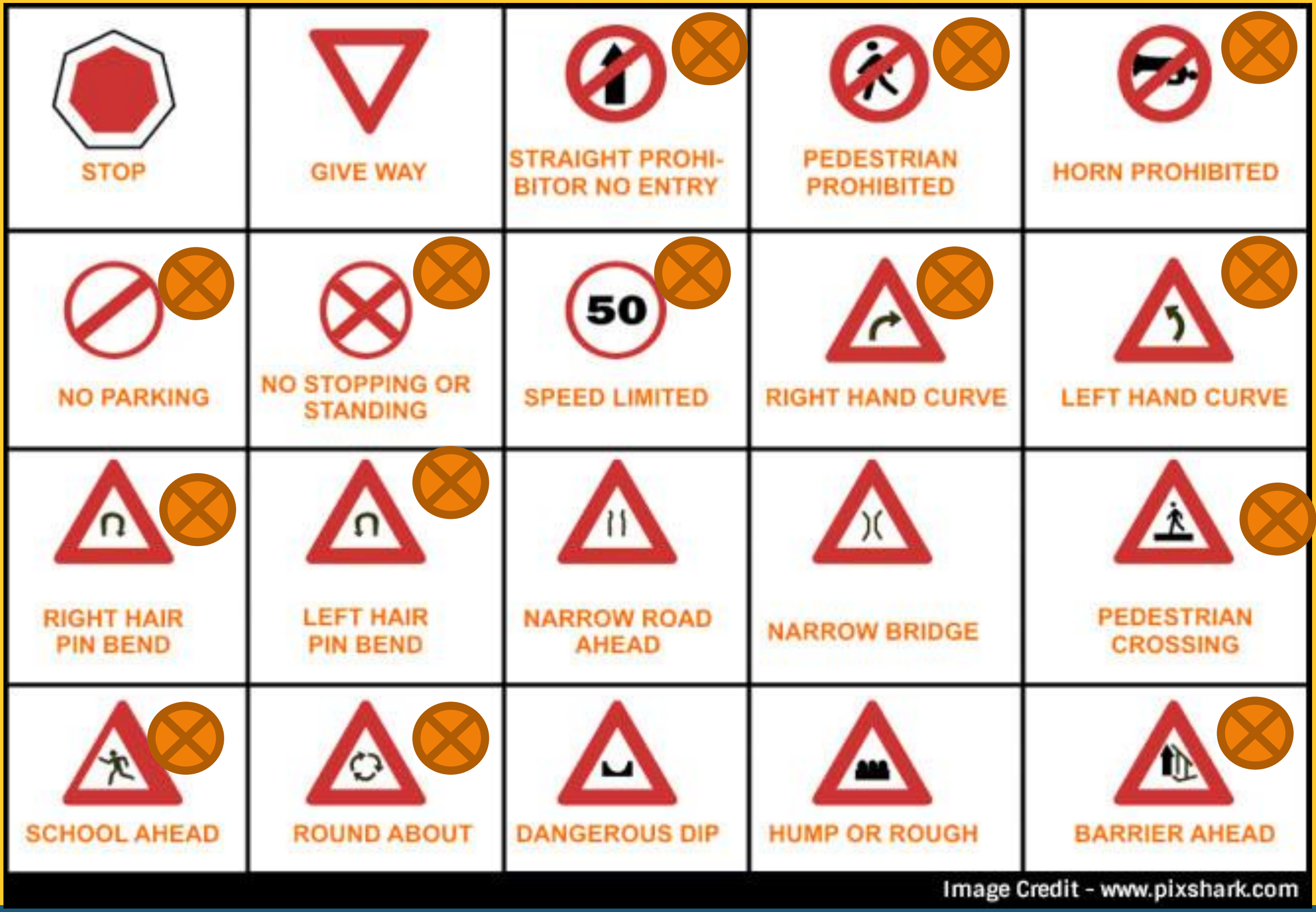
Symmetrical Shapes

- Multi Line Symmetry:



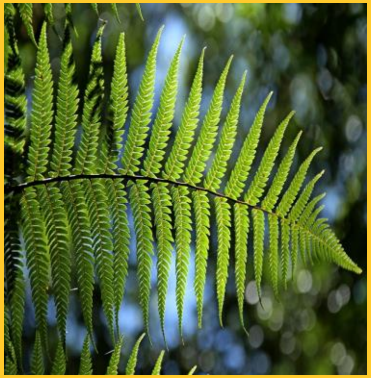
Symmetry Every Where

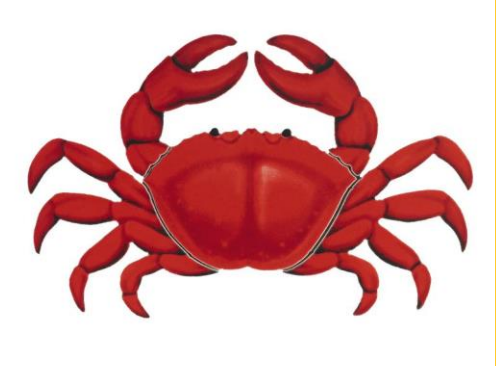
- Symmetry in Road Safety:



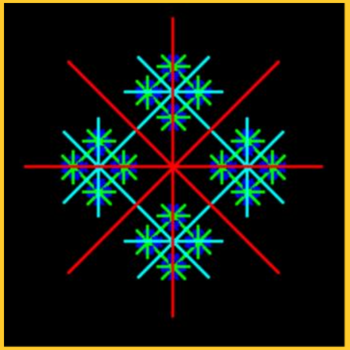
Symmetry Every Where

- Symmetry in Nature

Symmetry Everywhere



Types of Symmetry

- Line (Reflection) Symmetry
- Rotational Symmetry
- Line and Rotational Symmetry

Definition of Symmetry

- A symmetry of some mathematical structure is a transformation of that structure, of a special kind, that leaves specified properties of that structure unchanged.

- Condition – Only invertible transformations are permitted.

Definition of Symmetry

- A symmetry of a shape in the plane or space is a rigid motion of the plane or space that maps the shape into itself.

Visualization

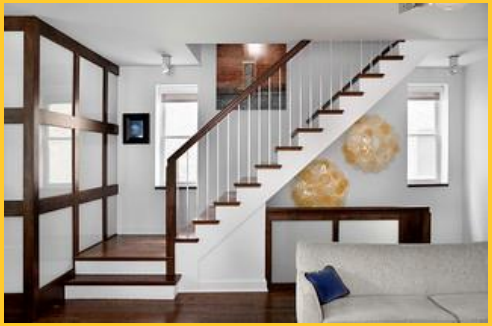
We live in a three dimensional world. The world is made of different types of object. These objects can be of different dimensions.

Some may be plane figures others may be solid shape. Same objects looks different when seen from different angles and distances. Hence understanding of visualization process or visualization technique is very important.

Visualization

SL	CLASS	CHAPTER	TITLE
1	III	1	Where to look from (Visualization and Pattern)
2	VII	15	Visualizing solid shapes

Visualization



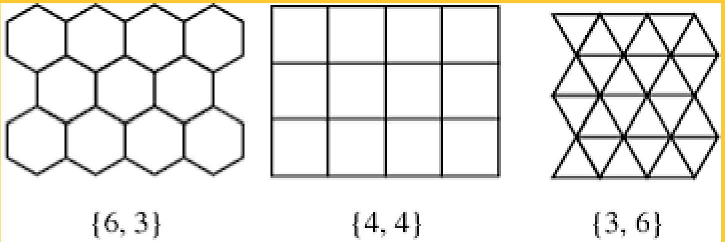
Tessellation or Tiling

Tessellation or Tiling

Definition: Tessellation is the process of covering a surface with shape object without any gap or overlaps

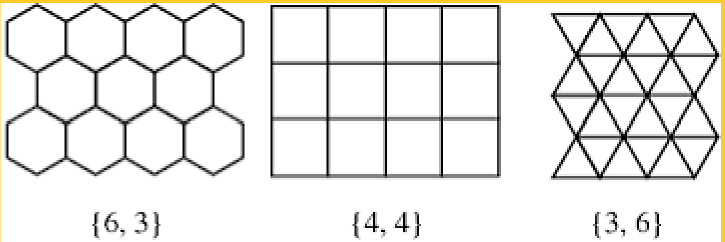
Type of Tessellations:
Regular,
Semi regular and
Random

Regular Tessellations



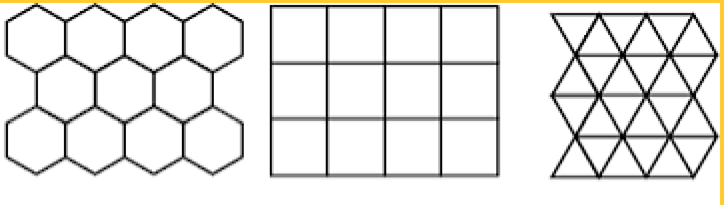
- Regular Tessellation is formed from three regular polygons: Triangles, Squares and Hexagonals
- Regular – the sides and angles are equivalent means the polygon is both equiangular and equilateral
- Congruent – Same shape and same size
- Similar – Same shape, different sizes

Regular Tessellations



- How to create Regular Tessellation:
- Take a vertex and the object
- As the regular tessellation must fill the plane at each vertex, the interior angle of the object must be exact divisor of 360 degree. Only three regular polygons meet this criteria. These polygons are triangle, square and hexagon.

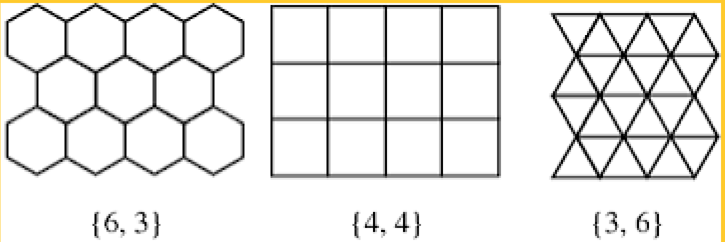
Regular Tessellations



{6, 3} {4, 4} {3, 6}

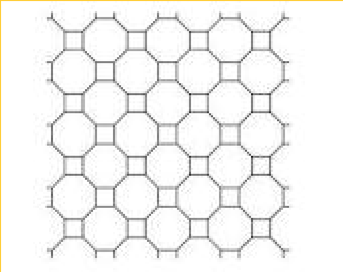
- Naming Convention:
- First choose a vertex and then look at one of the polygons that touches it.
- Calculate how many sides does it has.
- Now keep going around the vertex in either direction, finding the number of sides of the polygons till the starting polygon.

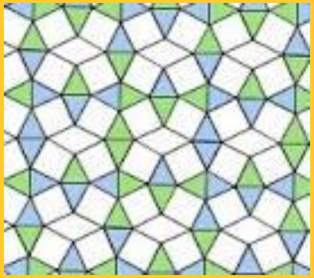
Regular Tessellations

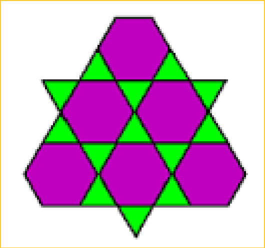


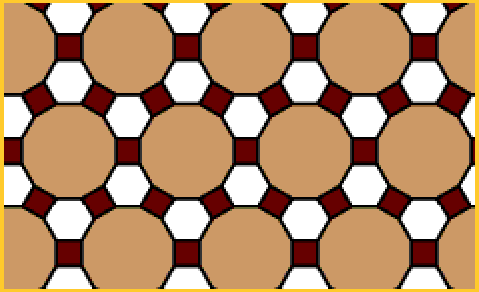
- Naming Convention:
- For a square – 4.4.4.4
- For a triangle – 3.3.3.3.3.3
- For a hexagon – 6.6.6

Semi Regular Tessellations







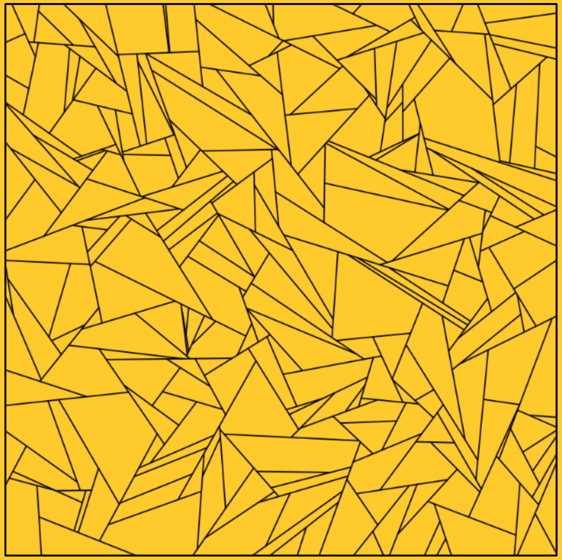


- Semi Regular Tessellation is formed by using different shape:
- Properties of Semi Regular Tessellation:
  - 1. It is formed by regular polygons
  - 2. The arrangement of polygons at every vertex is identical

Examples: 3.3.3.4.4, 3.3.4.3.4, 3.4.6.4, 3.6.3.6, 4.8.8, 3.12.12

Random Tessellations

In applied mathematics, a **Gilbert tessellation**[1] or **random crack network**[2] is a mathematical model for the formation of mudcracks, needle-like crystals, and similar structures. It is named after Edgar Gilbert, who studied this model in 1967.[3]
In Gilbert's model, cracks begin to form at a set of points randomly spread throughout the plane according to a Poisson distribution. Then, each crack spreads in two opposite directions along a line through the initiation point, with the slope of the line chosen uniformly at random. The cracks continue spreading at uniform speed until they reach another crack, at which point they stop, forming a T-junction. The result is a tessellation of the plane by irregular convex polygons.



Set Theory

- Sets- Collection of Objects
- Set of natural numbers, integers, rational numbers, real numbers represented by N, Z, Q, R etc
- Range – Domain

Representation of Sets:

1. Roster
2. Set builder

Symbols used in Set Theory

Symbol	Symbol Name	Meaning / definition	Example
{ }	set	a collection of elements	A = {3,7,9,14}, B = {9,14,28}
|	such that	so that	A = {x | x∈r, x>0}
A ∩ B	intersection	objects that belong to set A and set B	A ∩ B = {9,14}
A ∪ B	union	objects that belong to set A or set B	A ∪ B = {3,7,9,14,28}
A ⊆ B	subset	subset has fewer elements or equal to the set	{9,14,28} ⊆ {9,14,28}
A ⊂ B	proper subset / strict subset	subset has fewer elements than the set	{9,14} ⊂ {9,14,28}

Symbols

A ⊄ B	**not subset**	**left set not a subset of right set**	{9,66} ⊄ {9,14,28}
A ⊃ B	**proper superset / strict superset**	**set A has more elements than set B**	{9,14,28} ⊃ {9,14}
A ⊅ B	**not superset**	**set A is not a superset of set B**	{9,14,28} ⊅ {9,66}

Source-http://www.rapidtables.com/math/symbols/Set_Symbols.htm

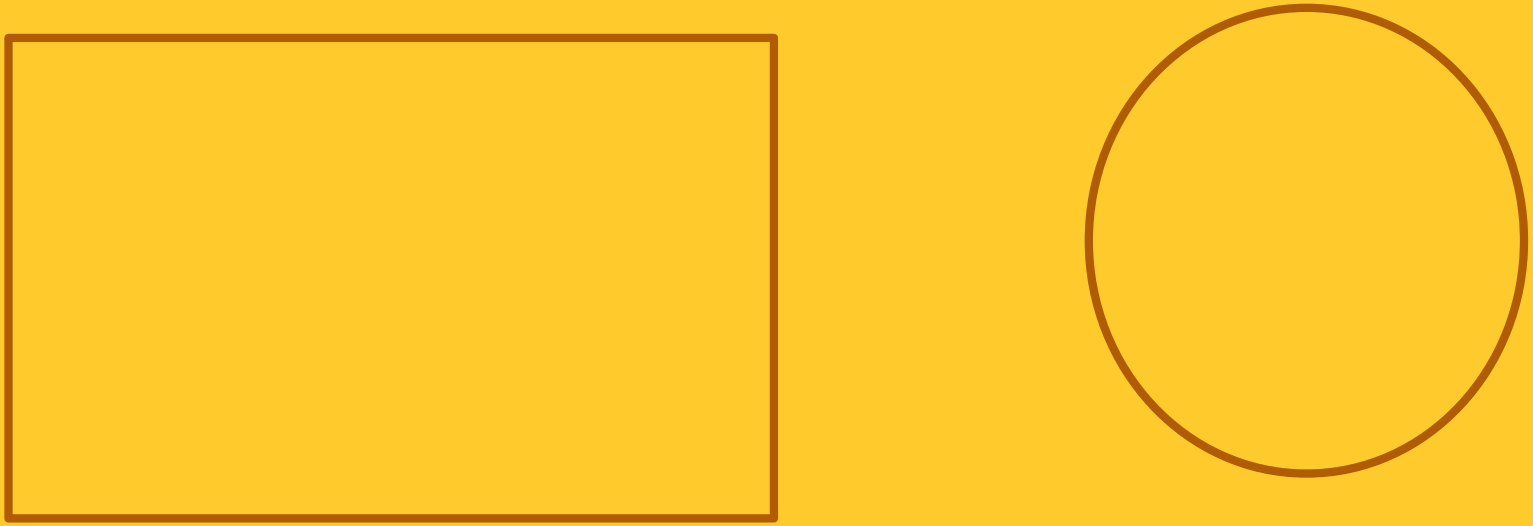
Type of Sets

1. Empty set
2. Finite and Infinite Set
3. Equal Set
4. Sub sets
5. Power set
6. Universal Set
7. Complimentary Set

Venn Diagram

Venn diagram are used to represent relationship between sets

Universal sets in general a rectangle and sub sets are circles.



Operations of Sets

1. Union: {1,2, 5, 7} U {5, 7, 9, 11}={1,2, 5, 7, 9, 11}

2. Intersection: {1,2, 5, 7} I {5, 7, 9, 11}={ 5, 7}

3. Difference: {1,2, 5, 7} D {5, 7, 9, 11}={1,2}

Function of Single Variables

Explicit Equations: $y = 7x^4 + 5x^3 + 2x^2 + 3x \quad + 4$

This is expressed as: y=[7 , 5, 2, 3, 4]

P1=polyval(y,2) >>170

r=roots(y)

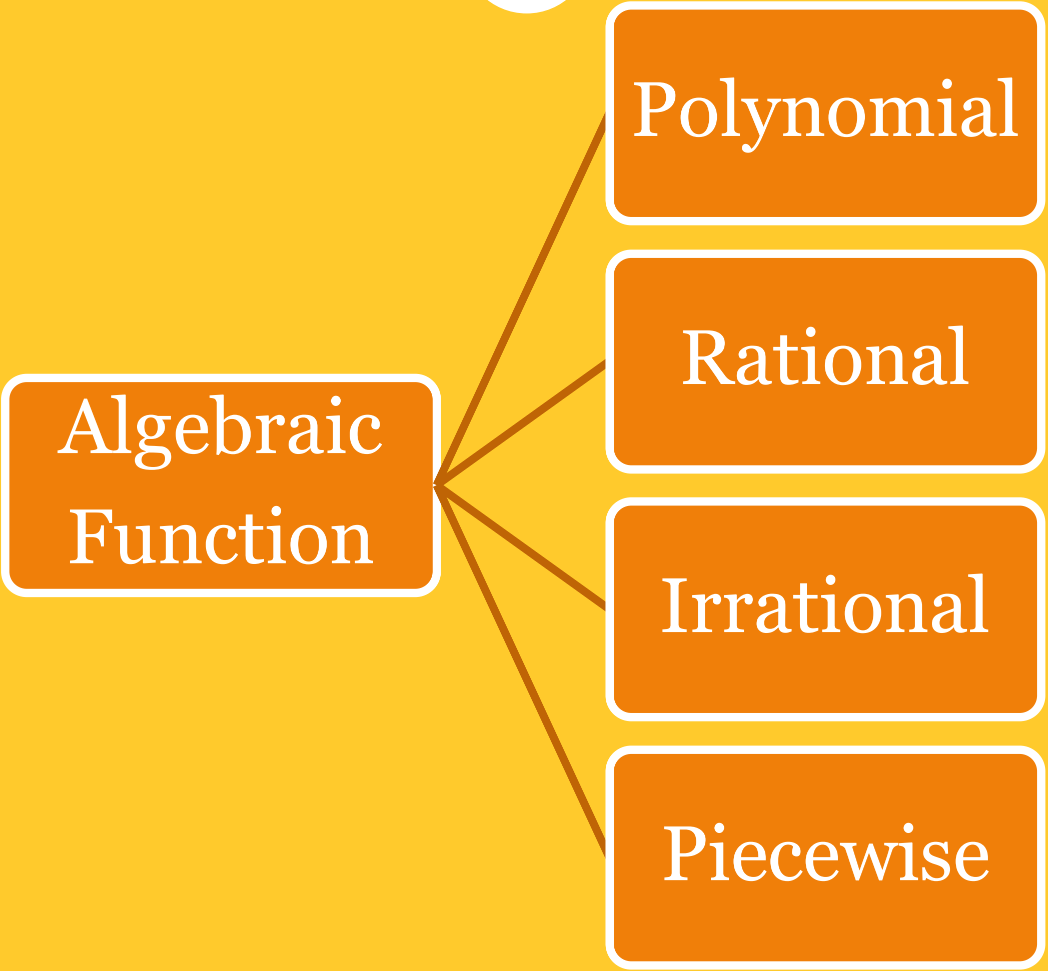
-0.7610 + 0.4761i
-0.7610 - 0.4761i
0.4038 + 0.7390i
0.4038 - 0.7390i

FUNTOOL

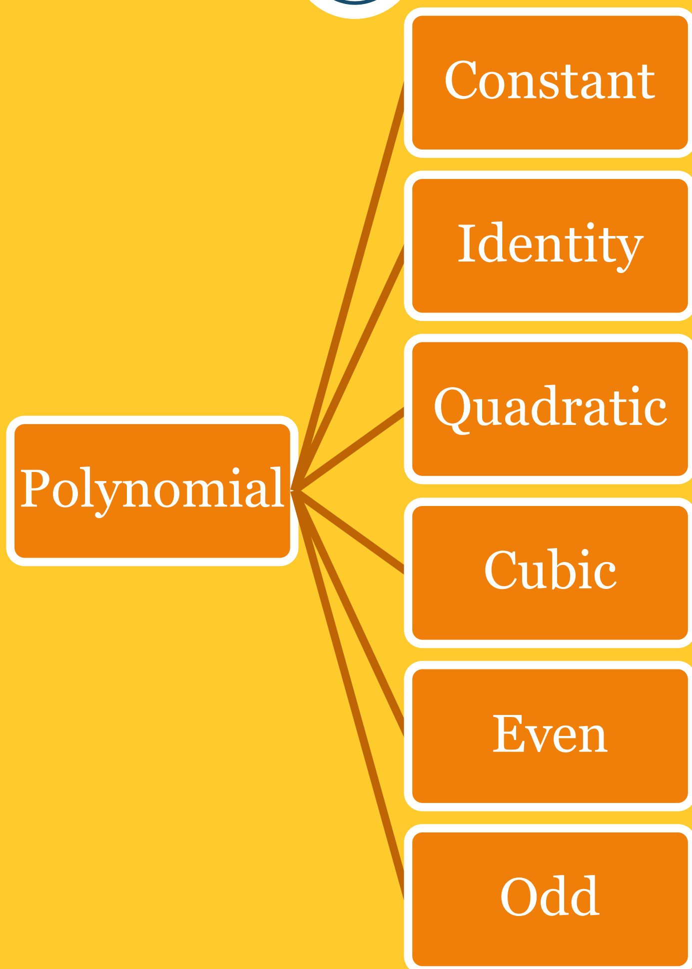
Different Types of Functions

- Function
  - Algebraic
  - Transcendental

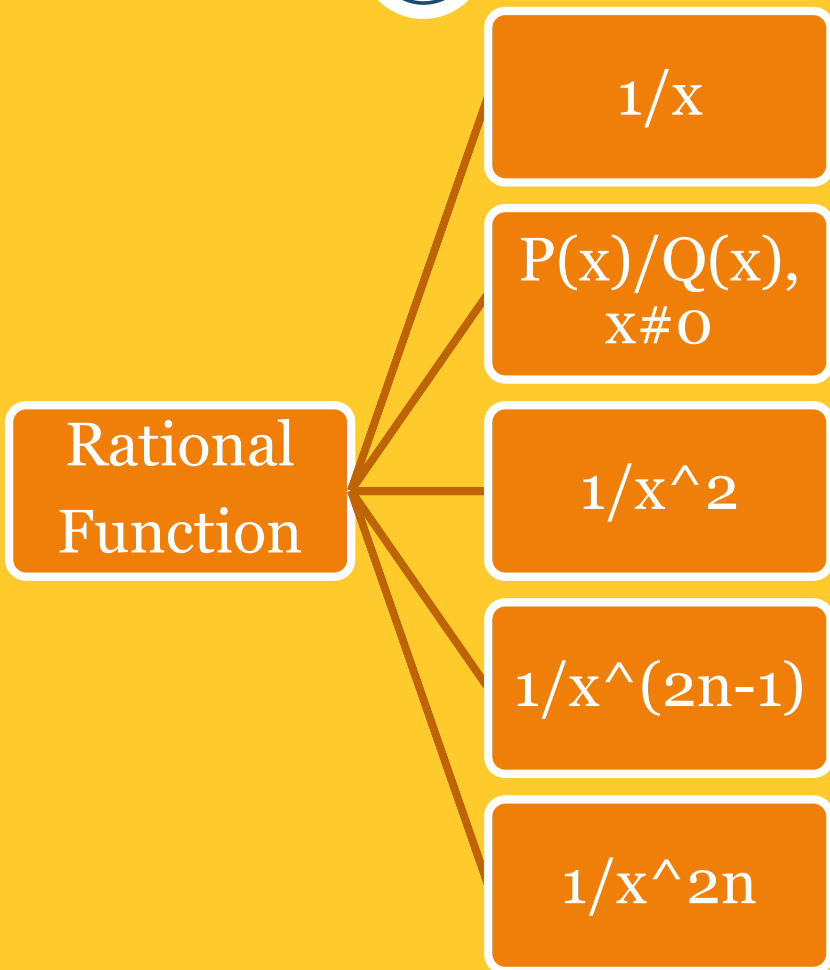
Different Types of Algebraic Functions



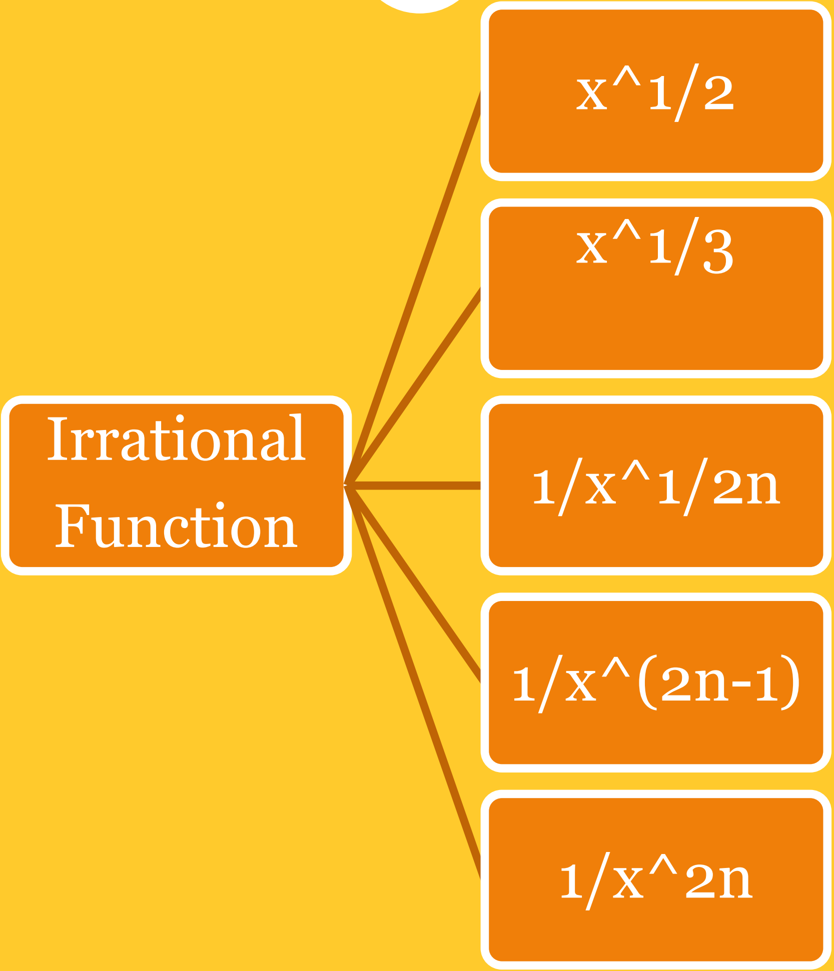
Different Types of Polynomial Functions



Different Types of Rational Functions



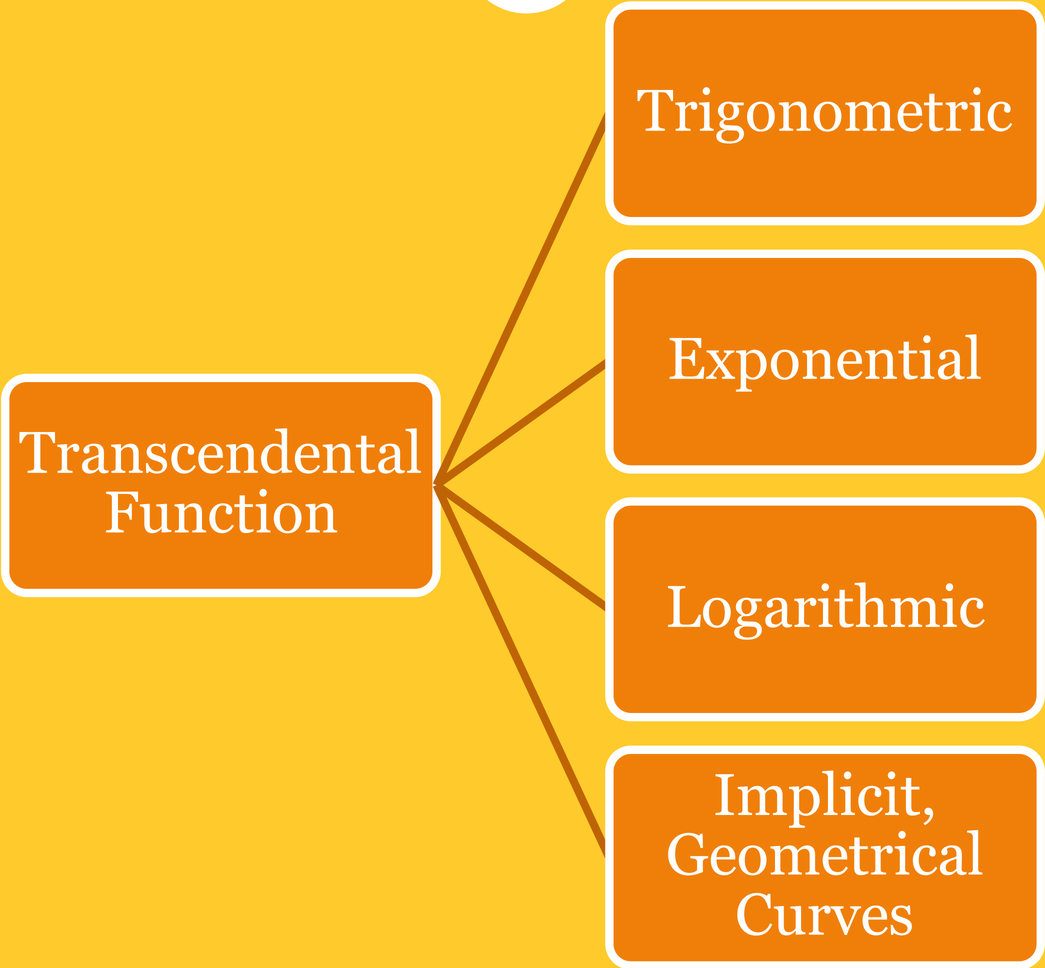
Different Types of Irrational Functions



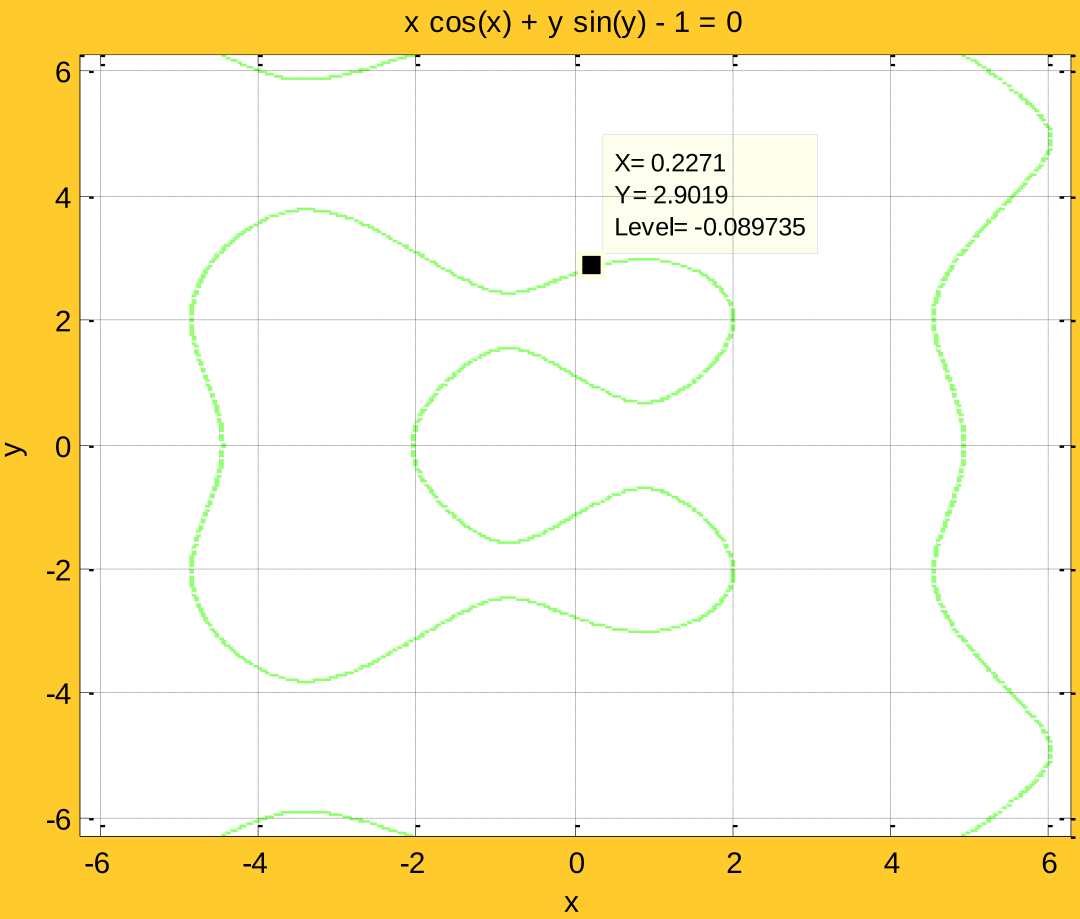
Different Types of Piecewise Functions

- Piecewise Function
  - Modulus or Absolute Value
  - Signum
  - Greatest Ineger
  - Fractional Part
  - Least Integer

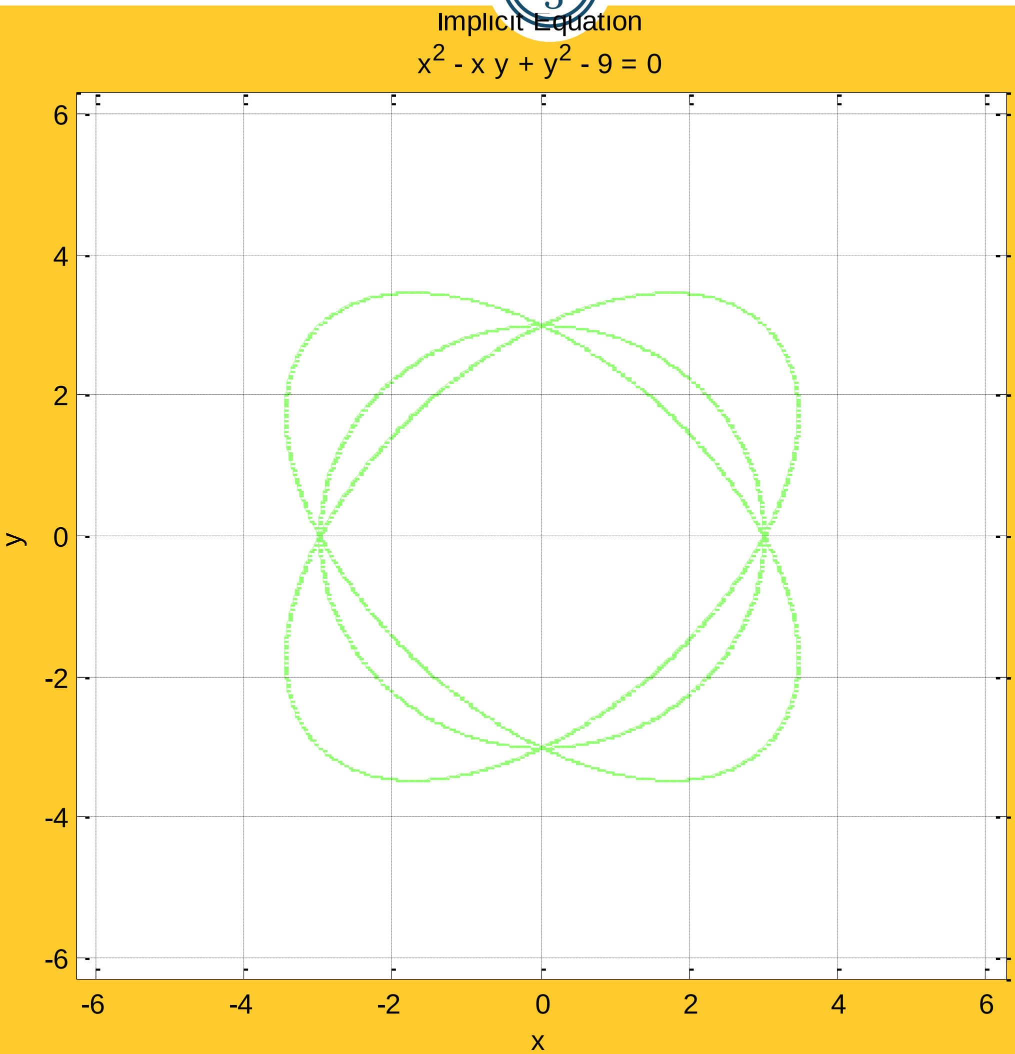
Different Types of Transcendental Functions



Implicit Function



Implicit Function
order of dependency not clear



Polynomials in Matlab

Polynomial: $y = 7x^4 + 5x^3 + 2x^2 + 3x + 4$

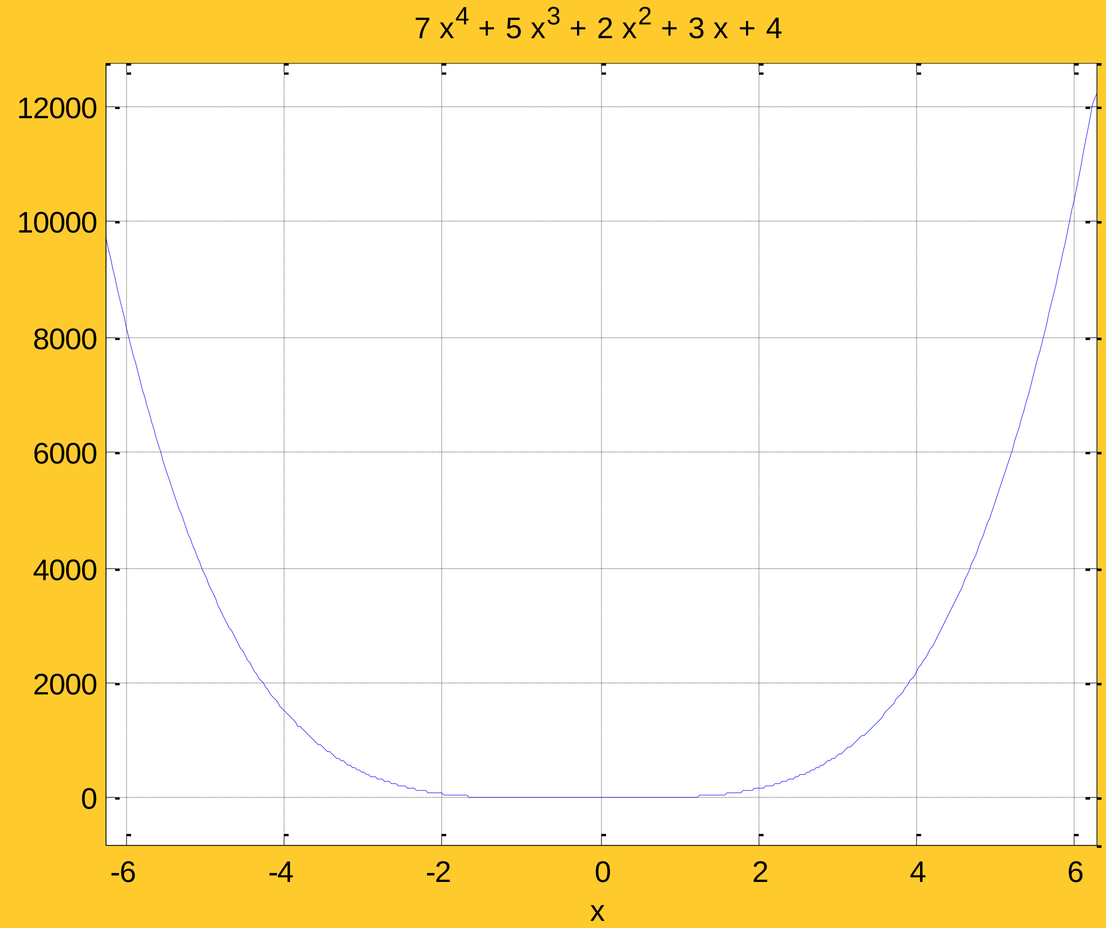
This is expressed as: y=[7 , 5, 2, 3, 4]

P1=polyval(y,2) >>170

r=roots(y)

-0.7610 + 0.4761i
-0.7610 - 0.4761i
0.4038 + 0.7390i
0.4038 - 0.7390i

Explicit Equation



Notes:
1. Shapes
2. Even /Odd
2. Roots and Sign Change

Matlab Commands for Polynomial

1. Finding the Polynomial when roots are given:

r=[3 5 6 7]

p=poly(r)

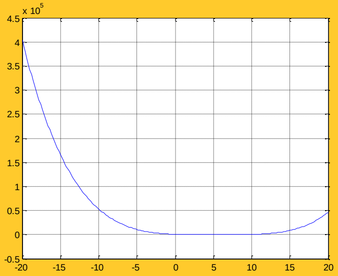
p = 1 -21 161 -531 630

2. Plotting the Polynomial

x=-20:.1:20

y=polyval(p,x)

Plot(x,y)



Operations in Polynomials

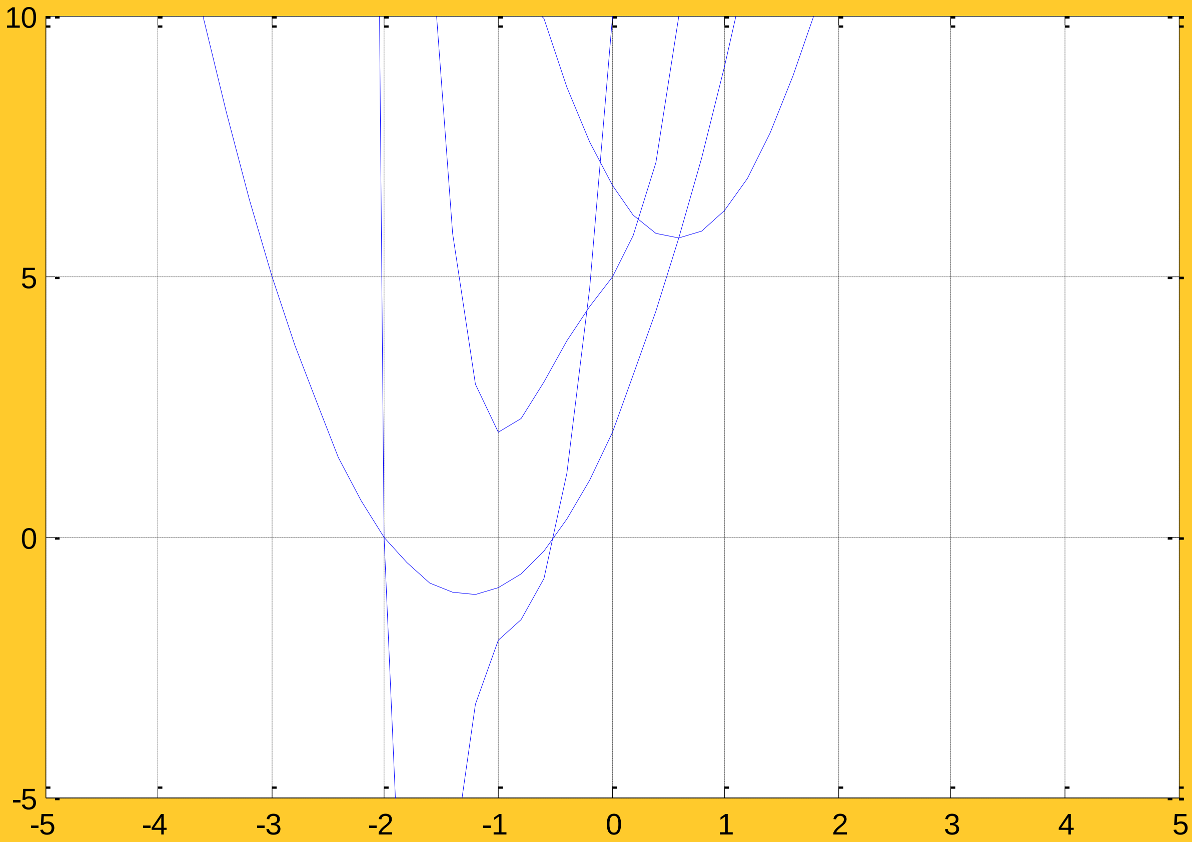
Addition of Polynomial (Requires Padding)

- p1=[6 8 2 3 5]
- p2=[2 5 2]
- p=p1+[0 0 p2]
- Result:
- p = 6 8 4 8 7
- p=p1-[0 0 p2]
- p = 6 8 0 -2 3

Operations in Polynomials

- Multiplication of Polynomial
- c=conv(p1,p2)
- c = 12 46 56 32 29 31 10
- Division of Polynomial
- d=deconv(p1,p2)
- [d,r]=deconv(p1,p2)
- d = 3.0000 -3.5000 6.7500
- r = 0 0 0 -23.7500 -8.5000

Polynomial Graphs of Different Functions



Geometrical Representation of Math…

- All these lines are formed with points.
- Math is finding a point (x, y) in line (space).
- More specifically, math is finding an element of a point (y) when other element (x) is given/known/free to assume.
- Conclusion: If we have control in every point in space, we can solve any problem, construct any figure.
- Engineers and scientists work on a dynamic world, so they should be equipped with tools to handle static and dynamic world situations.

•NOTE: Point can be an isolated point or a part of a line or plane or solid object.

Analytical Geometry: Point, Line and Equation

From M VYGODSY:

Equation, $y=mx+c$

Any point whose coordinate (x , y) satisfy the equation will lie on the same line.

Conversely for any point lying on line, the coordinates satisfy the equation.

By representing each point in the plane by its coordinates and each line by an equation that relates the running coordinates, we reduce geometrical problem into analytical problems. This is the basis of analytical geometry.

Old Age vs New Age

- When analytical Geometry developed, then there was no gadget for graphing and matrix method was not developed.

- Now we have easy graphing software and matrix methods are highly developed.

- We should take advantages of these two advancements for making math easy.

Equation and Matrix Relation

- Equation of a line:

5x+3y=9   can be expressed as [x y]*[5 ; 3]=9

- Linear Equations:

2x+5y=7

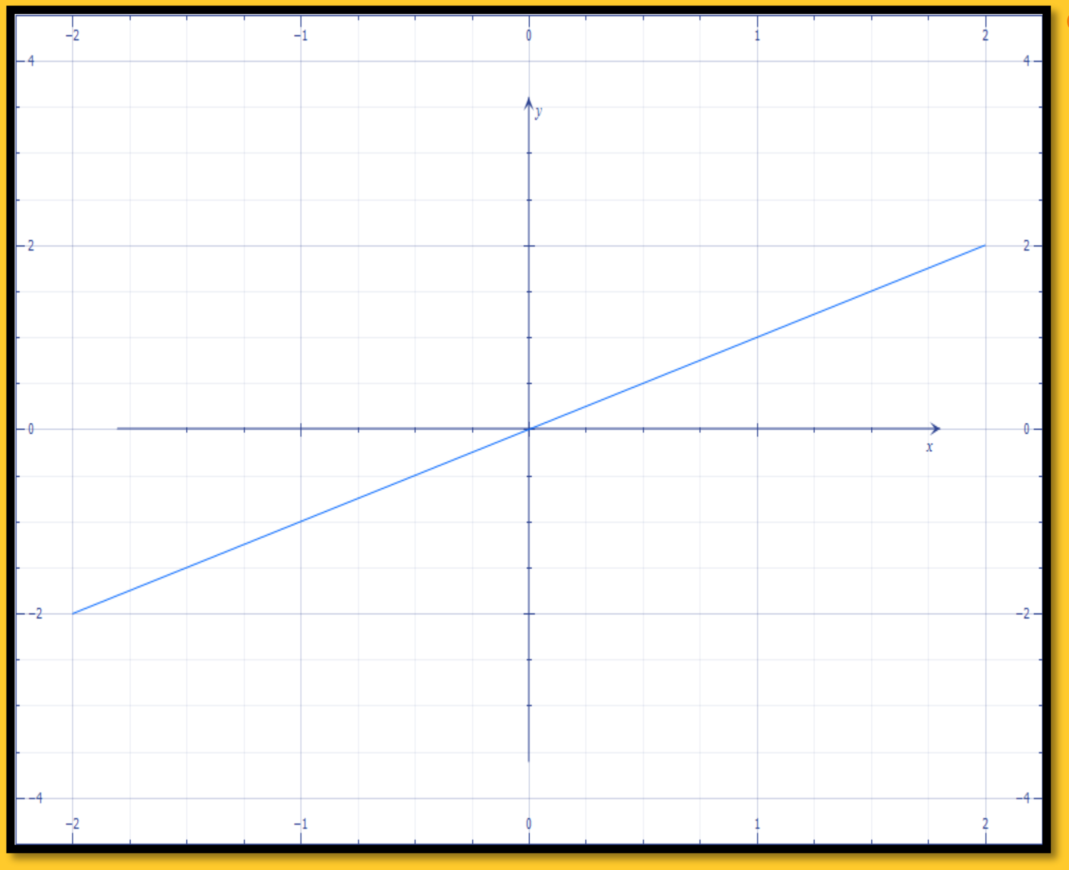
4x+9y=3

[x y]*[2 4; 5 9] = [7 3]

- Quadratic Equation:

[x y 1][1 2 3 ; 4 5 6; 1 2 4][x y 1]=0

How to Construct/Draw a Conic Section-
Explicit and Implicit Functions



- Few examples of explicit Functions are given below:

1. y=x

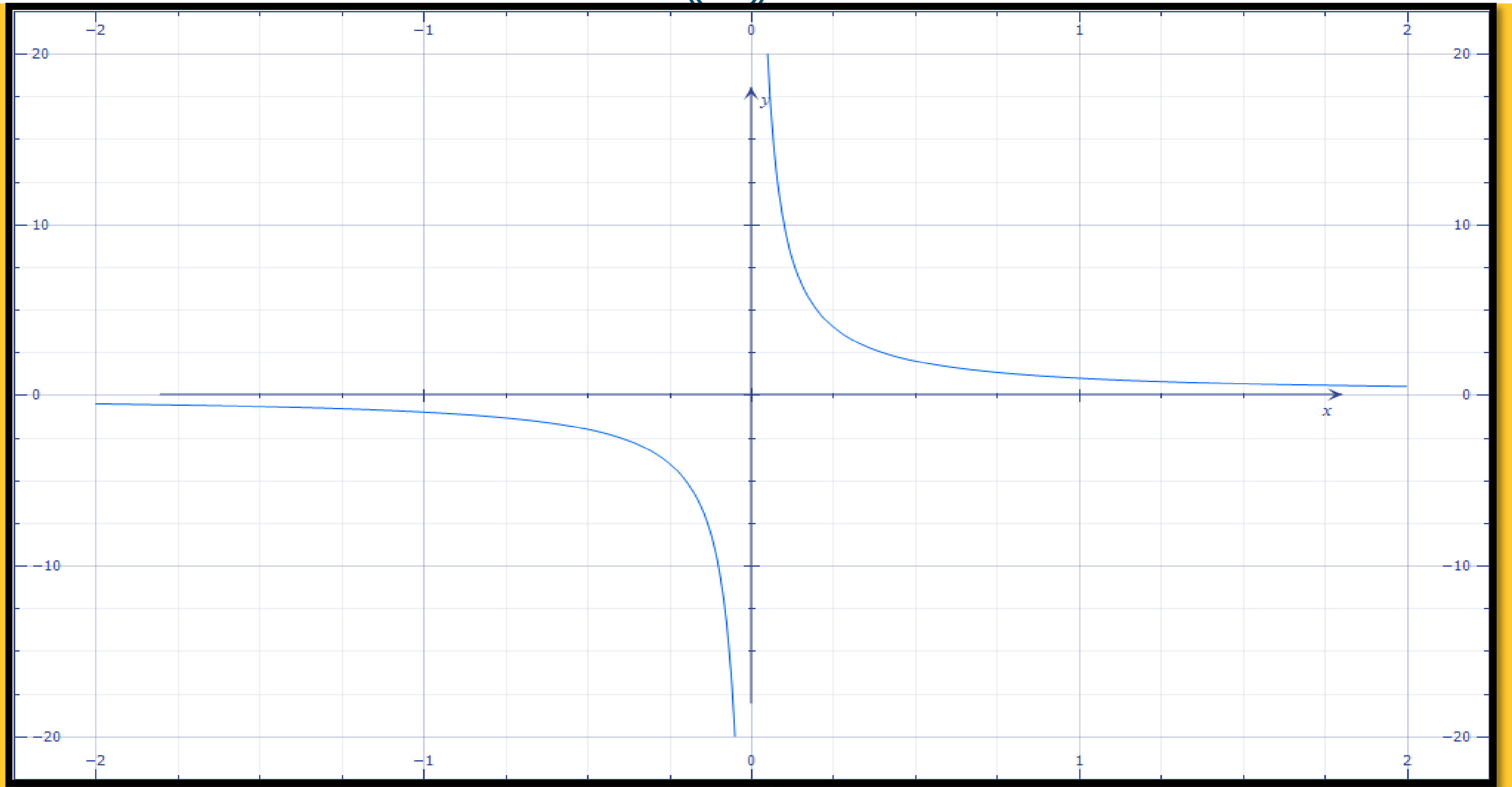
   y=Dependent variable

   x= Independent variable

Condition:

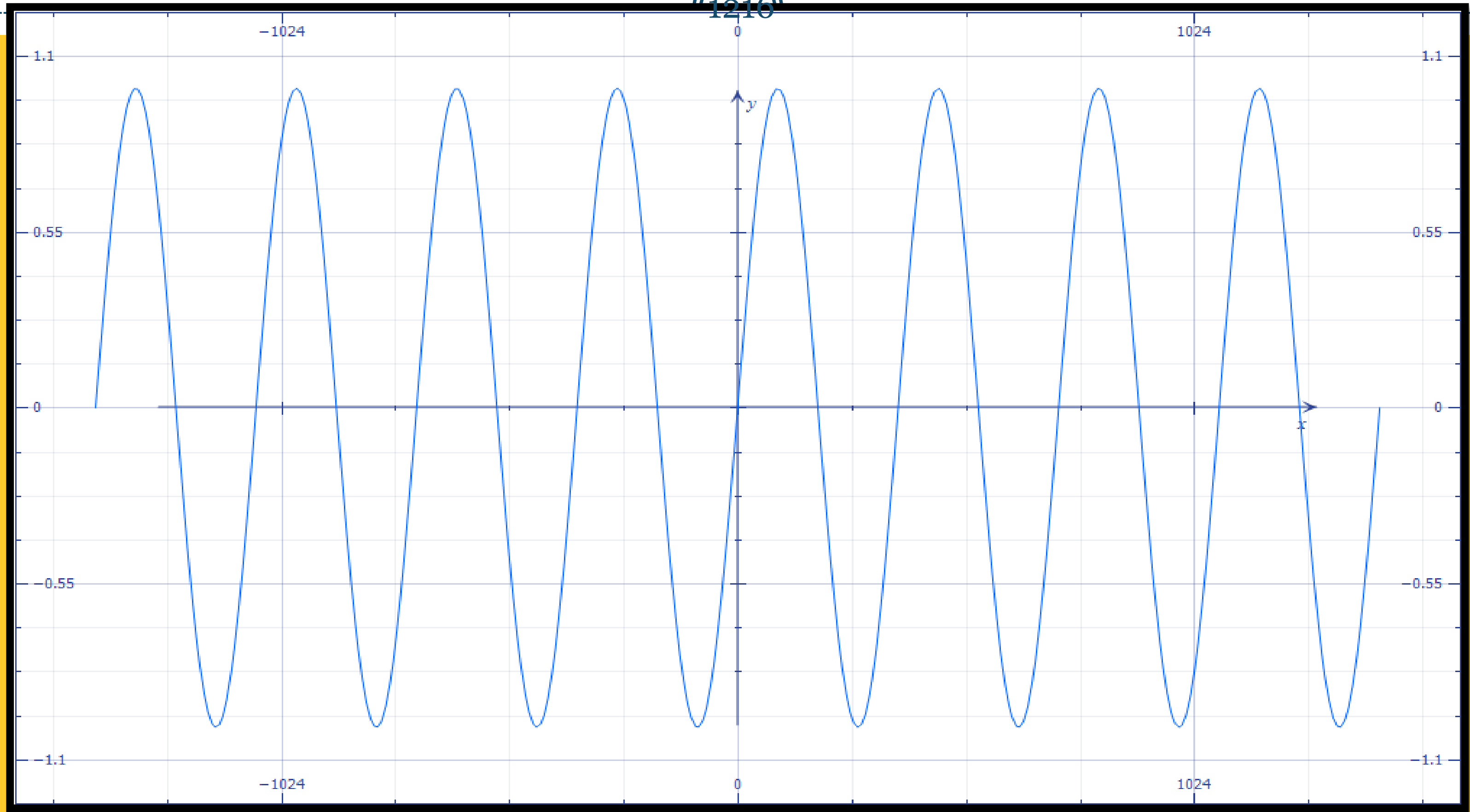
For each x there should be one and only one y

Implicit Functions



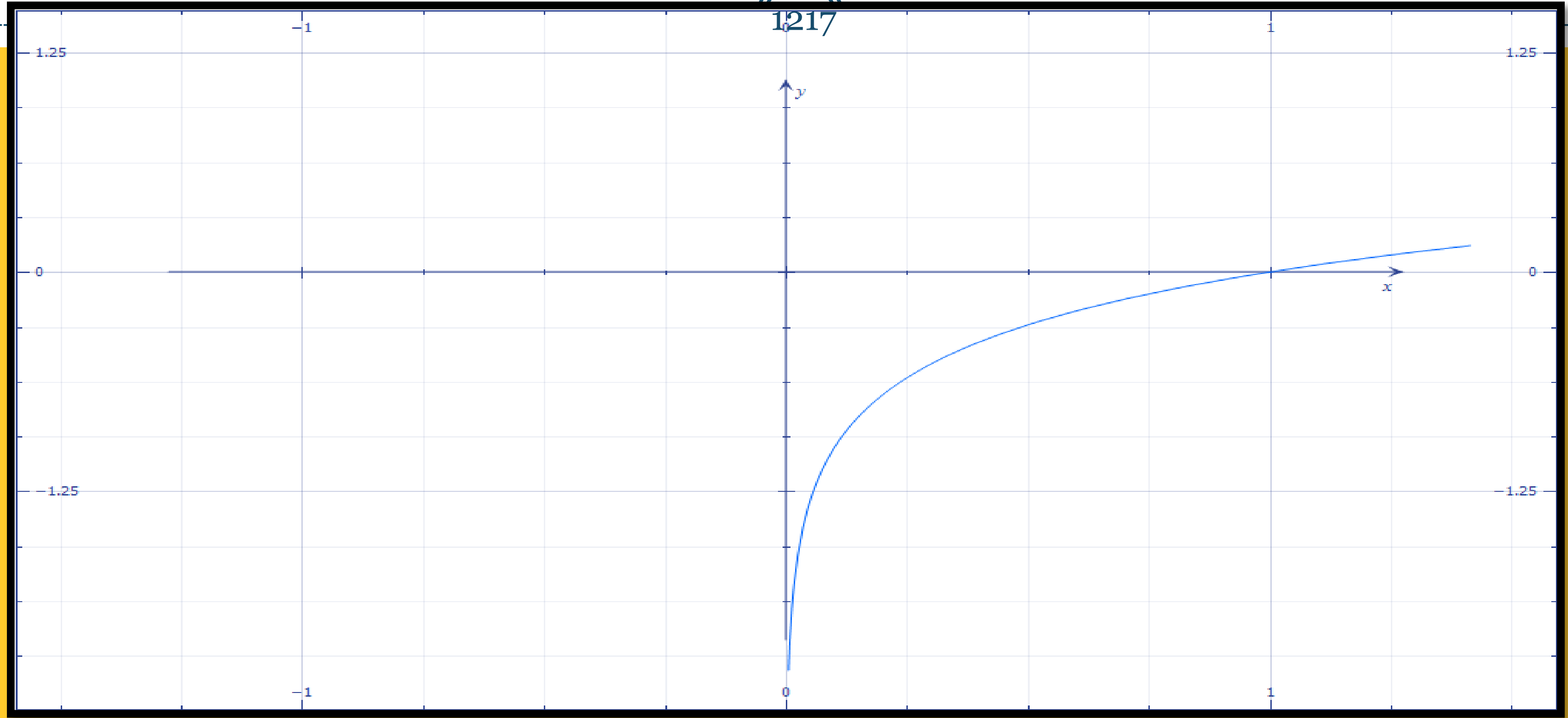
$y=1/x$

Explicit Functions



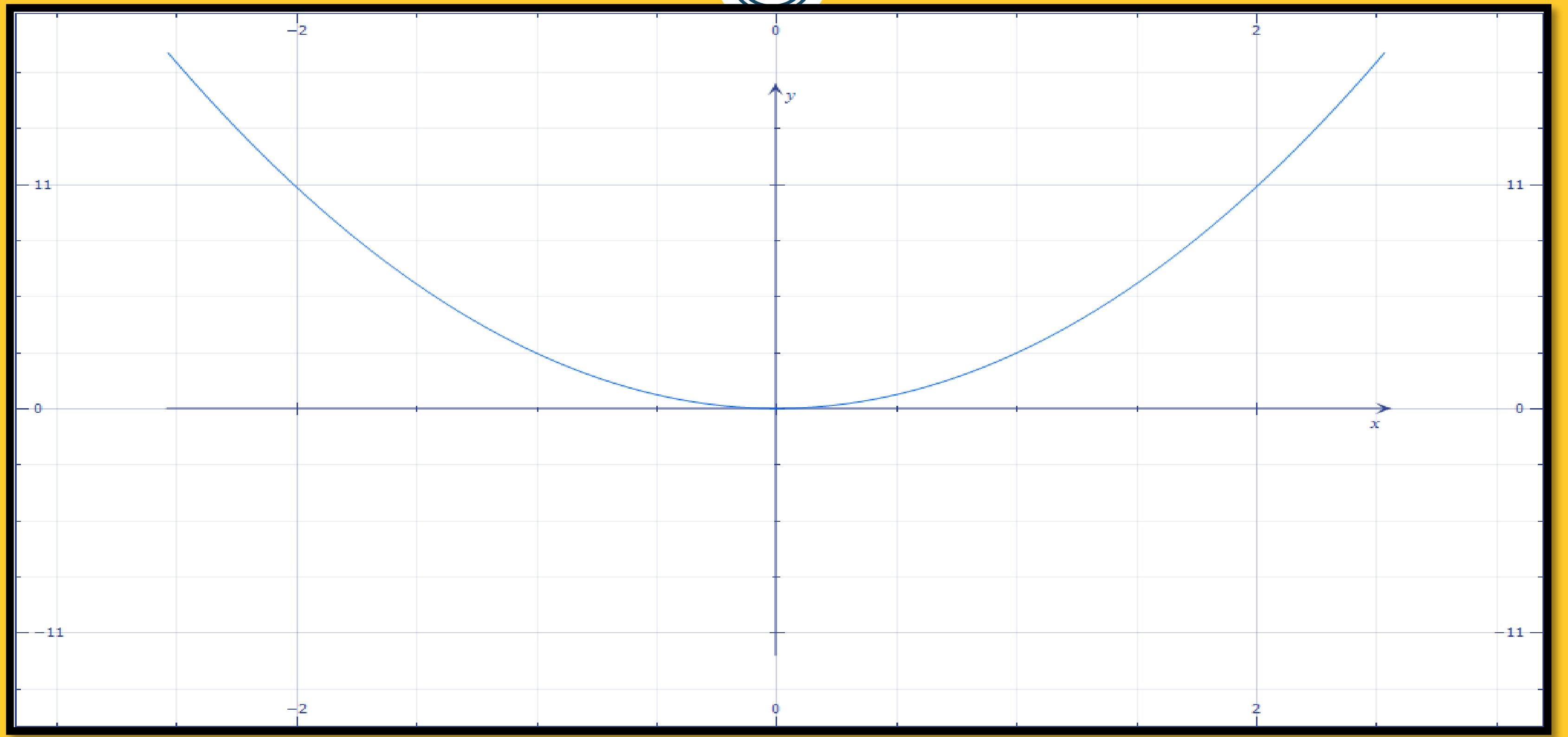
y=sin(x)

Transcendental Functions



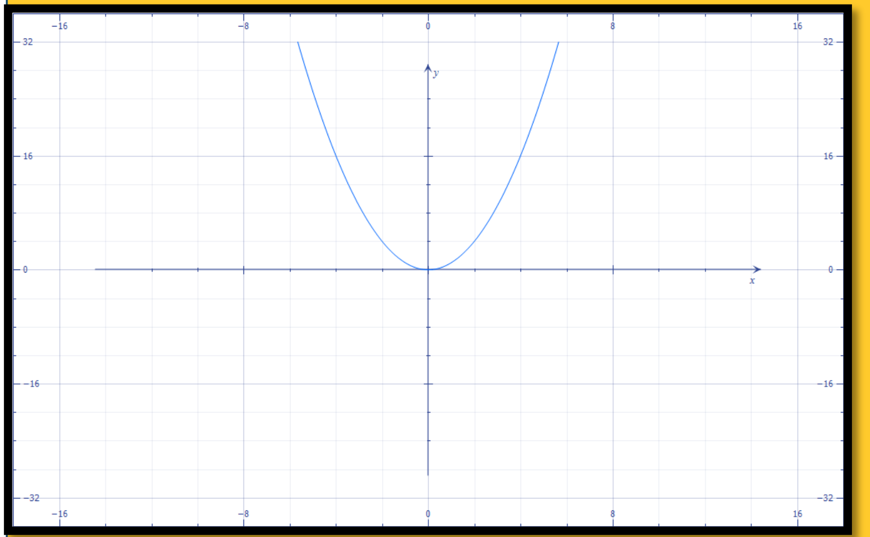
y=log(x)

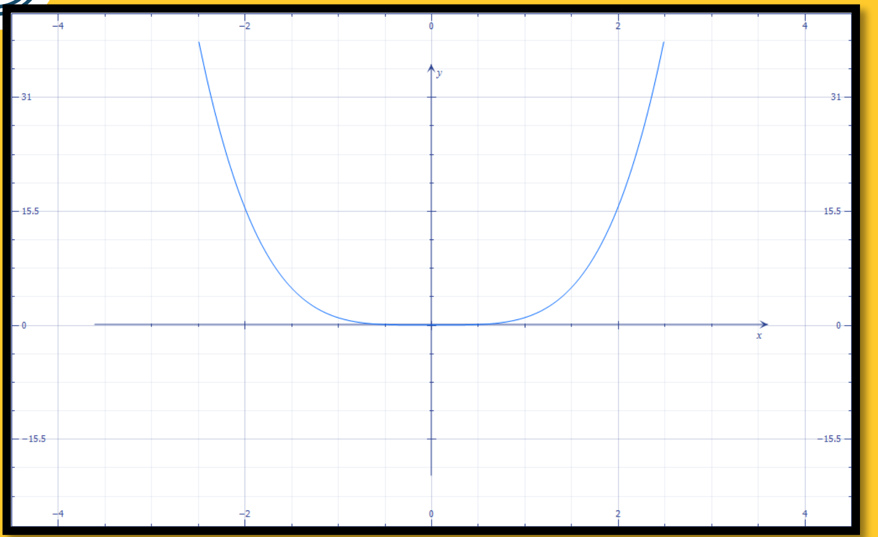
Explicit Functions



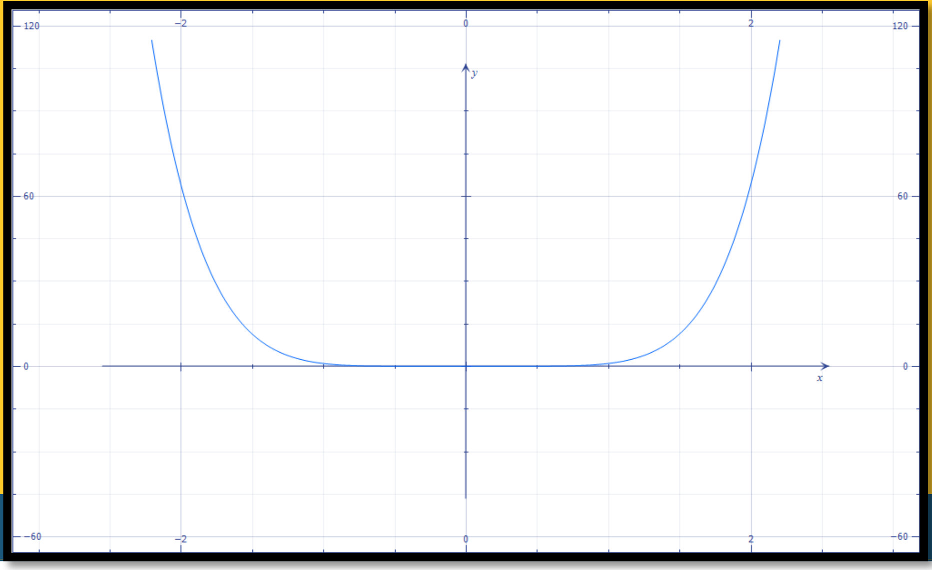
y=x^2

Even Functions- Compare and Observe !!!





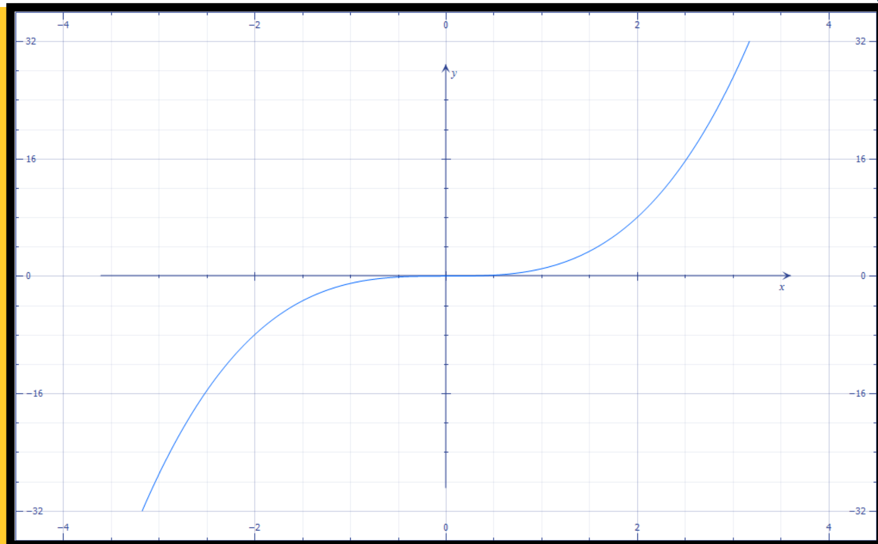
$y=x^2$

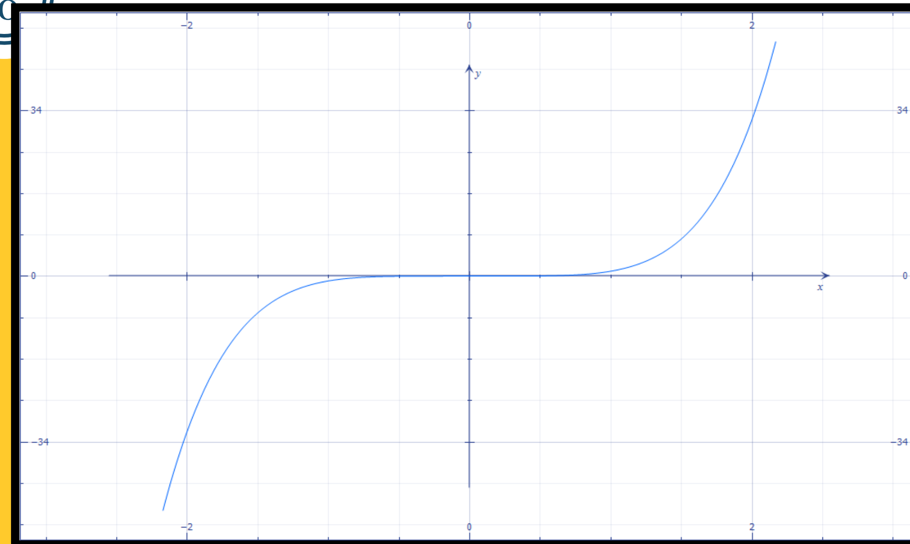


$y=x^4$

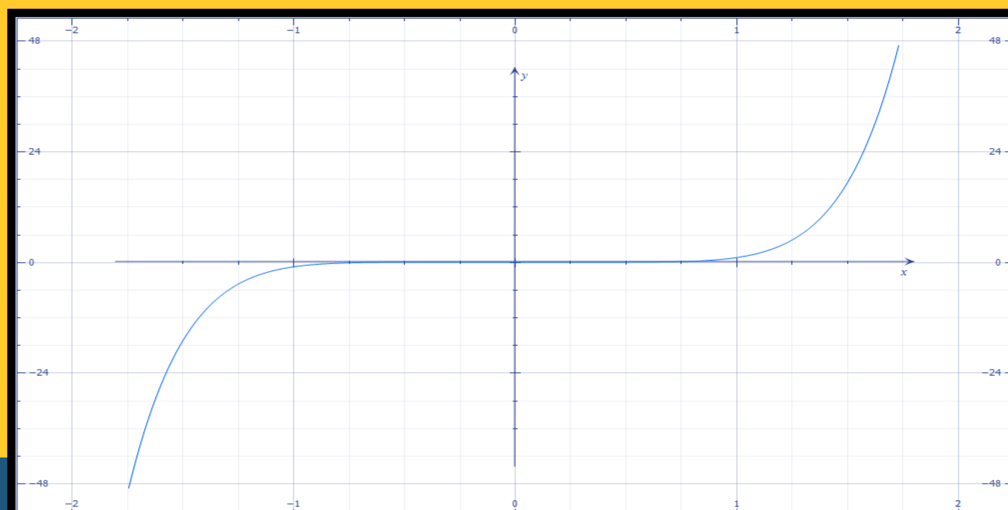
$y=x^6$

Odd Functions- Compare and Observe !!!





$y=x^3$

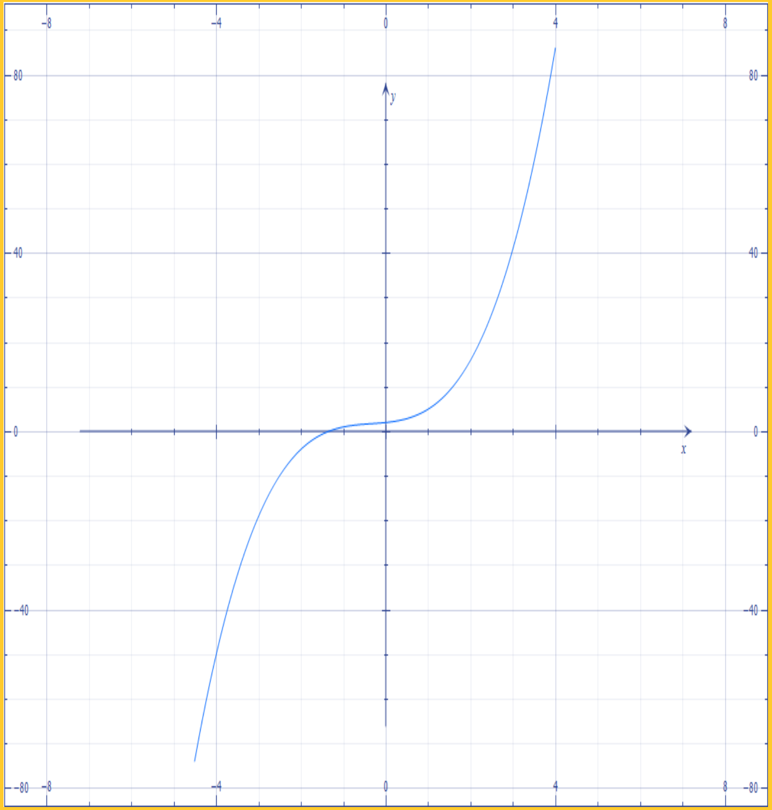


$y=x^5$

$y=x^7$

Explicit Functions

- All these graphs are examples of explicit equation as the dependent variable(y) can be expressed as a function of independent variable (x).
- These graphs and its shapes can be transformed by changing its coefficients- Examples as shown below:



$$y = x^3 + x^2 + x + c$$

Shifting the Curve in x-direction

- Can be shifted in x-direction by adding a constant to x.
- New Equation-> ynew=(x-+a)3+(x+-a)2+(x+-a)+c

Shifting the curve in y-direction

- The constant a is to be added to y-axis
- Ynew=(y+-a)

For stretching the graph in x-direction

- Can be achieved by multiplying ‘a’ with ‘y’

For compressing the graph in x direction

- Can be achieved by multiplying ‘a’ with ‘x’

For Reflection of Curve about x-axis

- Y is multiplied by -1 and for reflection of the curve about the y-axis, x is to be multiplied by -1

Implicit Equations

- Explicit equations give a formula to calculate y from x.
- Many functions in which x and y cannot be separated are called implicit functions.
- Implicit functions defines shapes through all values of x and y that satisfy the equation.
- Conic sections (circle, ellipse, parabola, hyperbola) are examples of implicit functions. (polynomial equations with power 2)

$$x^2 + y^2 = c^2$$

$$y = \pm\sqrt{c^2 - x^2}$$

Implicit Functions

$$x^2 + y^2 = c^2 \qquad y = \pm\sqrt{c^2 - x^2}$$

- In this case, there are two values of 'y' for a single value of 'x' which shows that the graph cannot be expressed as an explicit formula y=f(x).
- For these reasons, implicit equations cannot be drawn easily by tracking x.
- If we still want to draw a circle explicitly, we can first draw for a half circle for '+y' values corresponding to x and then draw another graph with '-y' values corresponding to same x values.
- Implicit equations can be drawn easily parametrically.

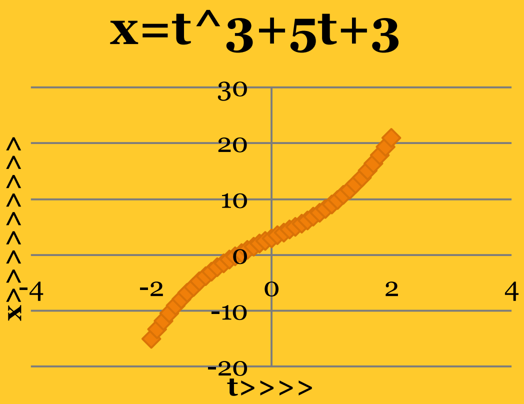
Implicit Functions

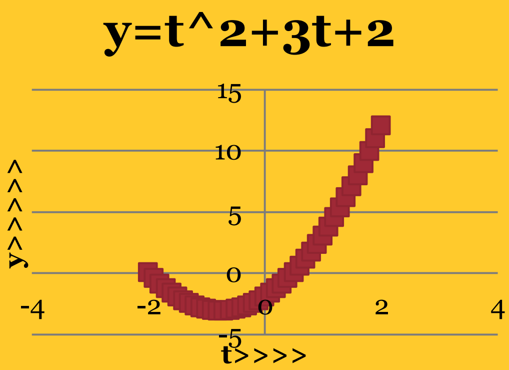
$$x^2 + y^2 = c^2 \qquad y = \pm\sqrt{c^2 - x^2}$$

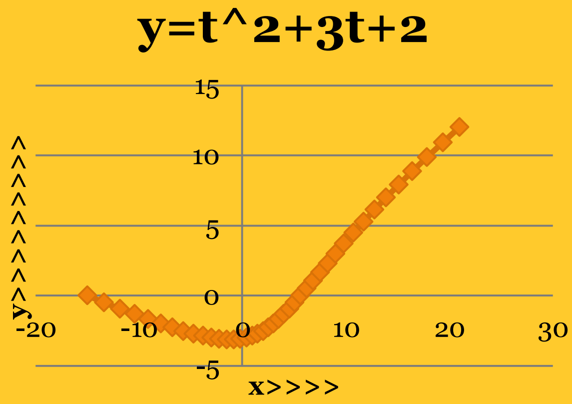
- The graph is first drawn for positive values of y and then negative values of y.
- This curves are not smoth.
- Parametric equations can help in this situations.

Parametric Equation

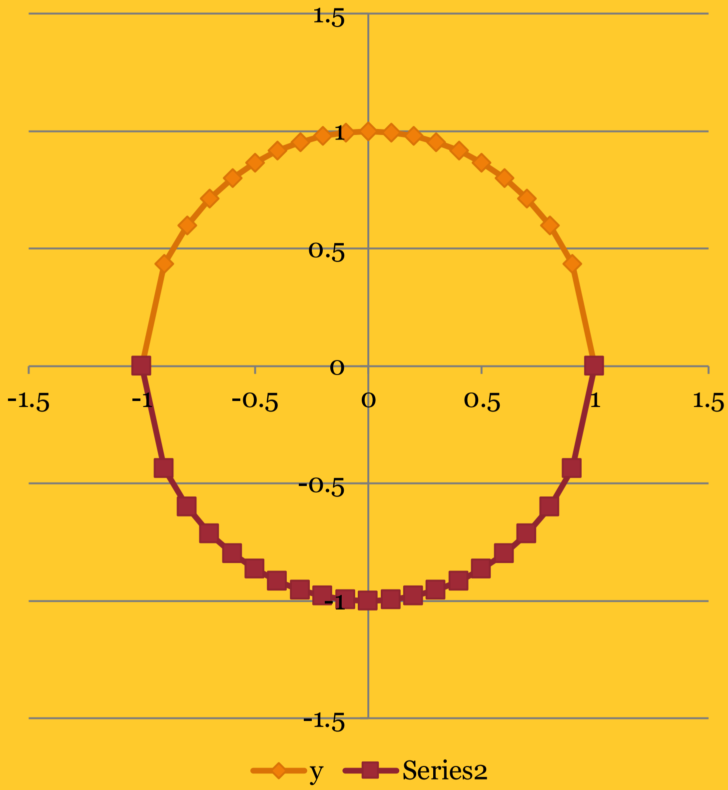
- The Cartesian coordinates (x,y) are dependent on a common parameter, t.
- Let, x=t^2+5t+3
- And y=t^2+9t+2
- cdass parametric equation 25012015.xlsx

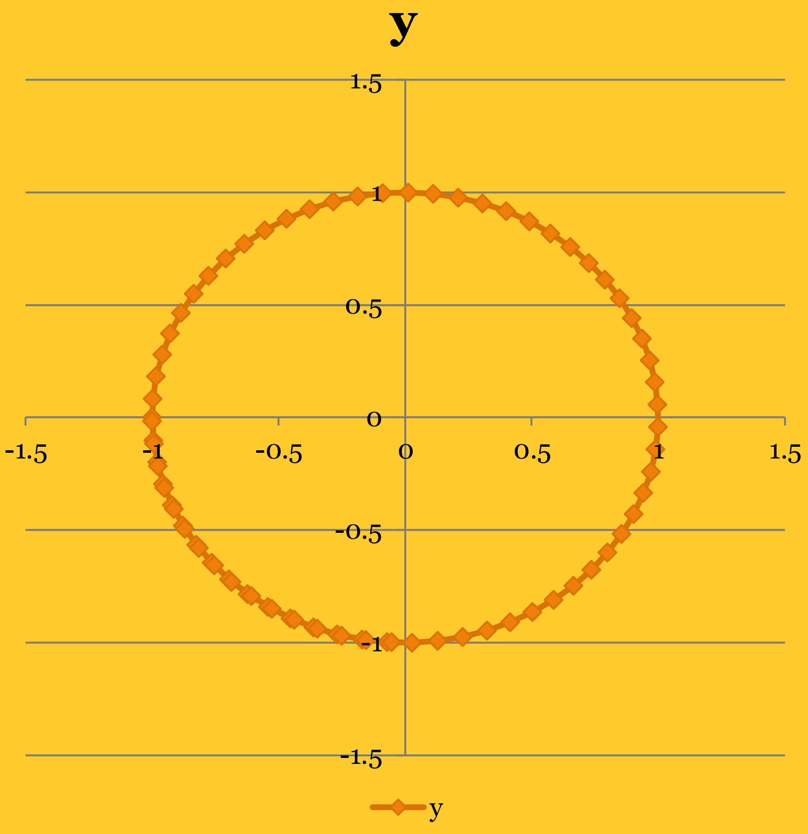




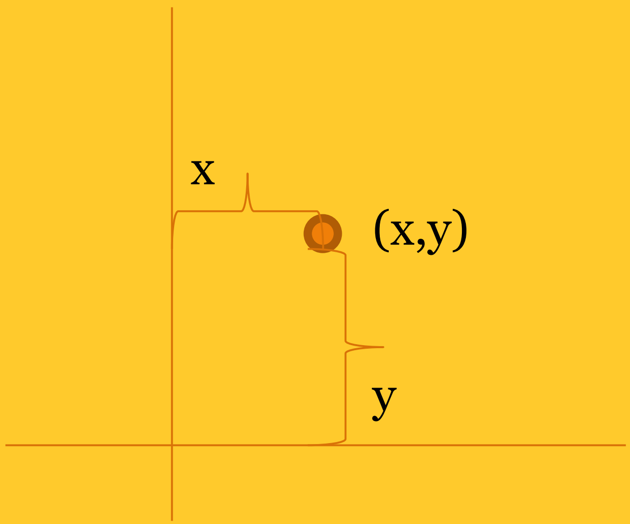


Implicit and Parametric Construction

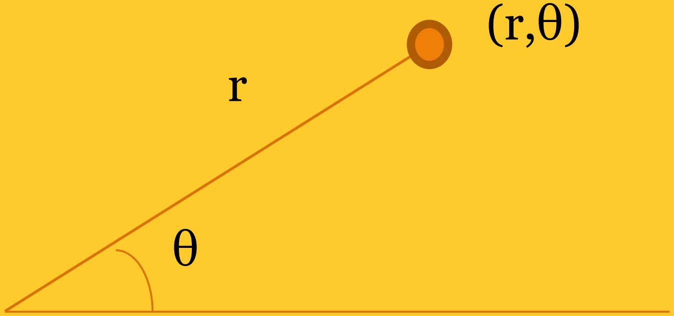




Polar Equations



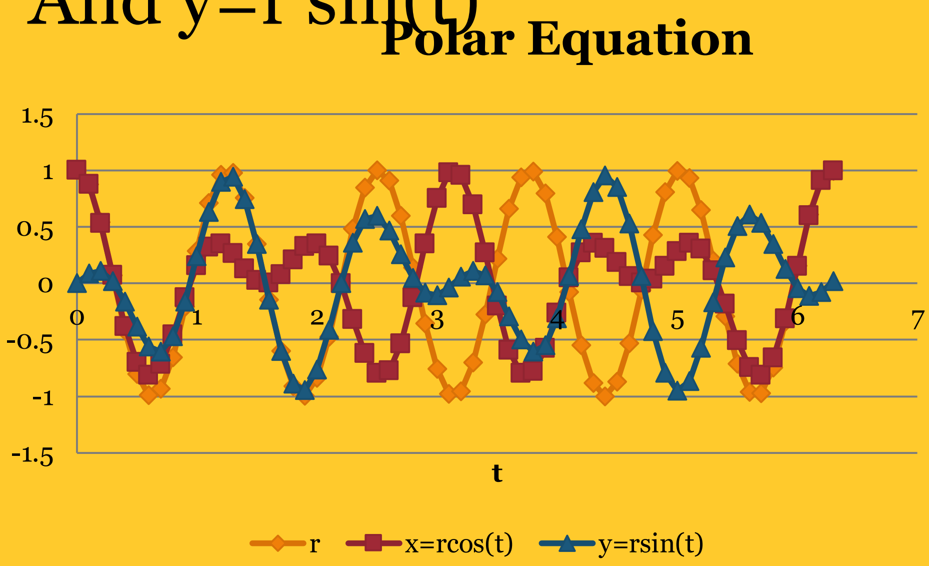
Cartesian Coordinates

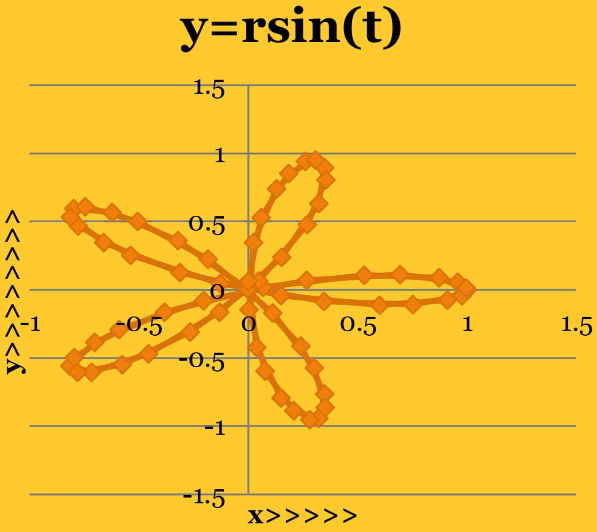


Polar Coordinates

Polar Equation

- The Cartesian coordinates (x,y) are dependent on a common parameter, r, t.
- r=sin5t
- Let, x=r cos(t)
- And y=r sin(t)





Laws of Symmetry

- Symmetry about x-axis:

Putting (r, -t) for (r,t) gives same equation

- Symmetry about y-axis:

Putting (r,pi-t) for (r, t) gives same equation

Symmetry about origin:

- Putting (-r, t) or (r, t+pi) for (r,t) gives same equation.

Symmetry of circle

- CIRCLE:

  Symmetric with origin: r=a, x=r*cost, y=r*sin(t)

  Symmetric with x-axis: r=a*cos(t)

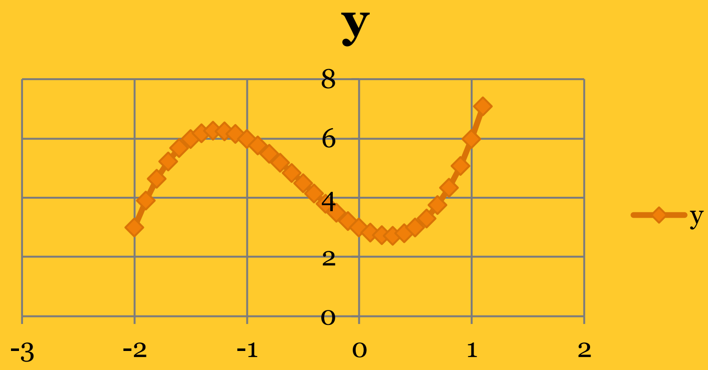
  Symmetry with y-axis: r=a*sin(t)

Few Polar Graphs

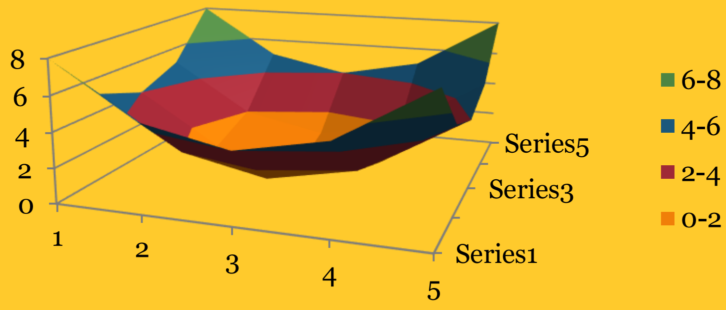
- Limancons: r=a+b*cos(t), r=a+b*sin(t)
- Cardoids: r=a+b*cos(t), a=b
- Rose or Petal curves: r=a*cos(n*t), if n is odd, then number of petal = n, if n=even, then petal = 2n
- Lemniscates: r^2=a^2*cos(2*t)
- Archimedes Spiral: r=k*t

Function to Two Variable

- Function of Single Variable represents a line, y=f(x)
- cdass function of two variable.xlsx



- Function of Two Variable represents a surface, z=f(x,y)



How to create a function of two Variable

- A function of two variable means it will have single range value and two domain value.

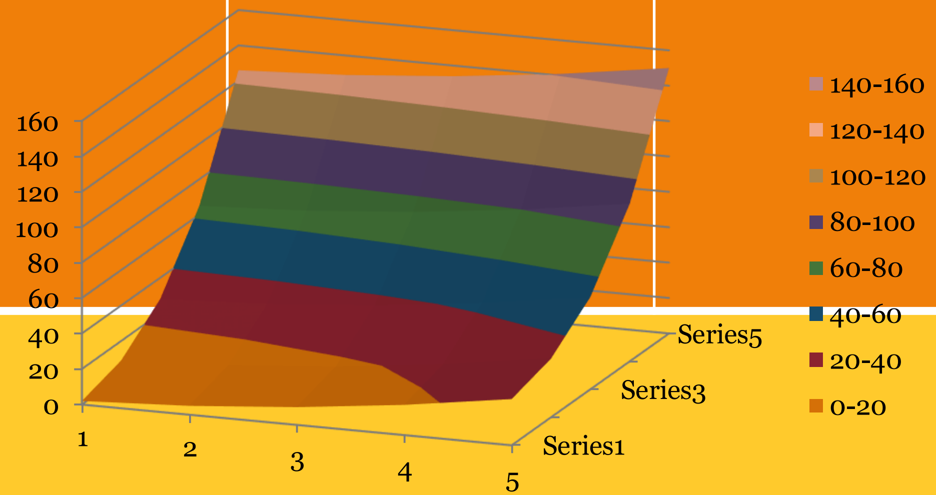
- Let, $z = x^2 + y^3$ is function of two variable. Here z is dependent on the values of x and y.
- To accommodate two domain, we are required to create a grid of two independent variables as shown in the next slide.

Function of Multiple Variable

- Function of multiple variable is simple extension of two variable case.
- Geometrical Interpretation or representation is difficult.

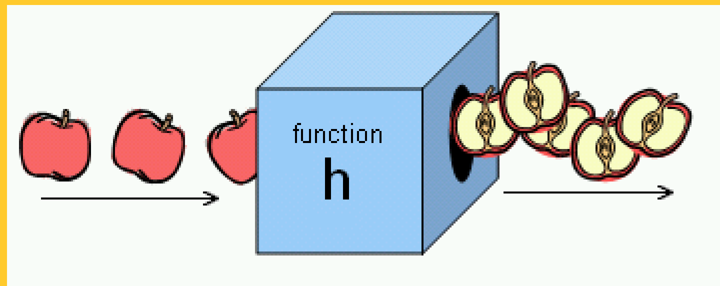
How to create a function of two Variable

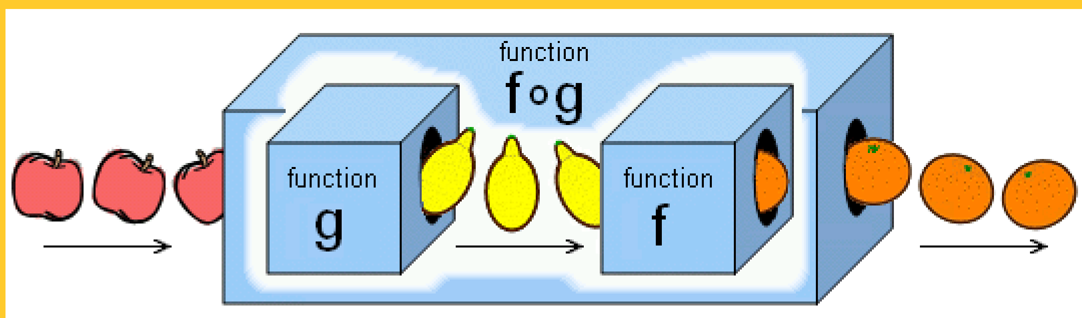
x	Y	z=x^2+y^3
1 2 3 4 5	1 1 1 1 1	2 5 10 17 26
1 2 3 4 5	2 2 2 2 2	9 12 17 24 33
1 2 3 4 5	3 3 3 3 3	28 31 36 43 52
1 2 3 4 5	4 4 4 4 4	65 68 73 80 89
1 2 3 4 5	5 5 5 5 5	126 129 134 141 150



Calculus

Function Operator Calculus





http://upload.wikimedia.org/wikibooks/en/d/d6/Composite Function_Box.PNG

DOMAIN, FUNCTION, SEQUENCE AND SERIES

>Any function can be represented by a series.

>y=x^2 can be represented by an AP series with a=1, d=2

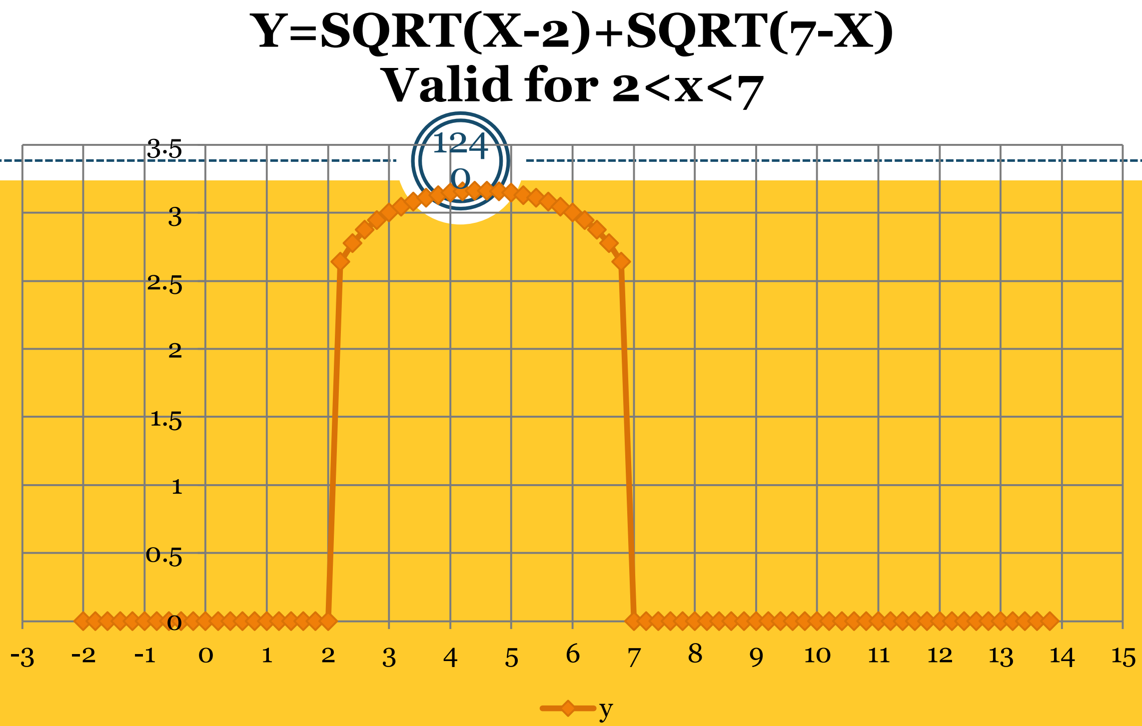
>General term of AP: a_n=2x-1

>But this Series is valid for the function with domain of only positive integers

d=2			
n	**a_n=a1+(n-1)*d**	**s_n**	**y=x^2**
1	**1**	**1**	**1**
2	**3**	**4**	**4**
3	**5**	**9**	**9**
4	**7**	**16**	**16**
5	**9**	**25**	**25**
6	**11**	**36**	**36**
7	**13**	**49**	**49**
8	**15**	**64**	**64**
9	**17**	**81**	**81**
10	**19**	**100**	**100**
11	**21**	**121**	**121**

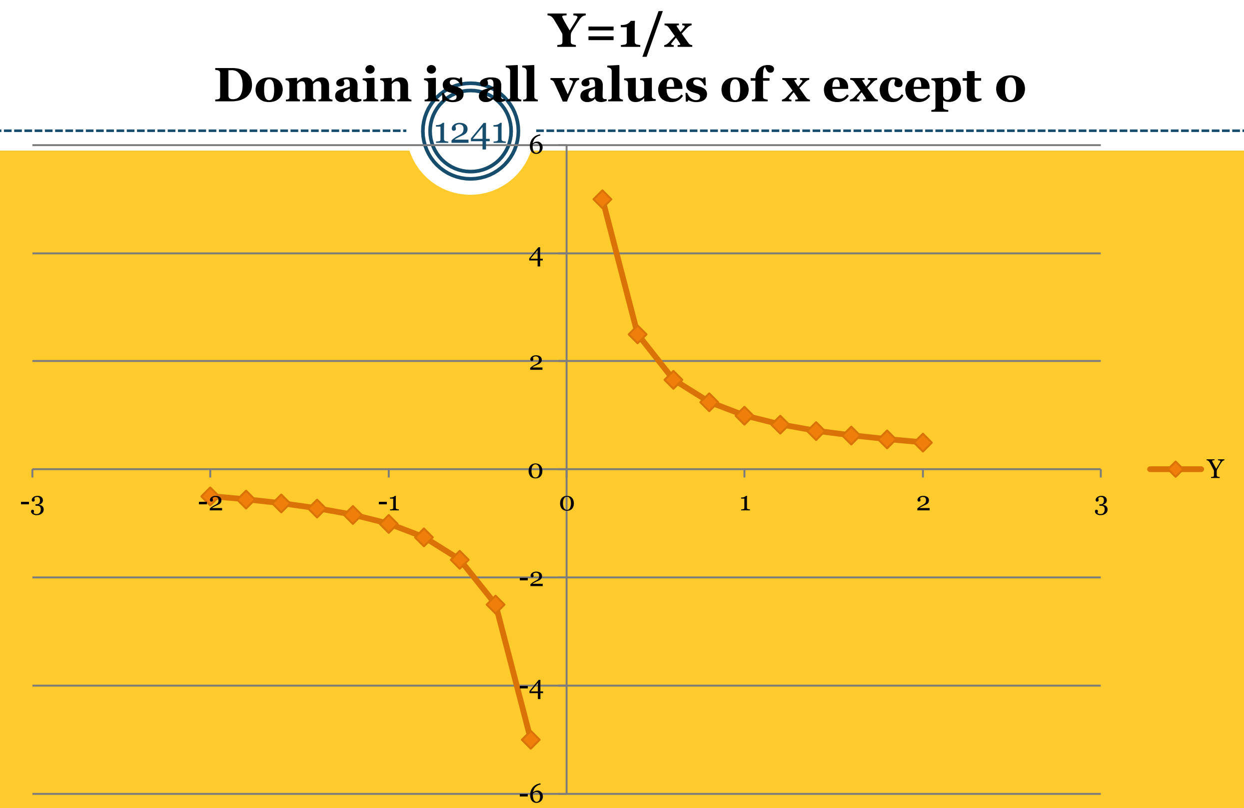
>**When a function is specified by a formula without any indication of domain, it can be assumed that the formula is valid for any value of the argument.**

The function is valid for domain 2 to 7



Excel file: cdass01 calculus series 11052014

The function y=1/x is not valid for x=0



Integral

>Integral: When the domain of a function is the collection of natural numbers, the function is called integral.

>Values of an integral function form a sequence or terms of a sequence

DIFFERENTIAL CALCULUS

By: Chanchal Dass

What is Differentiation?

- Differentiation is the process of calculating a derivative.
- The derivative of a function represents an infinitesimal change in the function with respect to other parameters.
- Simply it can be in terms of slopes or gradients.
- For all curves, the slope of the curve changes at each point.

Calculus

- What is the need of calculus
- The world is dynamic. Calculus is used to capture these dynamism.
- Mathematics is the language of Science*! Science deals with the growth, movements and changes. Calculus helps in these situations.

- What for we study calculus?

Calculus

- What are the topics of Calculus?

Topics of Calculus

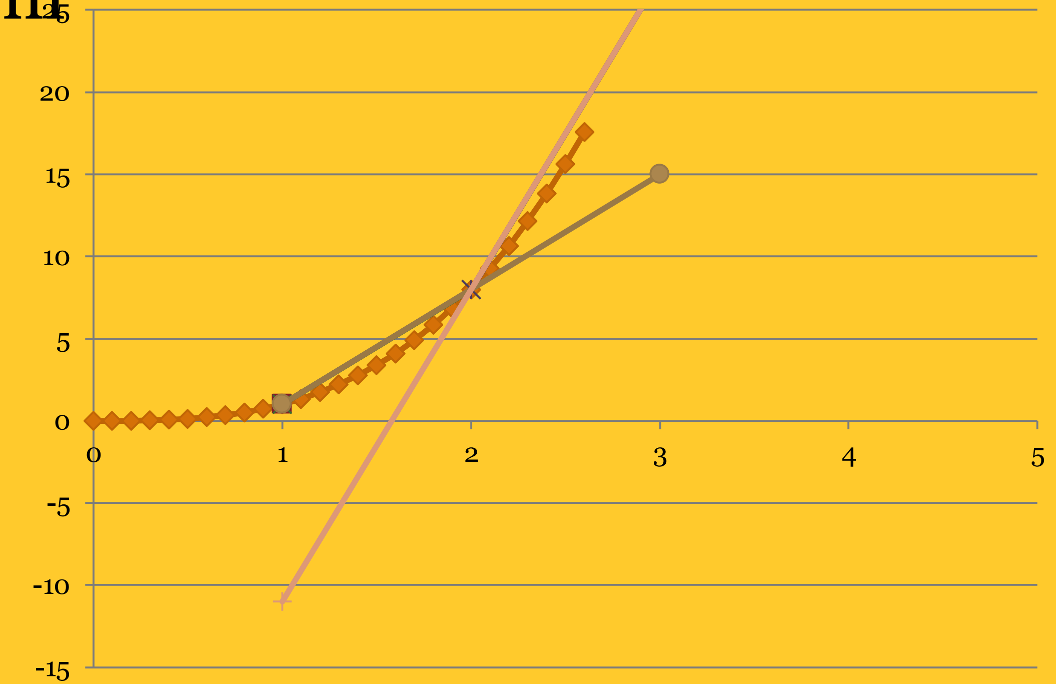
- Functions and Relations
- Limits
- Continuity
- Derivatives
- Differentiation
- Applications of Differentiations
- Integrals
- Applications of Integrals

Limits and Continuity

- Why we study Limits and Continuity?

Origin of Limit Theory

- Velocity Problem
- Tangent Problem



Tangent Problem

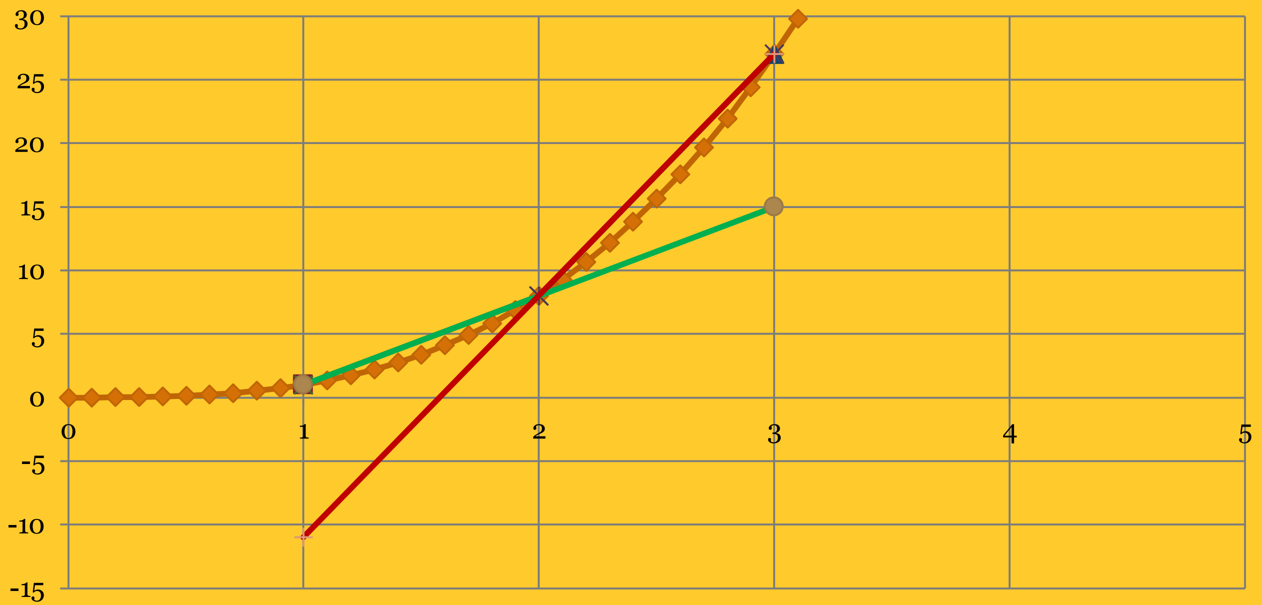
- Curve $y=x^3$
- To draw a tangent at point $x=2$
- Issue: With one point, we cannot draw a tangent

Limit

To draw a tangent at point x=2

- Issue: With one point, we cannot draw a tangent
- Solution: Take another point close to given point and calculate the slope, m as (y2-y1)/(x2-x1)

To draw a tangent at point x=2



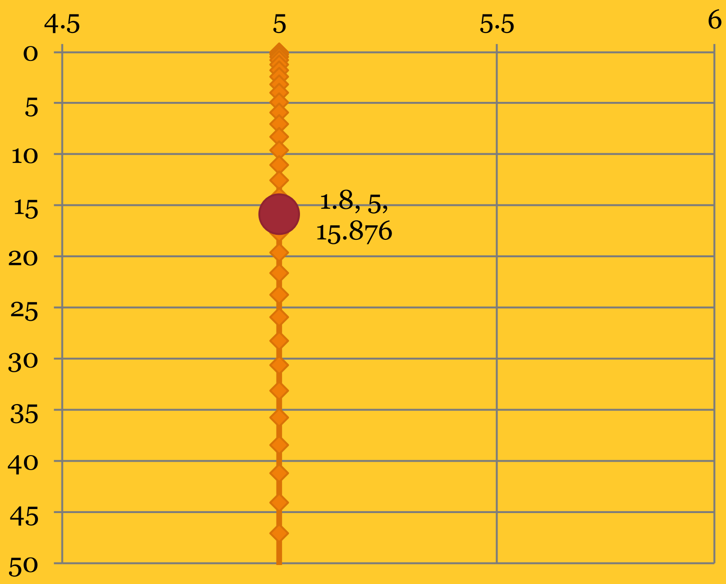
- These are closer points but does not represent the tangent
- For the tangent, we have to take more closer point and this gives rise to the theory of Limits.

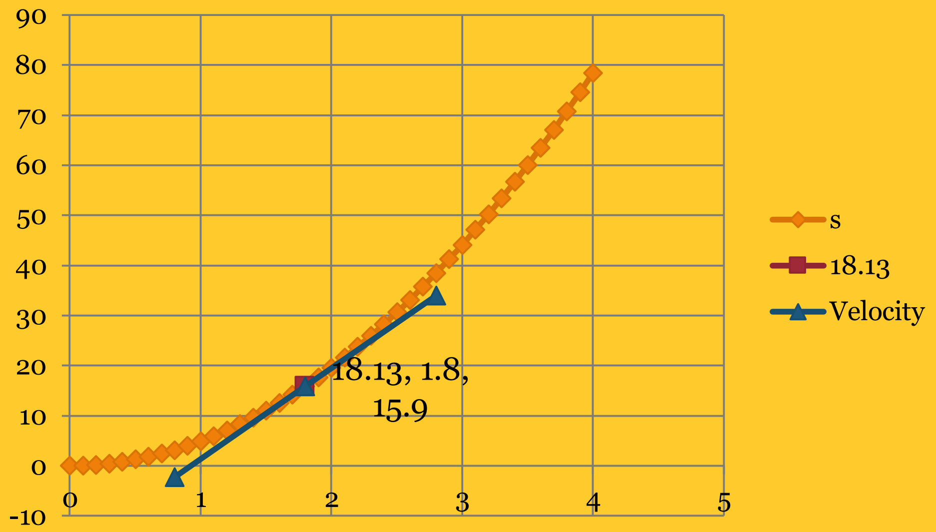
Limit : Tangent Problem

x approaches towards 2 from left and from right

Limit : Velocity Problem

Problem: Find the velocity of a falling body at time t.





The Limit of a Function

- When x and y is related, we want to know the value y tends to assume (L) when x approaches to a specific value (a)
- The answer to this question lies in the concept of limit.
- The number L is called the limit of the function y = f(x) as x tents to a.
- Symbolically, $\lim_{x \to a} f(x) = L$
- The limit of f(x) as x approaches a, equals L
- Note: The limiting value of a variable is that value, which the variable approaches all the time but never attains it.

Demonstration of Limit

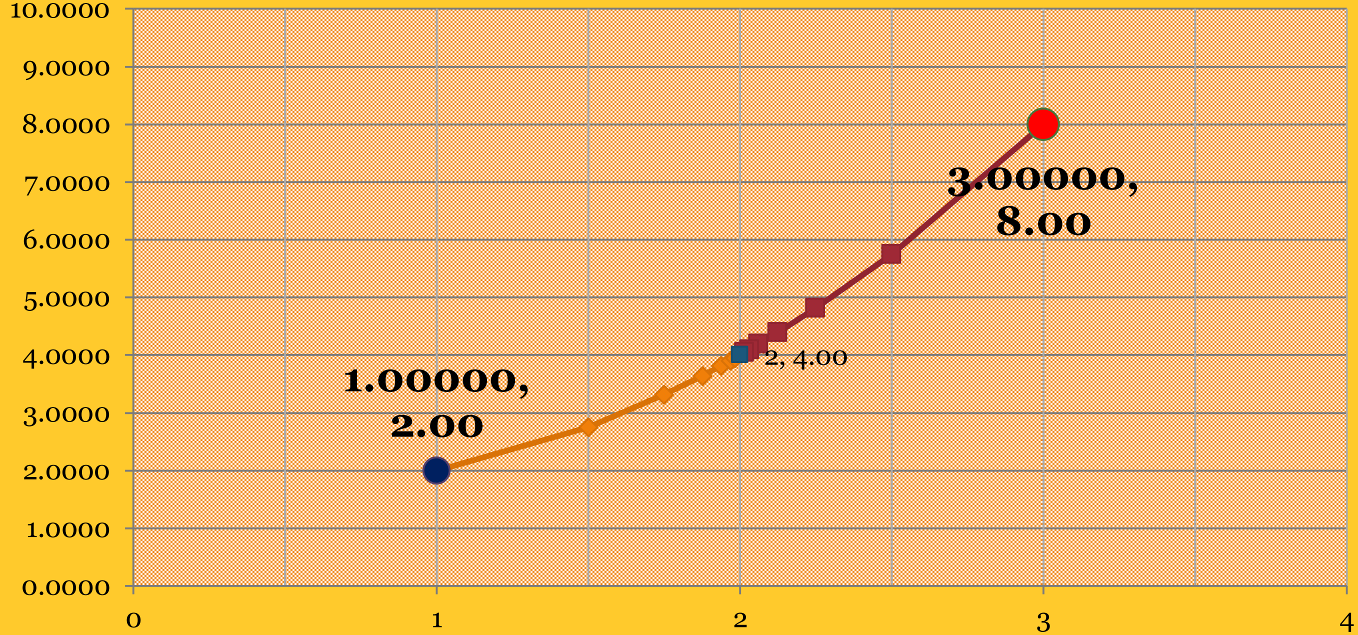
- For a function y=f(x), if the value y tends to the number L1 as x tends to a from the side of small values, then the number L1 is called left hand limit.

- If y tends to L2 as x tends to a from the side of larger values, then L2 is called the right hand limit.

-

Left Hand and Right Hand Limit

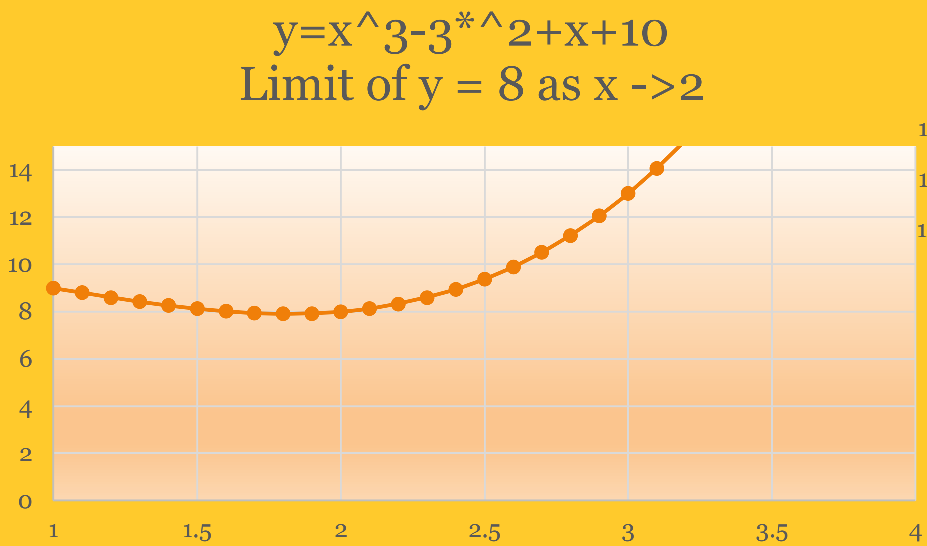
- Example y = x^2-x+2
- We are trying to find the Limit of y at x=2 as (1) x approaches from left at point 1 and (2) x approaches from right at point 3.

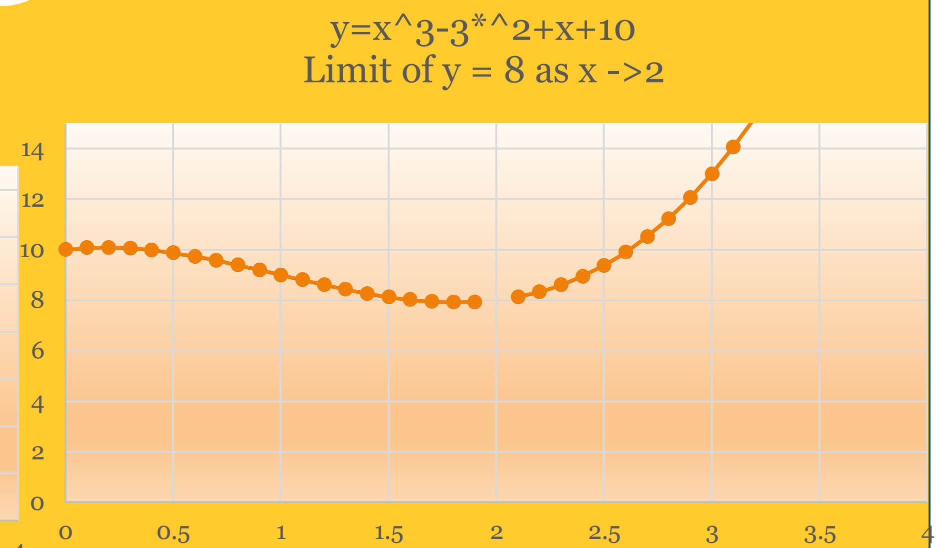


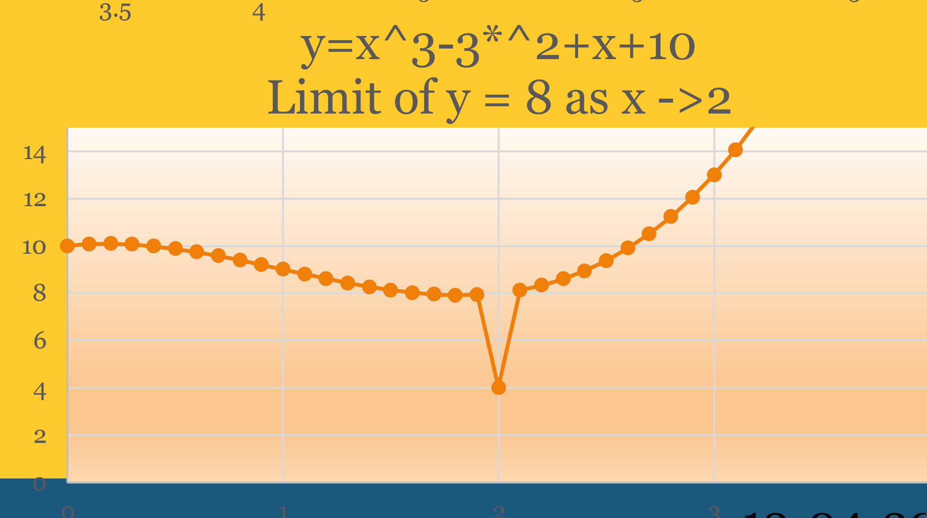
Mathematical Treatment of x approaches a

- Our function is y=x^2-x+2
- We are interested to vary the values of x as x approaches from left, 1, to 2 as well as a approaches from right , 3, to 2
- To draw the curve, we used following formulas:
- x1=1, xn=2
- x2=(x1+xn)/2
- x3=(x2+xn)/2 ....
- To draw the point, we inserted a slider whose values (v) are 0 to 100. We then calculated the value of t=v/100.
- To vary the values of x from 1 to 2, we used the formula x=x1*(1-t)+x2*t
- For demonstration, refer excel file: cdass limit 04092015 05082016

Examples of Limits







Limit of Function

- $y=\sin(x)/x$

2.4 The Precise Definition of a Limit

δ and ε Definition of Limit

- Till now we have used intuitive definition of a limit. The phrases like "x is close to 2" or "f(x) gets closer and closer to L" are vague.

- How close to a does x have to be so that f(x) differs from L by less than ε?
- The distance from x to a is |x-a| and the distance from f(x) to L is |f(x)-a|, so our problem is to find a number delta, δ, such that $|f(x)-L| < \varepsilon$ if $|x-a| < \delta$.

δ and ε Definition of Limit

- Here we are required to find out a number $\boldsymbol{\delta}$ for a given number $\boldsymbol{\varepsilon}$.
- Where $\boldsymbol{\varepsilon}$>|f(X)-L|>0 and $\boldsymbol{\delta} > |x - a| > 0$

Step to find δ *and for a given* ε

- Step-1: Solve the inequality |f(x)-L|< ε to find an open interval (a, b) containing xo on which the inequality holds for all x # 0.

- Step-2: Find a value of δ >0 that places the open interval (xo- δ, xo+ δ) centered at xo inside the interval (a, b).
- The inequality |f(x)-L|< ε will hold for all x#xo in this δ –interval.

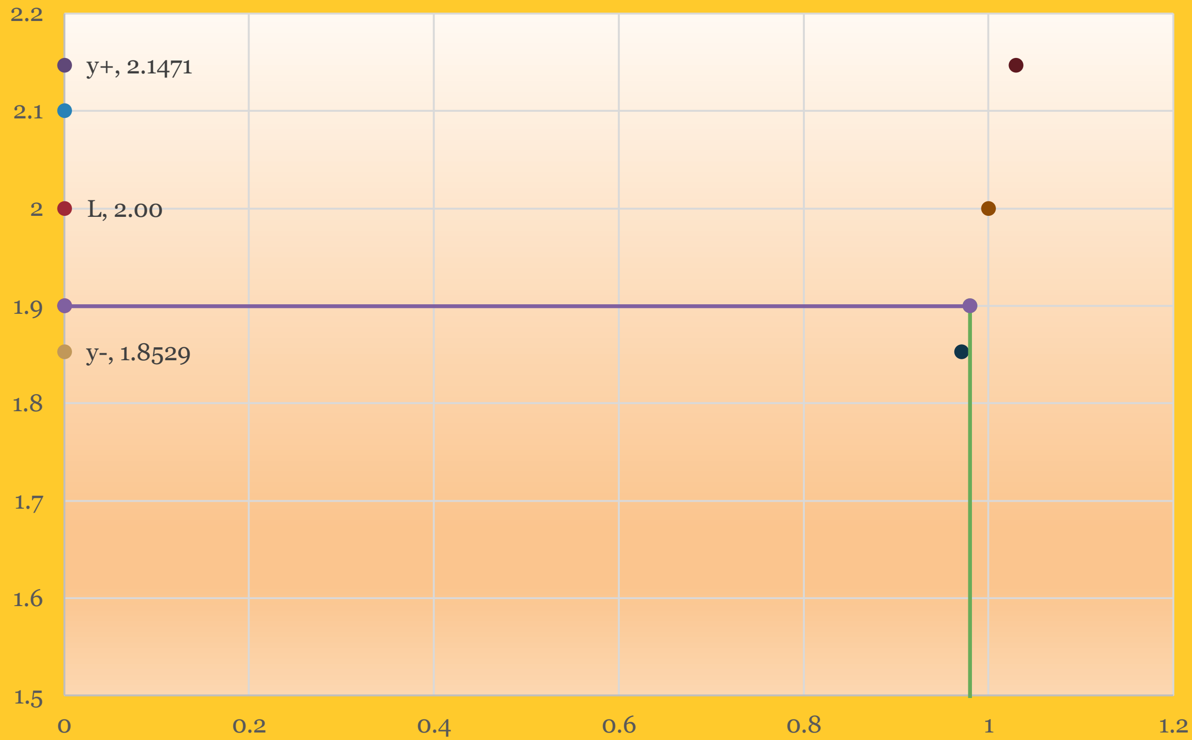
Example-2: $\lim_{x\to 1}(5x-3)=2$

- $Here, xo = 1, f(x) = 5x - 3, L = 2$
- Let us assume $\boldsymbol{\varepsilon}$=.1
- We are required to find the value of |x-1| <$\boldsymbol{\delta}$

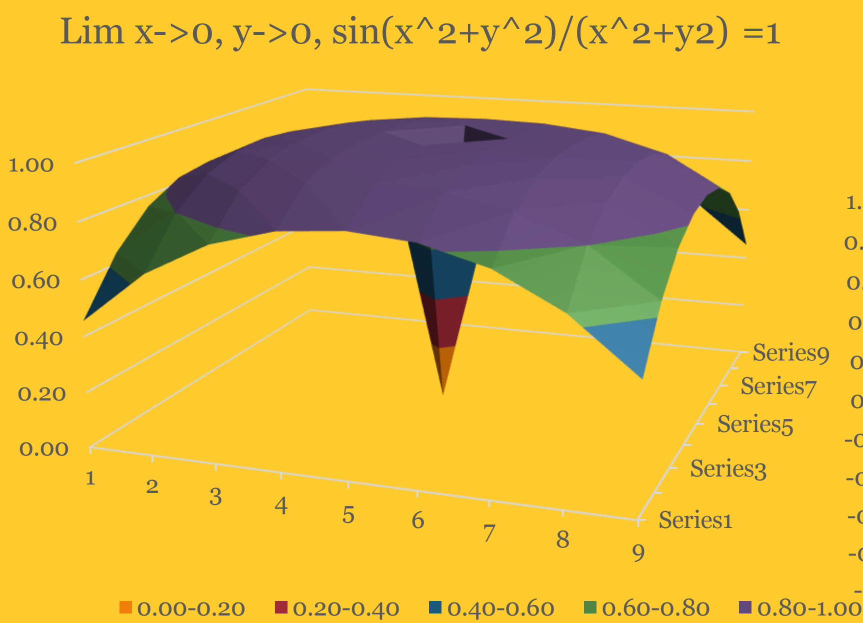
  for |f(x)-L|<$\boldsymbol{\varepsilon}$=.1
- |(5x-3)-2|< $\boldsymbol{\varepsilon}$
- |5x-5|< $\boldsymbol{\varepsilon}$
- 5|x-1|< $\boldsymbol{\varepsilon}$
- |x-1|< $\boldsymbol{\varepsilon}/5$
- Thus we can take **$\boldsymbol{\delta} = \boldsymbol{\varepsilon}/5$**

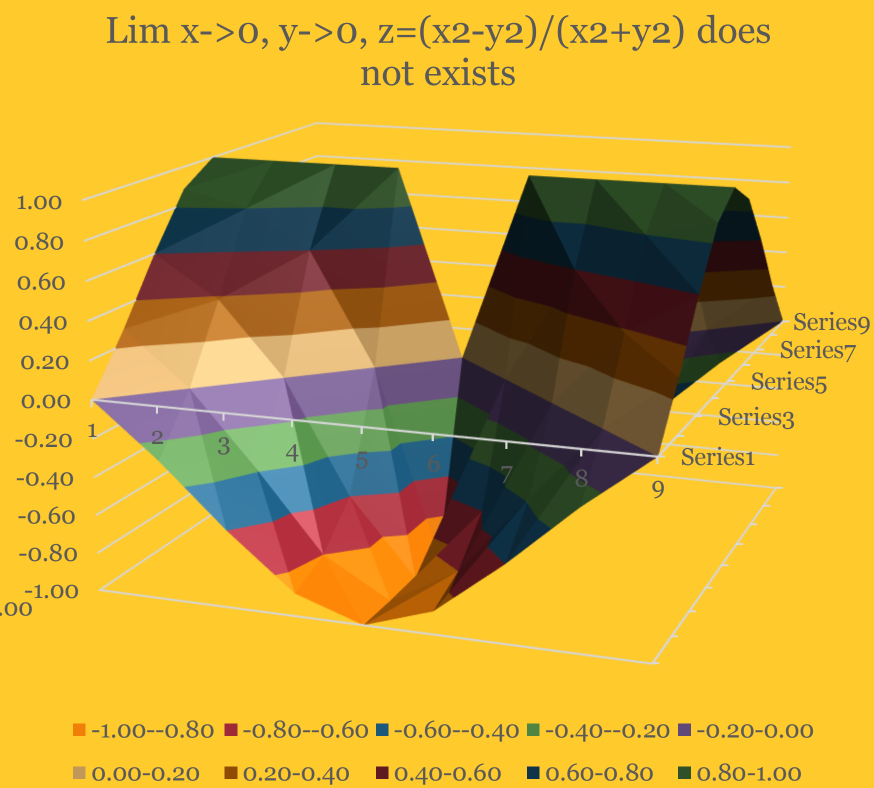
Example-2: $\lim_{x \to 1}(5x - 3) = 2$

- Graphical Solution:



Limit of Functions of 2 variable





Limit of Functions of 2 variable

- Definition: Limit of function z =f(x,y) as (x, y) approaches (a, b) is L and is written as
- $\lim_{(x,y)\to(a,b)} f(x,y) = L$

if for every number $\boldsymbol{\varepsilon}$ >0 there is a corresponding number $\boldsymbol{\delta > 0}$ such that 0<sqrt((x-a)^2+(y-b)^2)<$\boldsymbol{\delta}$ when |f(x, y)-L|< $\boldsymbol{\varepsilon}$

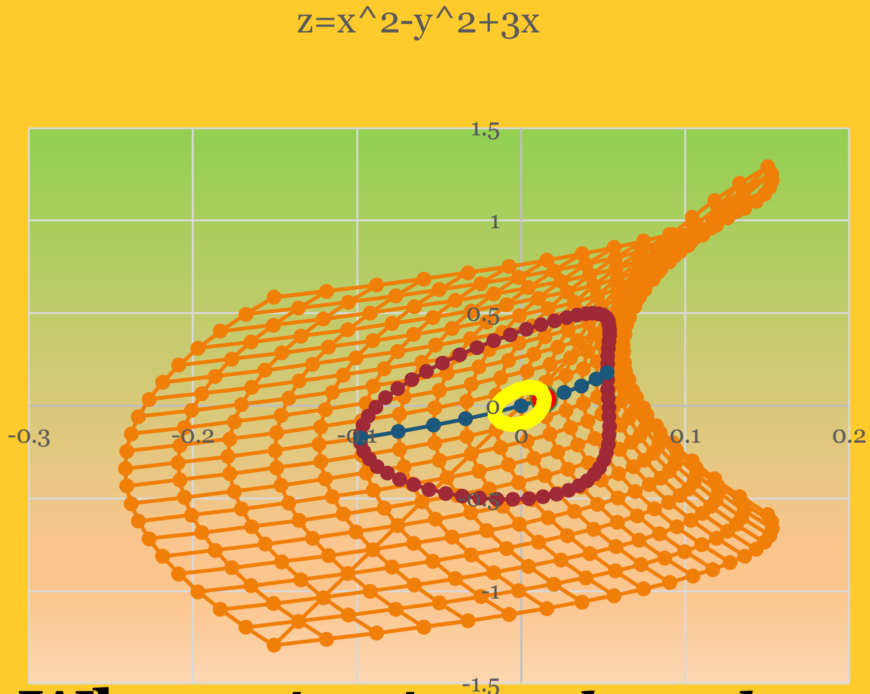
Limit of Functions of 2 variable

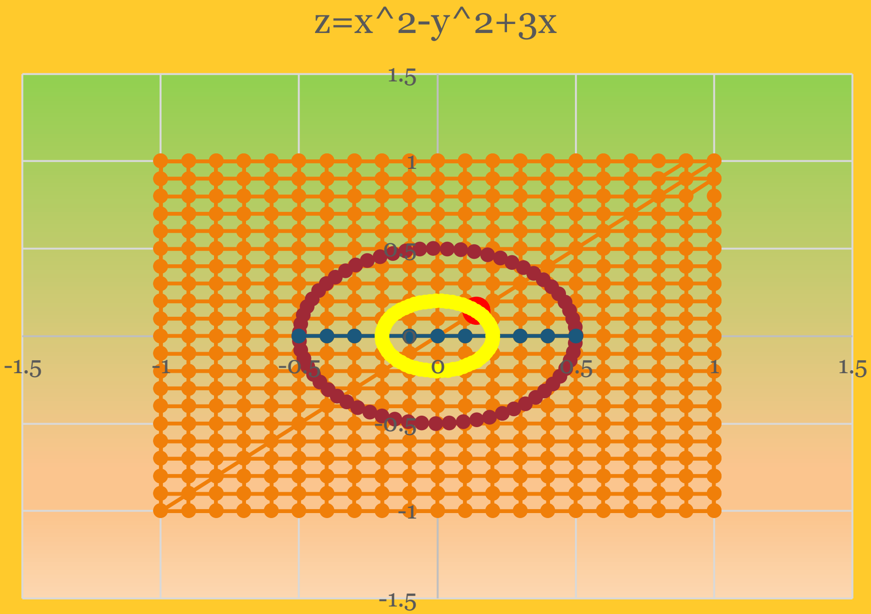
- Demonstration of $\lim_{(x,y)\to(a,b)} f(x,y) = L$

Note: sqrt((x-a)^2+(y-b)^2) is the distance between (x, y) and (a, b) and |f(x, y)-L| is the difference between the numbers f(x, y) and L.

Limit of Functions of 2 variable

- Demonstration of $\lim_{(x,y)\to(a,b)} f(x,y) = L$



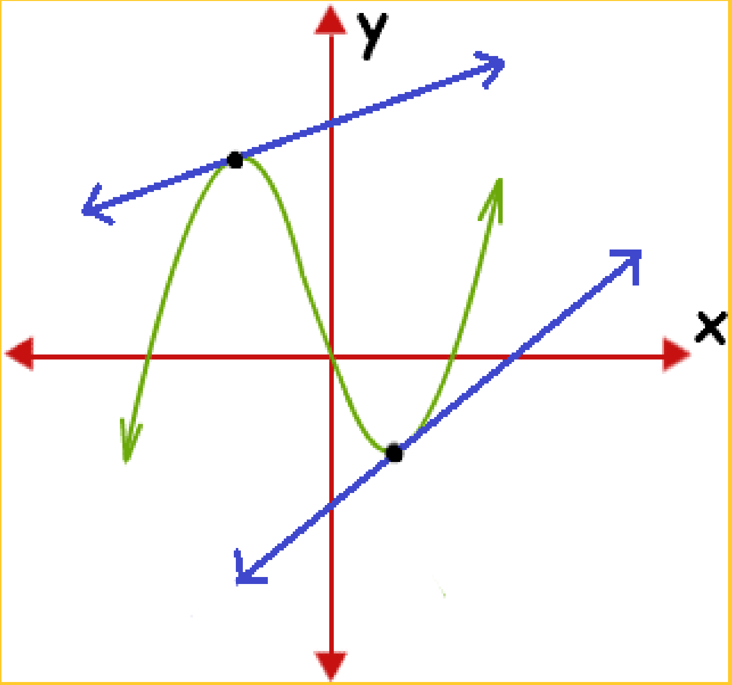


- **When ε *is given then the maximum value of δ* is given by the formula 0<sqrt(x-a)^2+(y-a)^2)< δ = r**

Limit of Vector Functions

Continued

- A,B are the tangents.
- It is clearly seen that the tangents vary at each point along the curve. This means that the tangents to the curve at various points varies and different.



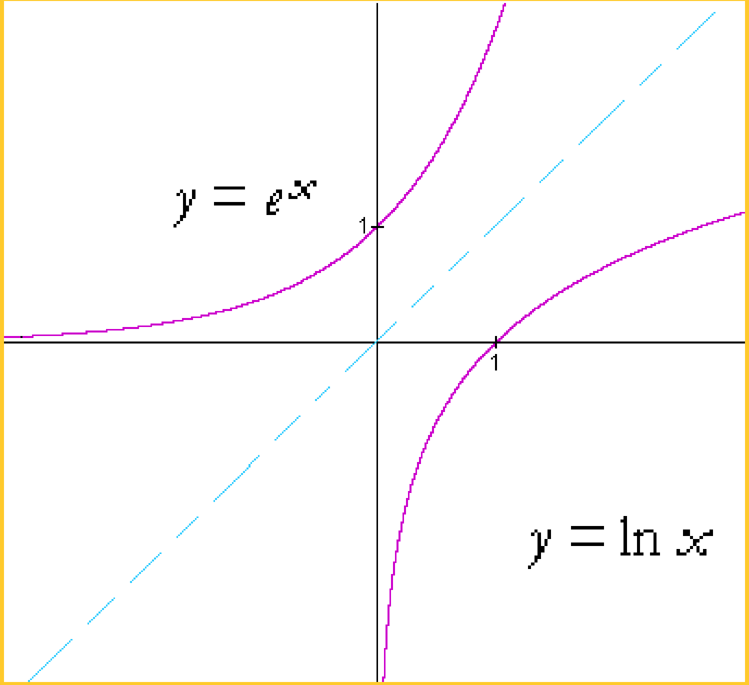
Notations used to denote a derivative:

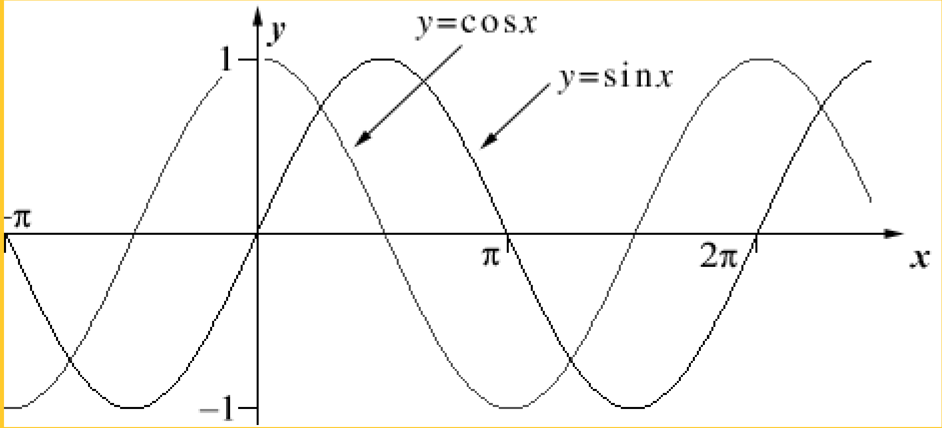
- $\dfrac{dy}{dx}$ or $f'(x)$ or $\dfrac{d}{dx} f(x)$

Important Derivatives

- $$\frac{d}{dx}(\sin x) = \cos x$$
- $$\frac{d}{dx}(\cos x) = -\sin x$$
- $$\frac{d}{dx}(x^n) = nx^{n-1}$$
- $$\frac{d}{dx}(\log x) = \frac{1}{x}$$
- $$\frac{d}{dx}(e^x) = e^x$$

Continued





Applications of derivatives

- Rate of Change of Quantities
- Increasing and Decreasing Functions
- Tangents and Normals
- Maxima and Minima
- Optimization
- Mean value Theorem
- Linearization

Rate of Change of Quantities

- In this section, we will find the rate of change of one quantity with another.

- Whenever one quantity y varies with another quantity x, satisfying some rule $y=f(x)$, then $f'(x)$ represents the rate of change of y with respect to x and $f'(x_o)$ represents the rate of change of y with respect to x at $x=x_o$.

Continued..

For example:

- If displacement of a particle is ‘s’ and given as a function of time t by

$$s=5t^2$$

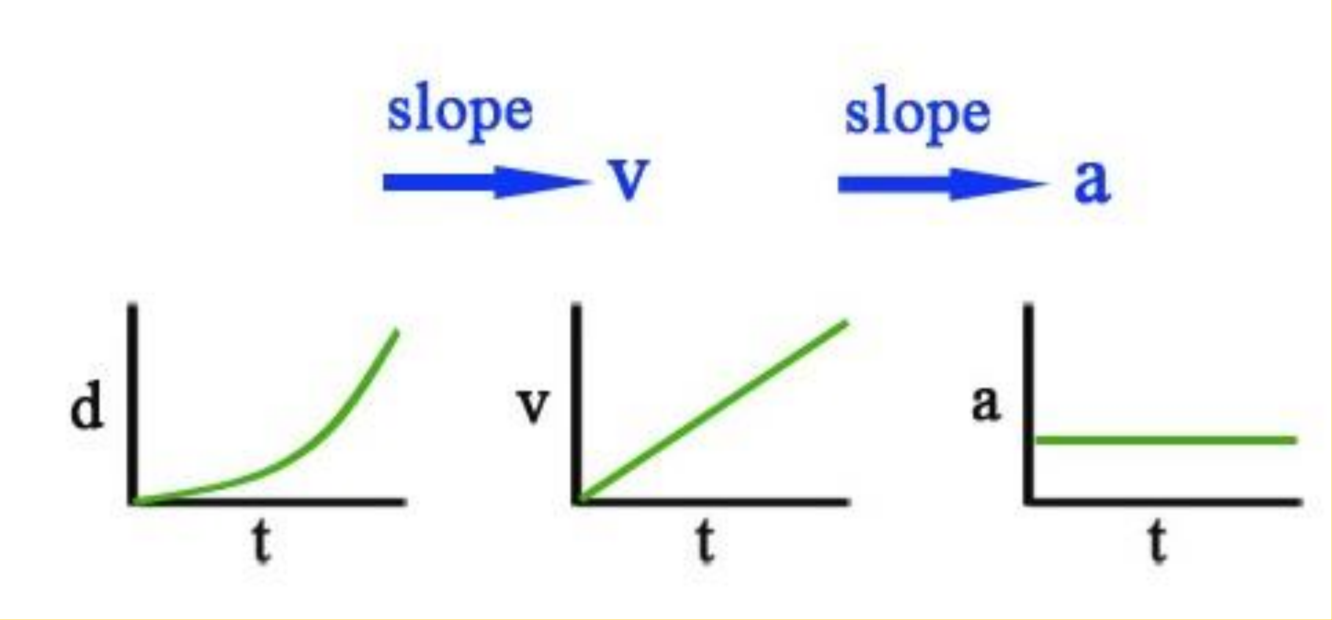
then the rate of change of s with respect to t (also known as velocity v) is given by

$$v=ds/dt=10t$$

and rate of change of v with respect to t (also known as acceleration a) is given by

$$a=dv/dt=10$$

Continued



Increasing and Decreasing Functions

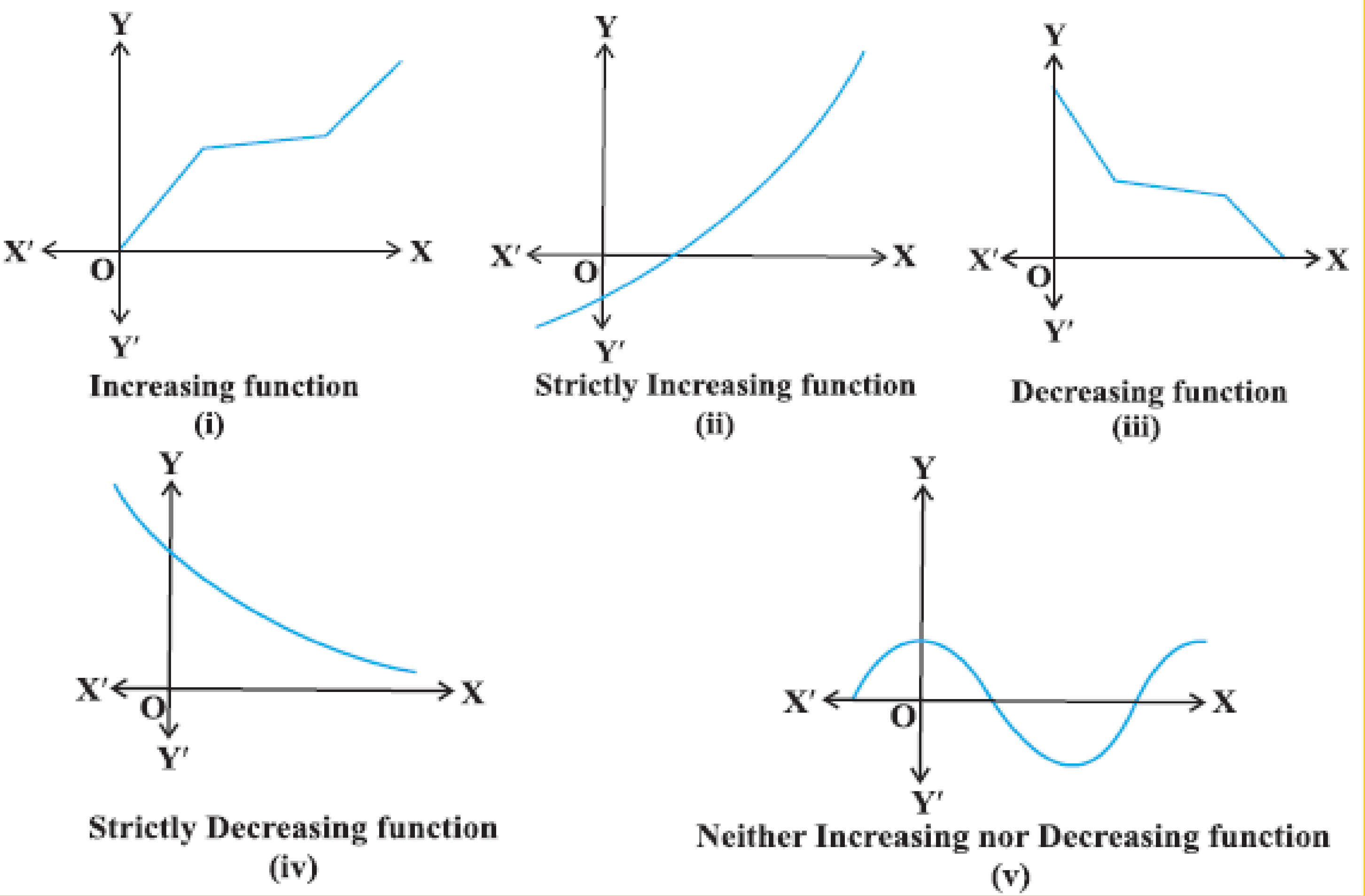
In this section, we will find out whether a function is increasing or decreasing with the help of differentiation.

Now let us consider an open interval I contained in the domain of an real valued function f. Then f is said to be:

Continued

- Increasing on I if $x1 < x2$ in $I \Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$.
- Strictly increasing on I if $x1 < x2$ in $I \Rightarrow f(x1) < f(x2)$ for all $x1, x2 \in I$.
- Decreasing on I if $x1 < x2$ in $I \Rightarrow f(x1) \geq f(x2)$ for all $x1, x2 \in I$.
- Strictly decreasing on I if $x1 < x2$ in $I \Rightarrow f(x1) > f(x2)$ for all $x1, x2 \in I$.

Continued


Y
X'
X
O
Y'
Increasing function
(i)
Y
X'
X
O
Y'
Strictly Increasing function
(ii)
Y
X'
X
O
Y'
Decreasing function
(iii)
Y
X'
X
O
Y'
Strictly Decreasing function
(iv)
Y
X'
X
O
Y'
Neither Increasing nor Decreasing function
(v)

Tangents and Normals

In this section, we will find the tangent line and normal line to a curve with the help of differentiation.

Tangents

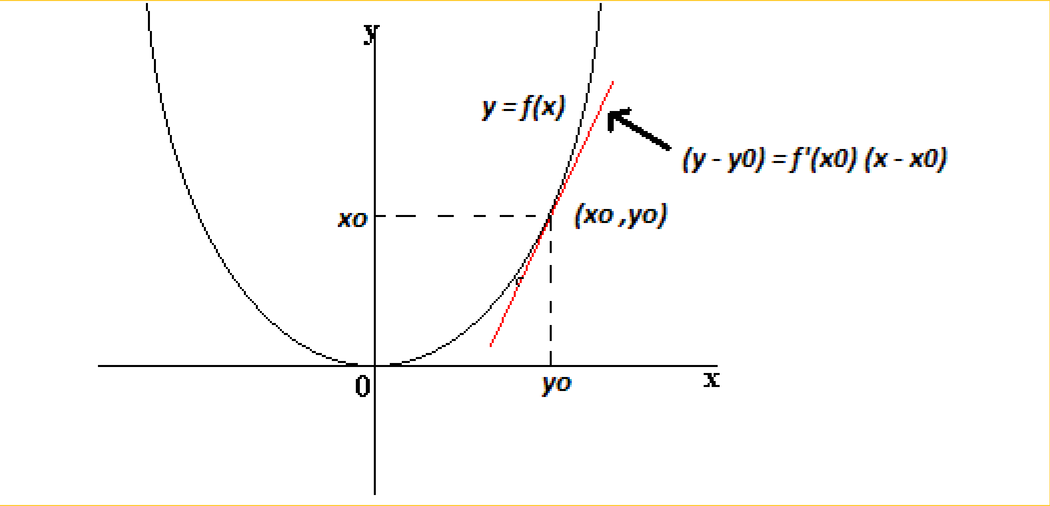
We know that the equation of a line passing through a given point (x_0, y_0) having slope m is given by

$$(y - y_0) = m(x - x_0)$$

Continued

Now we know that the slope of the tangent of the function $y = f(x)$ at point (x_0, y_0) is given by $f'(x_0)$. Therefore the equation of the tangent to the curve $y = f(x)$ at point (x_0, y_0) is give by

$$(y - y_0) = f'(x_0)\,(x - x_0)$$



Continued

Normals

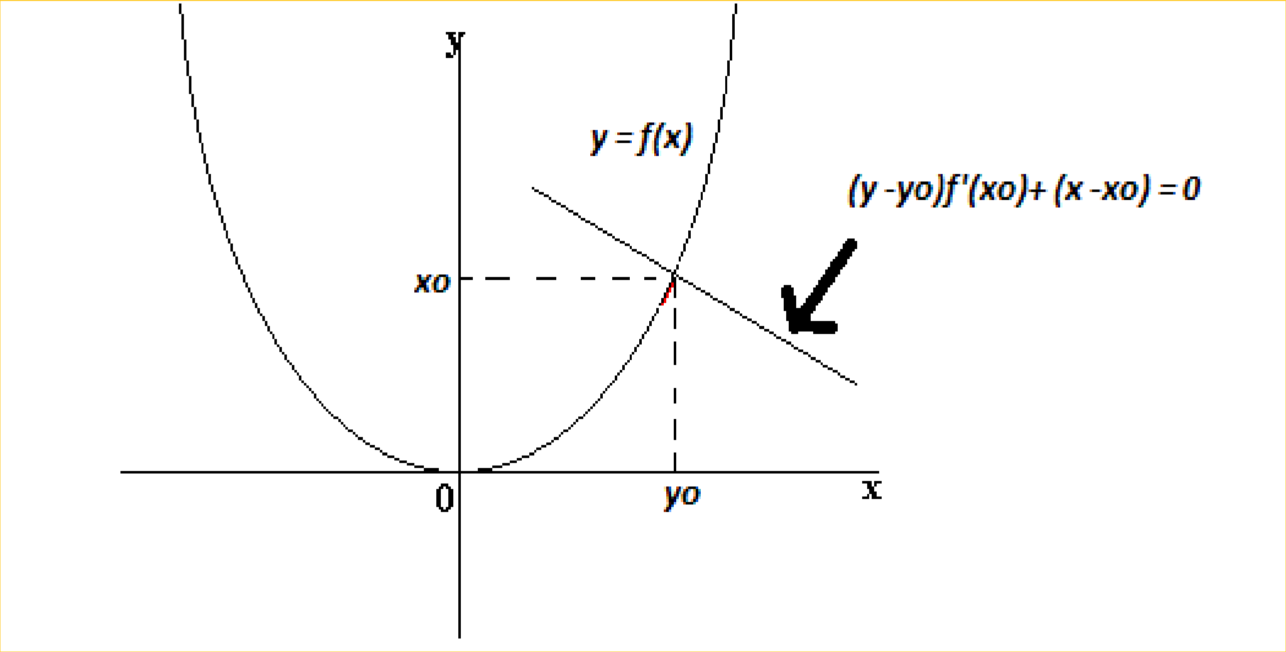
We know that the normal is perpendicular to the tangent, therefore slope of the normal to the curve $y = f(x)$ at point (x_0, y_0) is given by $-1/f'(x_0)$.

Therefore the equation of normal to the curve $y = f(x)$ at point (x_0, y_0) is given by

$$(y - y_0) = (-1/f'(x_0))(x - x_0)$$

i.e. $(y - y_0)f'(x_0) + (x - x_0) = 0$

Continued

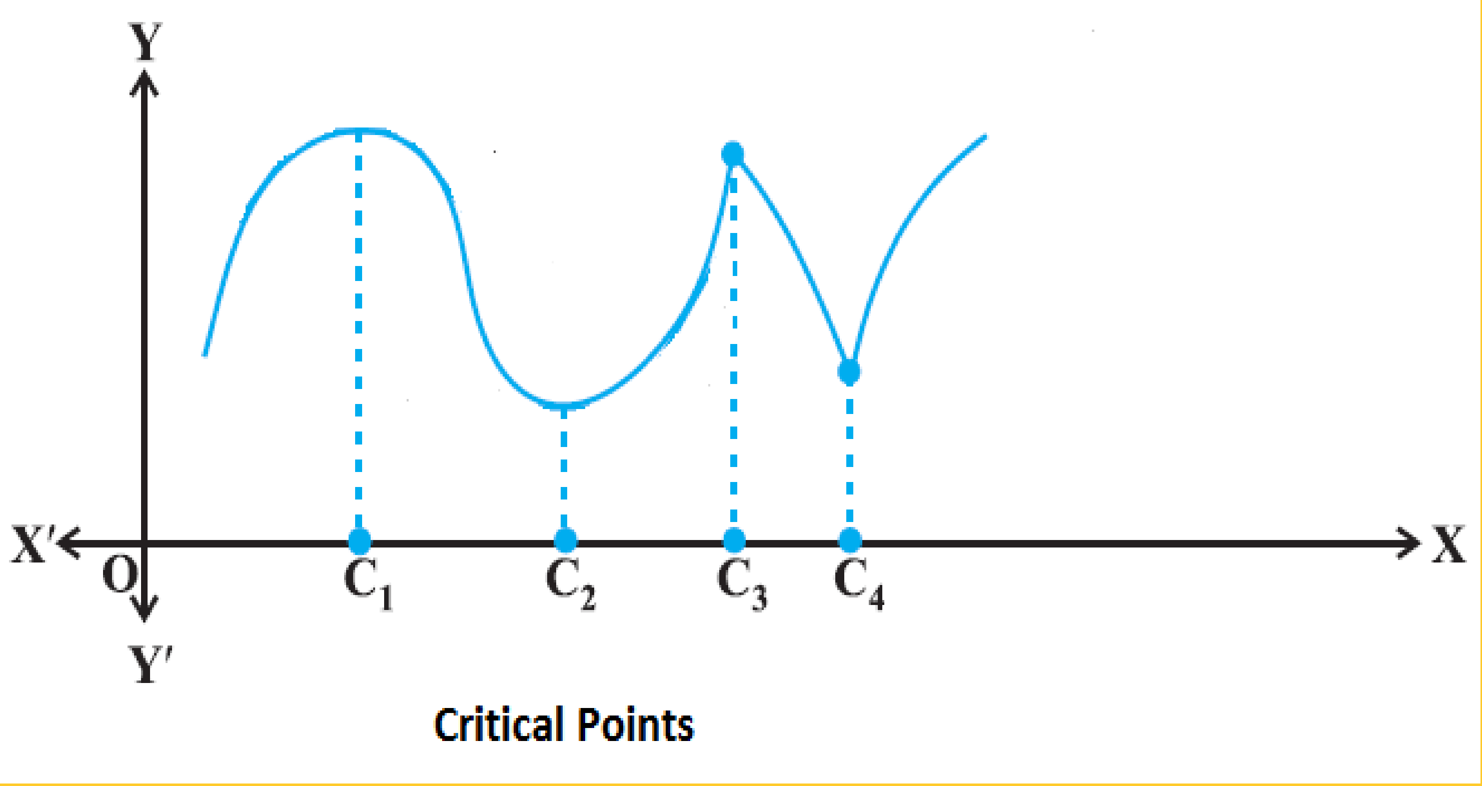


Maxima and Minima

In this section, we will find the maxima and minima of the function with the help of differentiation.
In terms of *differentiation* function attends it *maxima or minima* at *critical point*.

Critical Point
Let us consider a point $\boldsymbol{c}$ in the domain of a function $\boldsymbol{f}$ .If at this point $\boldsymbol{f'(c) = 0}$ or $\boldsymbol{f}$ is not differentiable, then the point is called a ***critical point*** of $\boldsymbol{f}$.

Continued



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Maxima and minima and be found out using two tests:

i. First derivative test
ii. Second derivative test

Using First Derivative Test

Let *f* be a function defined on an open interval *I*. Let *f* be continuous at a critical point **c** in *I*. Then

i. If $f'(x)$ changes *sign* from *positive to negative* as ***x*** increases through ***c***, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of **c**,

Continued

and $f'(x) < 0$ at every point sufficiently close to and to the right of **c**, then **c** is a point of *local maxima* and if $f(x) \leq f(c)$ for all $\boldsymbol{x} \in \boldsymbol{I}$ then **c** is called the *global maxima*.

(ii) If $f'(x)$ changes sign from negative to positive as $\boldsymbol{x}$ increases through **c**, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of **c**, and $f'(x) > 0$ at every point sufficiently close to and to the right of **c**, then **c** is a point of *local minima* and if $f(x) \geq f(c)$ for all $\boldsymbol{x} \in \boldsymbol{I}$ then **c** is called the *global minima*.

Continued

(iii) If $f'(x)$ does not change sign as $\boldsymbol{x}$ increases through **c**, then **c** is neither a point of local maxima nor a point of local minima. Infact, such a point is called *point of inflection*.

Using Second Derivative Test

Let f be a function defined on an interval I and $c \in$ I. Let f be twice differentiable at c. Then

(i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$

The value $f(c)$ is local maximum value of f.

Continued..

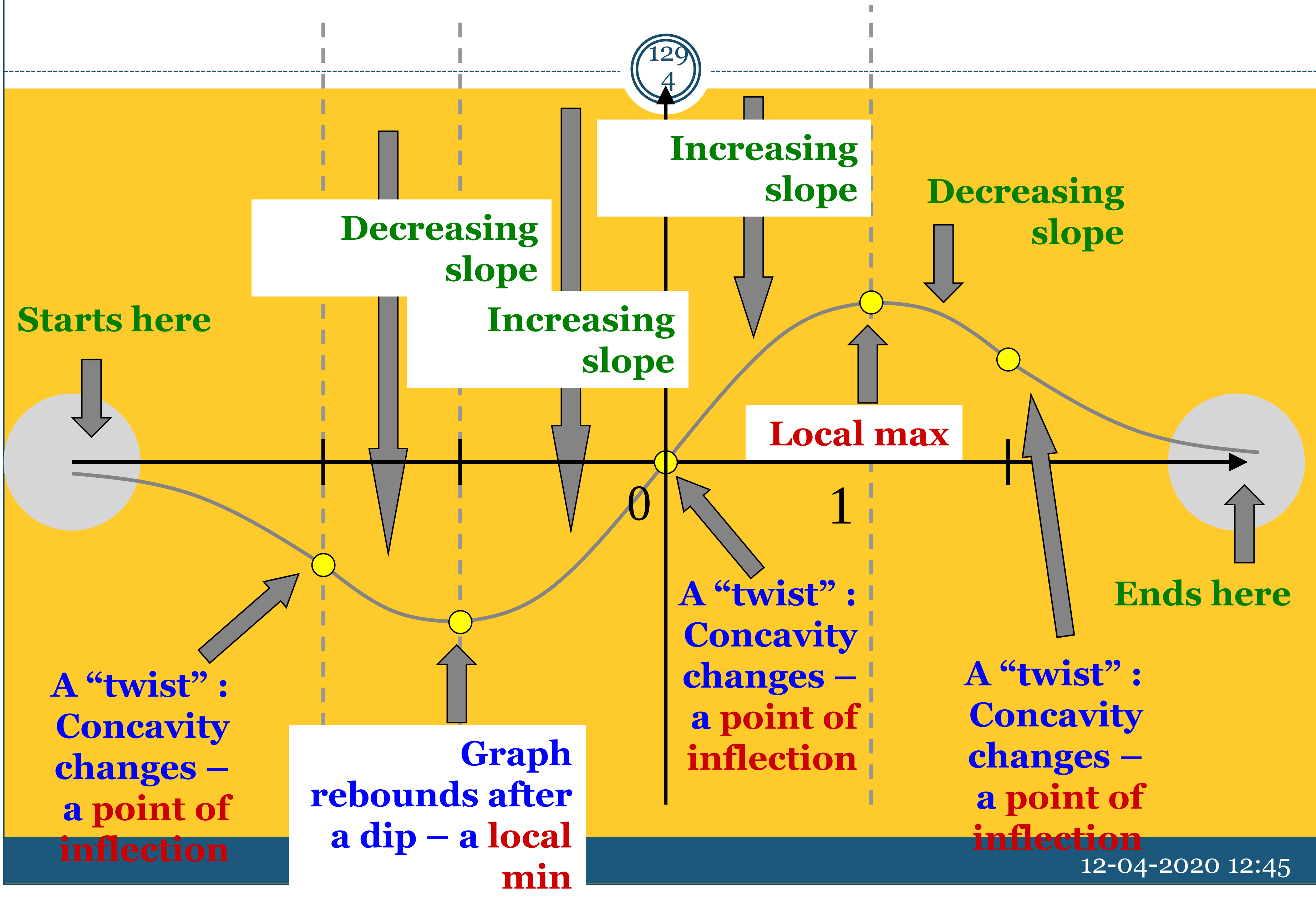
(ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$

In this case, $f(c)$ is local minimum value of f.

(iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.

In this case, we go back to the first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

Example: $f(x) = \frac{x}{1+x^2}$



Optimization

Optimization is the selection of a best element (with regard to some criteria) from some set of available alternatives.

In terms of *differentiation* it means to *maximizing or minimizing* a function.

This can be better explained with an example.

Continued

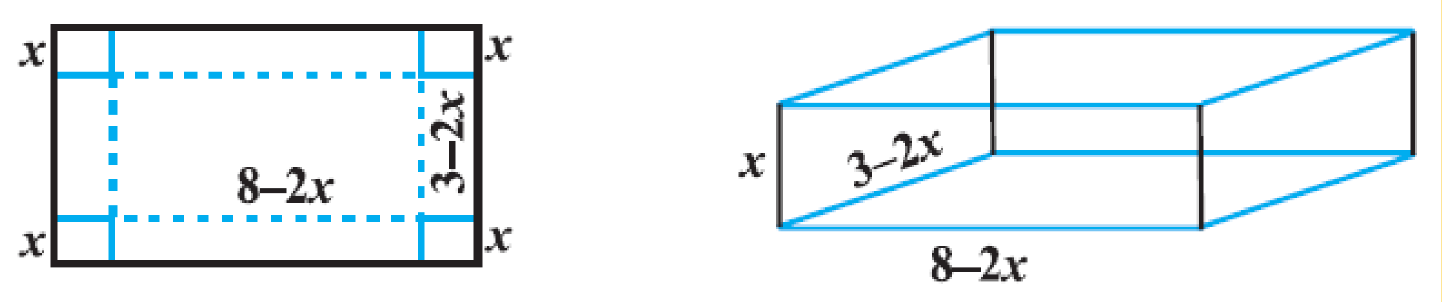
Let us consider a problem:

Q: Find the volume of the largest open top box that can be created using a 3 meters by 8 meters rectangular aluminium sheet, by cutting equal square from the edges and folding up the sides.

Sol: Let x metre be the length of a side of the removed squares. Then, the height of the box is x, length is $8 - 2x$ and breadth is $3 - 2x$ and the volume V(x) is given by

$$V(x) = x\,(3 - 2x)\,(8 - 2x)$$

$= 4x^3 - 22x^2 + 24x$



Therefore $V'(x) = 12x^2 - 44x + 24$

$$V''(x) = 24x - 44$$

Now $V'(x) = 0$ gives $x = 3, \frac{2}{3}$

But $x \neq 3$, Because it will be equal to the breadth.

Thus, we get $x = \frac{2}{3}$, Now $V''(\frac{2}{3}) = 24(\frac{2}{3}) - 44 = -28 < 0$

Continued..

Therefore $x = \frac{2}{3}$ is the point of maxima, which means by removing a square of side $\frac{2}{3}$ and folding the sides will give us maximum volume given by

$$V\left(\frac{2}{3}\right) = 4\left(\frac{2}{3}\right)^3 - 22\left(\frac{2}{3}\right)^2 + 24\left(\frac{2}{3}\right)$$

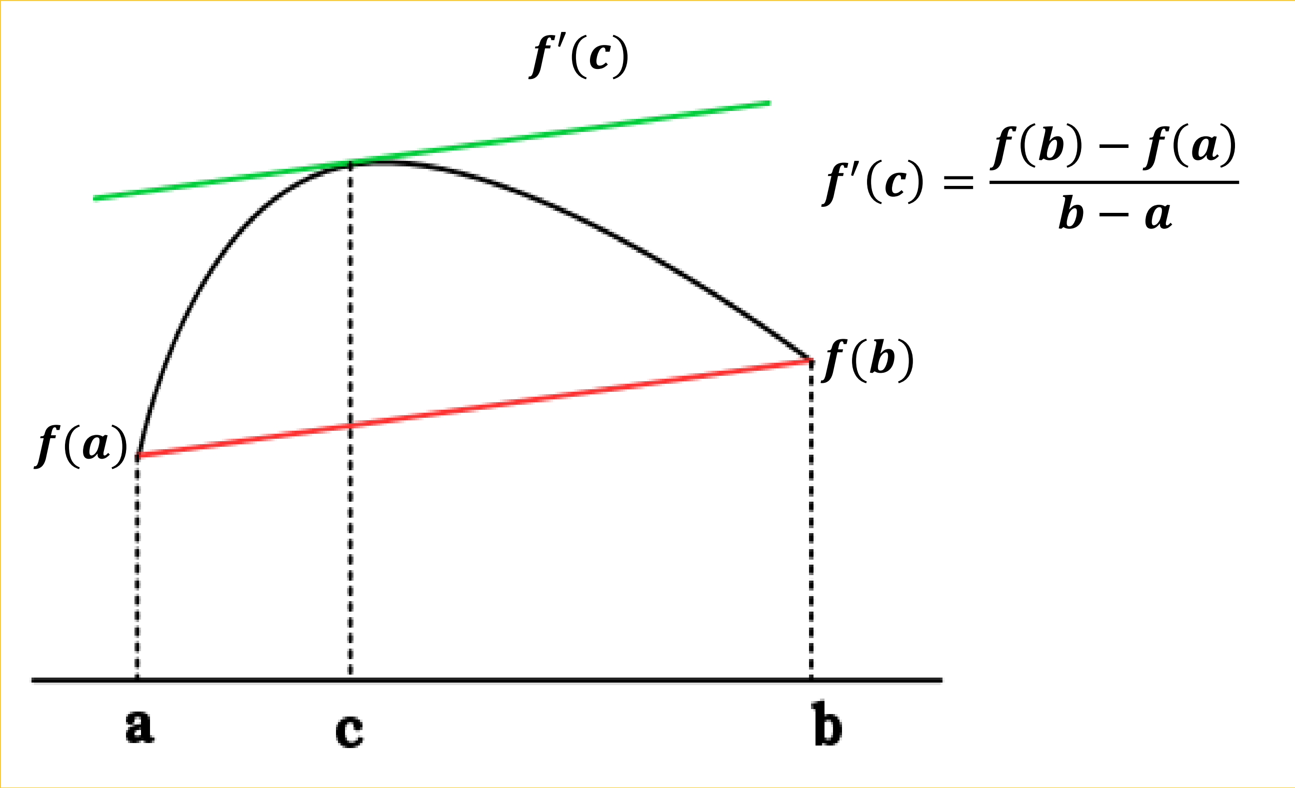
$$= \frac{200}{27} m^3$$

Mean Value Theorem

The Mean Value Theorem states that there is a point c in (a, b) such that the slope of the tangent at $(c, f(c))$ is same as the slope of the secant between $(a, f(a))$ and $(b, f(b))$. In other words, there is a point c in (a, b) such that the tangent at $(c, f(c))$ is parallel to the secant between $(a, f(a))$ and $(b, f(b))$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Mean Value Theorem



Q&A…

INTEGRAL CALCULUS

INTEGRATION & ITS APPLICATIONS

INTRODUCTION TO INTEGRATION

WHY TO INTEGRATE?

- Integration is a tool that helps us multiply changing quantities.
- USES:
  a) the problem of finding a function when its derivative is given
  b) the problem of finding the area bounded by the graph

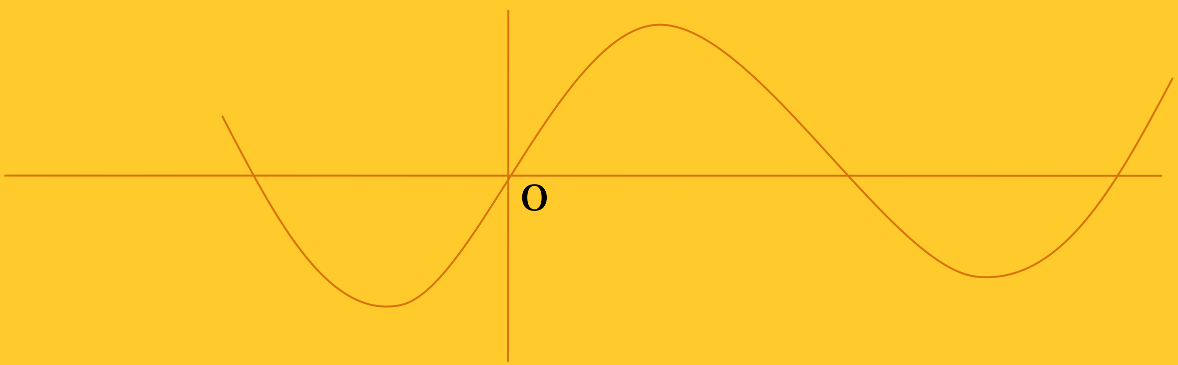
- Type of Integrals:

  - **Indefinite** and **Definite** integrals

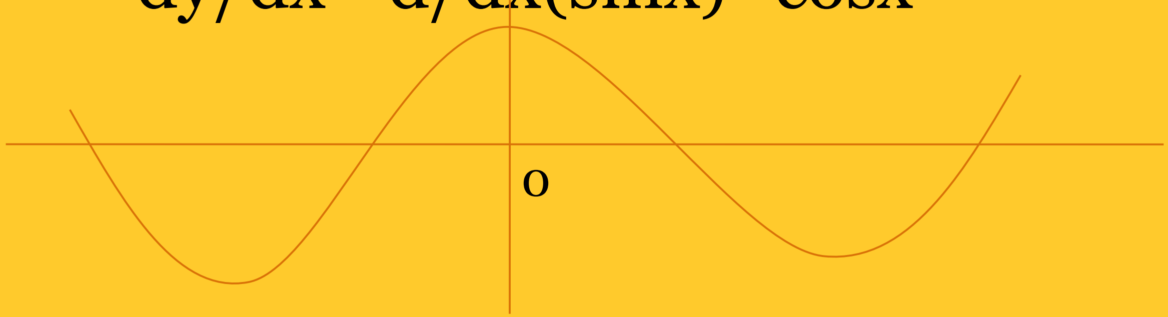
To understand this concept better, let’s see some examples.

- Example 1:

$y=\sin x$



$dy/dx = d/dx(\sin x)=\cos x$



Example 2

$$Y = x^3 + 5$$

$$dy/dx = 3x^2$$

$$\int 3x^2 \, dx = x^3 + C$$

- Which Shows that..
  - Integration is Anti-Differentiation
  - Indefinite Integral

What do we observe?

- Thus, integrals of the functions are not unique.
- Actually, there exist infinitely many integral of a function which can be obtained by choosing C arbitrarily from the set of real numbers.
- For this reason C is called arbitrary constant

SYMBOLS/ TERMS/ PHRASES & THEIR MEANINGS

Integration Symbol

upper limit of integration

$$\int_a^b f(x)\,dx$$

lower limit of integration

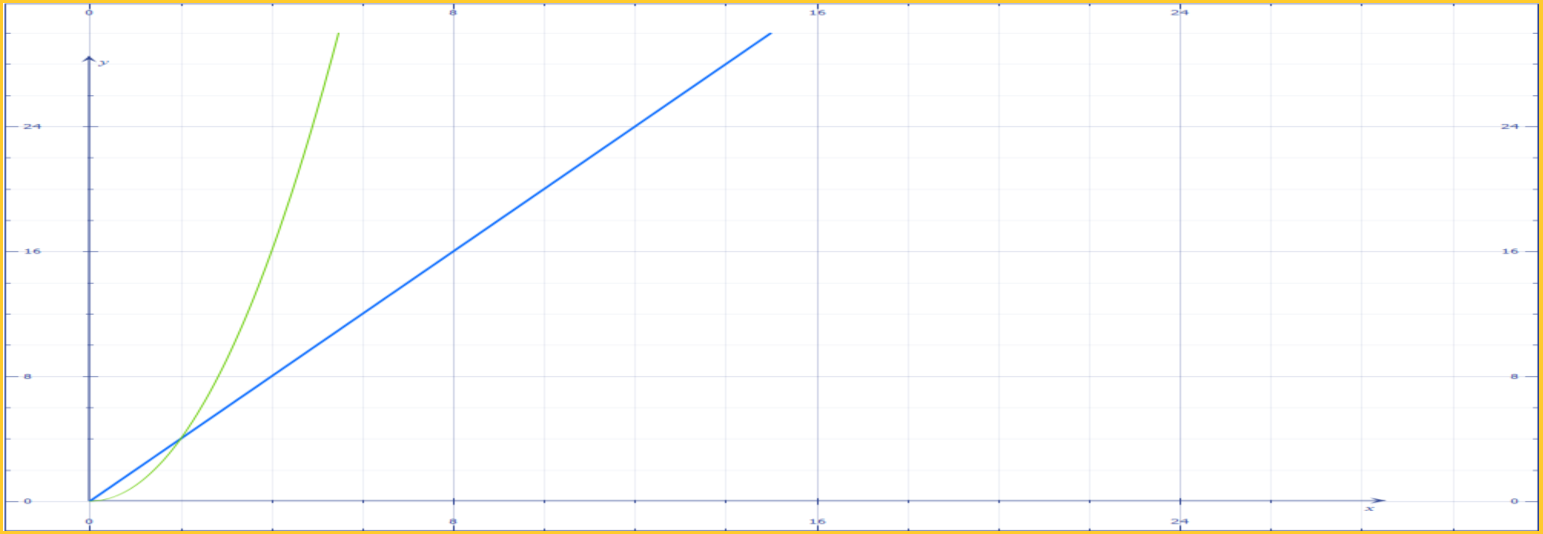
integrand

variable of integration

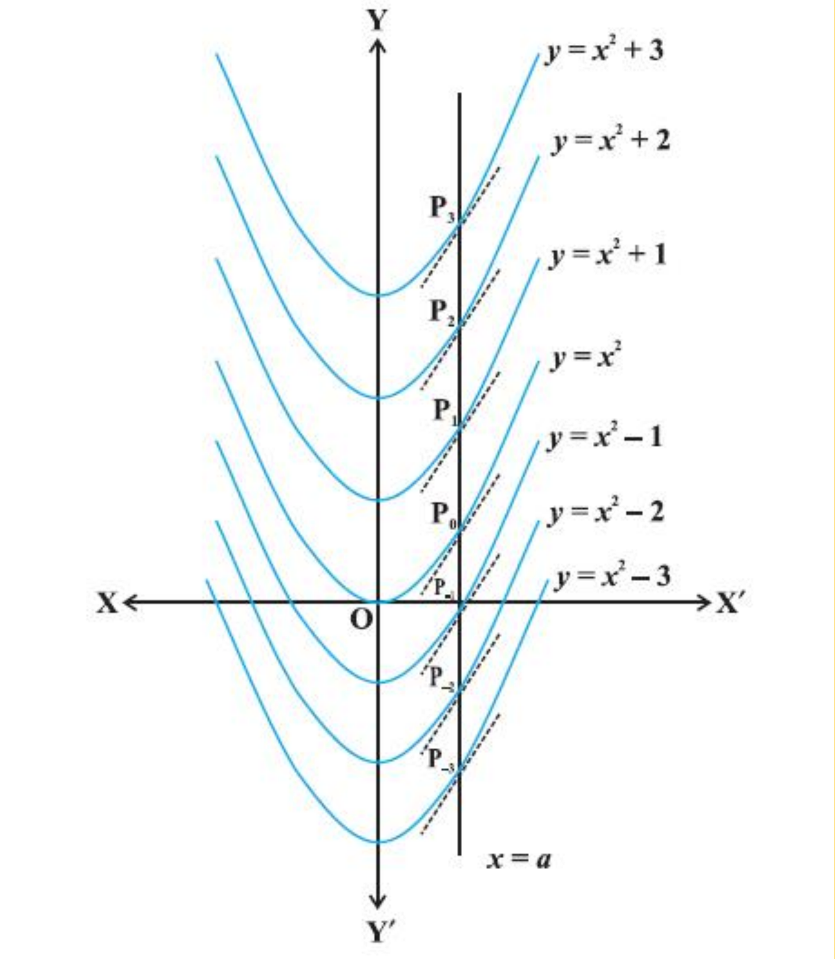
Geometrical Interpretation of Indefinite Integral

- Y=2x
- $\int 2x dx = 2 * x^2 / 2 = x^2$

- Integrating from 0 to 5, y=25

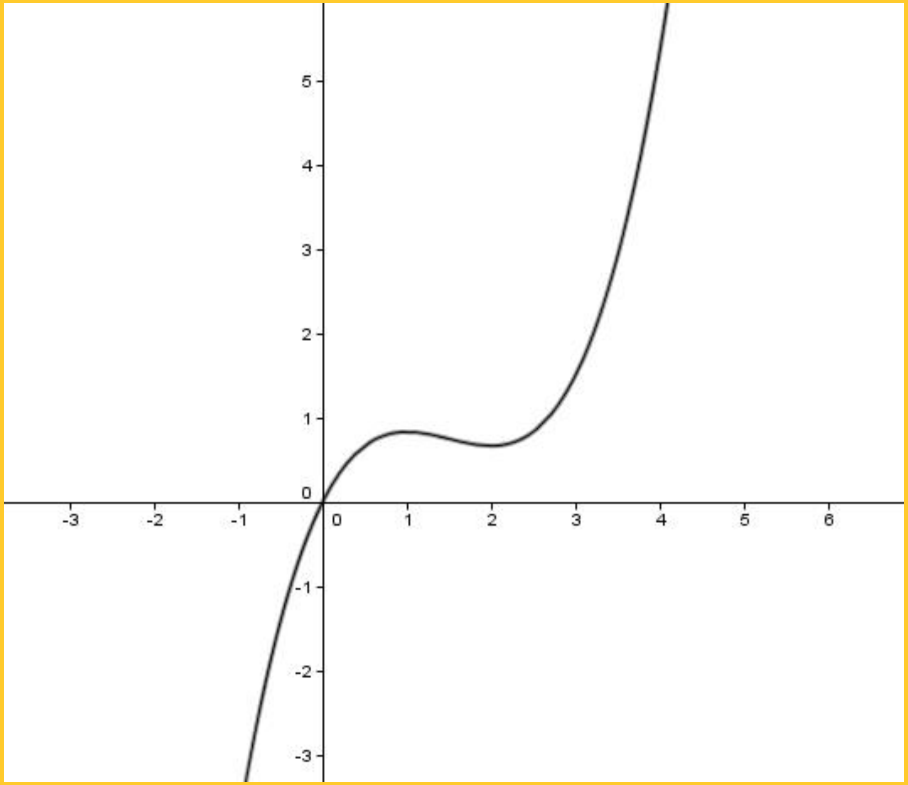


Geometrical Interpretation of Indefinite Integration



Using the Power rule, we can easily find the integrand of this function-

$$\int y\,dx = \int x^2 - 3x + 2\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 2x$$



Some more useful formulae in common use

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int e^x dx = e^x$$

$$\int \frac{1}{x} dx = \log x$$

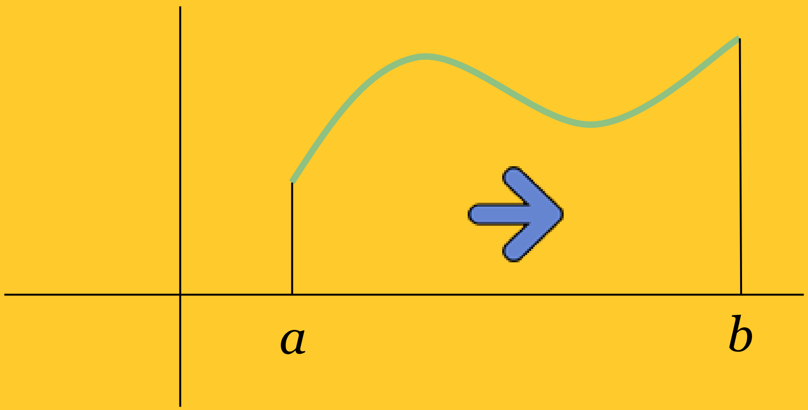
Methods of Integration

- Earlier function has been integrated by inspection.
- Some standard method of integration are:

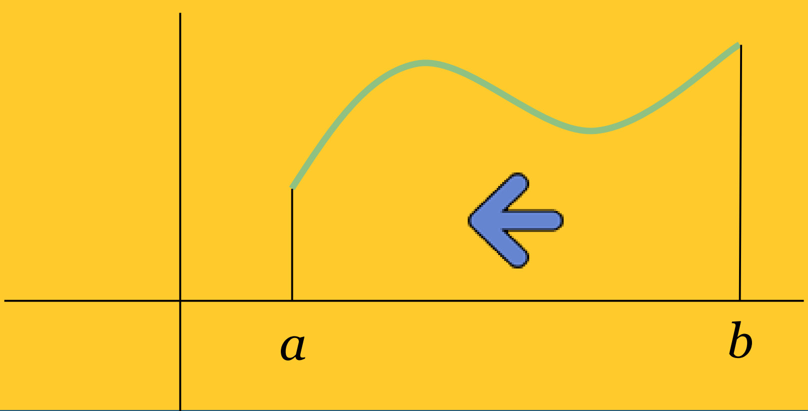
  - Integration by Substitution
  - Integration using Partial Fractions
  - Integration by Parts

Some Properties of Definite Integral

- Reversing the limits changes the sign.



$$\int_a^b f(x)dx = A$$

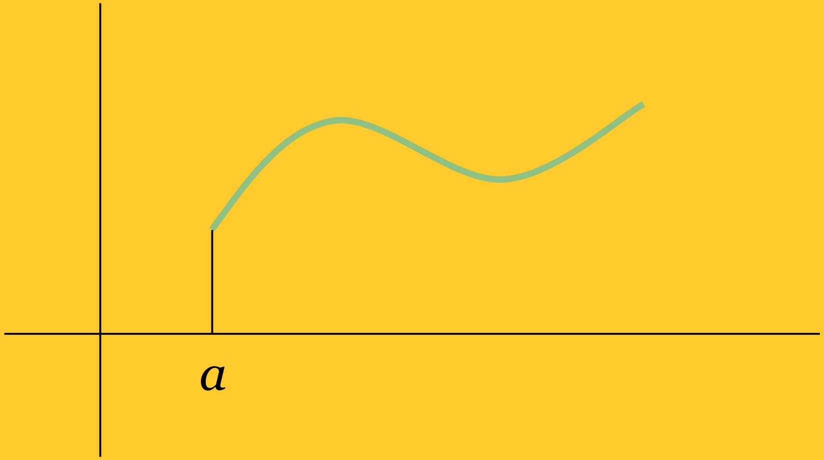


$$\int_b^a f(x)dx = -A$$

(Since the direction changes)

Some Properties of Definite Integral

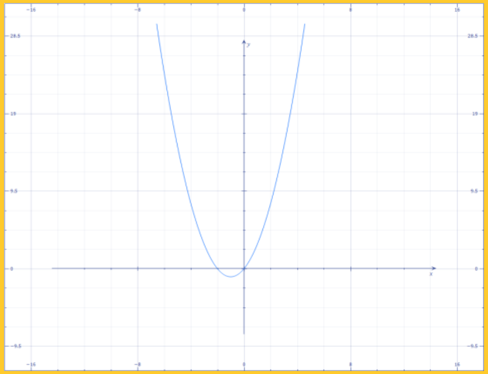
- If the lower limit and the upper limit are equal, then the integral is zero.



$$\int_a^a f(x)dx = 0$$

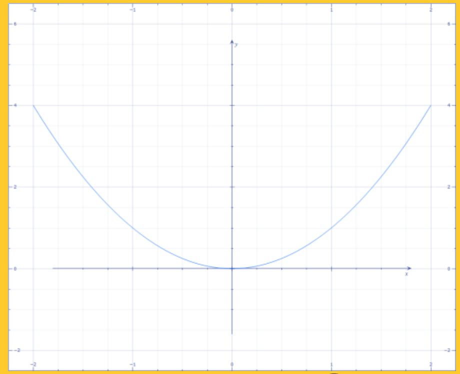
If the lower limit and the upper limit are equal, it means we have to find the area under a vertical straight line, which is obviously zero.

Let $y = x^2 + 2x$. Let us split and integrate this function.



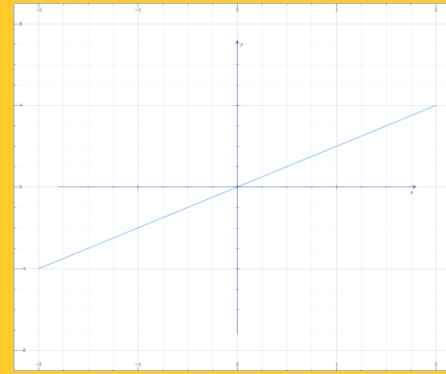
$y = x^2 + 2x$

=

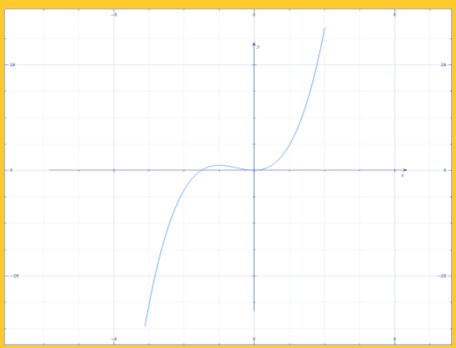


$y = x^2$

+



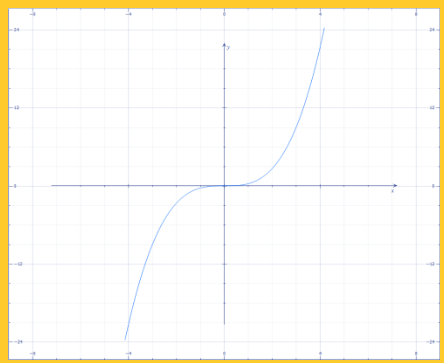
$y = 2x$



$\int y dx = \int (x^2 + 2x)\, dx$

$$y = \frac{x^3}{3} + x^2$$

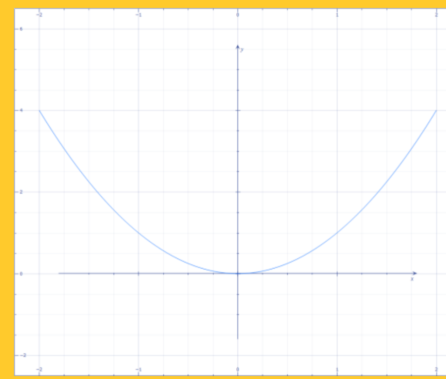
=



$\int y dx = \int x^2\, dx$

$$y = \frac{x^3}{3}$$

+



$\int y dx = \int 2x\, dx$

$$y = x^2$$

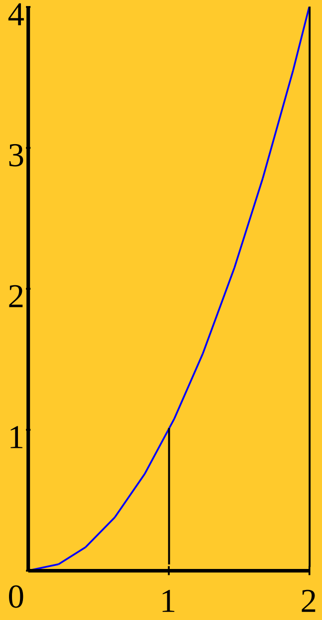
A few more properties of Definite Integrals

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$$

Area under the Curve

- Example: Find the area under the curve Y= x^2 from x=1 to x=2
- 

$$Area = \int_1^2 y dx = \int_1^2 x^2 dx = \frac{x^3}{3} = \frac{7}{3}$$

Q&A…

DIFFERENTIAL EQUATIONS

THE CONCEPT BEHIND..

What is a differential equation???

An equation which involves the derivatives of one or more independent variables w.r.t. one or more independent variables. For ex.

$$\frac{dy}{dx} = 5$$

Here, y'' is second derivative of y w.r.t. x
y' is first derivative of y w.r.t. x.
x^2y is function of x and y.

Put simply, *a differential equation states how a rate of change (a "derivative") in one variable is related to other variables.*

1st order differential equations

- 1st order ordinary differential equation is a relation between two variables and the first derivative of dependent variable w.r.t. the independent variable i.e. differential of *y* w.r.t. *x*.

$$y' = f(x, y)$$

for example,

$$\frac{dy}{dx} = x^2 + xy$$

- Here, dy/dx is the derivative of *y* w.r.t. *x* (slope), and $x^2 + xy$ is a function of *x* and *y*.

Solution to a simple equation

- We know that solution to an algebraic equation in one variable is the value(s) of the variable which can satisfy that equation.
  - For eg. The solution to the quadratic equation

$$x^2 - x - 6 = 0$$

is $x = 3$ or $x = -2$

Solution to a 'differential' equation

- In case of a differential equation, one would not get value(s) of the variables, rather a relation *between* the variables as a solution to the differential equation.
- Solving of a differential equation is by integration methods, hence it is an *algebraic equation* with arbitrary constants. Thus the general solution is a curve but its position on the *x-y plane* is not fixed.

Solution using slope fields

Another way to get solution curves for a *first-order differential equation* is by using the method of slope fields.

We consider a point on the plane (x_0,y_0). Then we find out y' at that point using the relation

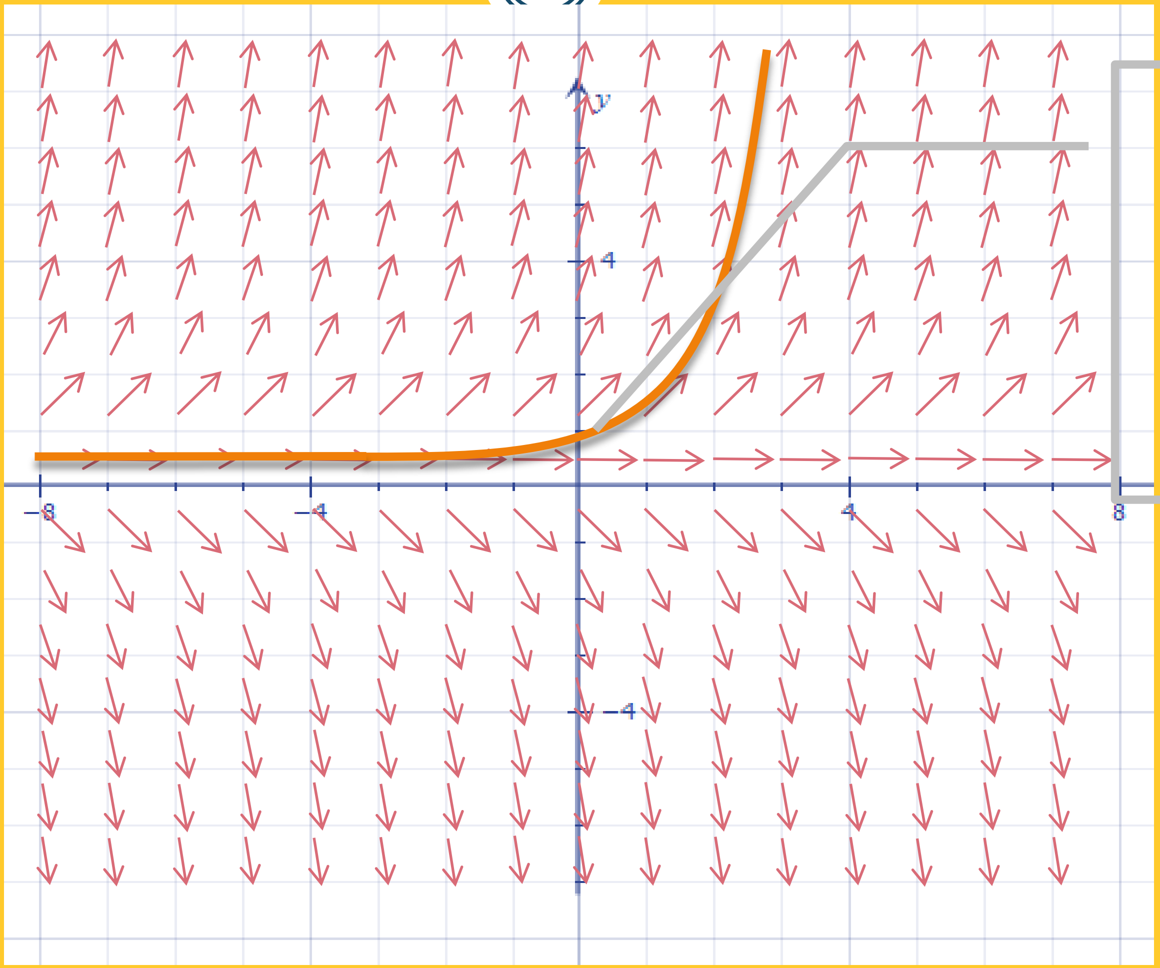
$$y'=f(x,y)$$

The solution curve will necessarily pass through the point (x_0,y_0) and will have slope equal to $y'=f(x_0,y_0)$ at that point.

We picturise this graphically by drawing short line segments of slope $f(x, y)$ at selected points (x, y) in the region of the xy-plane that constitutes the domain of f.

Each segment has the same slope as the solution curve through (x, y) and so is tangent to the curve there.

The resulting picture is called a slope field and gives a visualization of the general shape of the solution curves.



Solution curve passing through (0,1)
i.e. $y=e^x$

Differential Equation is dy/dx=y

There is another approach to this method.

Instead of considering many points to draw slope segments, we fix a value for y’ = k (constant) i.e. the slope of a segment. Then using the relation

$$y' = f(x,y)$$

we get

$$k = f(x,y)$$

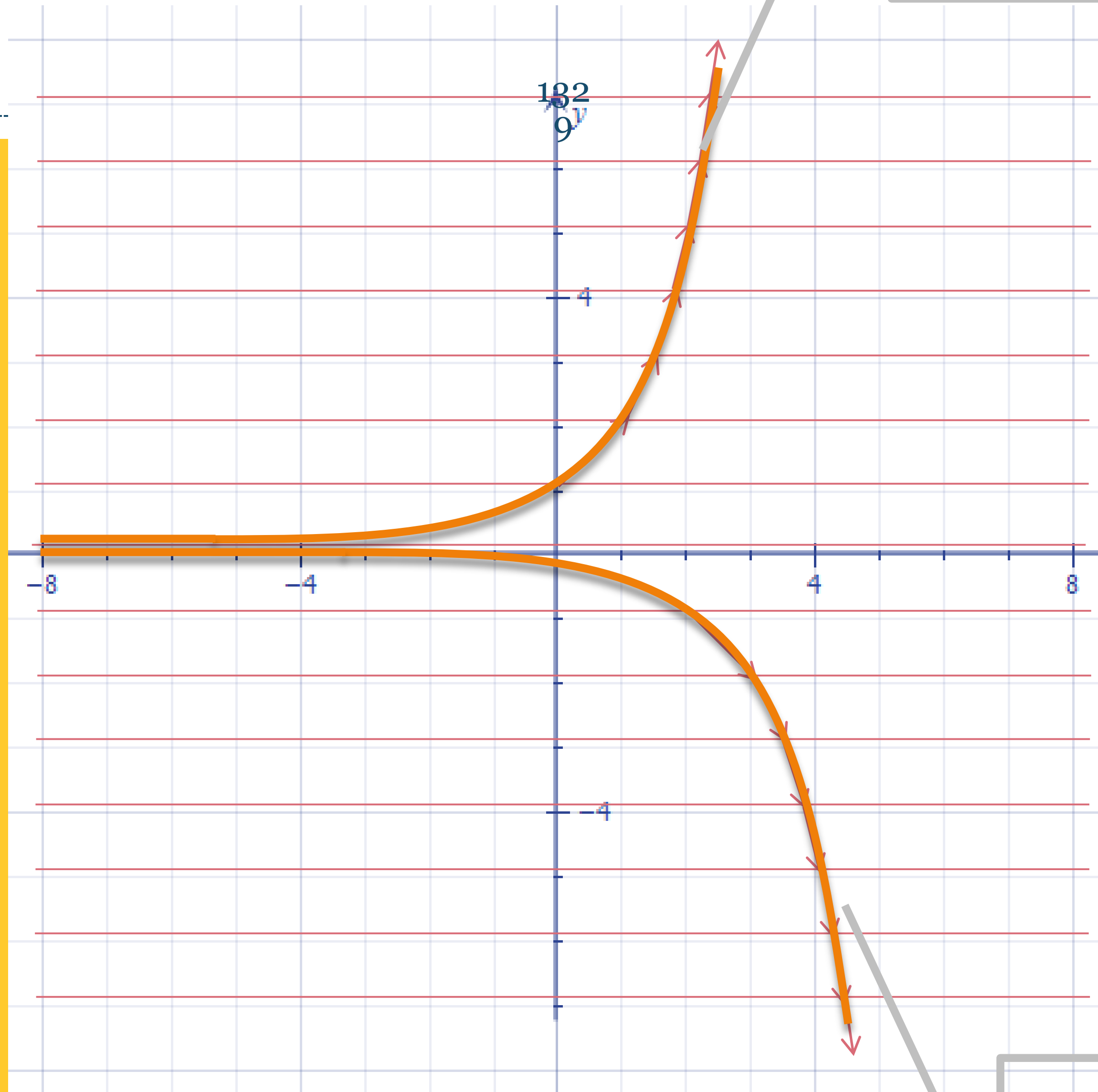
which represents a curve on the X-Y plane.

At the intersection of our solution curve with this isocline, the slope of solution curve must be equal to *k*.

So we start at a point on an isocline and draw a line segment up to next isocline with a slope equal to *k-value* of that isocline.

Continuing in this this manner, we can get a solution curve to our differential equation.

$y=e^x$



y = 7	slope constant = 7
y = 6	slope constant = 6
y = 5	slope constant = 5
y = 4	slope constant = 4
y = 3	slope constant = 3
y = 2	slope constant = 2
y = 1	slope constant = 1
y = 0	slope constant = 0
y = -1	slope constant = -1
y = -2	slope constant = -2
y = -3	slope constant = -3
y = -4	slope constant = -4
y = -5	slope constant = -5
y = -6	slope constant = -6
y = -7	slope constant = -7

Differential Equation is dy/dx=y

$y=-e^{x-2}$

Initial Conditions

- In many physical problems we need to find the particular solution that satisfies a condition of the form $y(x_o)=y_o$. This is called an *initial condition*, and the problem of finding a solution of the differential equation that satisfies the initial condition is called an *initial-value problem*.
- *Example:* Find a solution to $y^2 = x^2 + C$ satisfying the initial condition $y(0) = 2$.

$$2^2 = 0^2 + C$$

this implies $C = 4$

therefore, $y^2 = x^2 + 4$ is the particular solution

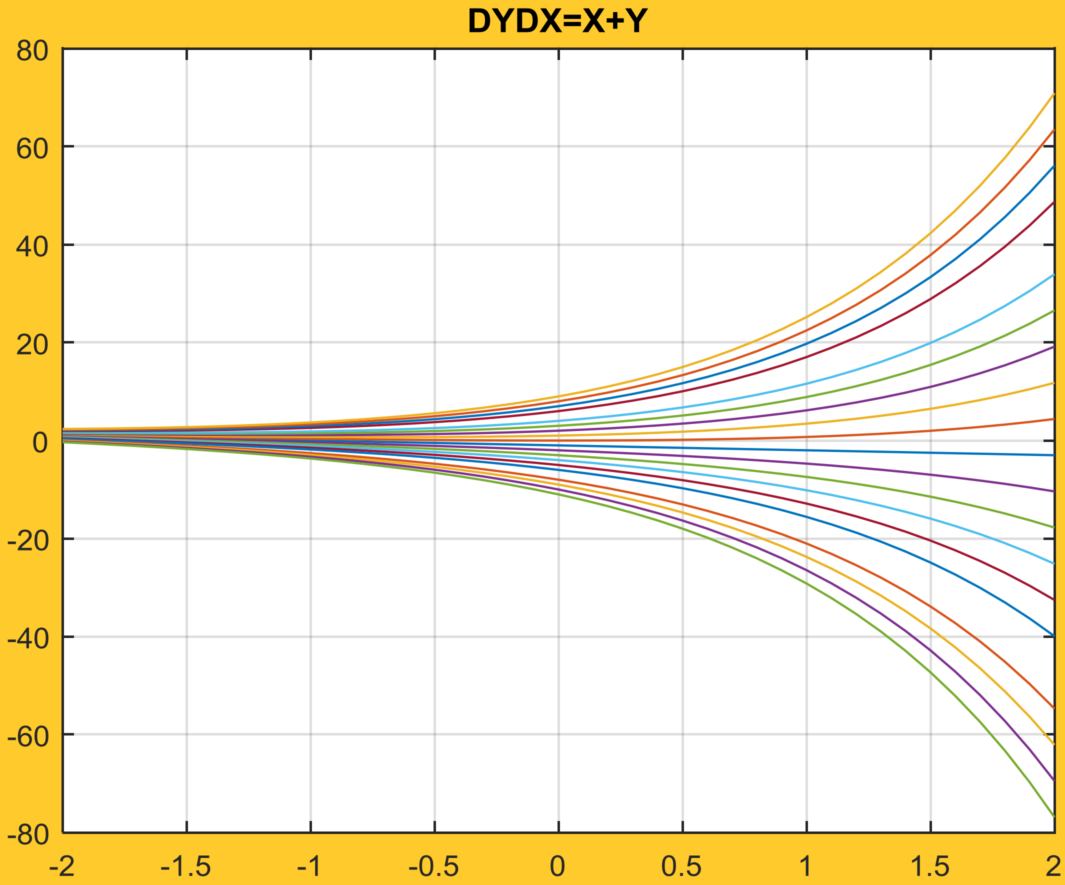
Solving DE in Matlab:: dy/dx=x+y

- syms y(x)
- ode = diff(y,x) = x+y
- ySol = dsolve(ode)
- C4=-10:1:10
- d=C4.*exp(x) - x - 1
- x=-2:.1:2
- y20=- x - 10.*exp(x) - 1
- y19=- x - 9.*exp(x) - 1
- y18=- x - 8.*exp(x) - 1
- y17=- x - 7.*exp(x) - 1
- y16=- x - 6.*exp(x) - 1
- y15 =- x - 5.*exp(x) - 1

Solving DE in Matlab:: dy/dx=x+y

- y7 =- x +7.*exp(x) - 1
- y8 =- x +8.*exp(x) - 1
- y9=- x +9.*exp(x) - 1
- y10 =- x +10.*exp(x) - 1
- plot(x,y1,x,y2,x,y3,x,y4,x,y5,x,y6,x,y7,x,y8,x,y9,x,y10, x,y11,x,y12,x,y13,x,y14,x,y15,x,y17,x,y18,x,y19,x,y20)
- grid

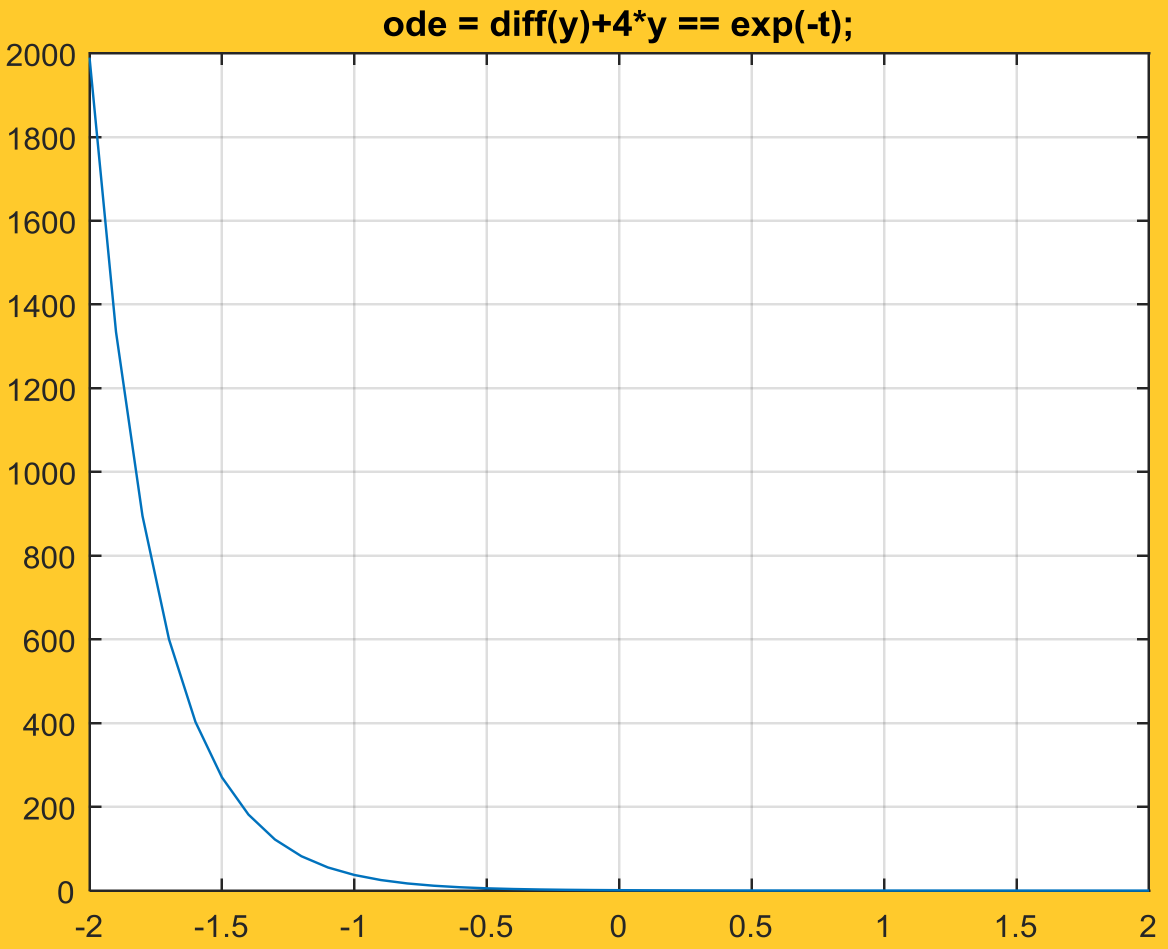
Solving DE in Matlab:: dy/dx=x+y



Initial Condition:: $\frac{dy}{dt}+4y(t)=e^{-t}$, $y(0)=1$

- syms y(t)
- ode = diff(y)+4*y == exp(-t);
- cond = y(0) == 1;
- ySol(t) = dsolve(ode,cond)
-
- t=-2:.1:2
- d=exp(-t)/3 + (2*exp(-4*t))/3
- plot(t,d)

Initial Condition:: $\frac{dy}{dt}+4y(t)=e^{-t}$, $y(0)=1$



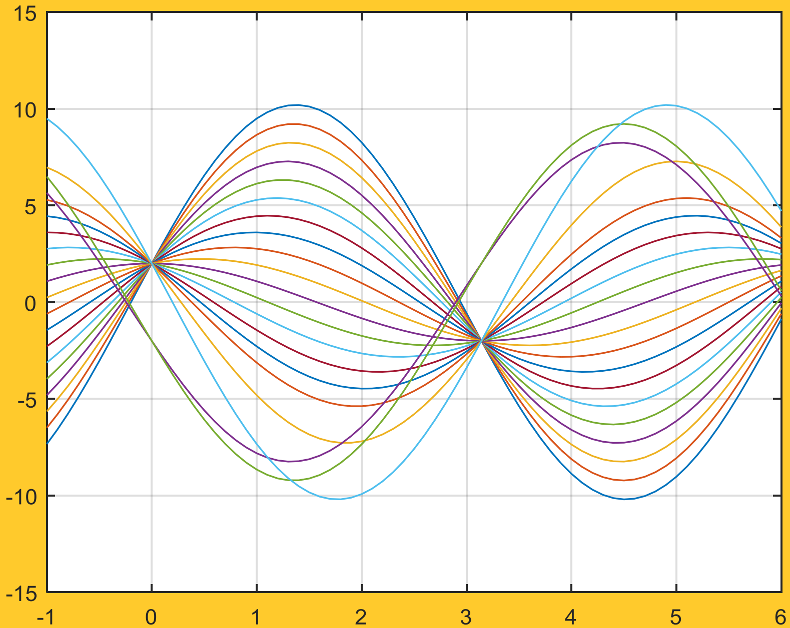
Solving DE in Matlab:: d2y/dx2=-y

- syms y(x)
- ode = diff(y,x,2) == -y
- ySol = dsolve(ode)
- C6=2
- C7=-10:1:10
- d =[2*cos(x) - 10*sin(x), 2*cos(x) - 9*sin(x), 2*cos(x) - 8*sin(x), 2*cos(x) - 7*sin(x), 2*cos(x) - 6*sin(x), 2*cos(x) - 5*sin(x), 2*cos(x) - 4*sin(x), 2*cos(x) - 3*sin(x), 2*cos(x) - 2*sin(x), 2*cos(x) - sin(x), 2*cos(x), 2*cos(x) + sin(x), 2*cos(x) + 2*sin(x), 2*cos(x) + 3*sin(x), 2*cos(x) + 4*sin(x), 2*cos(x) + 5*sin(x), 2*cos(x) + 6*sin(x), 2*cos(x) + 7*sin(x), 2*cos(x) + 8*sin(x), 2*cos(x) + 9*sin(x), 2*cos(x) + 10*sin(x)]
-
-
- x=-1:.1:6
- y20=2.*cos(x) - 10.*sin(x)
- y19=- 2.*cos(x) - 9.*sin(x)
- y18=- 2.*cos(x) - 8.*sin(x)
- y17=2.*cos(x) - 7.*sin(x)
- y16=2.*cos(x) - 6.*sin(x)
- y15= 2.*cos(x) - 5.*sin(x)
- y14= 2.*cos(x) - 4.*sin(x)
- y13= 2.*cos(x) - 3.*sin(x)
- y12= 2.*cos(x) - 2.*sin(x)
- y11=2.*cos(x) - 1.*sin(x)

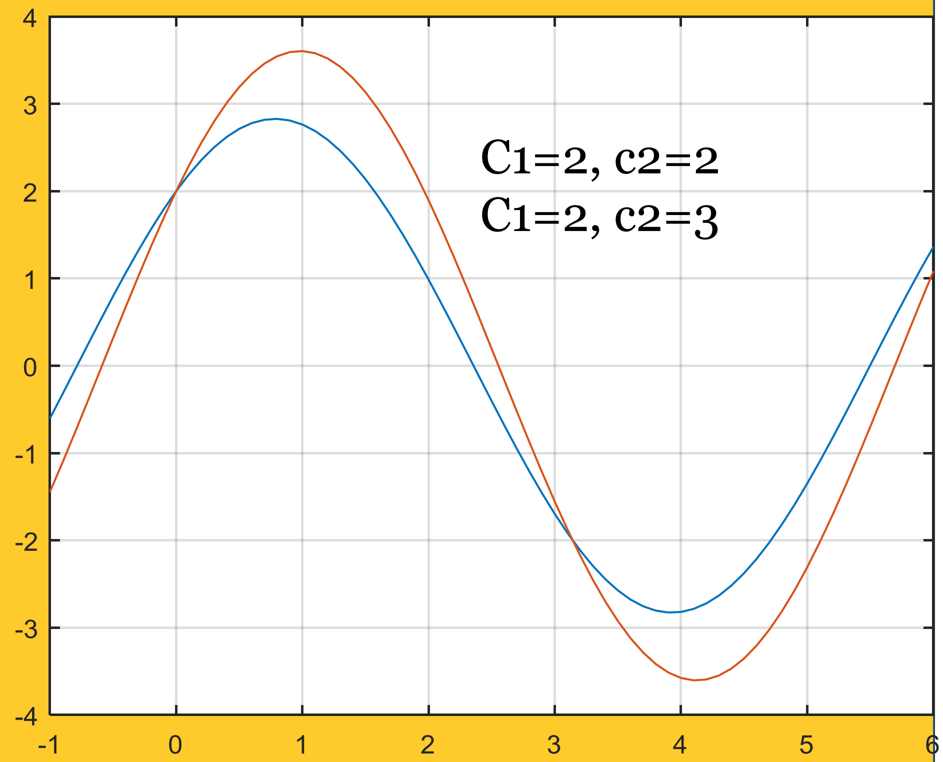
Solving DE in Matlab:: d2y/dx2=-y

- y10=2.*cos(x) - 0.*sin(x)
- y9=2.*cos(x) +1.*sin(x)
- y8= 2.*cos(x) +2.*sin(x)
- y7= 2.*cos(x) +3.*sin(x)
- y6= 2.*cos(x) +4.*sin(x)
- y5= 2.*cos(x) +5.*sin(x)
- y4=2.*cos(x) +6.*sin(x)
- y3=2.*cos(x) +7.*sin(x)
- y2= 2.*cos(x) +8.*sin(x)
- y1= 2.*cos(x) +9 .*sin(x)
- y0= 2.*cos(x) +10.*sin(x)
-
-
- plot(x,y0,x,y1,x,y2,x,y3,x,y4,x,y5,x,y6,x,y7,x,y8,x,y9,x,y10,x,y11,x,y12,x,y13,x,y14,x,y15,x,y17,x,y18,x,y19,x,y20)
-
- %plot(t,d(1),t,d(2), t,x3,t,x4,t,x5,t,x6,t,x7,t,x8,t,x9,t,x10,t,x11,t,x12,t,x13,t,x14,t,x15,t,x16,t,x17,t,x18,t,x19,t,x20)
- grid

d2y/dx2=-y



C1=2, c2=-10:10



$2x2d2ydx2+3xdydx-y=0.$

$2x2d2ydx2+3xdydx-y=0.$

2x2d2ydx2+3xdydx−y=0.

Resources and Reference

- Mathematical Elements for Computer Graphics

(2nd Edition) *by David F. Rogers and J. Alan Adams*

- NCERT Books
- Microsoft Maths Software
- Microsoft Excel
- www.ocw.mit.edu
- www.wikipedia.com

Sequence and Series

Very Important branch of Mathematics

Binomial Theorem

Why we study Binomial Theorem?????????????????????????

$(a+b)^2=a^2+2ab+b^2$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

>It is easy to calculate. But if we require to calculate $(a+b)^{59}$ or any other power of (a+b), how can we proceed???

>Binomial Theorem helps in these situations.

>Binomial theorem enable us to recognize the pattern hidden behind many mathematical problems.

Series

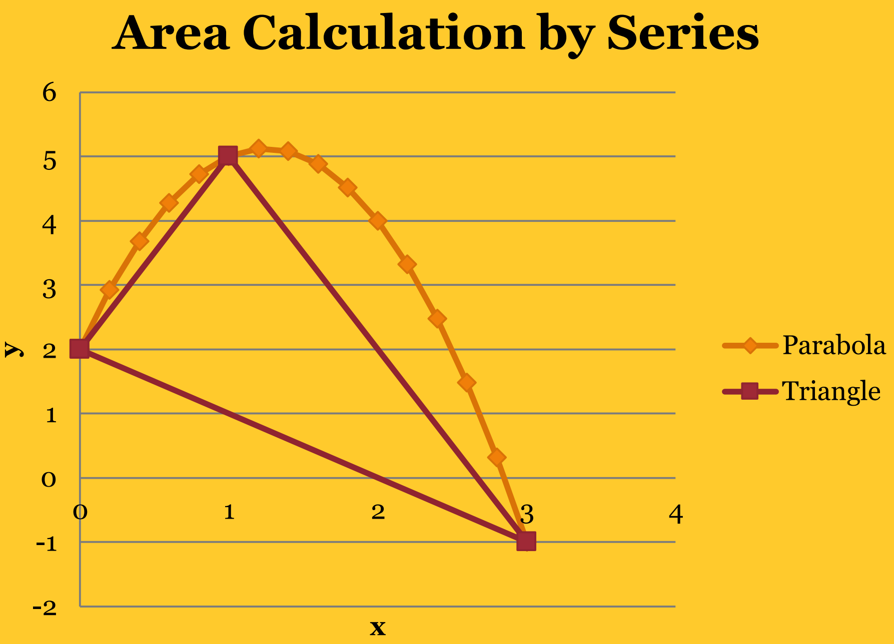
- History: **Archimedes of Syracuse** 287 BC – 212 BC
- In *The Quadrature of the Parabola*, Archimedes proved that the area enclosed by a parabola and a straight line is $4/3$ times the area of a corresponding inscribed triangle . He expressed the solution to the problem as an infinite geometric series with the common ratio $1/4$:
- If the first term in this series is the area of the triangle, then the second is the sum of the areas of two triangles whose bases are the two smaller secant lines, and so on. This proof uses a variation of the series $1/4 + 1/16 + 1/64 + 1/256 + \cdots$ which sums to $1/3$.
- Note: http://en.wikipedia.org/wiki/Archimedes

$\sum_0^n 1/4\hat{}n$

Archimedes Area Calculation

$a_n = 1/4^{(n-1)}$

n	p	a_n	s_n
1	0	1	1
2	1	0.25	1.25
3	2	0.0625	1.3125
4	3	0.015625	1.328125
5	4	0.003906	1.332031
6	5	0.000977	1.333008
7	6	0.000244	1.333252



Factorial
Permutation & Combination

>**Factorial** (n!) Definition: Factorial(**n**) is the product of all positive integers less than **n.**

n!=n*(n-1)*(n-2)*(n-3)-------3*2*1

>**Event**: A thing that happens or takes place.

>**Counting**: If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of occurrence of the event in the given order is m*n.

>**Permutation:** Definition: Counting the number of ways in which some or all objects can **be arranged** at a time.

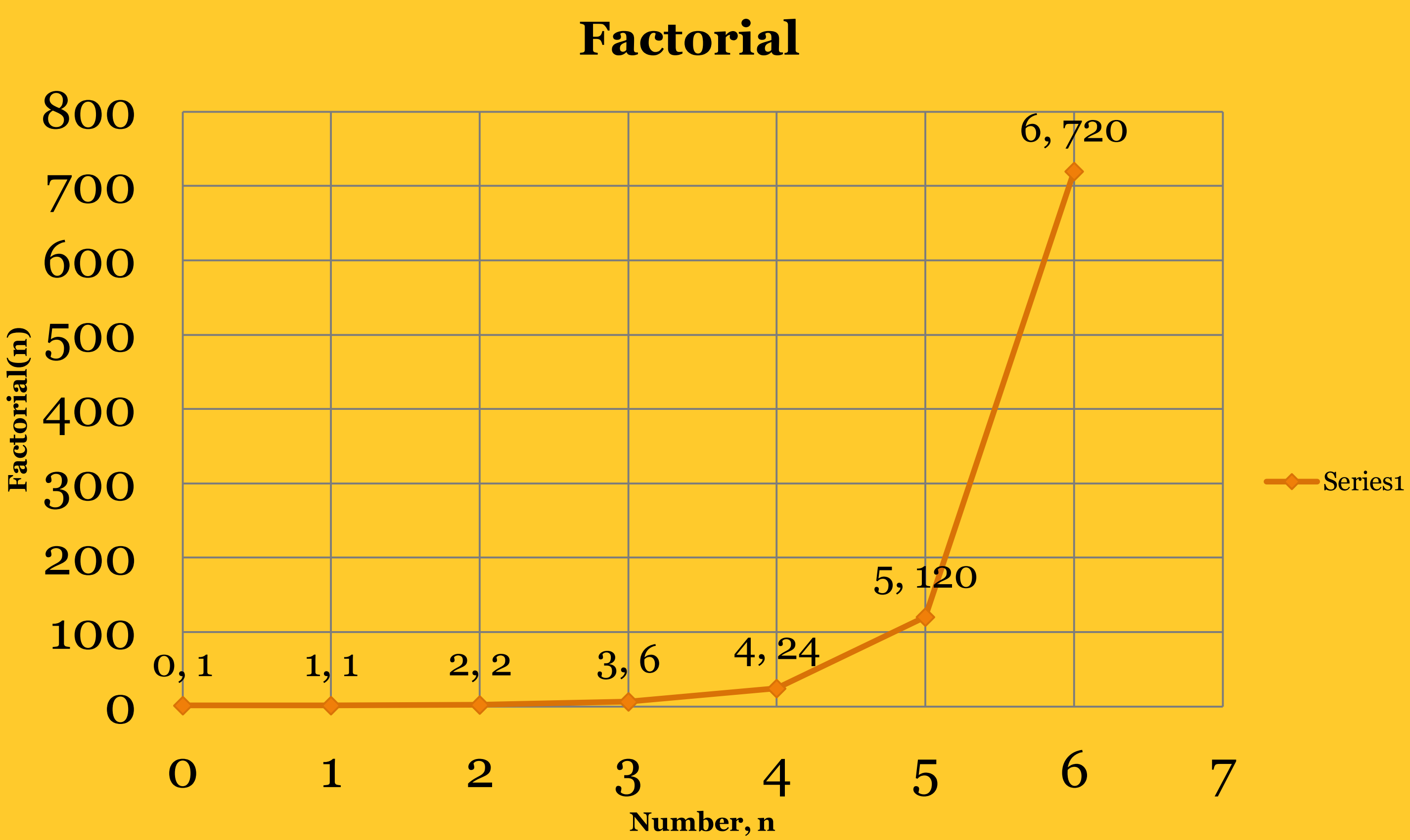
>**Permutation**, $^{n}P_{r}$ **=Factorial(n)/Factorial(n-r)**

>**Combination:** Definition: Counting number of ways in which fixed number of objects (r) can **be chosen** from (n) objects,

Combination, $^{n}C_{r}$ = **Factorial(n)/Factorial(n-r)*(Factorial(r)**

Factorial:

Factorial(n) is the product of all integers less than n



n	fact(n)
0	1
1	1
2	2
3	6
4	24
5	120
6	720

Factorial Permutation, Combination

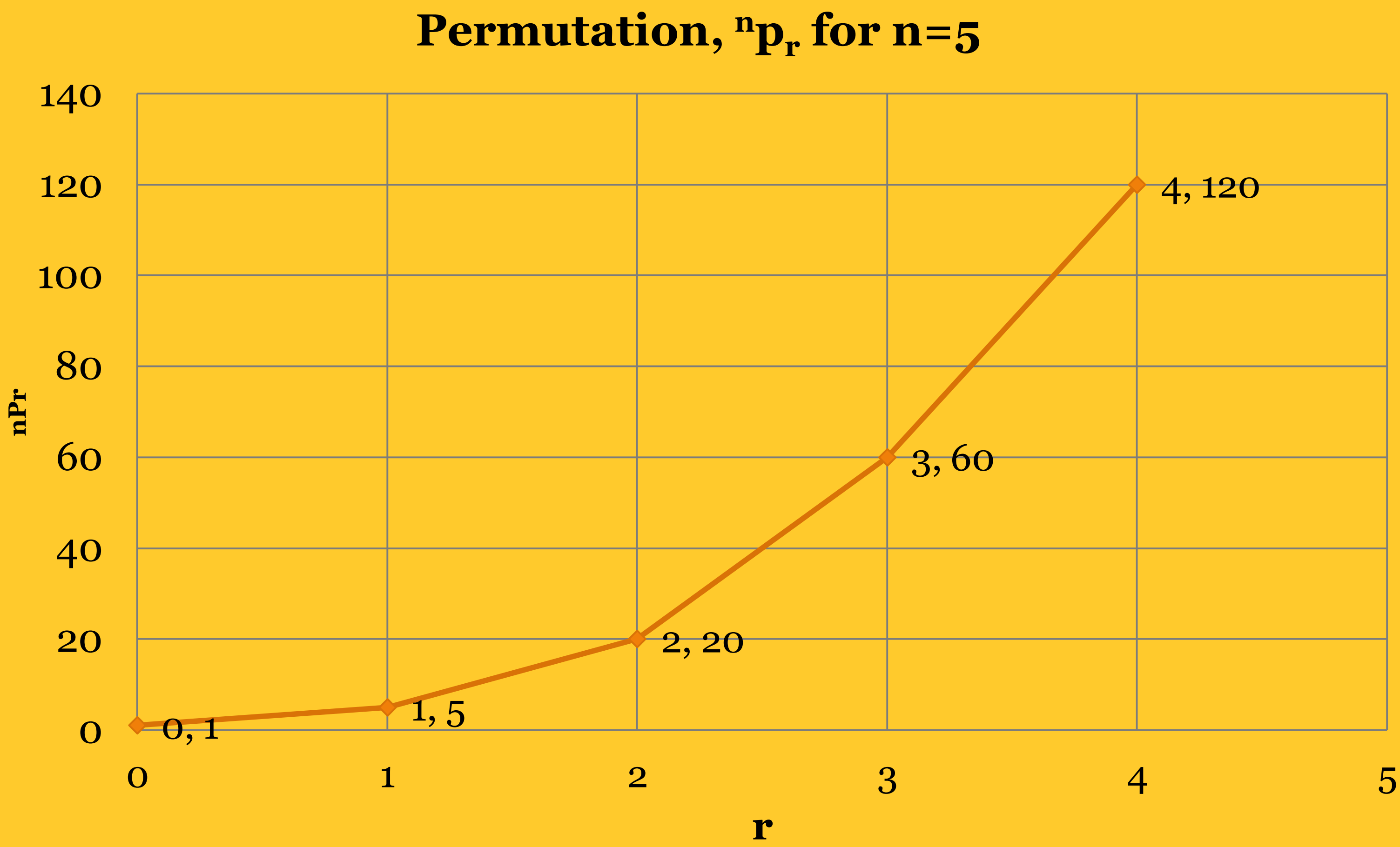
			n=	**5**		**n=**	**5**
Number(n)	**Factorial (n)**		**r**	**nPr=n!/(n-r)!**		**r**	**nCr=n!/(n-r)!r!**
0	1		0	1		0	1
1	1		1	5		1	5
2	2		2	20		2	10
3	6		3	60		3	10
4	24		4	120		4	5
5	120		5	120		5	1
6	720		6	#NUM!		6	#NUM!
7	5040		7	#NUM!		7	#NUM!
8	40320		8	#NUM!		8	#NUM!

With replacement

$5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3125$

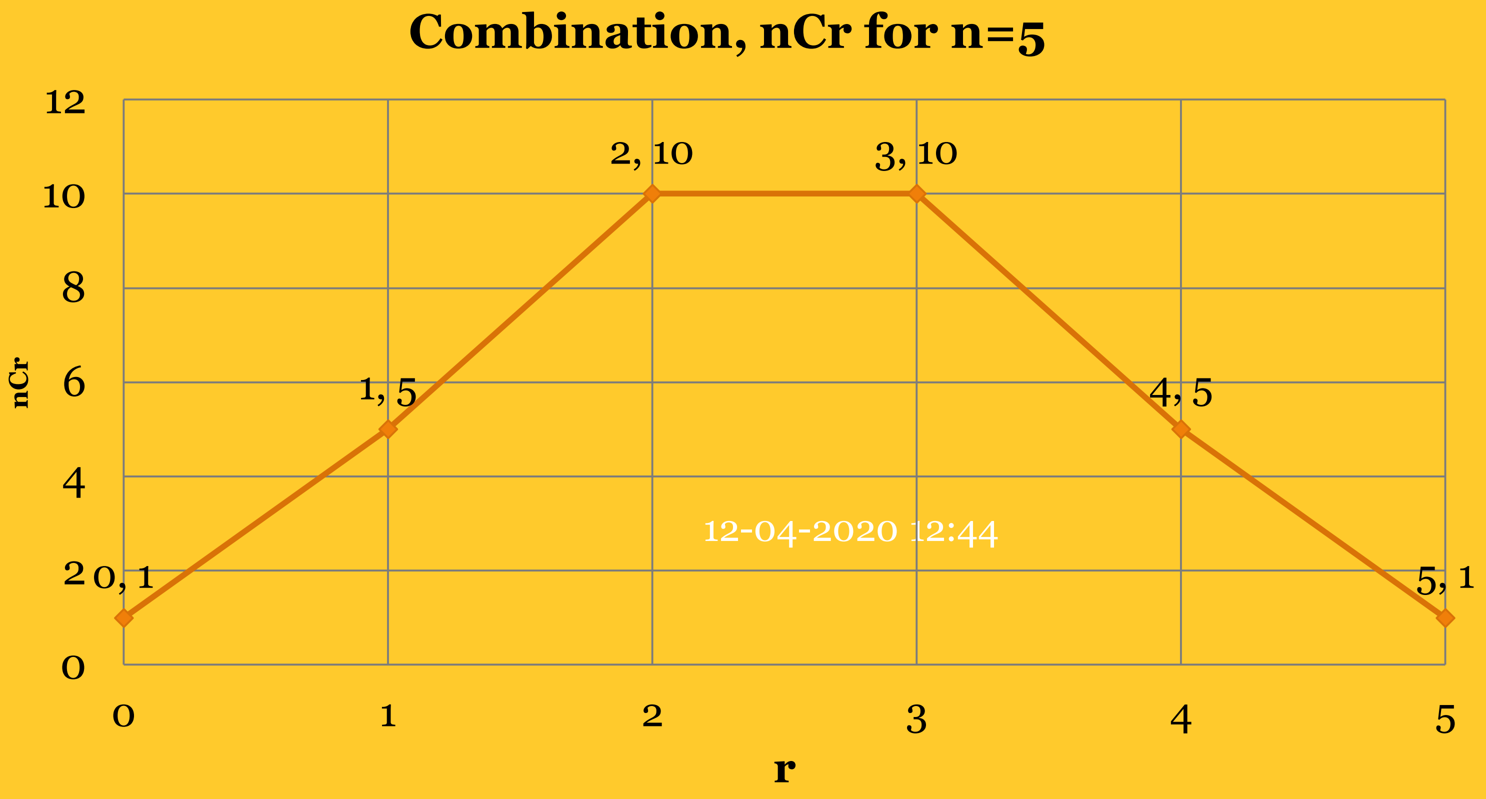
Permutation

$^{n}P_{r}$ =Factorial(n)/Factorial(n-r)



Combination

^nC_r = Factorial(n)/Factorial(n-r)*(Factorial(r)



Binomial Theorem

>Definition: Binomial theorem deals with the algebraic expression generated by the expansion of powers(n) to the binomial (a+b).

>The power (n) of the binomial can be 0, 1, 2, 3,……………

Case-1: Power(n=0): (a+b)^0 =1

Case-1: Power(n=1): (a+b)^1 =1, 1

Case-1: Power(n=2): (a+b)^2 =1, 2, 1

Case-1: Power(n=3): (a+b)^3 =1, 3, 3, 1

Case-1: Power(n=4): (a+b)^4 =1, 4, 6, 4,1

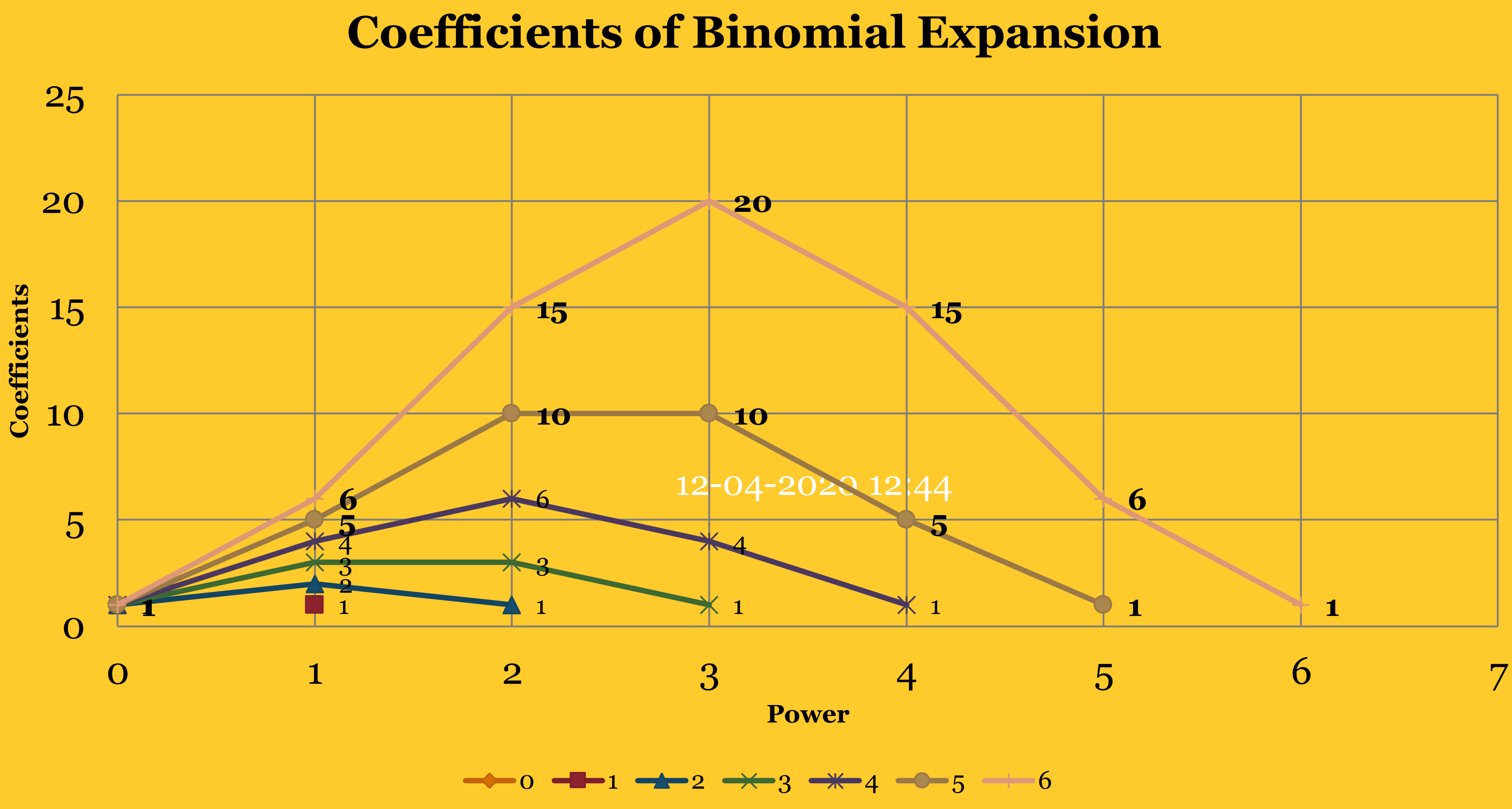
Case-1: Power(n=5): (a+b)^5 =1, 5, 10, 10, 5, 1

Case-1: Power(n=n): (a+b)^n = ${}^{n}c_{0}$, ${}^{n}c_{1}$, ${}^{n}c_{2}$, ${}^{n}c_{3}$, ${}^{n}c_{4}$…. ${}^{n}c_{(n-2)}$ ${}^{n}c_{(n-1)}$ ${}^{n}c_{n}$

(Only coefficients of the expansion considered)

Coefficients of Binomial Expansion

13



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Coefficients of Binomial Expansion

n=	7	10
r	nCr	nCr
0	1	1
1	7	10
2	21	45
3	35	120
4	35	210
5	21	252
6	7	210
7	1	120
8	#NUM!	45
9	#NUM!	10
10	#NUM!	1

>It is seen from the expansion that the coefficients of each term formed a sequence and the sum of the sequence forms a finite series.

>To understand the characteristics of the binomial theorem, it is better to understand the sequences and series.

>Binomial Expansion in sigma form:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Sequence, Series, AP, GP

>**Sequence**- Sequence is a collection of ordered objects.

>**Terms**- Each object in the sequence is called terms

Note-It is possible to express the rule which yields various terms of a sequence in terms of an algebraic formula.

>**Series**- Sum of the terms of sequence is called series.

>**Partial Sum**-When we add few terms of a sequence, we get Partial Sum.

>Series is expressed in sigma notation as, S= $\sum_{n=1}^{k} an$

Sequence, Series, AP

>**Arithmetic Progression** (AP) or Sequence is a sequence whose difference between consecutive terms is constant, called common difference.

>**Common Difference**, d=difference between consecutive terms.

>**General Terms of AP**, $a_n = a_1 + (n-1)*d$

>Note: If a constant is added/ subtracted/ multiplied/ divided to each term of an AP, the resulting sequence is also an AP.

>Let, first term=a, Last term=l, common difference=d, and no. Of terms=n, $Sn = \frac{n}{2}(a+l)$

$$l = a + (n-1)d$$

Then,

$$Sn = \frac{n}{2}(a + a + (n-1)d)$$ and $$Sn = \frac{n}{2}(2a + (n-1)d)$$

Term	1	2	3	4	5	6	7
a_n	3	5	7	9	11	13	15

Sequence, Series, AP

>**Arithmetic Mean(AM)**-AM is the number between two terms of a sequence. Let, c is the arithmetic mean between two terms a and b, then c-a=b-c,

Or, AM= $c = \frac{(a+b)}{2}$

>We can insert any number (n) between two terms. The formula is

$$an = a + \frac{n(b-a)}{n+1}$$

Sequence, Series, GP

>**Geometric Progression** (GP) is a sequence where each term except the first term bears a constant ratio to the term immediately preceding it.

>Common Difference,

$$r = \frac{an + 1}{an}$$

Term	1	2	3	4	5	6	7
Power	0	1	2	3	4	5	6
a_n	2	4	8	16	32	64	128

>

$$an = a * r^{(n-1)}$$

>

$$Sn = \frac{a(r^n - 1)}{(r - 1)}$$

Sequence, Series, GP

>**Geometric Mean(GM):** GM is a number, c, between two consecutive terms , a and c, of a GP is given by

$$c = \sqrt{ab}$$

>We can insert any number (n) between two terms. The formula is

$$b = ar^{n+1}$$

$$an = a(\frac{b}{a})^\wedge(n/(n+1)$$

Sequence, Series, AM, GM

>Relation between AM and GM:

AM=(a+b)/2

GM=✓ab

Observation: (AM-GM) is always positive, so AM is always greater than GM

Sequence, Series
Convergence, Divergence

What is the importance of the Study of the Series?????????????????????

Ans: Many functions can be expressed as infinite series of Polynomials. When a function can be expressed as a polynomial, it becomes easy to differentiate or integrate the function. Infinite series has many other uses also.

Term, n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
a_n	1	2	3	4	5	6	7	8
S_n	1	3	6	10	15	21	28	36

> We are concerned to understand at what condition the series converges or diverges.

Sequence, Series Convergence

➢The series is termed as convergent if its partial sum has a FINITE limit, L.

➢If partial sum tends to the infinity, then the series is called Divergent.

➢Partial Sum of some divergent series oscillates and called indeterminate series.

➢Necessary condition for Convergence – The general term, a_n , tend to 0. But this condition is not sufficient.

➢Positive series is one whose all terms are positive, is either divergent or convergent. It can not be indeterminate.

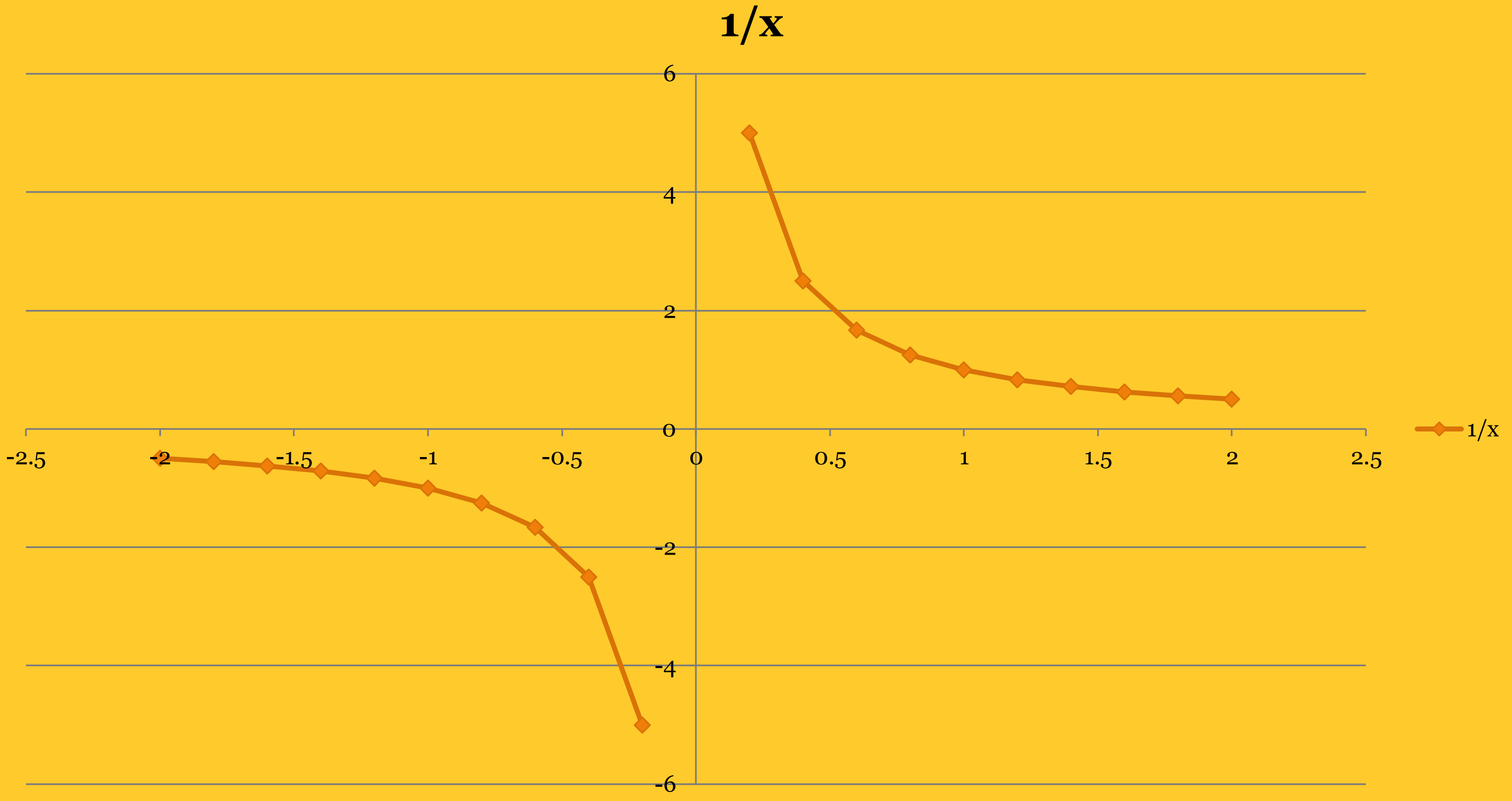
Sequence, Positive Series Convergence

D'Alembert's Theory- If the ratio of the following term to the preceding term tends to c, then, the series converges for $c<1$, diverges for $c>1$ and either diverge or converge for $c=1$.

>**Integral Test**-If every term of a positive series is less than its preceding term, then for convergence, we can consider improper integral. If improper integral converges, the series converges and vice versa.

In calculus, an **improper integral** is the limit of a definite integral as an endpoint of the interval(s) of integration approaches either a specified real number or ∞ or $-\infty$ or, in some cases, as both endpoints approach limits. An integral without upper and lower limits, called an indefinite integral.

Convergence



Sequence, Series
Absolute Convergence

A series converges definitely if the positive series composed of the absolute values of the term converges.

Sequence, Series
Effect of Rearrangement of Terms on Convergence

Rearrangement of terms of an absolute convergent series do not upset the series and the sum of the series remains unchanged.

Absolute convergent series follows commutative as well as associative laws.

Binomial Expansion

➢Finding coefficients of binomial expansion from Pascal's Triangle becomes difficult for expansion of binomials involving higher power

➢Binomial Expansion in sigma form:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Binomial Theorem:

$(a+x)^n = {}^nc_0 * a\text{^}(n) * x\text{^}0 + {}^nc_1 * a\text{^}(n-1) * x\text{^}1 + {}^nc_2 * a\text{^}(n-2) * x\text{^}2$.........

1. General Term, $a_n = {}^nc_r * a\text{^}(n-r) * x\text{^}r$,

2. This formula is valid for any positive integer

Binomial Series

➢A sequence $a_0, a_1, a_2, a_3,\ldots\ldots a_n$, having infinite number of terms is called infinite sequence and the sum of the sequence $a_0 + a_1 + a_2 + a_3\ldots\ldots + a_n$ is called infinite series.

Binomial Theorem: $(a+x)^n = {}^nc_0 * a^{\wedge(n)} * x^{\wedge 0} + {}^nc_1 * a^{\wedge(n-1)} * x^{\wedge 1} + {}^nc_2 * a^{\wedge(n-2)} * x^{\wedge 2}\ldots\ldots$

1. General Term, $a_n = {}^nc_r * a^{\wedge(n-r)} * x^{\wedge r}$,

2. This formula is valid for any positive integer

3. If a=1, the binomial expansion, can be written as:

$(a+x)^n = {}^nc_0 * a^{\wedge(n)} * x^{\wedge 0} + {}^nc_1 * a^{\wedge(n-1)} * x^{\wedge 1} + {}^nc_2 * a^{\wedge(n-2)} * x^{\wedge 2}\ldots\ldots$

$(1+x)^n = {}^nc_0 * 1^{\wedge(n)} * x^{\wedge 0} + {}^nc_1 * 1^{\wedge(n-1)} * x^{\wedge 1} + {}^nc_2 * 1^{\wedge(n-2)} * x^{\wedge 2}\ldots\ldots$

$(1+x)^n = {}^nc_0 + {}^nc_{1*} x^1 + {}^nc_2 * x^{\wedge 2}\ldots\ldots$

$(1+x)^n = 1 + n_* x^1 + n(n-1)/2! * x^{\wedge 2} + n(n-1)(n-2)/3! * x^{\wedge 3}\ldots\ldots$

$(1+x)^n = 1 + n_* x + n(n-1)/2! * x^{\wedge 2} + n(n-1)(n-2)/3! * x^{\wedge 3}\ldots\ldots$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

3. If a=1 and |x|<1 then the power series converges.

Binomial Series

- **Binomial Theorem is**

$(a+x)^n = {}^nc_0 * a^{(n)} * x^0 + {}^nc_1 * a^{(n-1)} * x^1 + {}^nc_2 * a^{(n-2)} * x^2$.........

>General term in the expansion:

$(a+x)^n = n(n-1)(n-2)(n-r+1) * a^{(n-r)} * x^r / r!$

>If a=1 and |x|<1 then the power series converges. In this case the power series can be written as :

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

Now let us consider the following cases:

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5$..............$= 1/(1+x)$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + x^5$..............$= 1/(1-x)$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5$..............$= 1/(1+x)^2$

$(1-x)^{-2} = 1 + x + x^2 + x^3 + x^4 + x^5$..............$= 1/(1-x)^2$

POWER SERIES

➢Binomial Series:

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

➢Pascal first established this formula for positive integers.
➢ Newton applied this formula for negative and fractional values of n.
➢ It is found that many functions can be expressed in terms of binomial series as given below:

$1/(1+x) = 1-x+x^2-x^3+x^4-x^5$...............
$1/(1-x)=1+x+x^2+x^3+x^4+x^5$.............
$1/(1+x)^2=1-2x+3x^2-4x^3+5x^4-6x^5$.............
$1/(1-x)^2=1+2x+3x^2+4x^3+5x^4+6x^5$.............

POWER SERIES

- Suppose we have a function as given below:

  $y=1/(1+x)$

- Its derivative is:

  $dy/dx= -1/(x + 1)^2$

We know that expansion of $y =1/(1+x)$ is given by:

- $y=+ 1 - x + x^2 - x^3+ x^4 - x^5$.

- If we termwise differentiate the above series, we get,

- $dy/dx= - 1 + 2*x - 3*x^2 + 4*x^3 - 5*x^4 +6*x^5$

- Again we know that :

$1/(1+x)^2=1-2x+3x^2-4x^3+5x^4-6x^5$.............

Hence, from binomial expansion, we can get the derivative of a function without differentiation of the function.

Taylor Series


```
syms x t
taylor(exp(-x))
    taylor(log(x),6,1)
    taylor(sin(x),  6)

    taylor(x^t,3,t)
```

Exp(-x) =- x^5/120 + x^4/24 - x^3/6 + x^2/2 - x + 1

Log(x) =x - (x - 1)^2/2 + (x - 1)^3/3 - (x - 1)^4/4 + (x - 1)^5/5 - 1

Sin(x) =x^5/120 - x^3/6 + x

X^t =(t^2*log(x)^2)/2 + t*log(x) + 1

POWER SERIES

Power Series: Power series
From Wikipedia, the free encyclopedia
In mathematics, a **power series** (in one variable) is an infinite series of the form

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c)^1 + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

Where a_n represents the coefficient of the nth term, c is a constant, and x varies around c(for this reason one sometimes speaks of the series as being *centered* at c). This series usually arises as the Taylor series of some known function.

In many situations c is equal to zero, for instance when considering a Maclaurin series. In such cases, the power series takes the simpler form

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

>Many functions as well as differential equations can be expanded as an infinite power series.

>The advantage of expanding the functions in power series is that the differentiation and integration becomes easier.

- DE's also become easier to solve.

- The power series is like a polynomial without an upper limiting power.

- A function can be expanded in power series if it can be differentiated infinitely.

EXPANSION OF FUNCTION TO POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

For expansion of function to a power series, it is required to find the coefficients of the terms of the series.
In binomial series, the coefficients are generated from the index(n).
In power series, this is done by putting x=0 and sequentially find the values of y by successive differentiation.

Step-1: Put x=0 in the infinite power series. This gives y(0)=a0 or a0=y(0) at x=0.

Step-2: Differentiate the series to get,
$f'(y)=a1+2a^2x+3a^3x^2+4a^4x^3+$........
Put x=0 to get a1=y'(0)
Step-3: Follow step-2 repeatedly.
The pattern is $a_n=y^n(0)/n!$

EXPANSION OF EXPONENTIAL FUNCTION TO POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

Given function: y=exp(x)

Step-1: Put x=0 in y=exp(x). This gives a0=y(0) = 1 at x=0.

Step-2: Differentiate the series to get, y'=f'(y)=a1+2a^2x+3a^3x^2+4a^4x^3+......

Put x=0 to get a1=y'(0)=1

Step-3: Follow step-2 repeatedly to get a2=1/2.1, a3=1/3.2.1, a4=1/4.3.2.1 etc

The pattern is $a_n=y^n(0)/n!$
exp(x)=1+x+x^2/2!+x^3/3!+....................

Source: Computer Graphics Through Key Mathematics-Huw Jones

%Power Series Expansion of Elementary functions

- %Power Series Expansion of Elementary functions
- syms y x
- exponential_function_expansion=taylor(exp(x))
- %x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1
- trigonometric_expansion_sin(x)=taylor(sin(x))
- %x^5/120 - x^3/6 + x
- trigonometric expansion cos(x)=taylor(cos(x))
- %- x^6/720 + x^4/24 - x^2/2 + 1
- It can be seen that exp(ix)=cos(x)+sin(ix)
- If x=pi, then exp(i*pi)=cos(pi)+sin(i*pi)=-1

(Important Note)

Power Series Expansion of Elementary functions


```
sinh=taylor(sinh(x))
%x^5/120 + x^3/6 + x
 cosh=taylor(cosh(x))
%x^4/24 + x^2/2 + 1
 tanh=taylor(tanh(x))
%(2*x^5)/15 - x^3/3 + x
 log1=taylor(log(1-x))
%- x^5/5 - x^4/4 - x^3/3 - x^2/2 - x
 log2=taylor(log(1+x))
%x^5/5 - x^4/4 + x^3/3 - x^2/2 + x
 rational=taylor((1+x)/(1-x))
%2*x^5 + 2*x^4 + 2*x^3 + 2*x^2 + 2*x + 1
 taylor(exp(-x^2))
% x^4/2 - x^2 + 1
```

Trigonometric Series

- In the year 1753, D Bernoulli introduced Trigonometric series for vibrating strings.
- A trigonometric series is given below:

- $S=a_0/2+a_1\cos(1^*x)+b_1\sin(1^*x)+a_2\cos(2^*x)+b_2\cos(2^*x)+\ldots\ldots\ldots\ldots\ldots\ldots$
- All the terms are periodic function with period 2*pi. This means that when x is increased by a multiple of 2*pi, all the terms retain their values.
- Genaral Expression:

$S=ao/2+a_1\cos(pi^*x/l)+b_1\sin(pi^*x/l)+a_2\cos(2^*pi^*x/l)+b_2\cos(2^*pi^*x/l)+\ldots\ldots\ldots\ldots\ldots\ldots$(Periodic function, period = 2l)

Orthogonality of System of functions

- Two functions f(x) and g(x) are said to be orthogonal in an interval (a,b) if the integral of the product f(x).g(x) taken between the limits a and b is zero.

- Some trigonometric identities:

Laplace Transformation

- Integral Function – The integral function is one whose domain is positive integral
- Power Series – a0x^0+a1x^1+a2x^2+a3x^3…….
- Radius of convergent of a power series – an/an+1<1

- If the power series converges then it represent a function.

- A discreet function defined as a(n) for positive integer n, expanded as power series, sigma (anx^n) the output is a continuous function of x. The variable n transformed to variable x

Laplace Transformation

- Laplace transformation is the continuous analog version of infinite power series (Discreet version).

- It start with a function defined for positive values of t and turns it into a function of s and this is called Laplace transform. Laplace transform is a transform. As the function of t comes in and a function of s comes out. The variable gets changed but for an operator, a function of x comes in and a function of s comes out.

LINEAR ALGEBRA…

Linear Algebra: Re-defined

- Earlier we were transforming a point with a Transformation Matrix.

$$\begin{bmatrix} x & y \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} x* & y* \end{bmatrix}$$

$$\begin{bmatrix} x* & y* \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} x & y \end{bmatrix}$$

- In linear Algebra, we are given the transformed point and the inverse of transformation matrix. We have to find out which is the original point.
- Inverse of the Transformation Matrix is the Coefficient Matrix.

MATRIX MULTIPLICATION

We can only multiply Matrix if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix.

$$\begin{bmatrix} -3 & 2 & 5 \\ 7 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -8 & 2 \\ 1 & 5 \\ 0 & -3 \end{bmatrix}$$

They must match.

Dimension: 2 x 3 3 x 2

The dimensions of your answer.

Example:

1. $$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}_{2 \times 3}$$

$$\begin{bmatrix} 2(3)+ -1(5) & 2(-9)+ -1(7) & 2(2)+ -1(-6) \\ 3(3)+4(5) & 3(-9)+4(7) & 3(2)+4(-6) \end{bmatrix}_{2 \times 3}$$

$$\begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}_{2 \times 3}$$

2. $\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

Dimensions: 2 x 3   2x2

They don't match so can't be multiplied together.

3. $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 2 \\ 2 & 6 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 8 \end{bmatrix}$ ⇒

$$x + 2y - z = 1$$
$$x + 3y + 2z = 7$$
$$2x + 6y + z = 8$$

INVERSE OF A MATRIX

- For a real number, the inverse is a:

$$x^{*}\, x^{-1} = 1$$

EXAMPLE:

- Inverse of 5 is 1/5 or 5^{-1} *or*

$$5 \times 1/5 = 1$$

- 1 is the identity element of real numbers.

Similarly, *AB=BA=I* (A and B are square matrix)

then *A* is said to be ***invertible,*** and *B* is called an ***inverse*** of *A*.

- If no such matrix *B* can be found, then *A* is said to be ***singular***.

NOTE

$A^{-1} \neq 1/A$ For a Matrix !!!

INVERSE

$$\mathbf{A}^{-1} \cdot \mathbf{A} = \mathbf{A} \cdot \mathbf{A}^{-1} = \mathbf{I} \quad \cdots (1)$$

- The inverse matrix A^{-1} of an $n \times n$ matrix A exists if and only if rank A = n.
- Such a matrix A is called a non-singular matrix. If it has no inverse, it is singular matrix.
- For singular matrix its Determinant DetA=|A|=0.

FINDING INVERSE

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \cdot \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$

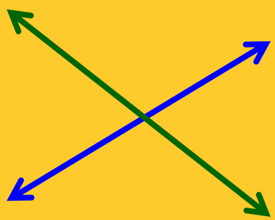
where: $A_{ij} = (-1)^{i+j} D_{ij}$

signed minor (cofactor) → A_{ij}; minor → D_{ij}

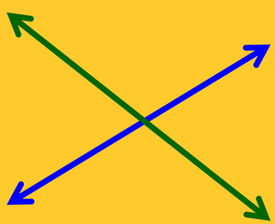
In Excel, Command is ' **minverse**'

Types of Solutions

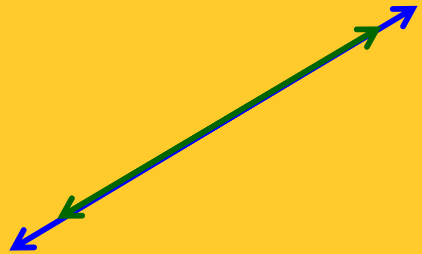
Consistent System



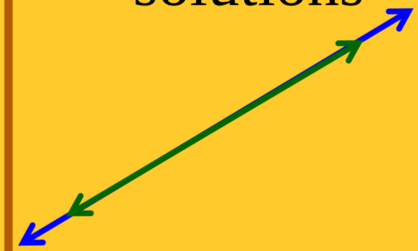
One solution



Consistent System

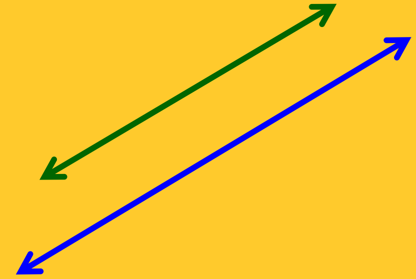


Infinite solutions



Inconsistent System

No solution



Co-efficient Matrix::Augmented Matrix

Co-efficient Matrix ::Augmented Matrix – a matrix that is used to solve a system of equations.

$$2x + y = 5$$
$$-4x + 6y = -2$$

$$x + y + z = 0$$
$$3x - 2y + 4z = 9$$
$$x - y - z = 0$$

Coefficient | Augmented matrix

$$\left[\begin{array}{cc|c} 2 & 1 & 5 \\ -4 & 6 & -2 \end{array}\right]$$

Coefficient | Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 3 & -2 & 4 & 9 \\ 1 & -1 & -1 & 0 \end{array}\right]$$

Using Matrix to Solve System of Equations

$$\left[\begin{array}{cc|c} 5 & -1 & 9 \\ 2 & 8 & 7 \end{array}\right]$$

System of Equations

$$5x - y = 9$$
$$2x + 8y = 7$$

$$\left[\begin{array}{ccc|c} 3 & 6 & -2 & -8 \\ 2 & 0 & 5 & 13 \\ 1 & 3 & -7 & 12 \end{array}\right]$$

System of Equations

$$3x + 6y - 2z = -8$$
$$2x + 5z = 13$$
$$x + 3y - 7z = 12$$

$$3x + 6y - 2z = -8$$
$$2x + 0y + 5z = 13$$
$$x + 3y - 7z = 12$$

Using Matrix to Solve Systems of Equations

The use of Elementary Row Operations is required when solving a system of equations using Matrix.

Elementary Row Operations

I. Interchange two rows.

II. Multiply one row by a nonzero number.

III. Add a multiple of one row to a different row.

Using Matrix to Solve Systems of Equations

The solution to the system of equations is complete when the augmented matrix is in Row Echelon Form.

Row Echelon Forms:

$$\left[\begin{array}{cc|c} 1 & 4 & 5 \\ 0 & 1 & 8 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 3 & -7 & 2 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 1 & 6 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 4 & 7 & 5 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Consistent or Inconsistent System?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

One Solution: Consistent System

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

No Solution: Inconsistent System

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Infinite Solutions: Consistent System

Eigen Values and Eigen Vectors

Definition: A nonzero vector **x** is an ***eigenvector*** (or *characteristic vector*) of a square matrix **A** if there exists a scalar **λ** such that **Ax = λx.** Then **λ** is an ***eigenvalue*** (or *characteristic value*) of **A**.

Note: The zero vector can not be an eigenvector even though

A0 = λ0. But **λ = 0** can be an eigenvalue.

GEOMETRIC INTERPRETATION

•An $n \times n$ matrix A multiplied by $n \times 1$ vector x results in another $n \times 1$ vector y=Ax. Thus A can be considered as a transformation matrix.

•In general, a matrix acts on a vector by changing both its magnitude and its direction.

•However, a matrix may act on certain vectors by changing only their magnitude, and leaving their direction unchanged (or possibly reversing it). These vectors are the eigenvectors of the matrix.

•A matrix acts on an eigenvector by multiplying its magnitude by a **factor**, which is positive if its direction is unchanged and negative if its direction is reversed. This **factor** is the eigenvalue associated with that eigenvector.

PROPERTIES OF EIGEN VECTOR

Definition: The trace of a matrix A, designated by tr(A), is the sum of the elements on the main diagonal.

Property 1: The sum of the eigenvalues of a matrix equals the trace of the matrix.

Property 2: A matrix is singular if and only if it has a zero eigenvalue.

Property 3: The eigenvalues of an upper (or lower) triangular matrix are the elements on the main diagonal.

Eigen Vector, Eigen Value, Transformation Matrix

- Eigen Vector: When we transform a vector by a matrix multiplication, except few, all vectors changes its magnitude and direction. The vectors which do not change the direction after matrix multiplication is called eigen vector.
- When we multiply an eigen vector by a matrix, it is equivalent to the multiplication of the vector by a number called eigen value.
- The eigen value tells us whether the vector get stretched or shrunk or reversed or remained unchanged.

Number of Eigen Value

- Maximum Number of Eigen Values is equal to its rank.
- Let the transformation matrix is, a is a 2x2 matrix, hence the number of eigen value is 2.

0.8	**0.2**
0.3	**0.7**

- The eigen value of this matrix is l1=1 and l2=.5
- The eigen vector of this matrix is v1=[0.6 0.4] and v2=[1 -1]

Number of Eigen Value

- Let the transformation matrix is, a=[1 6 -1; 2 -1 -1; 1 0 1]

- It is a 3x3 matrix and the eigen value will be 3.

- The eigen values are l1=0, l2=-4 and l3=3
- The corresponding eigen vectors are,
- v1=[1 6 -13], v2=[-1 2 1], v3=[2 3 -2]

The Power of a Matrix, a^n

- Eigen values give an easy way to calculate the power of a matrix.
- Let matrix, a=[1 6 -1; 2 -1 -1; 1 0 1]

And the matrix formed by the vector, v1=[1 6 -13], v2=[-1 2 1], v3=[2 3 -2]

P=[1 6 -13; -1 2 1; 2 3 -2]

And P^{-1} =[

-0.08333	-0.32143	0.380952
0	0.285714	0.142857
-0.08333	0.107143	0.095238

Diagonal Matrix

- Diagonal Matrix is formed by-

- $D=P\,A\,P^{-1}$

- A can be written as $A= P^{-1}\,D\,P$

- Diagonal Matrix is formed by eigen values. Hence, it can be written as A^n=l^n
- Hence, it can be written that, v1*A^n=v1*l1^n, v2*A^n=v2*l2, v3*A^n=v3*l3^n

Computation of Eigen Value

- For a square matrix A of the order n, l is an eigen value if and only if there exists a non-zero vector v such that va=vl.

- It can be rewritten as v(a-Il)=0
- This equation has one solution if and only if det(a-Il)=0.
- Example, a=[1 -2; -2 0]
- The equation becomes, |1-l*-2; -2 0*l|=0

Computation of Eigen Value

- The characteristics equation is (1-l)(0-l)-4=0
- l1=(1+sqrt(17))/2
- l2=(1-sqrt(17))/2
- A 2x2 matrix has only two eigen values.

- In general for a square matrix of order n will not have more than n eigen values.

Computation of Eigen Value

- Let a=[a b; c d], the characteristics polynomial is given by |a-l b; c d-l|=0 or (a-l)(d-l)-bc=0 or l^2-(a+d)l+(ad-bc)=0

- The sum of diagonal elements (a+d) is called trace and denoted as tr(A) and the number (ad-bc) is the determinant of A. So the characteristics polynomial of A can be written as l^2-tr(A)+det(A)=0

Computation of Eigen Value

- But calculation of characteristic equation manually is very difficult even for a 3x3 matrix and for a matrix higher than 4x4, one should not even try to calculate eigen values manually.

- What to do? Matlab provides an easy way-
- B=poly(a)
- R=roots(B)
- For plotting the characteristic equations in matlab:
- x=-2:.1:2
- y=polyval(b,x)
- Plot(x,y)

Computation of Eigen value and eigen vectors

- Matlab provides easy options for calculating eigen value and eigen vectors:

- [v, d]=eig(a)

- A=[1 6 -1; 2 -1 -2; 1 0 -1]
- [v,d]=eig(A)

Diagonalization of matrices

- A matrix is diagonizable if it is similar to a diagonal matrix. If the order of the matrix is n and have n distinct eigen values then A is diagonalizable.

- If P is the matrix formed by the n eigen vectors, then the matrix PDP^{-1} is a diagonal matrix.

Complex Eigen Value

- The eigen values can be real numbers as well as complex numbers.

- A=[3 4; -2 -1]
- Eigen values are 1+2i and 1-2i

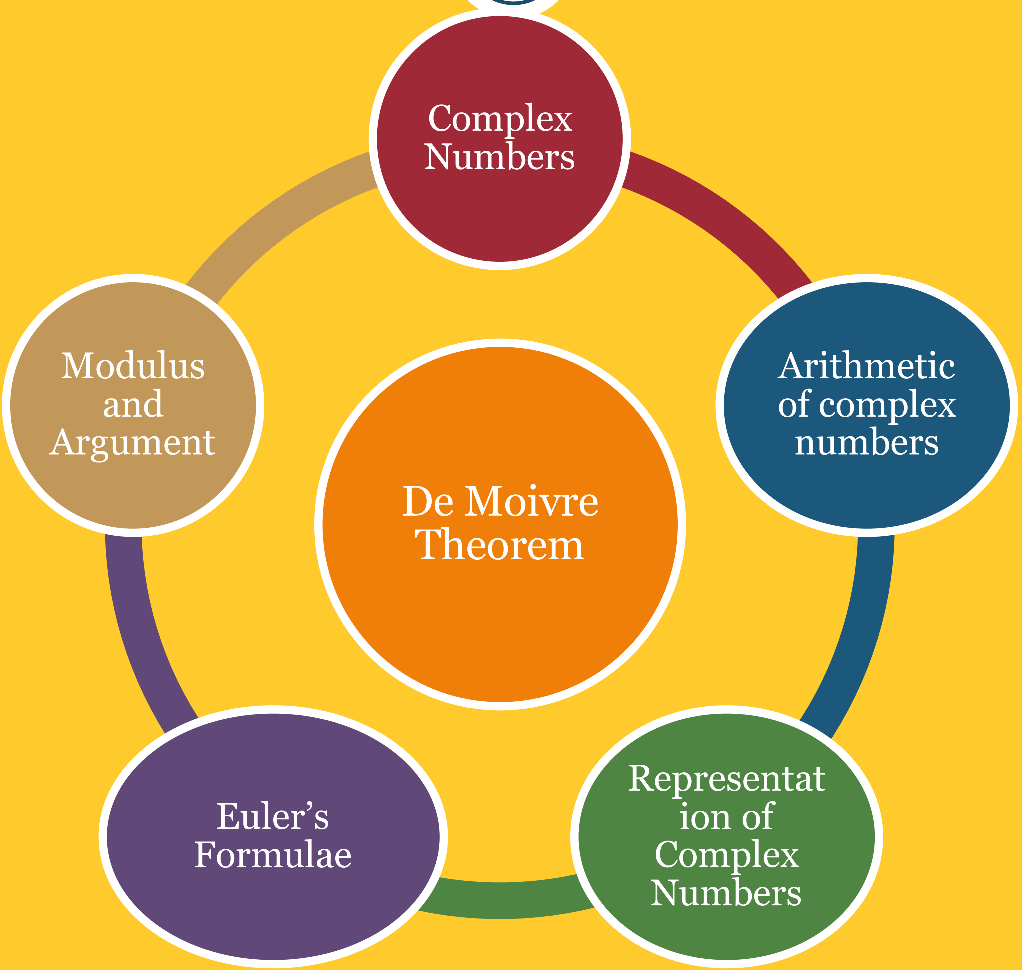
- Note: Symmetric matrices will have real eigen value.

Complex are Analysis

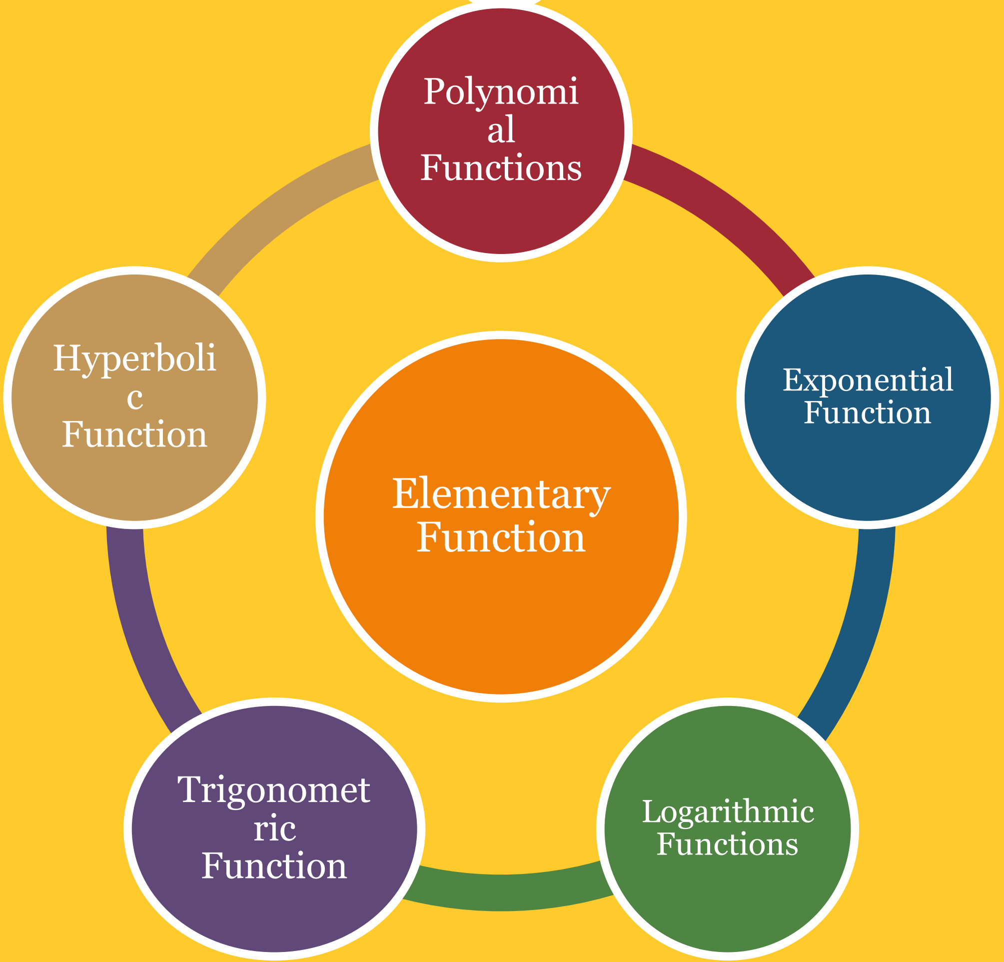
Complex Analysis

Contents

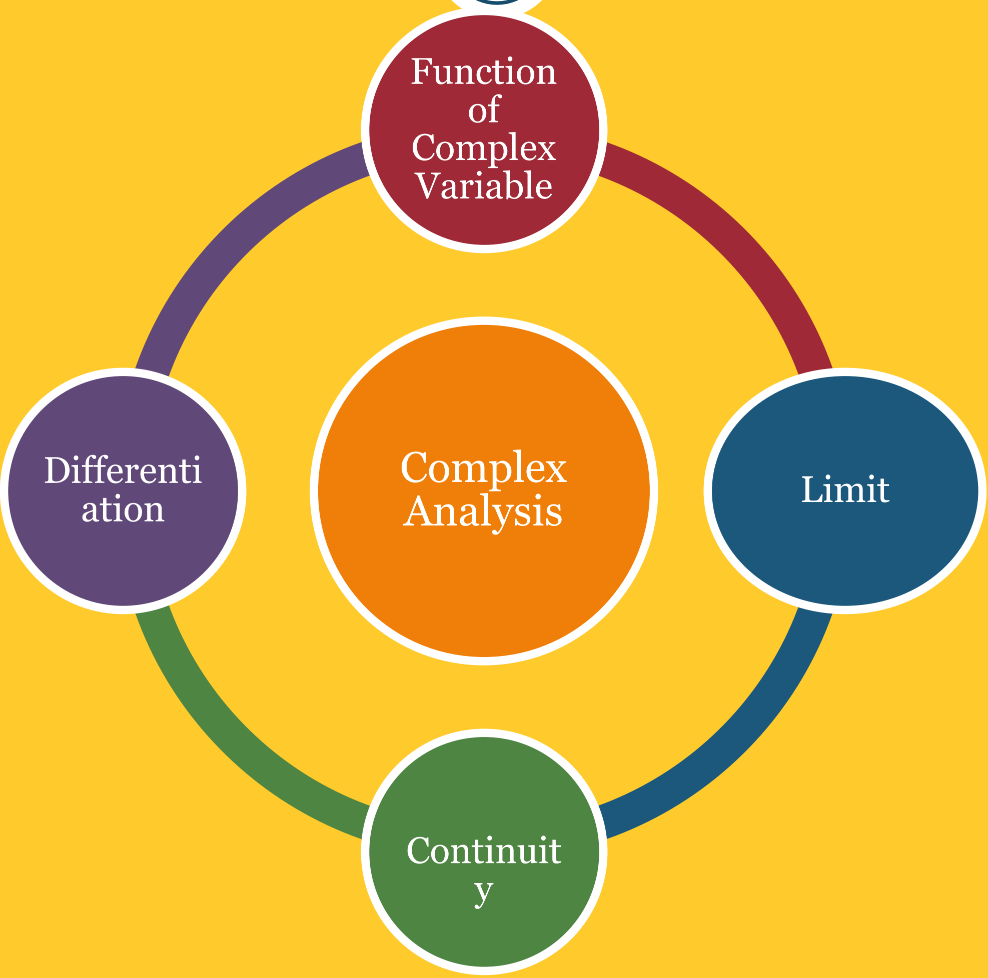
Complex Numbers



Complex Functions



Complex Analysis



Evolution of Complex Number

- One of Curved lines is represented by a quadratic Equation: **y=ax^2+bx+c**

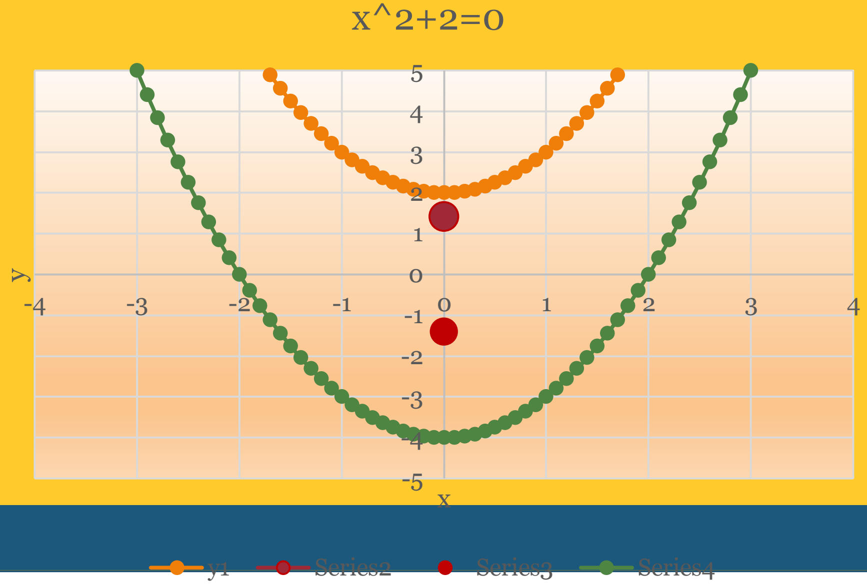
- As the highest power of x is 2, it should have 2 roots.

- The roots are given by the formula, $r = \frac{-b \pm \sqrt{(b^2 - 4 * a * c)}}{2 * a}$

- When 4*a*c become larger than b^2, then there will be no real root, this leads to evolution of imaginary numbers.
- When imaginary numbers added to real numbers, complex numbers are formed.

Demonstration Origin of Complex Number
Solve x^2+2=0

- Here, a=1, b=0, c=2 and b^2-4ac =-2 is negative.
- No real solution
- Roots are: $+\sqrt{2}\,i$ and $-\sqrt{2}\,i$



History

Students when grow up, faces two major problem:

- **Division by zero and**
- **Finding Square root of negative numbers**

Problem was first recorded in India

- **Rafeal Bombelli [1526-1572] proposed**
- **Girolamo Cardano [1501-1576] first used complex number** $\sqrt{-1}$
- **Rene Descartes [1596-1650] introduced Cartesian coordinate**
- **Leonhard Euler [1707-1783] introduced symbol i for** $\sqrt{-1}$
- **Carl Friedrich Gauss [1777-1855] introduced the term complex number**
- **Jean Robert Argand [1768-1822] introduced complex plane**

i is the imaginary number and represents $\sqrt{-1}$

Pre-requisite to understand Complex Analysis

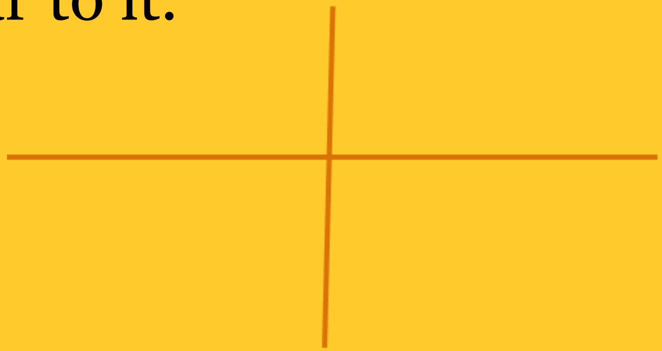
1. REAL NUMBERS
2. REAL NUMBER LINE
3. REAL FUNTION
4. REAL DOMAIN
5. REAL RANGE
6. CARTESIAN COORDINATE SYSTEM
7. COMPLEX NUMBERS
8. COMPLEX PLANE
9. COMPLEX FUNCTION
10. COMPLEX DOMAIN
11. COMPLEX RANGE

Complex Numbers

- Complex numbers are expressed as x+yi

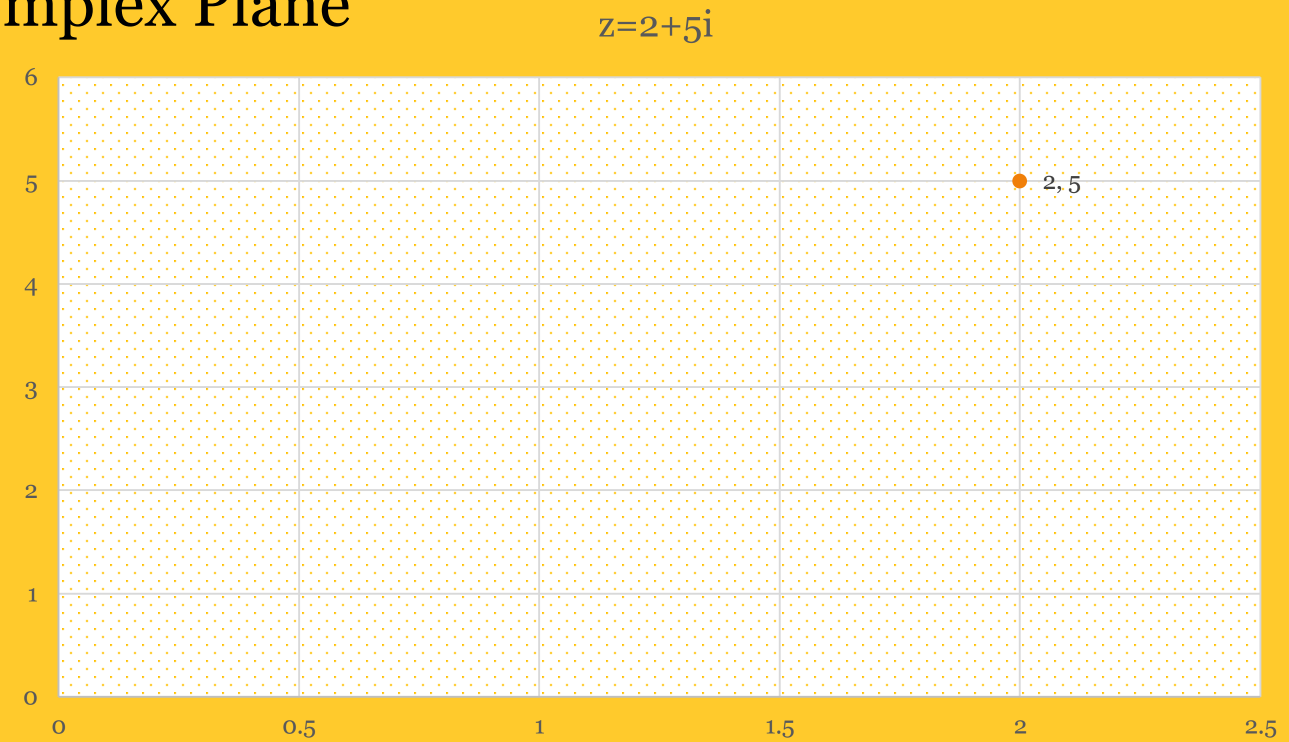
- Though it is called as a complex number it is not placed in the number line.

- Complex numbers are placed in a complex plane which is formed by horizontal axis represent real number line and an imaginary axis perpendicular to it.



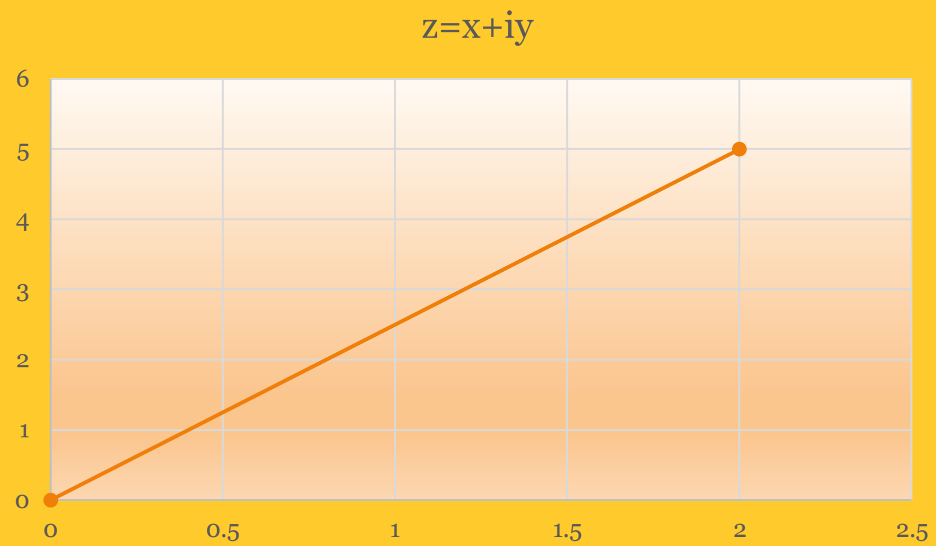
Complex Numbers

- Complex numbers are expressed as x+yi
- Argand Diagram
- Complex Plane



Complex Numbers: Vector Representation

- A complex number is an ordered pair (x, y) of real number x and y and represented by z=x+iy



Complex Number, Modulus, Argument, Conjugate

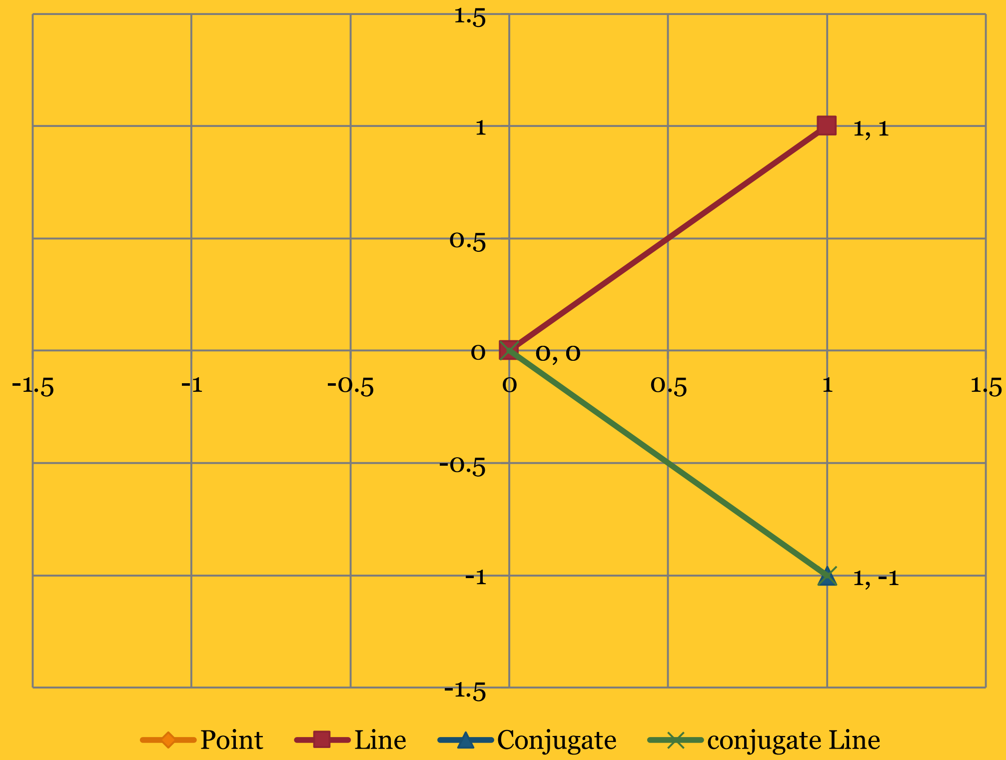
If z=x+yi, then z*=x-yi is its conjugate

Modulus=

$$|z| = \sqrt{a^2 + b^2}$$

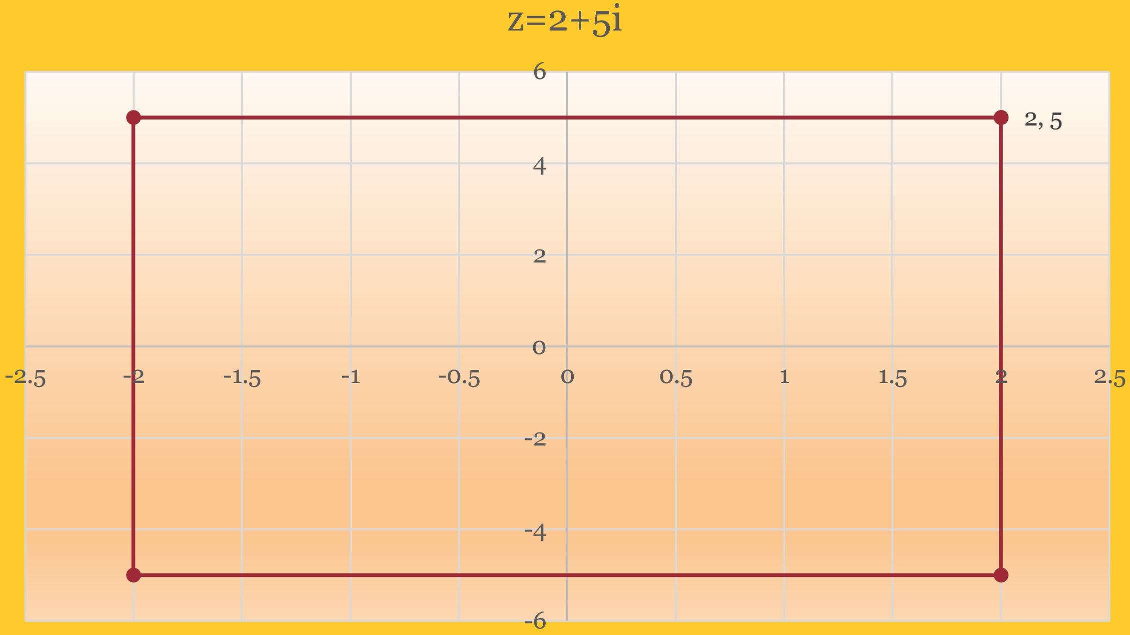
Excel Command: imabs(z)

Argument= t=atan2(y/x)



Different Complex Numbers same modulus

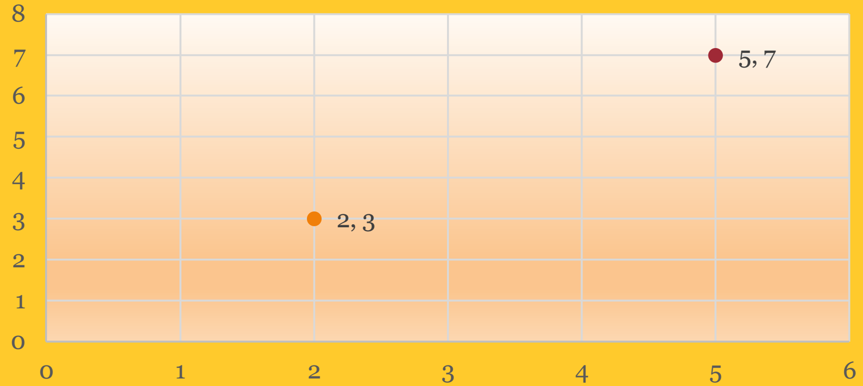
Use atan2 for arguments



IMABS(2,5),
Sqrt(2^2+5^2)=5.385165
ATAN2(2/5)=1.19029 rad or 68.19859 Degree

Equality

Equality: Two complex numbers are equal if their real parts and imaginary parts are equal,
z1=2+3i, z2=5+7i, hence, z1 not equal to z2.



Excel Commands for Complex Numbers and their Operations

Operations	Command
1. Convert from Real to Complex	Complex(1,8)
2. Convert from Complex to Real	Imreal(z)
3. Convert from Complex to Imaginary	Imaginary(z)
4. Get absolute value of z	Imabs(z)
5. Get Conjugate of z	Imconjugate(z)
6. Get Angle or Argument of z	Imargument(z)
7. Add z1, z2	Imsum(z1,z2)
8. Subtract z1,z2	Imsub(z1,z2)
9. Multiplication z1,z2	Improd(z1,z2)
10. Division z1,z2	Imdiv(z1,z2)

Five Main Operations

Operations	Command
Addition	Imsum(z1,z2)
Subtraction	Imsub(z1-z2)
Multiplications	Improduct(z1,z2)
Division	Imdiv(z1,z2)
Exponentiations	Imexp(z)

Matlab Commands

abs	**Absolute value and complex magnitude**
angle	Phase angle
complex	Create complex array
conj	Complex conjugate
cplxpair	Sort complex numbers into complex conjugate pairs
i	Imaginary unit
imag	Imaginary part of complex number
isreal	Determine whether array is real
j	Imaginary unit
real	Real part of complex number
sign	Sign function (signum function)
unwrap	Correct phase angles to produce smoother phase plots

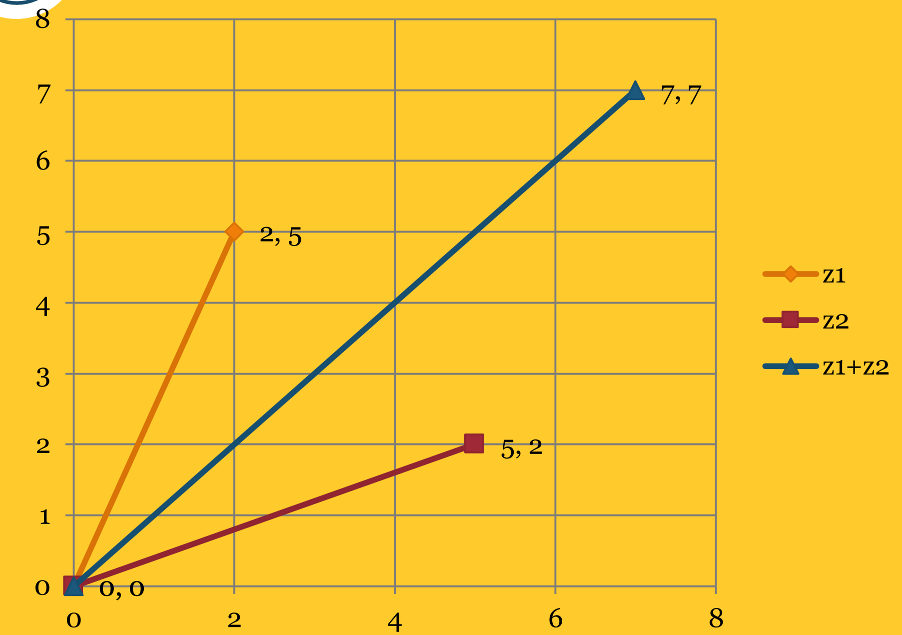
Laws of binary operations

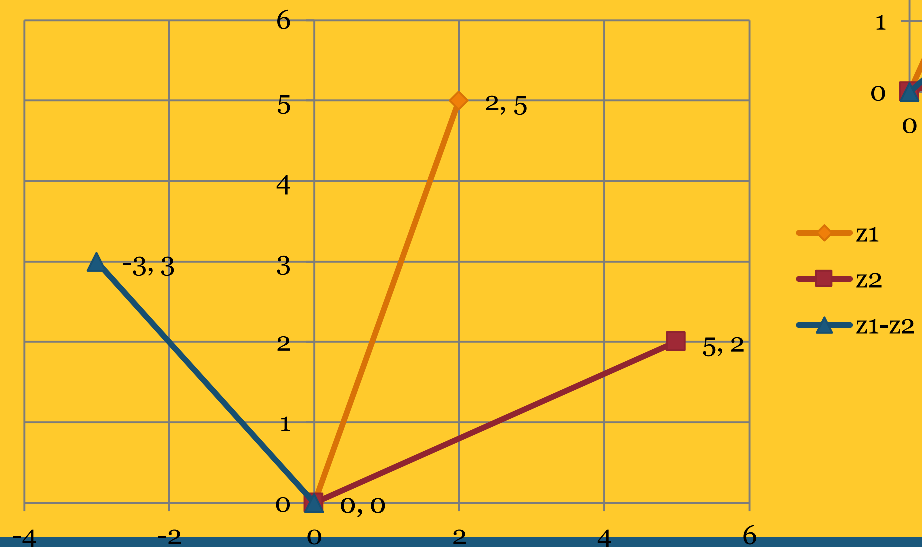
- Complex numbers are expressed as $z=x+iy$
- Closure Law, z1+z2=z, z1*z2=z
- Additive identity, z+0=0+z=z
- Multiplicative Identity, z*1=z
- Additive Inverse, z+(-z)=-z+z=0
- Multiplicative Inverse z*1/z=1
- Associativity, z1+(z2+z3)=(z1+z2)+z3, z1*(z2*z3)=(z1*z2)*z3
- Commutative Law, z1+z2=z2+z1, z1*z2=z2*z1
- Distributive Law, z1(z2+z3)=z1z2+z1z3

Binary Operations of Complex Number

Addition:
Ex. (2+5i)+(5+2i)=7+7i

Subtraction:
Ex. (2+5i)-(5+2i)=-3+3i





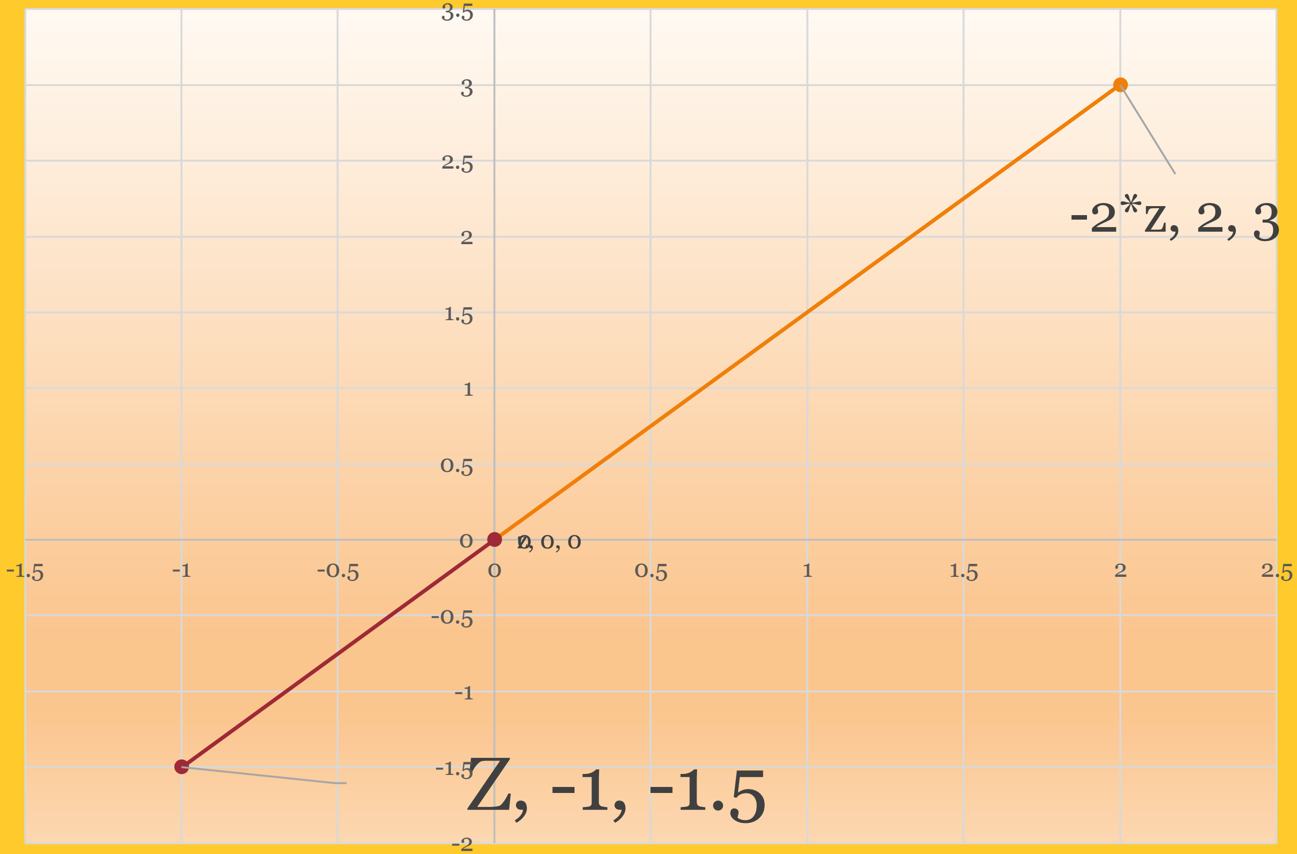
z1=x1+i y1,   z2=x2+i y2

z1+z2=(x1+x2)+i(y1+y2)

Addition follows Parallelogram Law

Scalar Multiplication

cz =c*(x+iy)=cx+i cy)



Multiplication and Division

Let,

$z1=x1+i\ y1$ $z2=x2+i\ y2$

Multiplication:

$z1*z2=(x1x2-y1y2)+i(x1y2+x2y1)$

Division:(Multiply denominator Conjugate of z2)

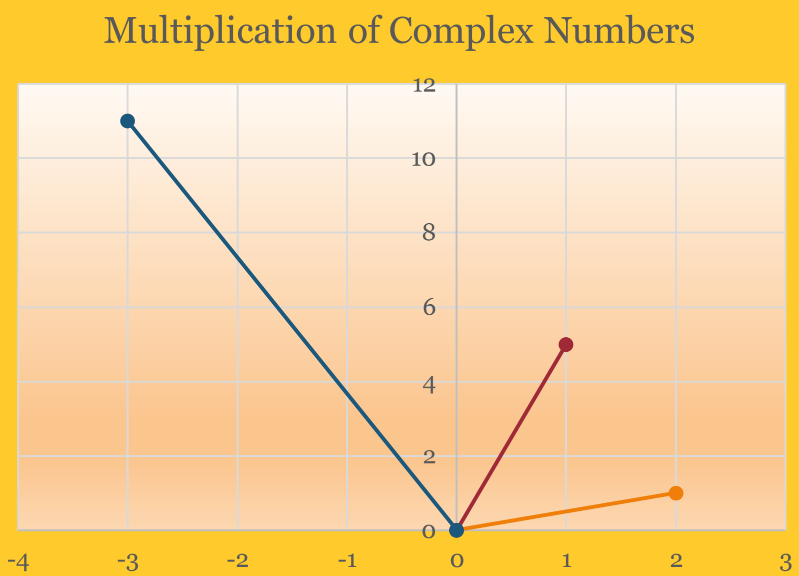
$z1/z2=(x1x2+y1y2)/(x2^2+y2^2)+i\ (x2y1-x1y2)/(x2^2+y2^2)$

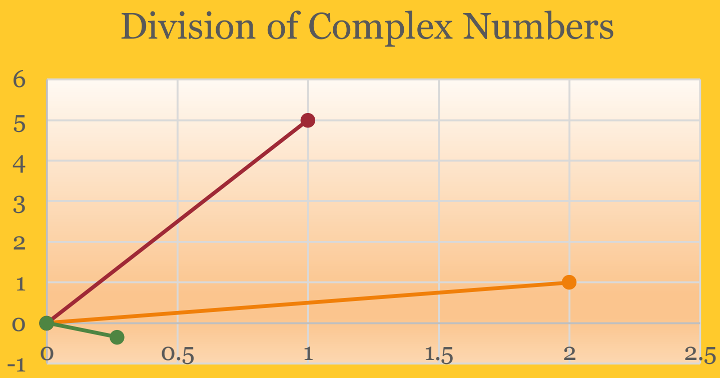
Multiplication and Division

Multiplication:
a = 2.0000 + 1.0000i
b = 1.0000 + 5.0000i

a*b = -3.0000 +11.0000i

Division:
a/b= 0.269230769230769
-0.346153846153846i



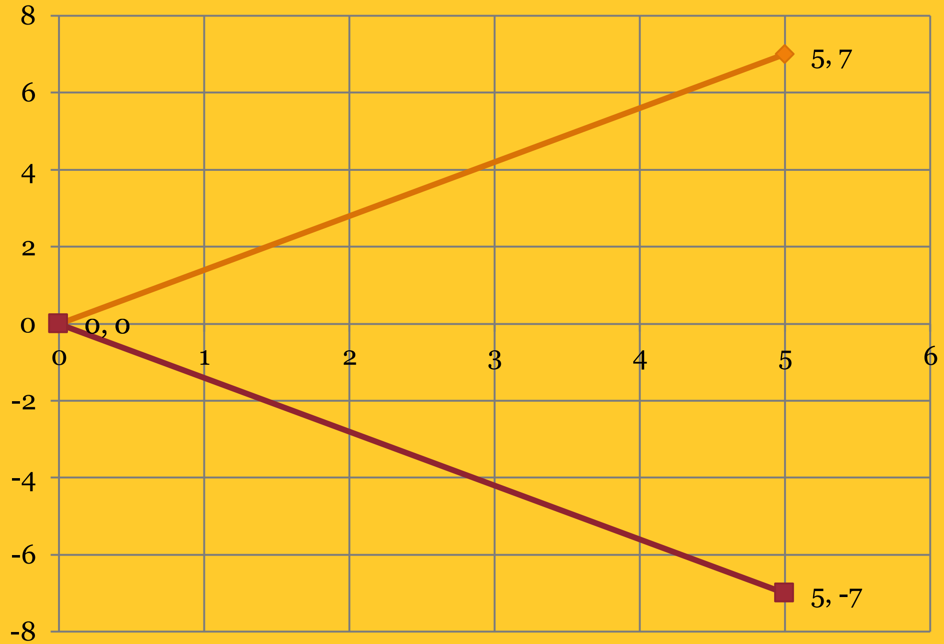


Conjugate of a complex Number

Multiplication of z and its cojugte
If z=a+bi, then z*=a-bi is its conjugate

z*z*=74

$$|z| = \sqrt{a^2 + b^2}$$



Multiplication of Complex Conjugate

1. Multiplication of z with its complex conjugate produces a scalar (x^2+y^2).
2. This property is used to simplify a rational numbers, z1/z2.

Multiplication of Complex Numbers

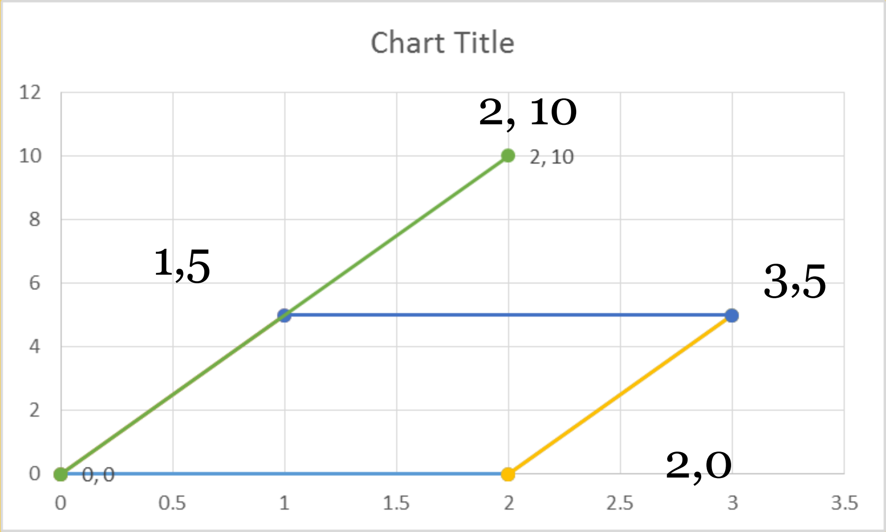
1. Multiplication of two complex numbers produces a complex number.

1. The real part is equal to the dot product and imaginary part is equal to the area formed by the complex numbers.

Multiplication of Complex Numbers

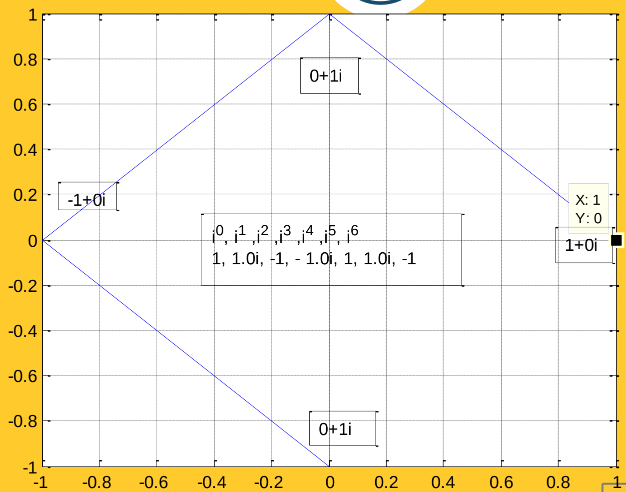
1. The real part is equal to the dot product and imaginary part is equal to the area formed by the complex numbers.

Real part=2=dot product, imaginary part=area of the rectangle formed by the complex numbers



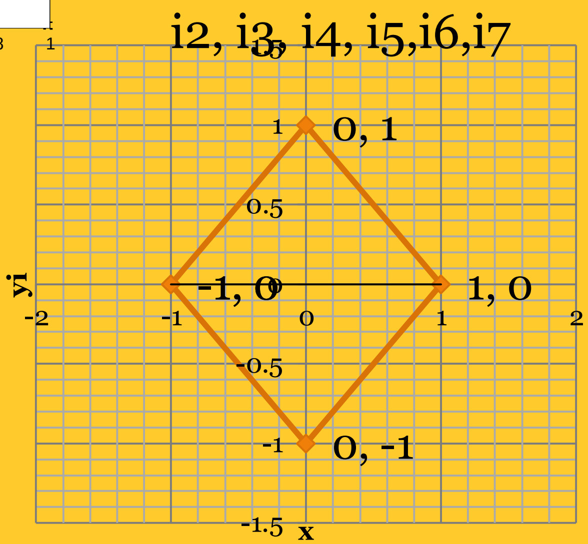
Complex Multiplication as Rotor, i*i=-1

i^2	-1
i^3	-i
i^4	1
i^5	i
i^6	-1
i^7	-i
i^8	1



Multiply (1+0i) by i produces 0+i

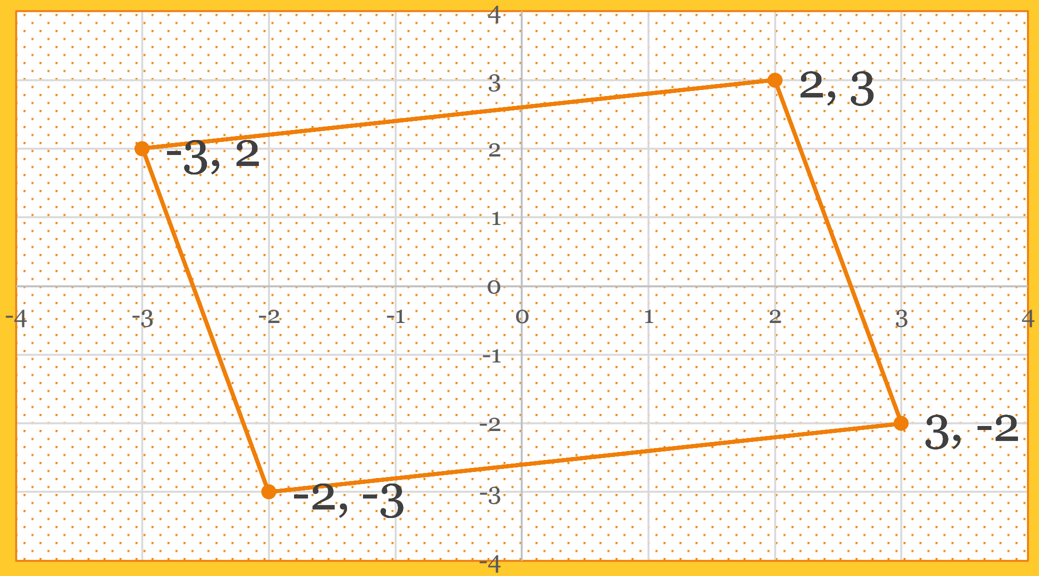
Multiply (0+i) by i produces -1



Complex Multiplication as Rotor, z=2+3i*i^n

n	i^n	z*i^n	real	imaginary
1	Z*i	-3+2i	-3	2
2	Z*i^2	-2-3i	-2	-3
3	z=*i^3	3-2i	3	-2
4	Z*i^4	2+3i	2	3
5	Z*i^5	-3+2i	-3	2
6	Z*i^6	-2-3i	-2	-3

z=2+3i



Polar Form of Complex Number

x=r*cos(t)

y=r*sin(t)

z=x+iy

z=r*cos(t)+r*i*sin(t)

z=r*(cos(t)+i*sin(t)

r=Modulus or Absolute Value or Length from origin or Imabs(z)

r=|z|=sqrt(x^2+y^2)

t=Angle from positive x-axis or Argument or imarg(z)

t=arc tan(y/x) or atan2(x/y) or tan^-1(y/x)

Principal Argument

- Let, z=x+iy
- We can calculate r=|z|=imabs(z)
- For angle, t=imarg(z)
- A complete rotation around the origin leaves a complex number unchanged, there are many choices which could be made for t by circling the origin any number of times.
- Hence, z is multivalued and t can take multiple values.
- Here t is expressed as t = t+2 * pi * n, n=0,+-1,+-2,+-3….
- The value of t that lies between –pi to pi is called principal value or principal argument.

Multivalued z=1+2i

n	t	degree	r	x	y
0	1.107149	63.43495	2.236068	1	2
1	7.390334	423.4349	2.236068	1	2
2	13.67352	783.4349	2.236068	1	2
3	19.9567	1143.435	2.236068	1	2
4	26.23989	1503.435	2.236068	1	2
5	32.52308	1863.435	2.236068	1	2

Multiplication of complex numbers

- Polar form of complex numbers are very useful in analysing multiplication and division.
- This is the basis of Euler's Formulae
- It helps in deriving De Moivre's Theorem
- It helps in finding roots of a complex number

Multiplication with Polar Form

Let, $z1=r1(\cos(t1)+i \sin(t1)$

And $z2=r2(\cos(t2)+i \sin(t2)$

Then, $z1*z2= (r1(\cos(t1)+i \sin(t1))*(r2(\cos(t2)+i \sin(t2))$

$z1*z2=r1r2[(\cos(t1)*\cos(t2)-\sin(t1)*\sin(t2)+$
$i(\sin(t1)*\cos(t2)+\cos(t1)*\sin(t2)]$

$z1*z2=r1*r2[\cos(t1+t2)+i \sin(t1+t2)$

Multiplication with Polar Form

$z1 = r1\cos(t1) + i\, r1 \sin(t1)$

$z2 = r2\cos(t2) + i\, r2 \sin(t2)$

$z1 * z2 = (r1\cos(t1) + i\, r1\sin(t1)) * (r2\cos(t2) + i\, r2\sin(t2))$

$z1 * z2 = r1\, r2\cos(t1)\cos(t2) + r1\, r2\, i\cos(t1)\sin(t2) + i\, r1\, r2\sin(t1)\cos(t2) + i^2\, r1\, r2\sin(t1)\sin(t2)$

$z1 * z2 = r1\, r2\, (\cos(t1)\cos(t2) - \sin(t1)\sin(t2)) + i\, r1\, r2\, (\cos(t1)\sin(t2) + (\sin(t1)\cos(t2))$

$z1 * z2 = r1\, r2\cos(t1+t2) + i\, r1\, r2\sin(t1+t2)$

$z1 * z2 = r1\, r2\, (\cos(t1+t2) + i\sin(t1+t2)$, $z = r\,(\cos(t) + i\sin(t))$

Note: Multiplication of two complex number produces a complex number whose length is equal to product of two lengths and angle is equal to the sum of two angle

De Moivre Theorem

- $z^n = (x + i y)^n$
- $z^n = (r \cos(t) + i r \sin(t))^n$
- $z^n = r^n * (\cos(t) + i \sin(t))^n$
- $z^n = r^n(\cos(nt) + i \sin(nt))$

Here Rule of Multiplication applied. In complex multiplication, the absolute values are multiplied and angles are added. This is De Moivre Theorem.

De Moivre Theorem

- z^n = (x + i y)^n
- z^n = (r cos (t) + i r sin(t))^n
- z^n = r^n * (cos (t) + i sin(t))^n
- z^n = r^n(cos (nt) + i sin(nt))

- Example 8.3.2* Let z = 1 – i. Find z^10 .
- r=$\sqrt{2}$, t=atan2(y/x)=1/-1=-0.7854

Here Rule of Multiplication applied. In complex multiplication, the absolute values are multiplied and angles are added.

Principal Argument

- Let, z=x+iy
- We can calculate r=|z|=imabs(z)
- For angle, t=imarg(z)
- A complete rotation around the origin leaves a complex number unchanged, there are many choices which could be made for t by circling the origin any number of times.
- Hence, z is multivalued and t can take multiple values.
- Here t is expressed as t = t+2 * pi * n, n=0,+-1,+-2,+-3….
- The value of t that lies between –pi to pi is called principal value or principal argument.
- This finding helps in finding the roots of a complex number

Finding the root of i

Polar multiplication gives an interesting way of finding roots of a complex number

Complex number is closed with respect to extracting roots

Finding the roots i^(1/n)

Let, i^(1/n)=x+i y

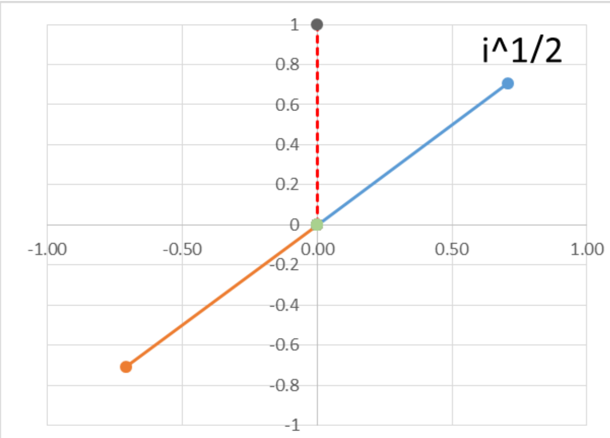
Hence, i = (x+ i y)^n

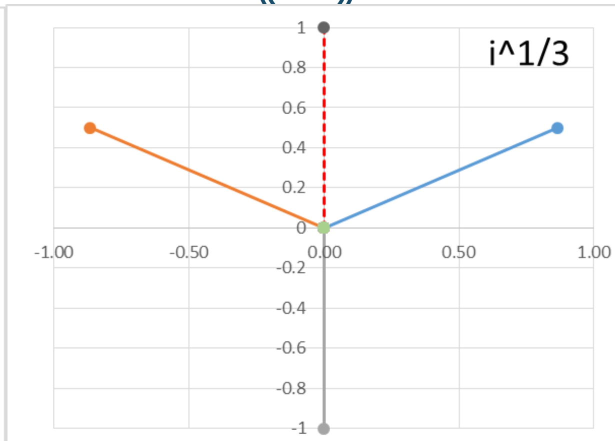
i can be expressered as (1 , pi()/2)^n

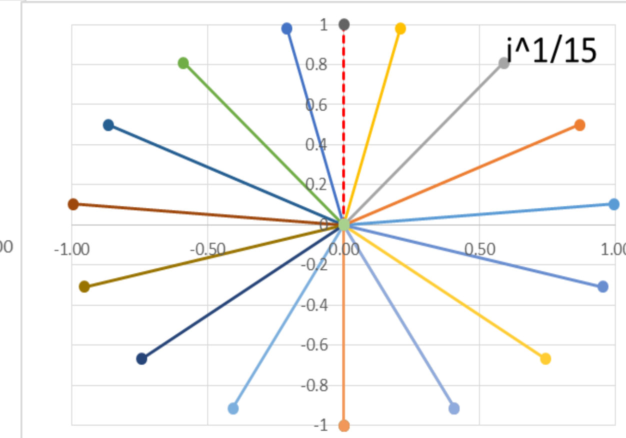
roots=(1^n, n*t)

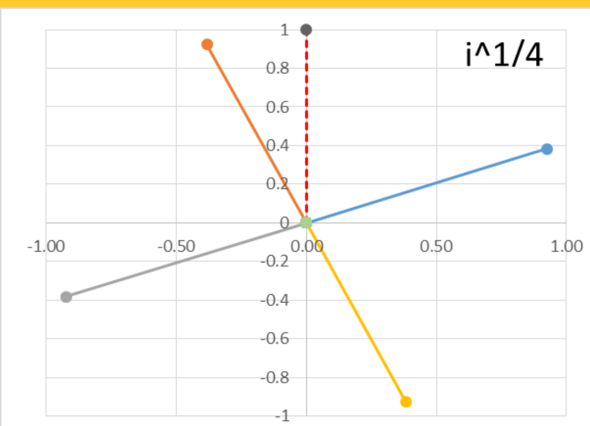
Hence, r=1, n*t=pi()/(2*n)+2*pi()/n*k, where, k=1,2,3....

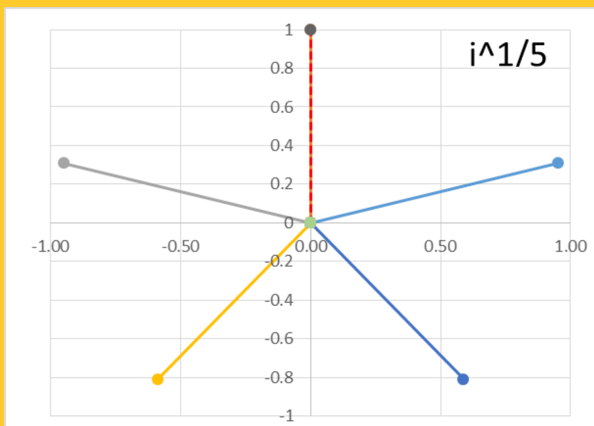
Finding the root of i

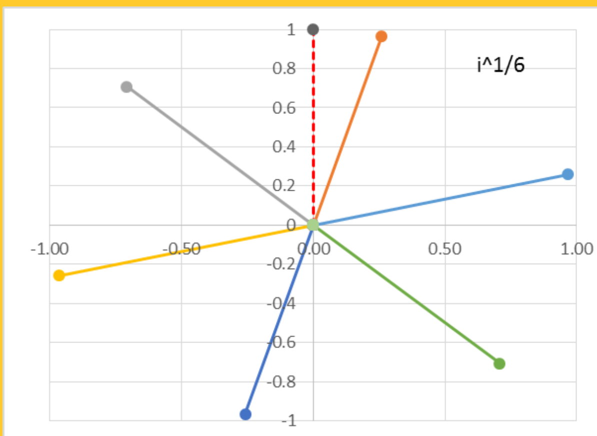












Exponential Form of Complex Numbers

- This is another way of denoting a complex number: the exponential form.
- Complex number notation established from the intimate connection between the exponential function and the trigonometric functions.

- By using the exponential form, many calculations, particularly multiplication and division of complex numbers, become easier than when expressed in polar form.

Origin of Exponential Form

- The function: y=e^x can be expressed by series expansion of as:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

- Similarly y=sin(x) and y=cos(x) can be expressed as
- $y=\sin(x)=x-x^3/3!+x^5/5!-x^7/7!+$.........
- $y=\cos(x)=1-x^2/2!+x^4/4!-x^6/6!$.........

Series Expansion of Sin(x), Cos(x), Exp(x)

The formulas are generated from power series expansion:

The Power Series is given below:

$$y = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 \ldots\ldots$$

If the coefficients are appropriately chosen to diminish, successive terms gets smaller and smaller and series approaches a limiting value. When this series converges, it represents a function.

Series Expansion of Sin(x), Cos(x), Exp(x)

The co-efficients are chosen by differentiating the series and equating it to 0.

The Power Series is given below:

$$y = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 \ldots\ldots$$

Note: Our objective is to get the values of a0, a1, a2.. so that the series converges. When the series converges it represent a function. We use calculus to find the values of the coefficients.

Series Expansion of Sin(x), Cos(x), Exp(x)

The co-efficients are chosen by differentiating the series and equating to 0.

The Power Series is given below:

$$y = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \ldots\ldots$$

Putting x=0, we get, a_0=y

Differentiating, and equating to 0, we get, a_1=y'(0)

Differentiating and equating to 0, we get, a_2=y''(0)/2.1

Differentiating and equating to 0, we get, a_3=y'''(0)/2.3

General term of co-efficient becomes, $a_n=y^n(0)$/fact(n)

Series Expansion of Exp(x)

General term of co-efficient becomes,

$a_n = y^n(0) / fact(n)$

The Power Series is given below:

$$y = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \ldots\ldots$$

The Power Series now transformed like :

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \ldots\ldots$$

Original Function was, y=exp(x)

Putting, y(0)=1 and other coefficients, we get,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

This is shown in next slide.

Series Expansion of Exp(x)

y=exp(x), @x=0, exp(0)=1, y(0)=1

y'=exp(x), @x=0, exp(0)=1, y'(0)=1

y"=exp(x), @x=0, exp(0)=1,y"=1

y'''=exp(x), @x=0, exp(0)=1, y'''=1

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \ldots\ldots$$

General term of co-efficient becomes,

$a_n = y^n(0) / \text{fact}(n) = 1/\text{fact}(n)$

Original Function, y=exp(x) becomes,

$y = \exp(x) = 1 + x + x^2/2! + x^3/3! + x^4/4! + x^5/5! + x^6/6 + x^7/7! \ldots\ldots\ldots$

Series Expansion of Sin(x)

y=sin(x), x=0, sin(0)=0, y(0)=0

y'=cos(x), x=0, cos(x)=1, y'(0)=1

y''=-sin(x), x=0, -sin(0)=0,y''=0

y'''=-cos(x), x=0, -cos(0)=-1, y'''=-1

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \dots..$$

Original Function, y=sin(x) becomes,

$y=\sin(x)=x-x^3/3!+x^5/5!-x^7/7!\dots\dots\dots$

Series Expansion of Cos(x)

y=cos(x), x=0, cos(0)=1, y(0)=1

y'=-sin(x), x=0, -sin(0)=0, y'(0)=0

y''=-cos(x), x=0, -cos(0)=-1, y''=-1

y'''=sin(x), x=0, sin(0)=0, y'''=0

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \ldots..$$

Original Function, y=cos(x) becomes,

$y=\cos(x)=1-x^2/2!+x^4/4!-x^6/6!\ldots\ldots\ldots$

Series Expansion of Sin(x), Cos(x), Exp(x)

$y=\exp(x)=1+x+x^2/2!+x^3/3!+x^4/4!+ x^5/5!+x^6/6!+x^7/7!.........$

$y=\sin(x)=x-x^3/3!+x^5/5!-x^7/7!.........$

$y=\cos(x)=1-x^2/2!+x^4/4!-x^6/6!.........$

Note:

1. It is seen that terms of series expansion of sin(x) and cos(x) are same as exp(x) except the sign of the terms.
2. This observation is cleverly used to find the exponential form of z

Series Expansion of sin(ix)

$y=\sin(x)=x-x^3/3!+x^5/5!-x^7/7!.......$

$y=\sin(ix)=ix-(ix)^3/3!+(ix)^5/5!-(ix)^7/7!....$

$y=\sin(ix)=i(x-i^2x^3\,/3!+i^4x^5\,/5!-i^6x^7\,/7!.....$

$y=\sin(ix)=i(x-x^3\,/3!+x^5\,/5!-x^7\,/7!.....)$

$y=\sin(ix)=i(\sin(x))$

As i^2, i^6,.......=-1, i^4=1

Series Expansion of cos(ix)

$y=\cos(x)=1-x^2/2!+x^4/4!-x^6/6!.........$

$y=\cos(ix)=1-(ix)^2/2!+(ix)^4/4!-(ix)^6/6!......$

$y=\cos(ix)=1+(x)^2/2!+(x)^4/4!+(x)^6/6!.......$

As i^2, i^6,.......=-1, i^4=1

Series Expansion of Exp(ix)=cos(x)+i sin(x)

$y=\exp(ix)=1+ix+(ix)^2/2!+(ix)^3/3!+(ix)^4/4!+(ix)^5/5!+(ix)^6/6!+(ix)^7/7!\ldots\ldots\ldots$

$y=\exp(ix)=1+ix+(-1)x^2/2!+(-1)\ ix^3/3!+x^4/4!+x^5/5!+(-1)x^6/6!+(-1)\ ix^7/7!\ldots\ldots\ldots$

$y=\exp(x)=1+x^2/2!+x^4/4!+x^6/6!+ix-ix^3/3!+ix^5/5!-ix^7/7!\ldots\ldots\ldots$

$y=\exp(x)=1-x^2/2!+x^4/4!-x^6/6!+i(x-x^3/3!+x^5/5!-x^7/7!\ldots\ldots\ldots$

$\exp(ix)=\cos(x)+i\sin(x)=z$

Hence, $z=\cos(x)+i\sin(x)=e^{ix},$ x=angle

And $z=r(\cos(x)+i\sin(x)=r\,e^{ix}$

Exponential Form of Complex Numbers

Euler's Equation:

e^(i*t) = cos(t)+i sin(t)
z=x+i y
z=r*(cos(t)+i sin(t))
z=r*e $^{(i*t)}$
Now, if t=pi()
Then, z=e $^{i\ pi()}$, cos(pi)+i sin(pi)=-1

This formula was found by RICHARD FEYMAN at the age of 14.

Relation between Trigonometry, Exponential and imaginary numbers

exp=taylor(exp(x))

%x^6/720+x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

sine=taylor(sin(x),7)

%x^5/120 - x^3/6 + x

cos=taylor(cos(x),7)

%- x^6/720 + x^4/24 - x^2/2 + 1

exp(i*x)=cos(i*x)+sin(i*x)=cos(x)+isin(x)

exp(i*pi)=cos(i*pi)+I sin(i*pi) = cos(pi)+isin(pi)=-1

(From Taylor expansion, putting x=ix, i^2=1)

Note: Exp(i*pi)= cos(pi)+i sin(pi) links five most important symbols in mathematics

Relation between Trigonometry, Exponential and imaginary numbers

- Exp(i*pi)= cos(pi)+i sin(pi) links five most important symbols in mathematics
- $e^{(i*x)=}$ cos(x)+i sin(x)
- z=x+iy=r(cos(t)+i sin(t)) = re^{it}

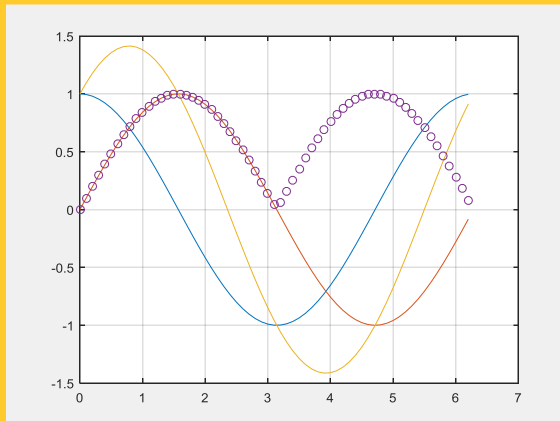
```
t=0:.1:2*pi
x=cos(t)
y=sin(t)
et=exp(t)
eti=exp(i*t)
plot(t,x,t,y)
sit=sin(i*t)
hold on
plot(t,x+y)
grid
```

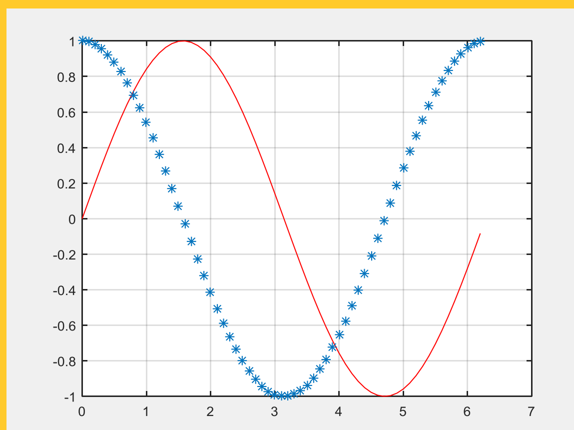
```
plot(t,imag(sit),t,et,'o')
grid
figure
plot(t,imag(eti),'-
r',t,real(eti),'*')
grid
figure
plot(t,abs(eti))
grid
```

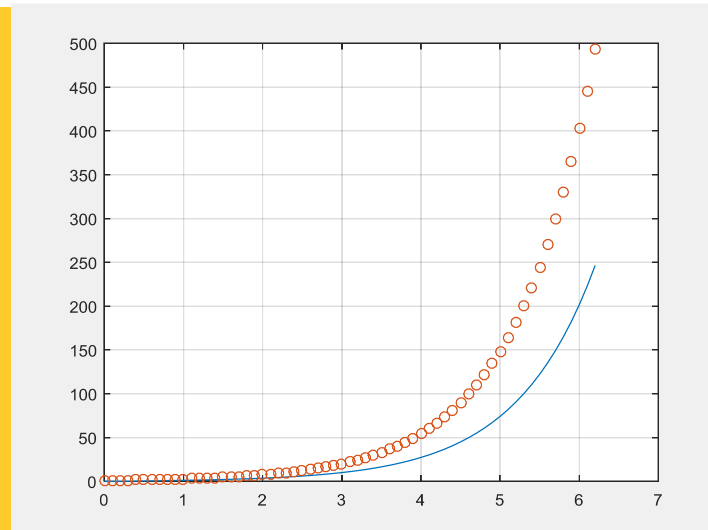
Relation between Trigonometry, Exponential and imaginary numbers

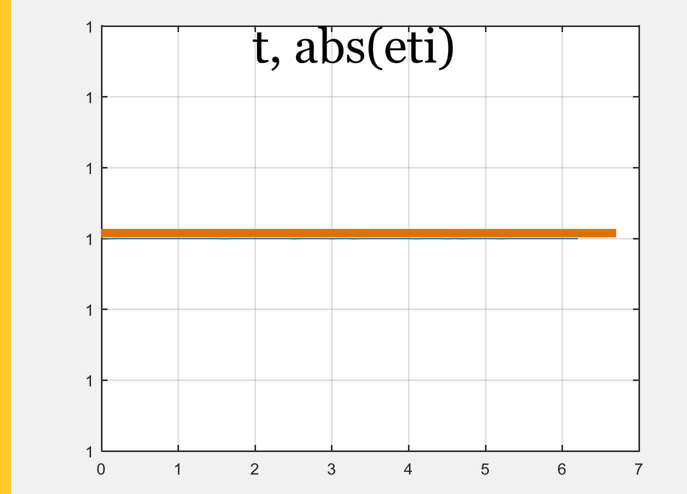
- $z=x+iy=r(\cos(t)+i\sin(t)) = re^{it}$

x=cos(t)
y=sin(t)
plot(t,x,t,y)
sit=sin(i*t)
plot(t,x+y)
plot(t,imag(sit),t,et,'o')
plot(t,imag(eti),'-r',t,real(eti),'*')
plot(t,abs(eti))









Problems of Exponential Form

	Find the complex number expression of the following					
		Angle, t	r	x	y	z=
Prob-1	e^(i*pi()/4)	0.785398	1	0.707107	0.707107	0.707106781186548+0.707106781186547i
Prob-2	e^-i	-1	1	0.540302	-0.841477	0.54030230586814-0.841470984807897i
Prob-3	e^i*pi()	3.141593	1	-1	1.23E-16	-1+1.22514845490862E-16i
Prob-4	3*e^1pi()/6	0.523599	3	2.598076	1.5	2.59807621135332+1.5i

Complex Functions
Limits
Differentiations

Complex Functions

Curves as Domain:

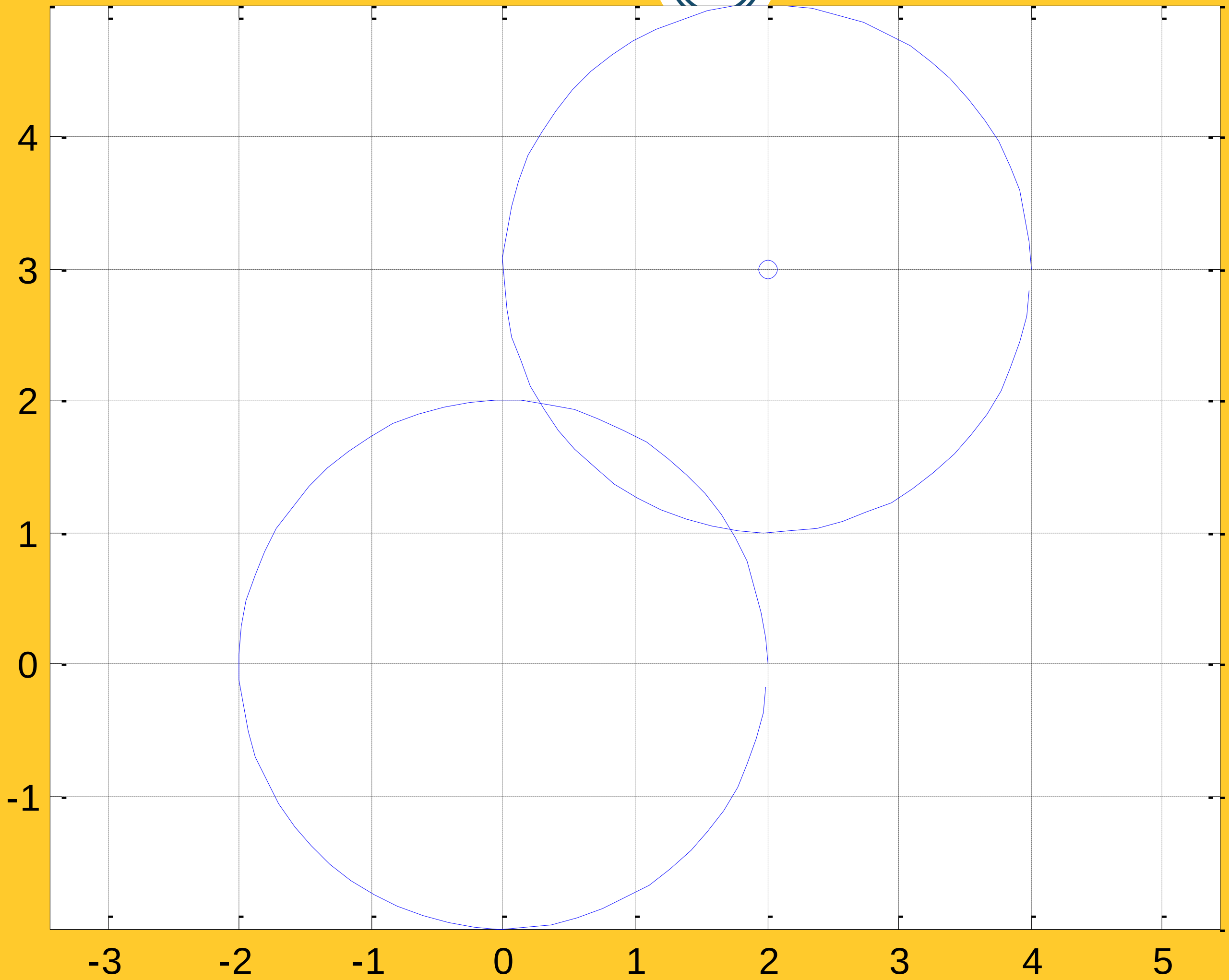
```
r=2
t=0:.1:2*pi
x=r*cos(t)
y=r*sin(t)
z=x+i*y
plot(x,y)
a=2+3*i
w=z+a

plot(real(w),imag(w))
```

```
hold on

plot(real(a),imag(a),'o')
 axis equal
  grid
  distance=abs(w-a)
```

Curve



Curve and Region

- Curve - $|z-a|=r$
- Open Circular Disc - $|z-a|<r$
- Closed Circular Disc - $|z-a|<=r$
- Exterior of circular Disc - $|z-a|>r$
- Annulus - $r1<|z-a|<r2$
-

Complex Function: Limit, Derivative

- Function: A function defines a rule which assign to each z in S a unique complex number w.
- The function is defined as, $w=f(z)$
- S is called domain and the set of complex numbers which w assumes as z varies is called range
- The real part (u) and imaginary part (v) of a complex number is a function of x and y.

Example of functions of complex variable

Let the function w=z^2+3z and z=x+i y

w=(x+i y)^2+3(x+i y)

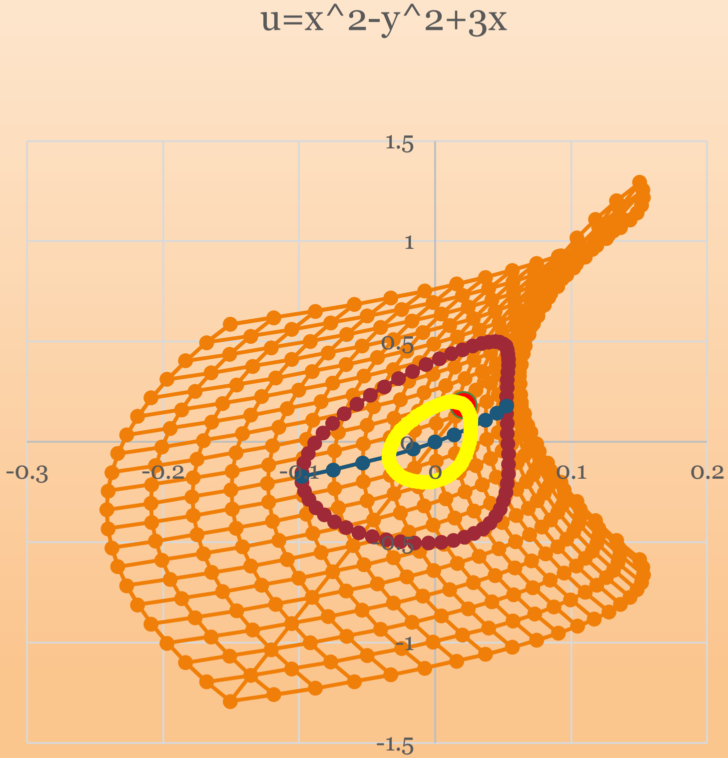
w=x^2+(i y)^2+2*x*i *y+3*x+3* i *y

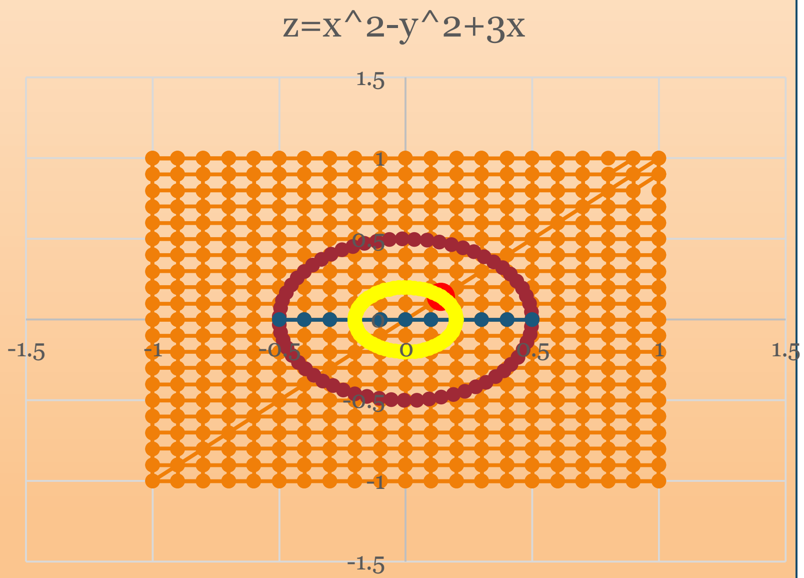
w=x^2-y^2+2*i*x*y+3*x+3*i*y

w=(x^2-y^2+3*x)+i*(2*x*y+3*y)

Here, u= x^2-y^2+3*x, v= 2*x*y+3*y

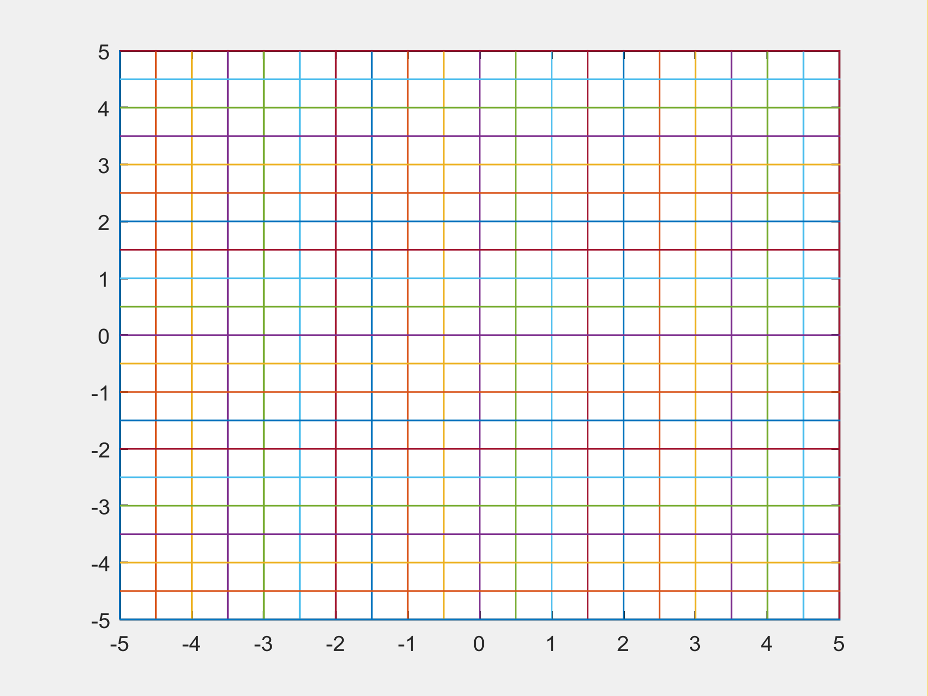
Plots of w=f(z)=u(x,y)+i*v(x,y)





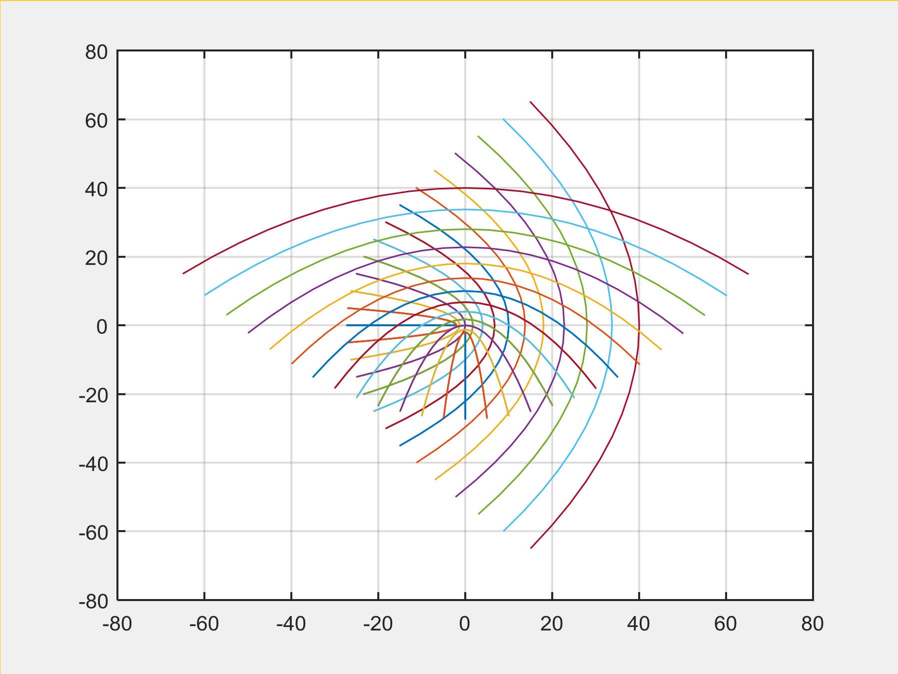
Complex Functions with Domain and Range

```
%For Creating GRID or Domain
x=-5:.5:5
y=x
[xx,yy]=meshgrid(x,y)
plot(xx,yy,yy,xx)
```

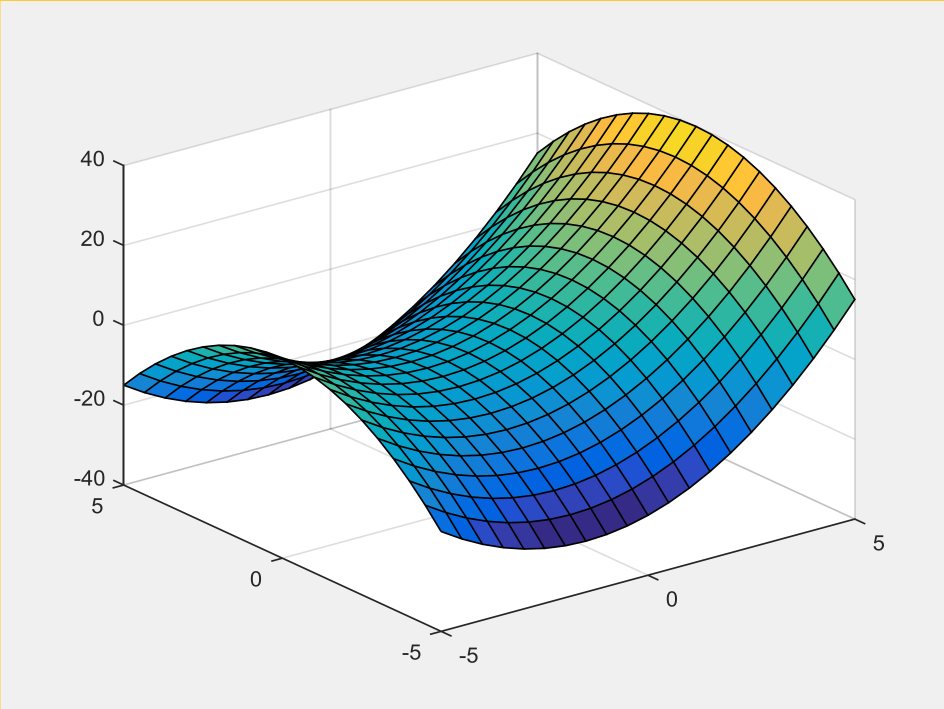


MATLAB COMMANDS FOR COMPLEX ANALYSIS

```
z=complex(xx,yy)
fz=z.^2+3.*z
refz=real(fz)
imfz=imag(fz)
 figure
 plot(refz,imfz,imfx,refz)
grid
```

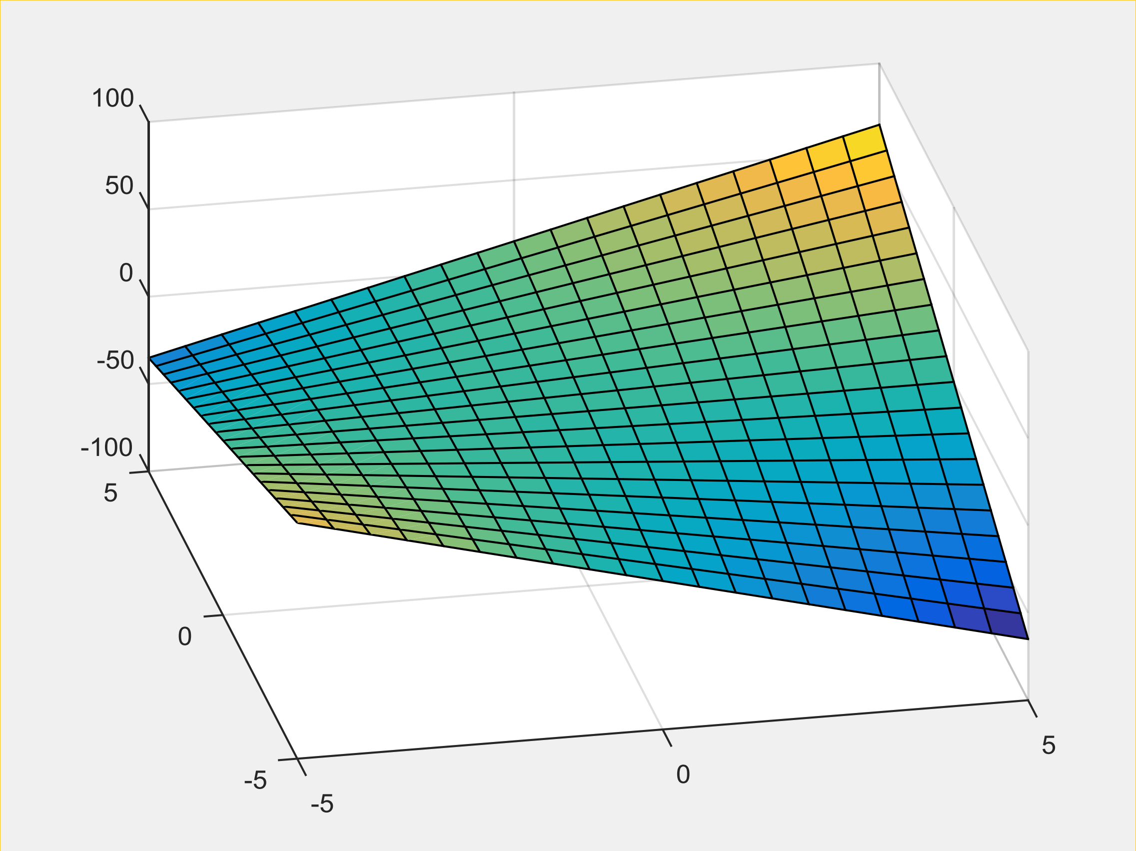


%Plotting real part of z
u=xx.^2-yy.^2+3*xx
surf(xx,yy,u)



MATLAB COMMANDS FOR COMPLEX ANALYSIS

```
%Plotting imaginary part of z
v=2*xx.*yy+3*yy
surf(xx,yy,v)
```



Limit of Functions of Complex variable

- Definition: A function w =f(z) is said to have the limit L as z approaches zo and if for very number ε >0 there is a corresponding number $\delta > 0$ such that |z-zo|< δ, |f(z)-L|< ε
- The problem can be posed in the following way:
- How close to zo does z have to be so that w=f(z) differs from L by less than ε.
- The distance from x to zo is |z-zo| and the distance from f(z) to L is |f(z)-L|.

Limit of Functions of Complex variable

- Let us fix the value of $|f(z)-L|<\varepsilon$
- Let, $\varepsilon = 0.2$.
- So our problem is to find out δ so that $|z-z_0|< \delta$.

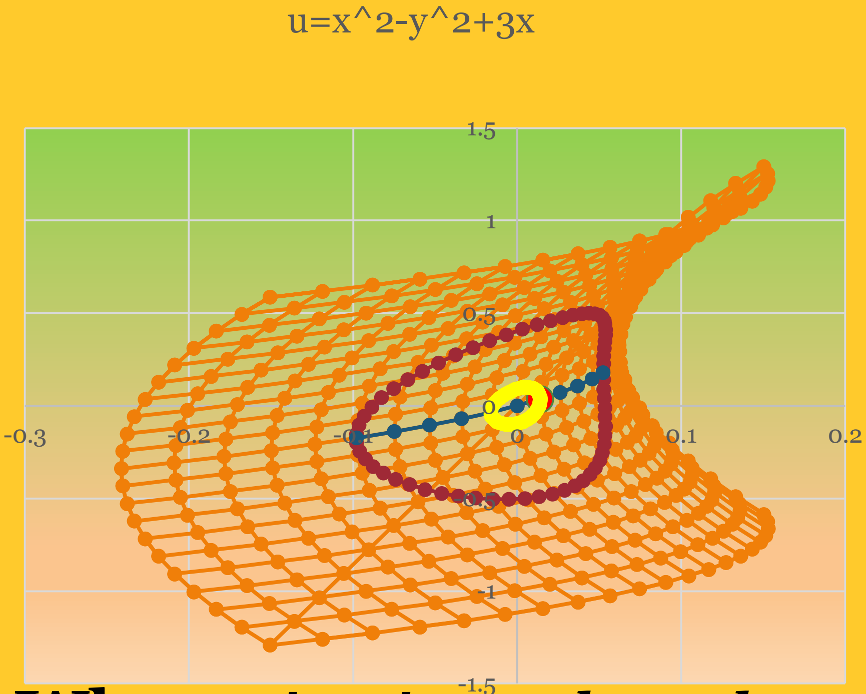
Limit of Functions of 2 variable

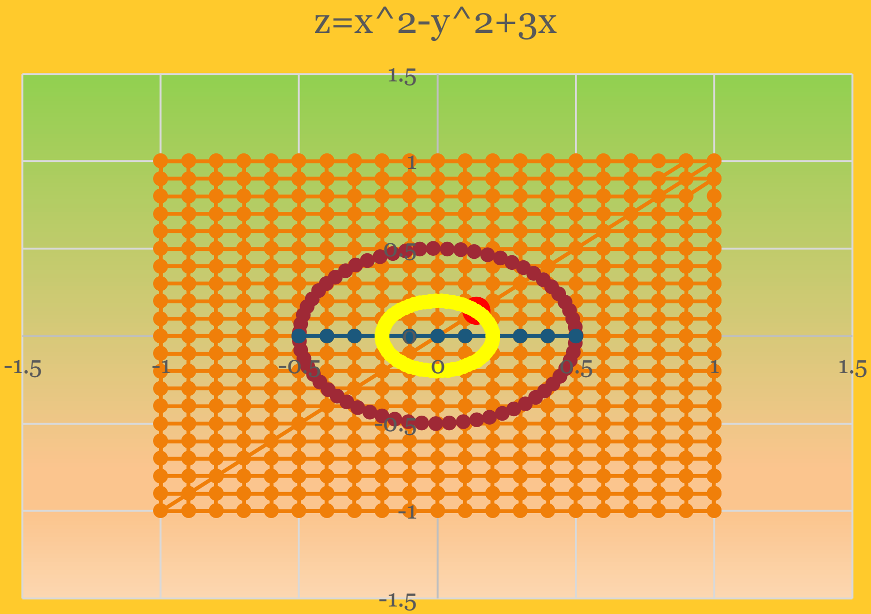
- Demonstration of $\lim_{(x,y)\to(a,b)} f(x,y) = L$

Note: sqrt((x-a)^2+(y-b)^2) is the distance between (x, y) and (a, b) and |f(x, y)-L| is the difference between the numbers f(x, y) and L.

Limit of Functions of 2 variable

- Demonstration of $\lim_{(x,y)\to(a,b)} f(x,y) = L$

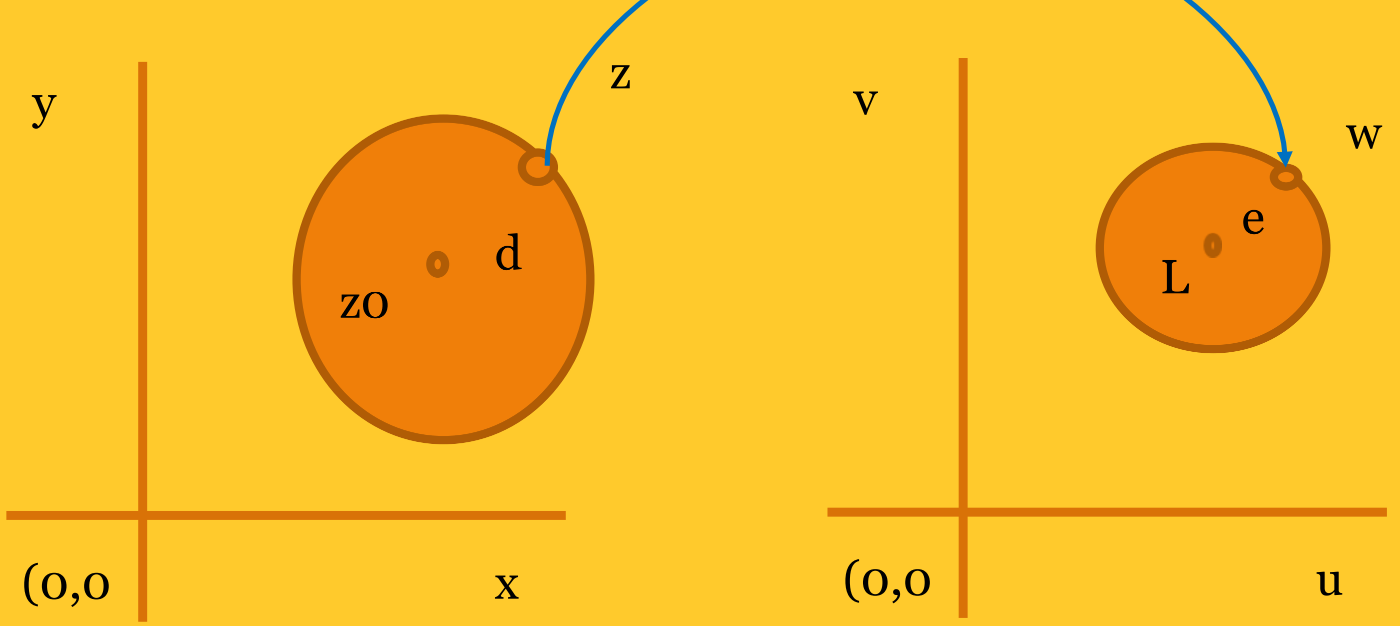




- **When ε *is given then the maximum value of δ* is given by the formula 0<sqrt(x-a)^2+(y-a)^2)< *δ* = r**

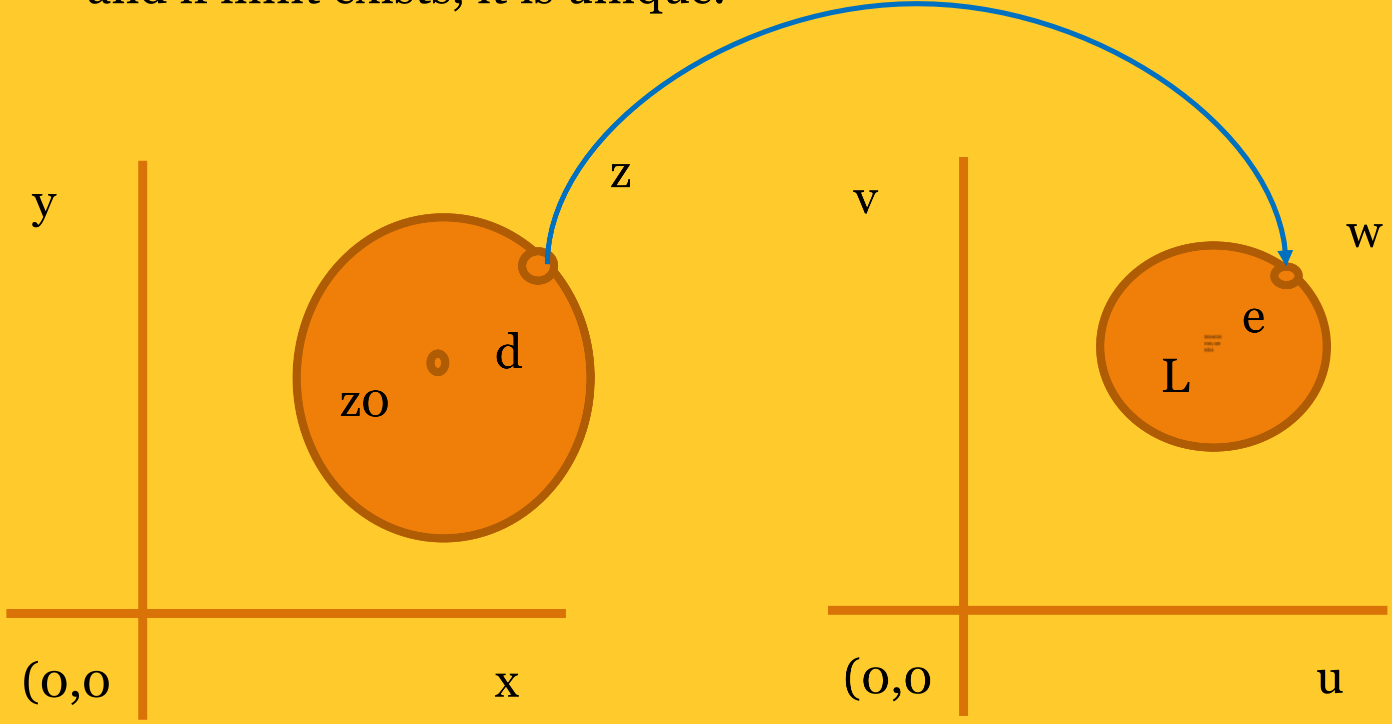
Limit

A function w=f(z) said to have the limit L as z approaches z0 if for every positive e we can find a real number d such that for all values |z-z0|<d, |w-L|<e



Limit

It indicates that the value w as close as desired to L for all z which are very close to zo. It is expressed as $\lim_{z \to z0} w \to L$. z can approach from any direction and if limit exists, it is unique.



Continuity

A function is said to be continuous at z=z0, if w is defined at and

$$\lim_{z \to z0} f(z) \to f(z0)$$

Differentiability

A function is said to be differentiable at a point z=z0, if $\lim_{z\to 0}\left(\frac{f(z0+dz)-f(z0)}{dz}\right)$ exists.

Note: All the rules of real differential calculus continue to hold for complex functions

Differentiation

Example-Find the derivative of the function

$w=z^2+3z$, and $z=x+i\,y$

$w=(x+i\,y^2+3(x+i\,y)$

$w=x^2+(i\,y)^2+2*x*i*y+3*x+3*i*y$

$w=x^2-y^2+2*i*x*y+3*x+3*i*y$

$w=(x^2-y^2+3*x)+i*(2*x*y+3*y)$

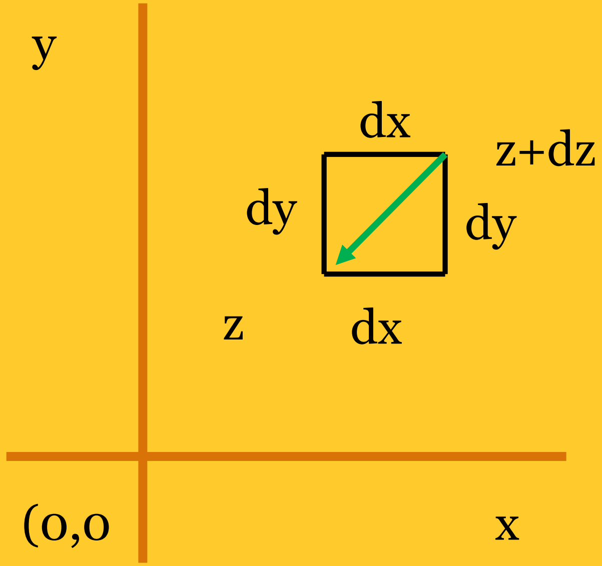
Here, $u= x^2-y^2+3*x$, $v= 2*x*y+3*y$

Now $\frac{dw}{dz} = \lim_{dz\to 0}\left(\frac{f(z0+dz)-f(z0)}{dz}\right)$

Differentiation

Here, u= x^2-y^2+3*x, v= $2*x*y+3*y$

$$\text{Now } \frac{dw}{dz} = \lim_{dz\to 0}\left(\frac{f(z0+dz)-f(z0)}{dz}\right)$$



We have, dz=dx+idy
We can make dz=0, by first making dy=0 and then dx=0.
When we put dy=0, dz=dx and the equation becomes:

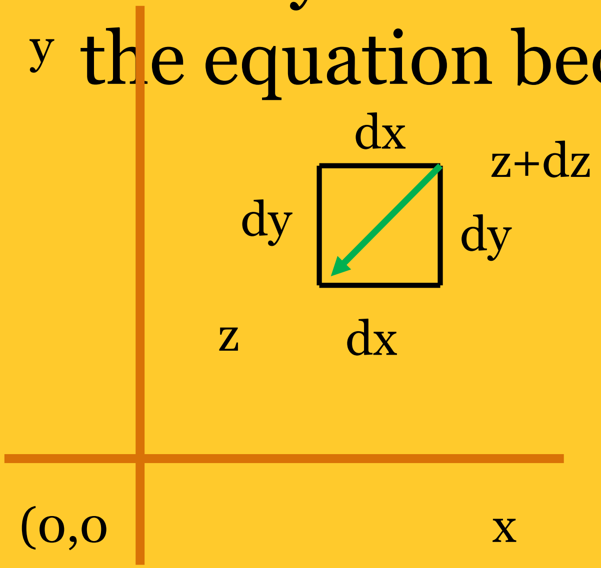
$$\frac{dw}{dz} = \frac{dw}{dx} = (2x+3)+2yi$$

Differentiation

We have, u= x^2-y^2+3*x, v= $2*x*y+3*y$

and dz=dx+idy

We can make dz=0, by first making dx=0 and then dy=0. When we put dx=0, dz=idy and the equation becomes:

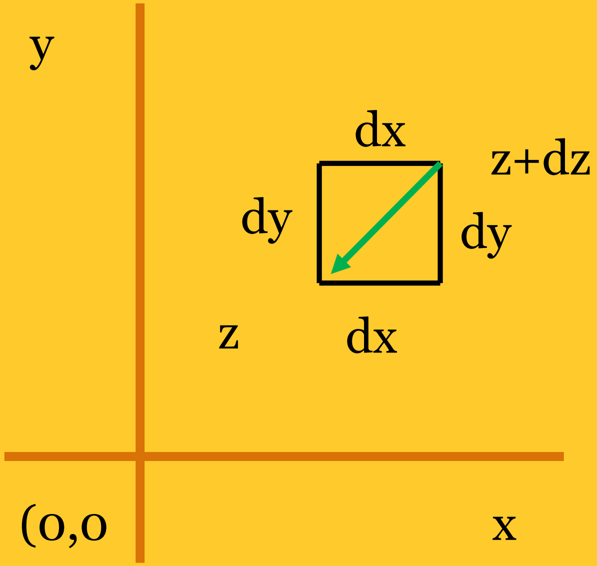


$$\frac{dw}{dz} = \frac{dw}{idy} = 2yi + (2x+3), \text{ as } 1/i=-i$$

Differentiation

$$\frac{dw}{dz} = \frac{dw}{dx} = (2x+3)+2yi$$

$$\frac{dw}{dz} = \frac{dw}{idy} = 2yi + (2x+3), \text{ as } 1/i=-i$$



It can be seen that dw/dz involves four partial derivatives:
du/dx, du/dy, dv/dx, dv/dy

Differentiation

$$\frac{dw}{dz} = \frac{dw}{dx} =(2x+3)+2yi$$

$$\frac{dw}{dz} = \frac{dw}{idy} =2yi +(2x+3), \text{ as } 1/i=-i$$

It can be seen that dw/dz involves four partial derivatives:
du/dx, du/dy, dv/dx, dv/dy

dw/dz=du/dx+idv/dx
dw/dz=du/dx-idu/dy
dw/dz=dv/dy+idv/dx
dw/dz=dv/dy-idu/dy

Differentiation with Matlab

dw/dz=du/dx+idv/dx
dw/dz=du/dx-idu/dy
dw/dz=dv/dy+idv/dx
dw/dz=dv/dy-idu/dy

```
syms x y z w
 z=x+i*y
dz/dx=diff(z,x)=1
dz/dy=diff(z,y)=i
w=z²    =(x + y*1i)^2

dw/dx =2*x + y*2i
dw/dy = x*2i - 2*y
u= x^2 - y^2
v= x*y*2i
```

```
u= x^2 - y^2
v= x*y*2
dudx = 2*x
dudy = -2*y
dvdx = y*2
dvdy = x*2
```

Cauchy-Riemann Equation

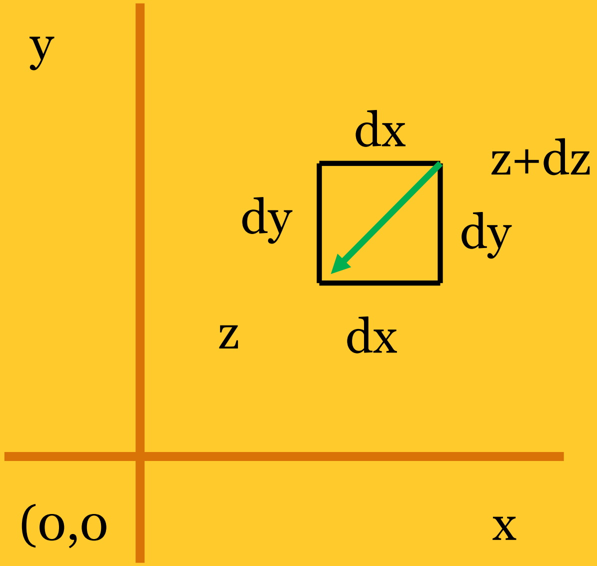
1. du/dx=dv/dy
2. du/dy=-dv/dx

Differentiation with Matlab

dw/dz=du/dx+idv/dx
dw/dz=du/dx-idu/dy
dw/dz=dv/dy+idv/dx
dw/dz=dv/dy-idu/dy

Cauchy-Riemann Equation

It is seen that dw/dz involves four partial derivatives: du/dx, du/dy, dv/dx, dv/dy. Cauchy Riemann equation establishes a relationship among the partial derivative.

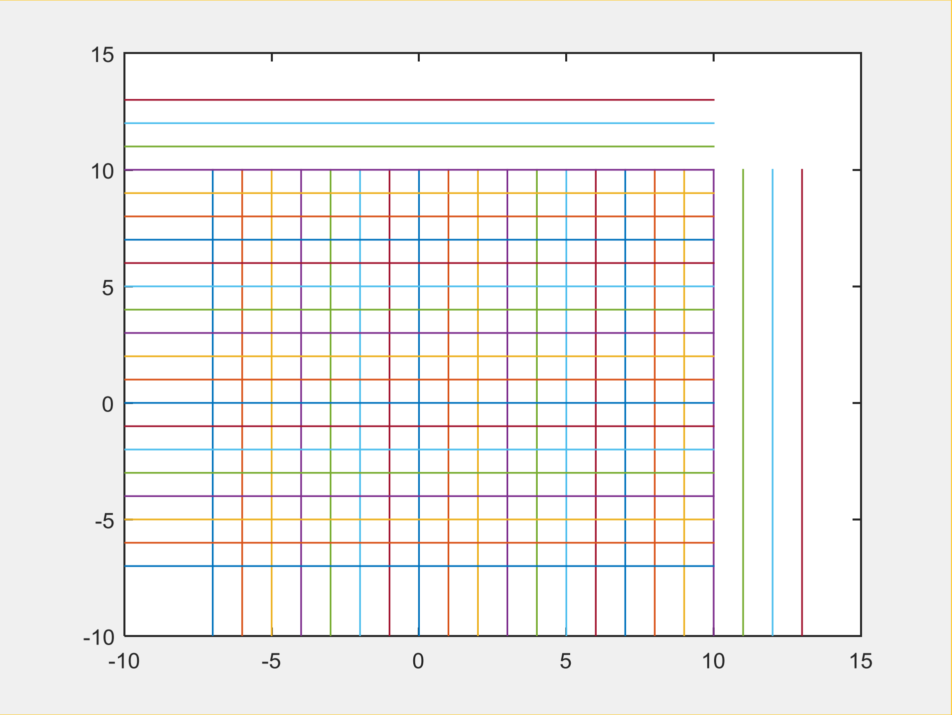


Cauchy-Riemann Equation

1. du/dx=dv/dy
2. du/dy=-dv/dx

Example: $w=z^2+3z$

```
%Plotting derivative of u and v of z
dudx = 2*xx + 3
 dvdx = 2*yy
 figure
plot(dudx,dvdx,dvdx,dudx)
grid
```



Analiticity

A function f(z) is said to be analytic in a domain D if f(z) is defined and differentiable at all points.

Note- Analytic and holomorphic are same meaning.

Cauchy Riemann equation helps in finding analiticity of a function.

Cauchy Riemann Equation

```
z = x + y*1i
w = (x + y*1i)^3
dwdx = 3*(x + y*1i)^2
dwdy = (x + y*1i)^2*3i
w1 = x^3 + x^2*y*3i - 3*x*y^2 - y^3*1i
u = x^3 - 3*x*y^2
v = x^2*y*3i - y^3*1i
dudx = 3*x^2 - 3*y^2
dudy = -6*x*y
dvdx = x*y*6i
dvdy =x^2*3i - y^2*3i
```

Cauchy Riemann Equation

u = x^3 - 3*x*y^2
v = x^2*y*3i - y^3*1i
dudx = 3*x^2 - 3*y^2
dudy = -6*x*y
dvdx = x*y*6i
dvdy =x^2*3i - y^2*3i
du/dx+idv/dx=3(x^2-y^2)-6xyi
dv/dx+idv/dy=6xyi+3(x^2-y^2)
As this is complex number, and LHS=RHS, then the real and imaginary are equal
du/dx=dv/dy and du/dy=-dv/dx

Cauchy Riemann Equation

$w=z^3$

Cauchy Riemann Equation in Polar form

In polar form, z=r*{cos(t)+i sin(t)}

- $e^{(i*x)=}$ cos(x)+i sin(x)
- z=x+iy=r(cos(t)+i sin(t)) = re^{it}
- z=re^{it}
- Now, z is the function of r and t and for differentiation of z w r t r or t, we apply product rule.

Cauchy Riemann Equation in Polar form

In polar form, z=r*{cos(t)+i sin(t)}

- $z=re^{it}$
- dz/dr=r dz/dr+t dz/dr, and,
- dz/dt=r dz/dt+ t dz/dt

- $dz/dr=r\ e^{it} + t\ r\ e^{it}$
- $dz/dt=r\ e^{it} +t$

Example C R Equation in Polar form

```
%Polar C R Equation
%Example Kachot 13a page 84 w=z^3
syms x y u v w z r t
 z=r*exp(i*t)
w=z^3
 dwdr=diff(w,r)=3*r^2*exp(t*3i)
dwdrt=diff(w,t)=r^3*exp(t*3i)*3i

+50.3504i
```

Example C R Equation in Polar form

z=r*exp(i*t), w=z^3

dwdr=diff(w,r)=3*r^2*exp(t*3i)

dwdrt=diff(w,t)=r^3*exp(t*3i)*3i

t=.3, r=3

z = 2.8660 + 0.8866i

w = 16.7835 +21.1498i

az = 3

aw = 27.0000

dwdr = 16.7835 +21.1498i

dwdt = -63.4495 +50.3504i

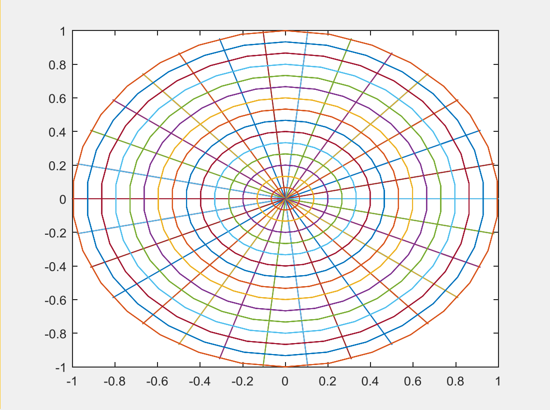
dwdt1 = -63.4495 +50.3504i

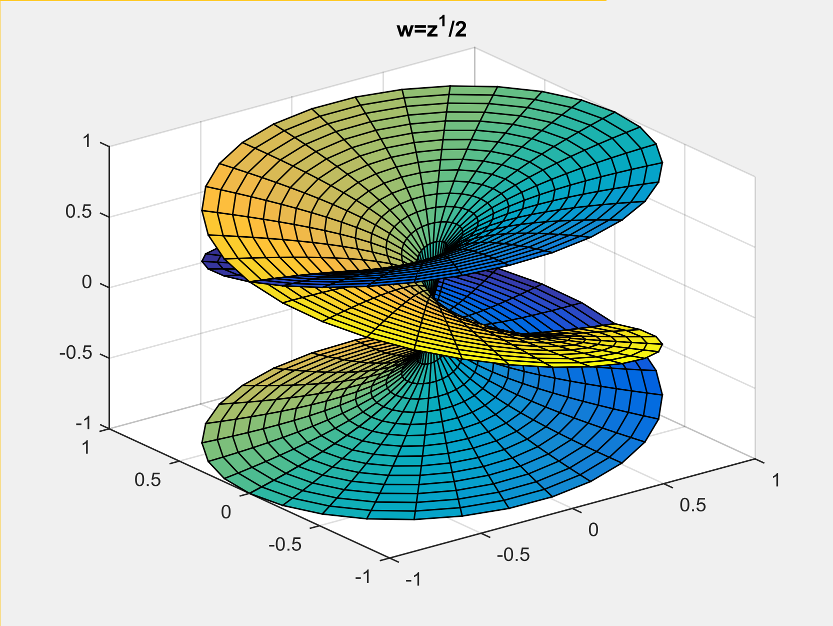
Example C R Polar Cartesian relation

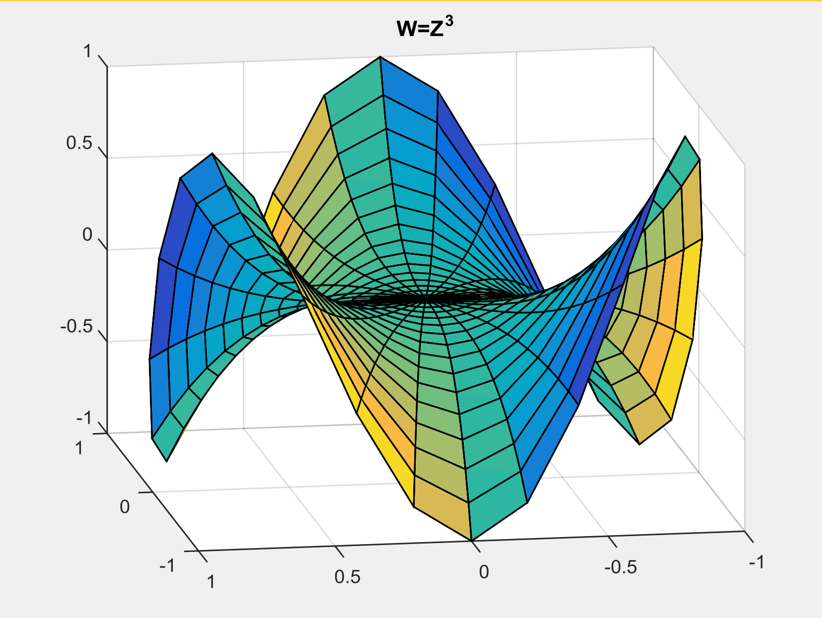
```
x=1
y=1
z=x+i*y
w =(x + y*1i)^3
az=abs(z)
aw=abs(w)
dwdx =3*(x + y*1i)^2
dwdy =(x + y*1i)^2*3i
dwdy1=i*dwdx
```

```
x = 1, y = 1
z = 1.0000 + 1.0000i
w = -2.0000 + 2.0000i
az =    1.4142
aw =    2.8284
dwdx =  0.0000 + 6.0000i
dwdy =    -6
dwdy1 =    -6
```

Example C R Equation in Polar form







Cauchy Riemann Equation in Polar form

In polar form, z=r*{cos(t)+i sin(t)}
Here, z is a function of r and t.
f(z)=u(r,t)+i v(r,t)
du/dr=1/r*(dv/dt) and dv/dr=-1/r(du/dt)

Example: $w=z^3$

dw/dz=dw/dr=r^3*{cos(3t)+i sin(3t)}
=3 r^2 {cos(3t)+i sin(3t)}
As t is constant and we are differentiating with respect to r.

Example: Cauchy Riemann Equation

Function, w=z^2

x	y	z	w=z^2	real	imag	u=x^2 -y^2	v=2xy	du/dx	dv/dy	du/dy	dv/dx
2	5	2+5i	-21+20i	-21	20	-21	20	4	4	-4	4

Hence, du/dx=dv/dy and du/dy = -dv/dx

Cauchy Riemann Equation in Polar form

Example: $w=z^3$

$dw/dz=dw/dr=r^3*\{cos(3t)+i\ sin(3t)\}=3\ r^2$
$cos/(3t)+$
Keeping t as constant and we are differentiating with respect to r.

$dw/dz=dw/dt=r^3*\{cos(3t)+i\ sin(3t)\}=3\ r^2$
$cos(3t)+ 3\ r^2\ sin(3t)i$
As t is constant and we are differentiating with respect to r.

Cauchy Riemann Equation in Polar form

Example: $w=z^3$

$dw/dz=dw/dt=r^3*\{\cos(3t)+i\sin(3t)$
Keeping r as constant and we are differentiating with respect to t.

$dw/dz=dw/dt=r^3*\{-\sin(3t)*3+i\cos(3t)*3\}$
$=3\ r^3\ \{\cos(3t)-\sin(3t)i\}$

$=3\ r^2\ \{\cos(3t)+i\sin(3t)\}$

Cauchy Riemann Equation in Polar form

Example: $w=z^3$

$dw/dz=dw/dr=r^3*\{\cos(3t)+i\sin(3t)\}=3r^2$ cos/(3t)+

As t is constant and we are differentiating with respect to r.

$dw/dz=dw/dt=r^3*\{\cos(3t)+i\sin(3t)\}=3r^2\cos(3t)+3r^2\sin(3t)i$

As t is constant and we are differentiating with respect to r.

Laplace Equation

The derivative of an analytic function f(z)=u(x,y)+I v(x,y) is itself analytic. From this fact it is established that u and v will have continuous partial derivative of all order. The mixed derivatives of these functions are equal.

$$\frac{\partial^2 u}{\partial x dy} = \frac{\partial^2 u}{dy dx} \qquad \frac{\partial^2 v}{\partial x dy} = \frac{\partial^2 v}{dy dx}$$

From the above facts and differentiating C R equation, we get

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{dy dx} \qquad \frac{\partial^2 u}{\partial y^2} = -\frac{\partial^2 v}{dx dy}$$

$$\frac{\partial^2 u}{\partial x dy} = \frac{\partial^2 v}{dy^2} \qquad \frac{\partial^2 u}{dx \partial y} = -\frac{\partial^2 v}{dx^2}$$

This finding give Laplace’s Equations.

Laplace Equation

The real and imaginary part of a complex function fz=u(x,y)+ i v(x,y) that is analytic in a domain D are solutions of Laplace Equations-

$$\nabla^2 = \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{and} \qquad \nabla^2 = \frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$$

in D and have continuous second partial derivatives I D.

Harmonic Function

Elementary Complex Functions

1. Algebraic Functions
    A. Power function
    B. Polynomial functions
    C. Rational Functions
    D. Roots

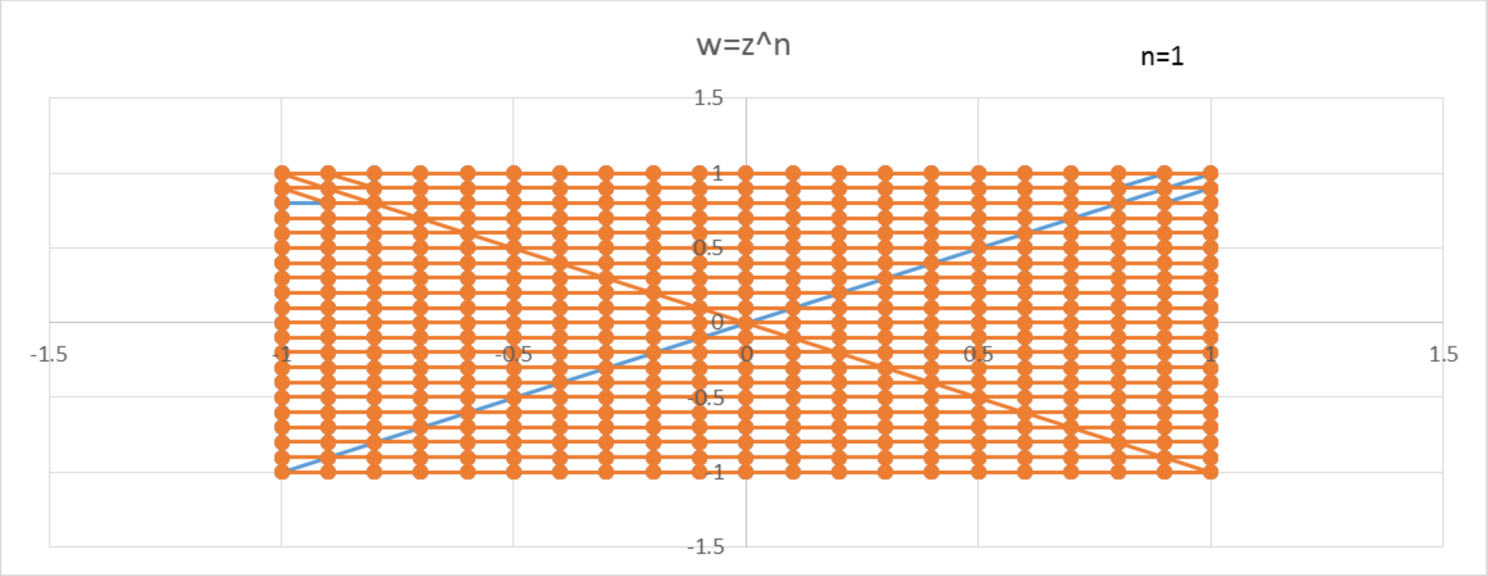
2. Exponential Functions

3. Trigonometric Functions
4. Hyperbolic Functions
5. Logarithmic Functions

Power Functions, w=z^n

Power function: $w=z^n$:n=natural numbers

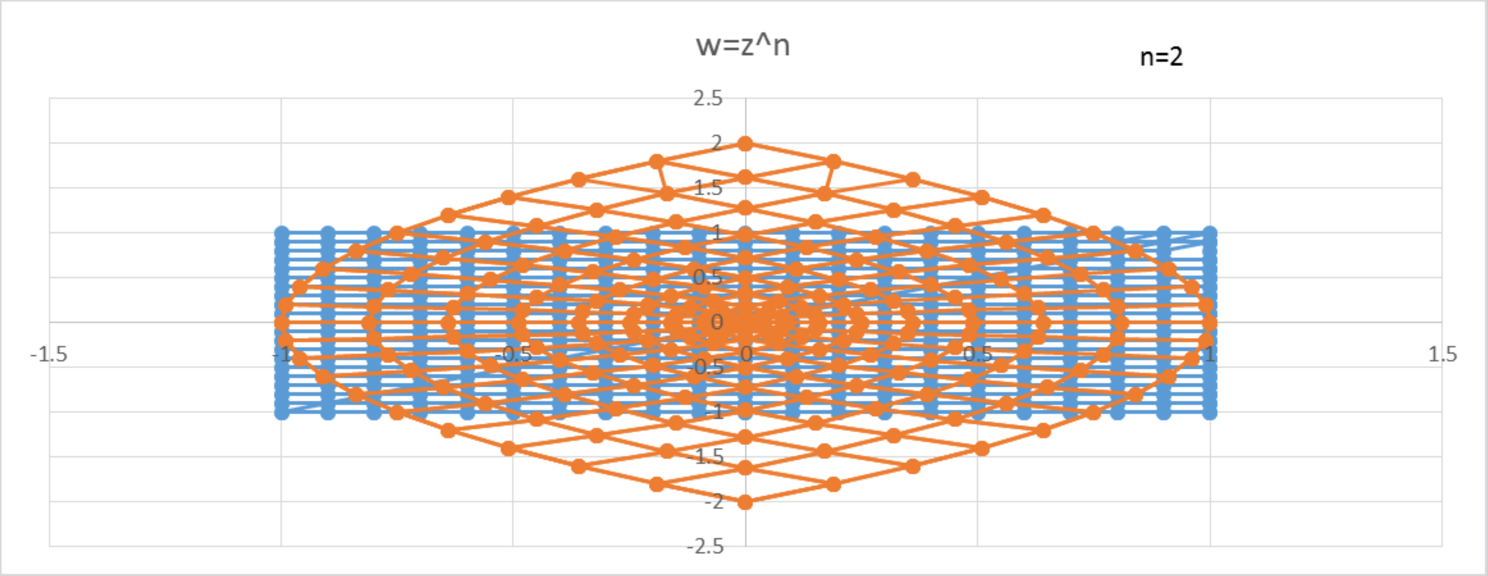
w=z^n, n=1



Power Functions, w=z^n

Power function: $w=z^n$:n=natural numbers

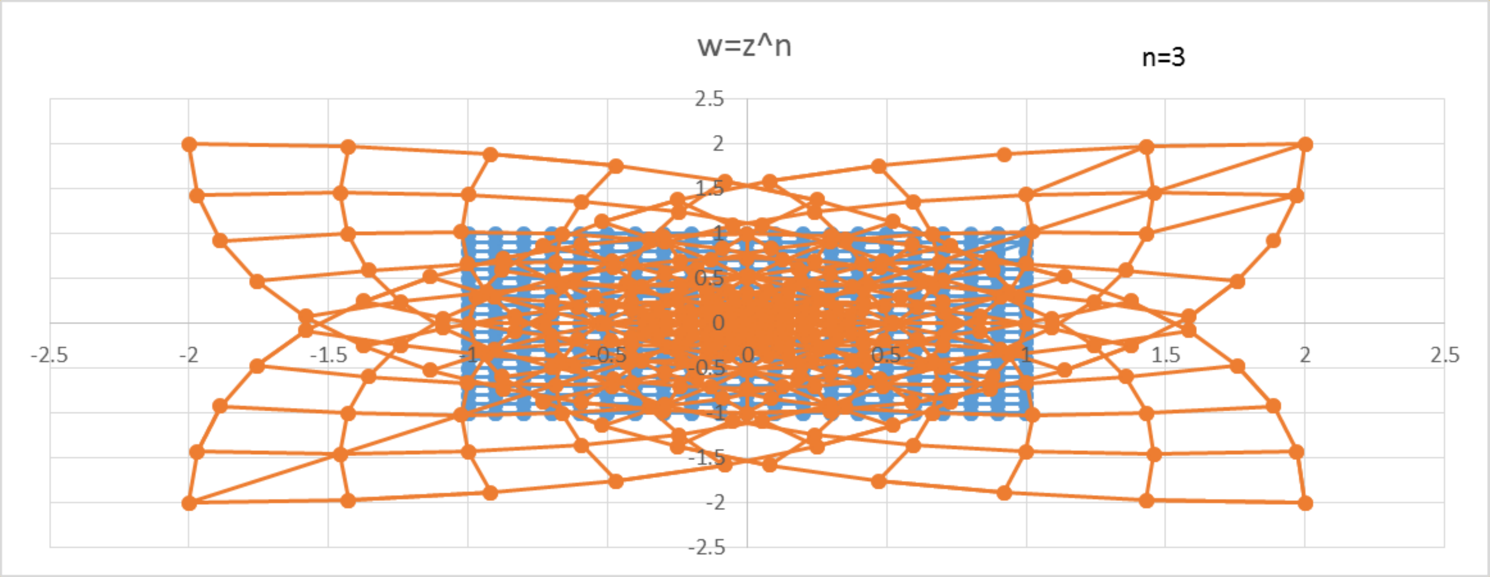
w=z^n, n=2



Power Functions, w=z^n

Power function: $w=z^n$:n=natural numbers

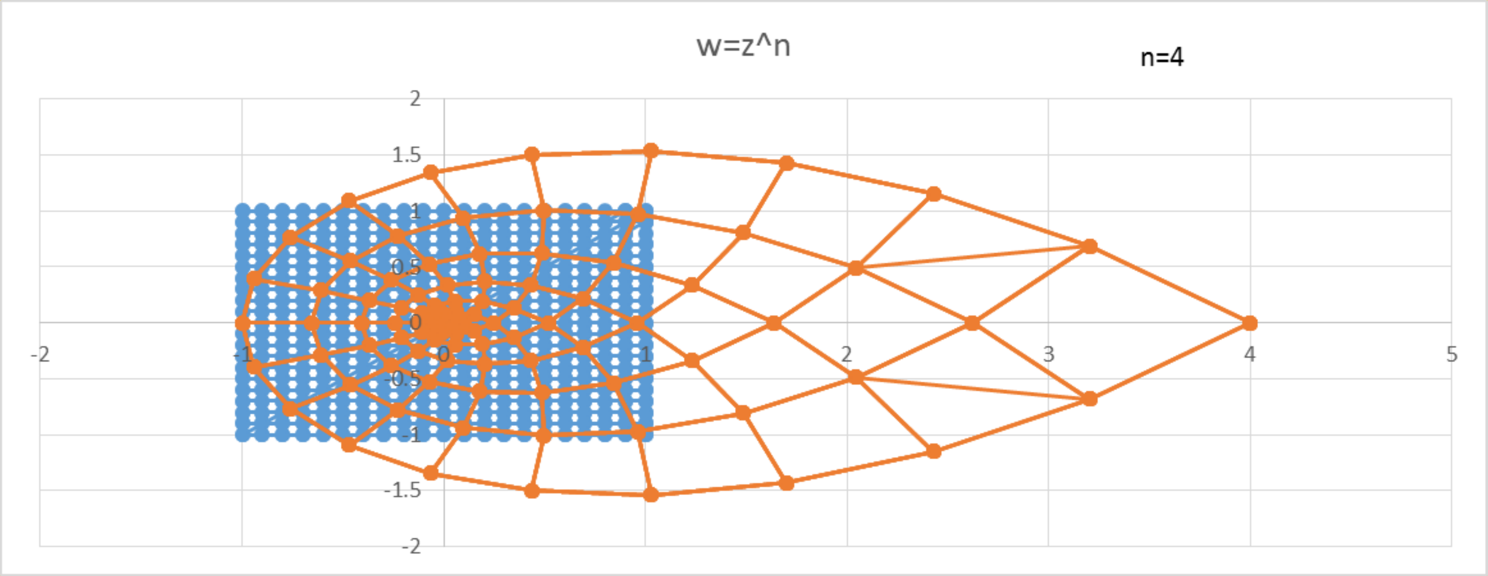
w=z^n, n=3



Power Functions, w=z^n

Power function: $w=z^n$:n=natural numbers

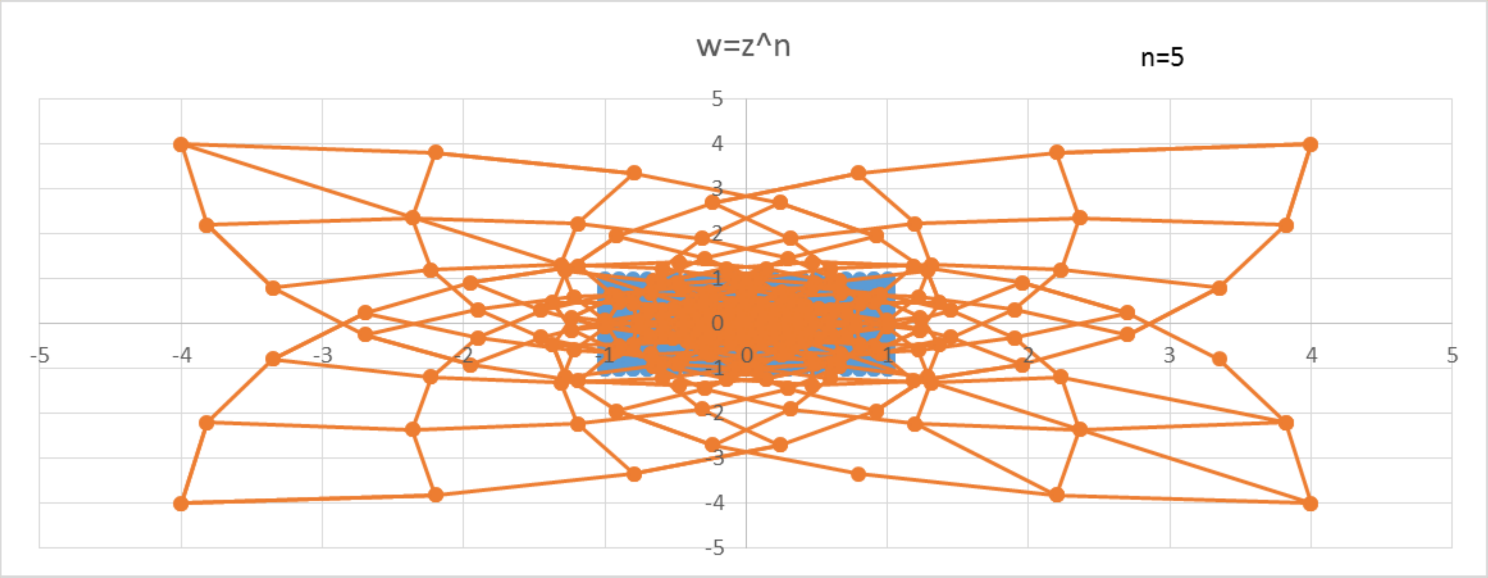
w=z^n, n=4



Power Functions, w=z^n

Power function: $w=z^n$:n=natural numbers

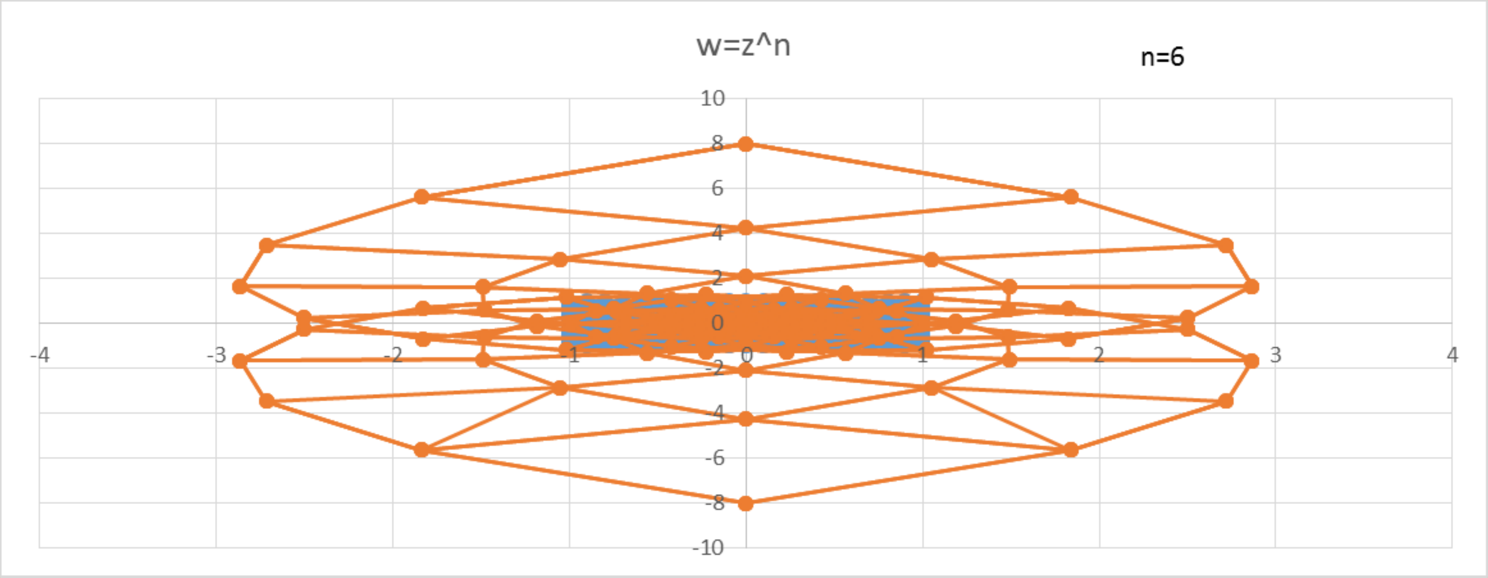
w=z^n, n=5



Power Functions, w=z^n

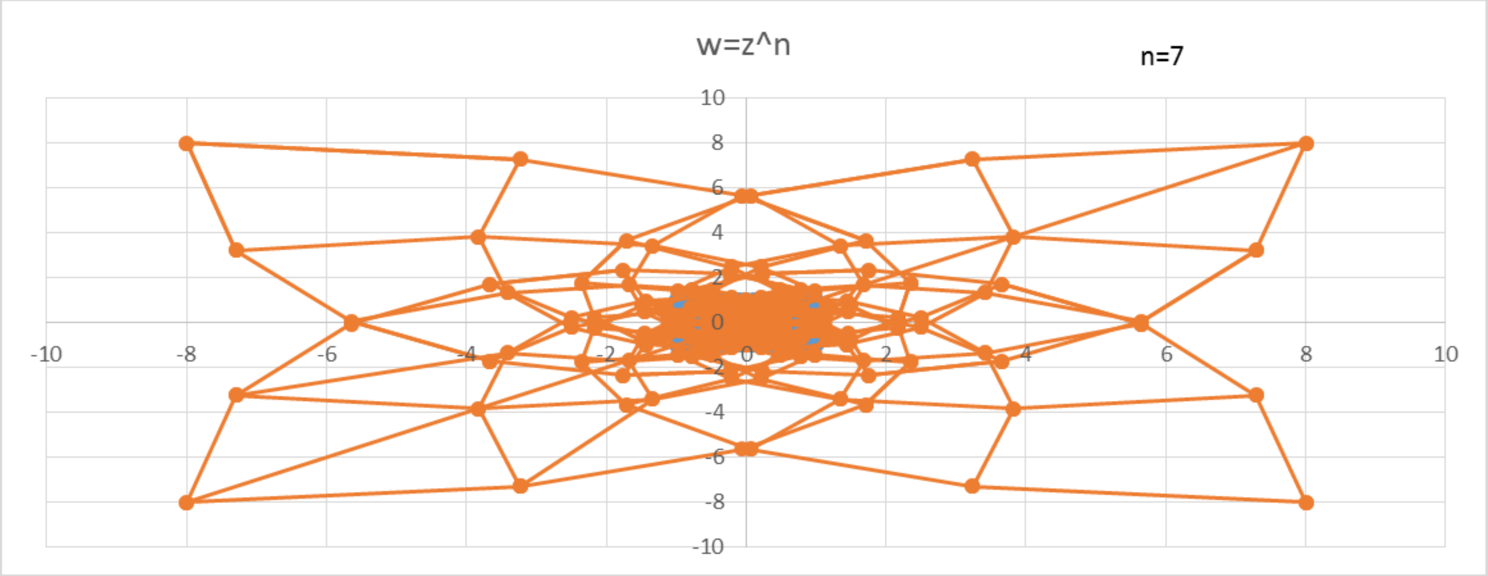
Power function: $w=z^n$:n=natural numbers

w=z^n, n=6



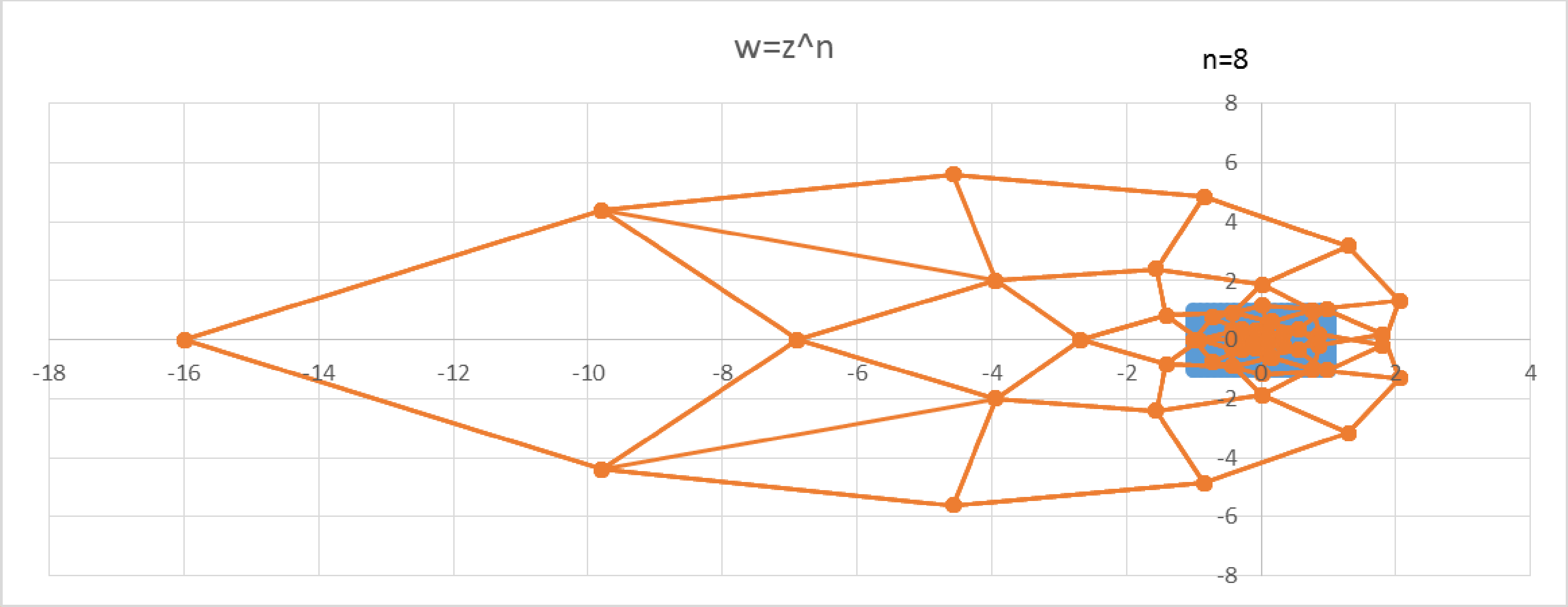
Elementary Complex Functions

Power function: $w=z^n$:n=natural numbers



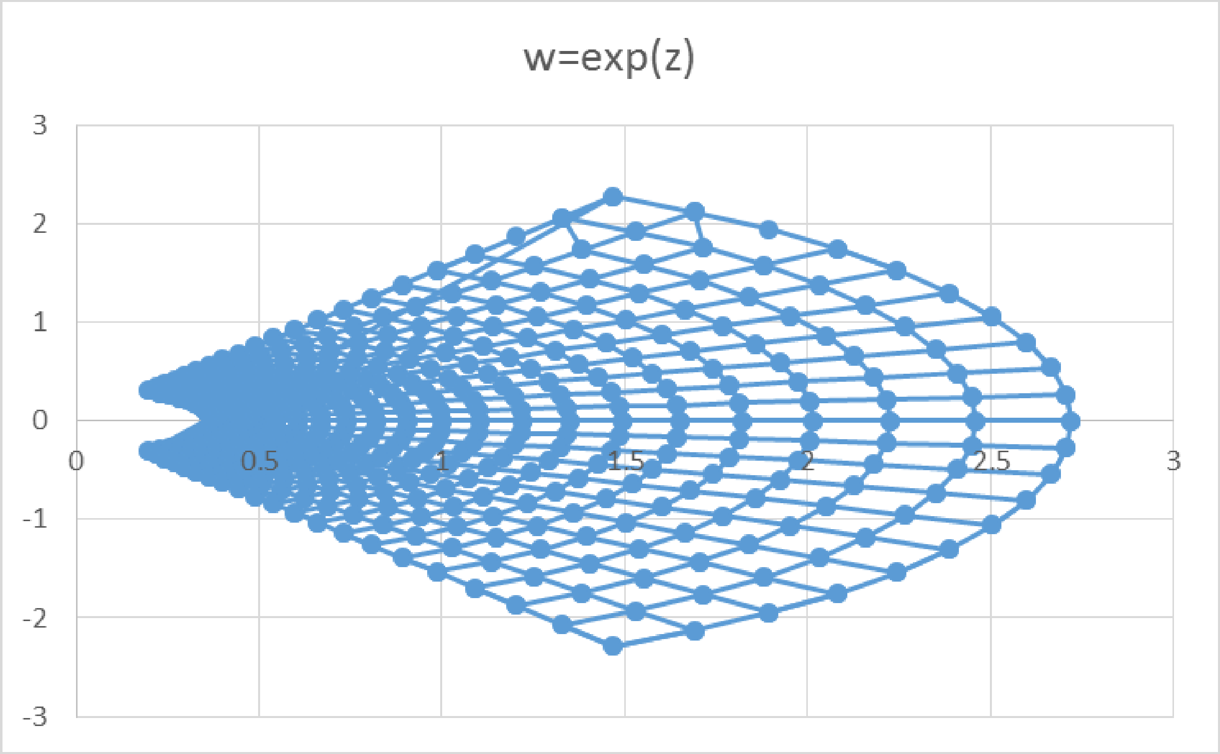
Elementary Complex Functions

Power function: $w=z^n$:n=natural numbers



Exponential Functions

Power function: $w=e^z$ $:z=x+iy$



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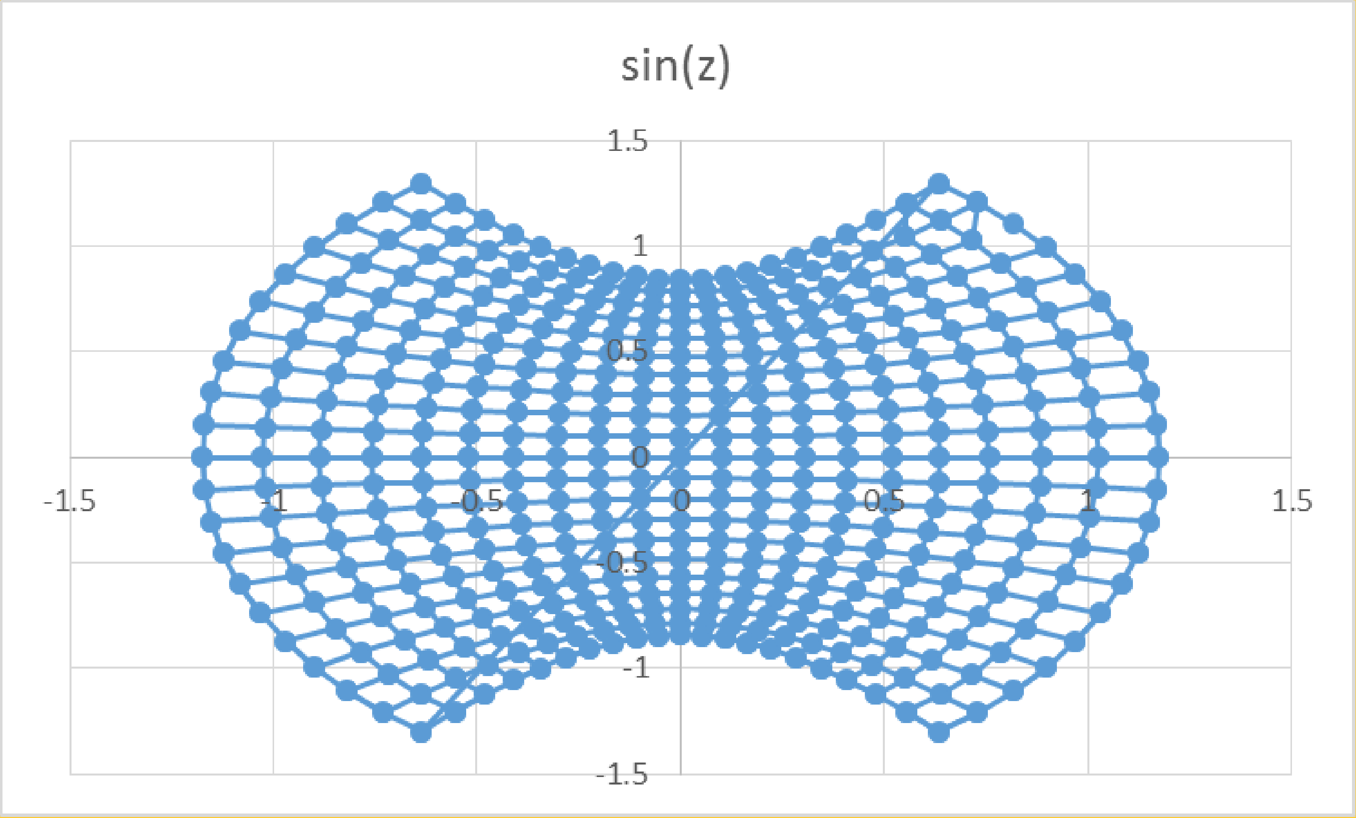
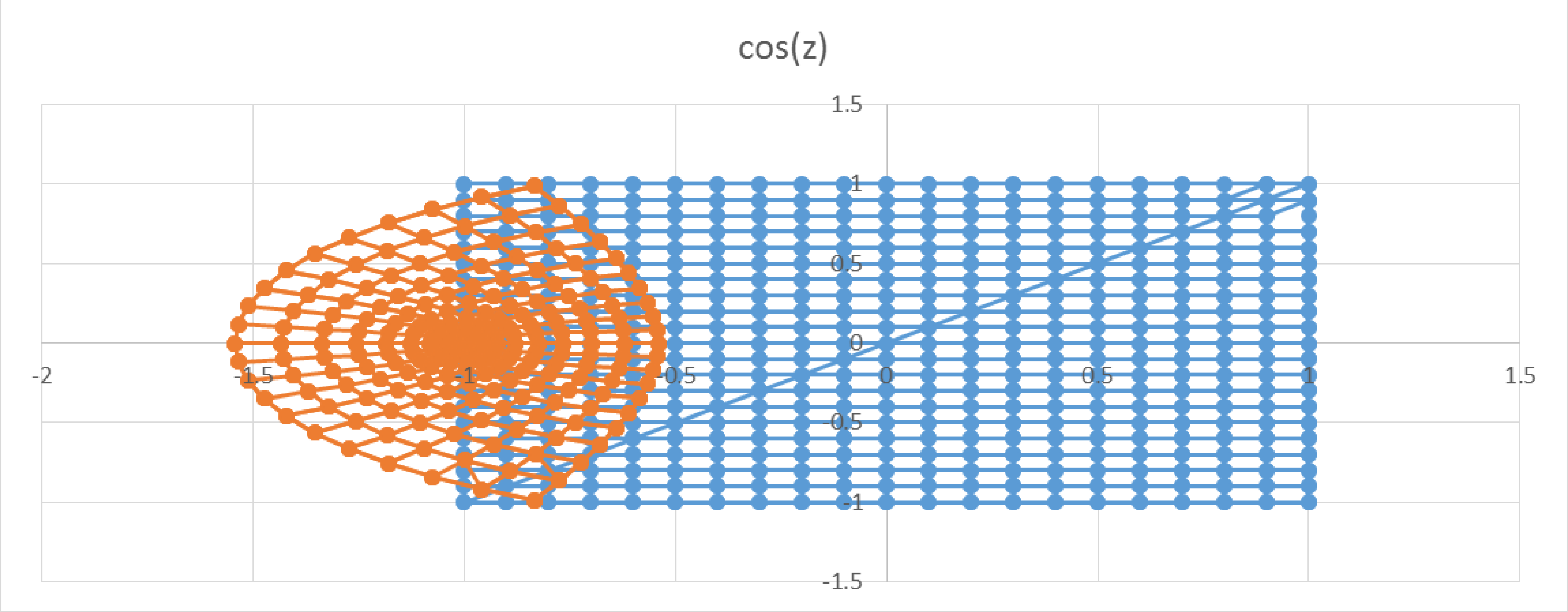
Trigonometric Functions

Sin Function: w=sin(z) :z=x+iy



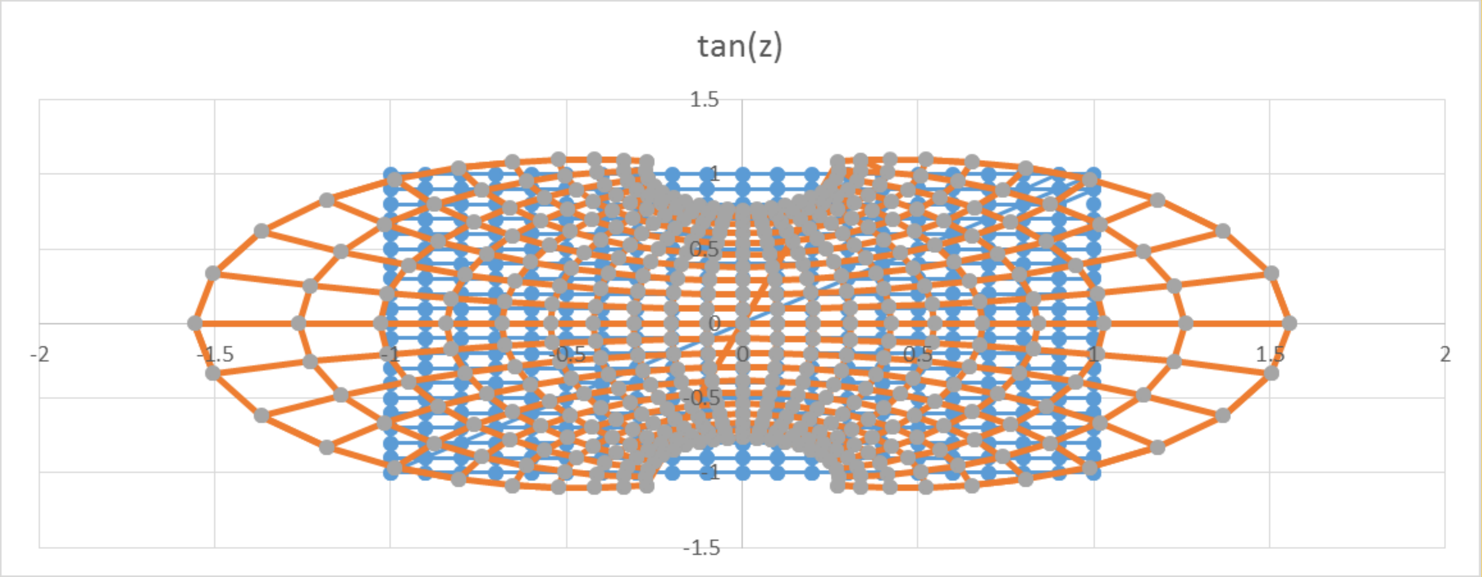
Trigonometric Functions

cos Function: w=cos(z) :z=x+iy



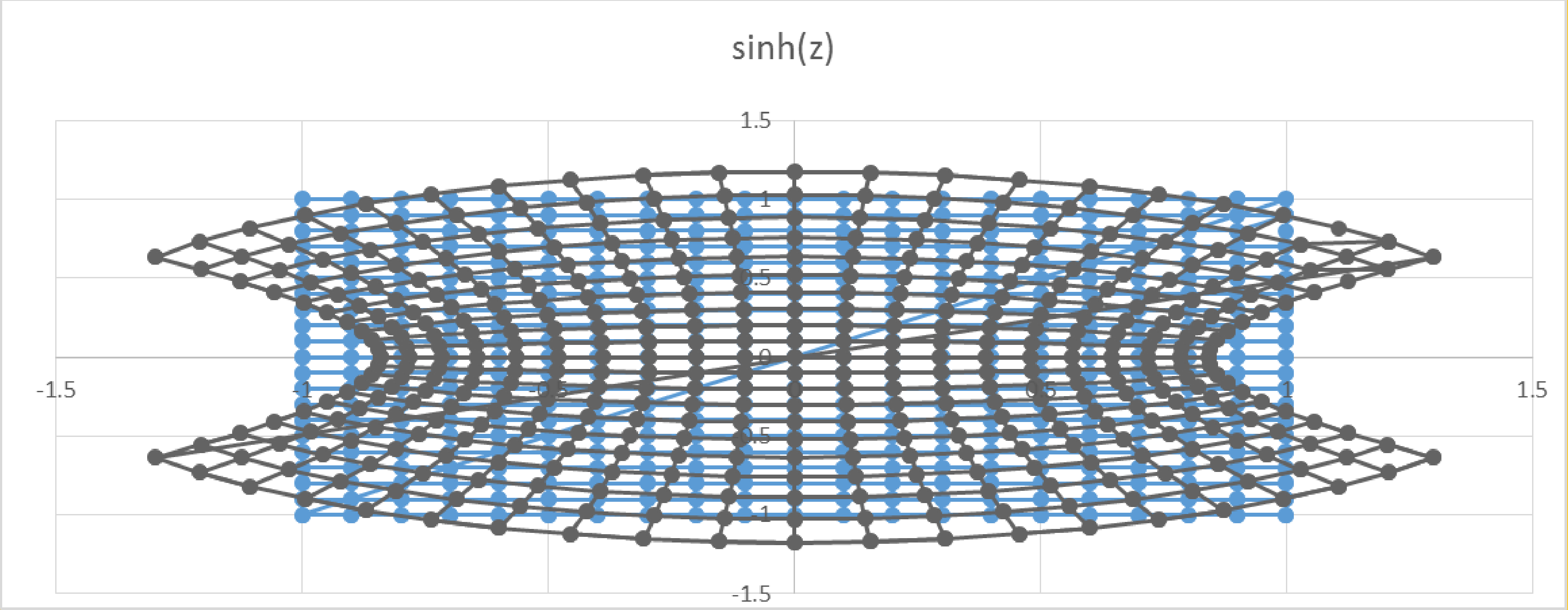
Trigonometric Functions

tan Function: $w=\tan(z)$:$z=x+iy$



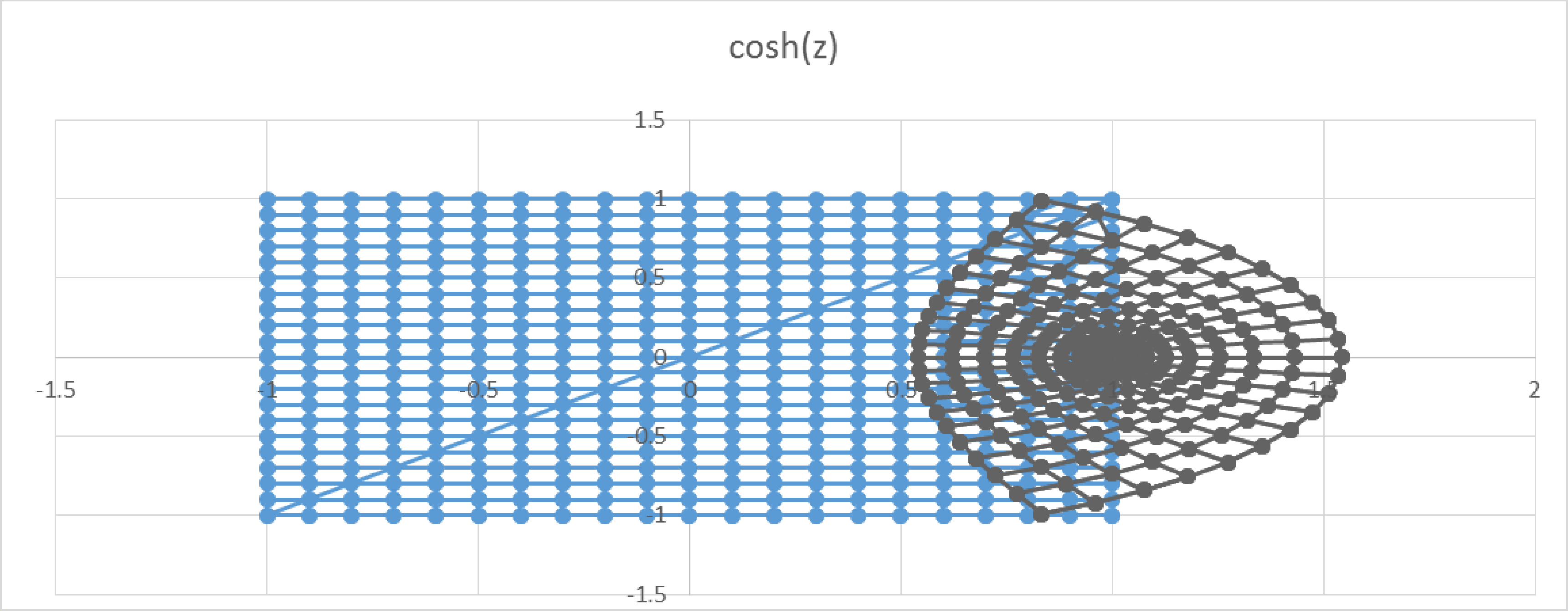
Trigonometric Functions

sinh(z) Function: w=sinh(z) :z=x+iy



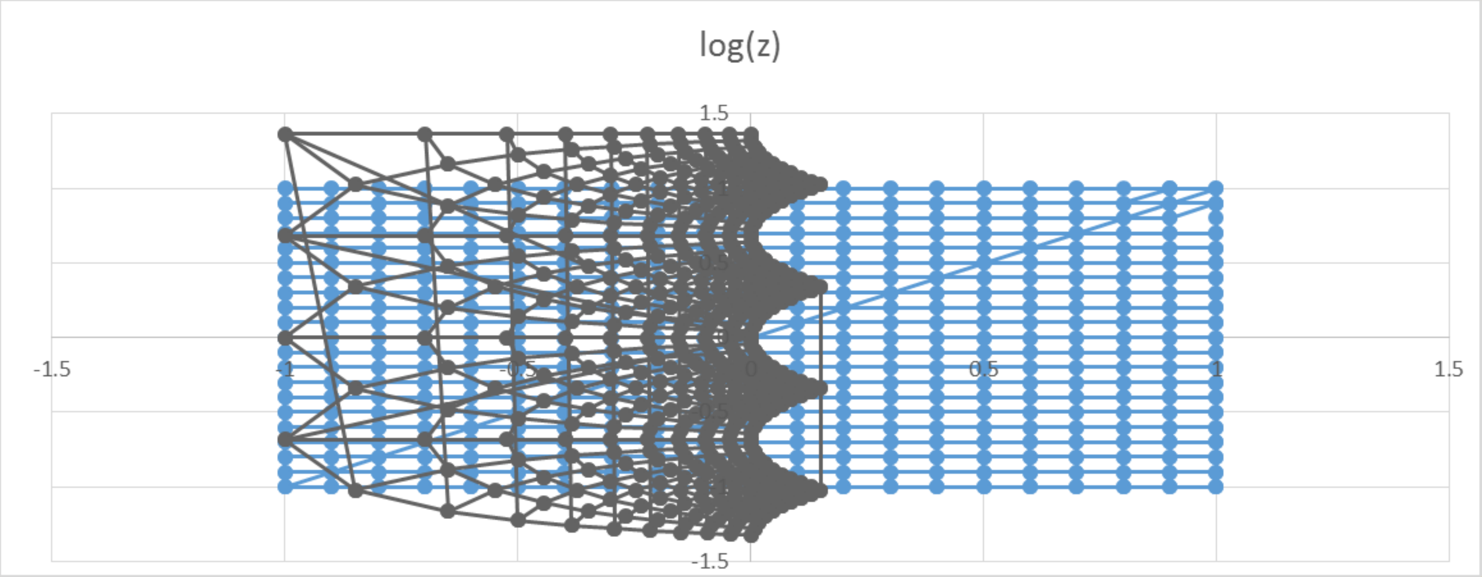
Trigonometric Functions

cosh(z) Function: w=cosh(z) :z=x+iy



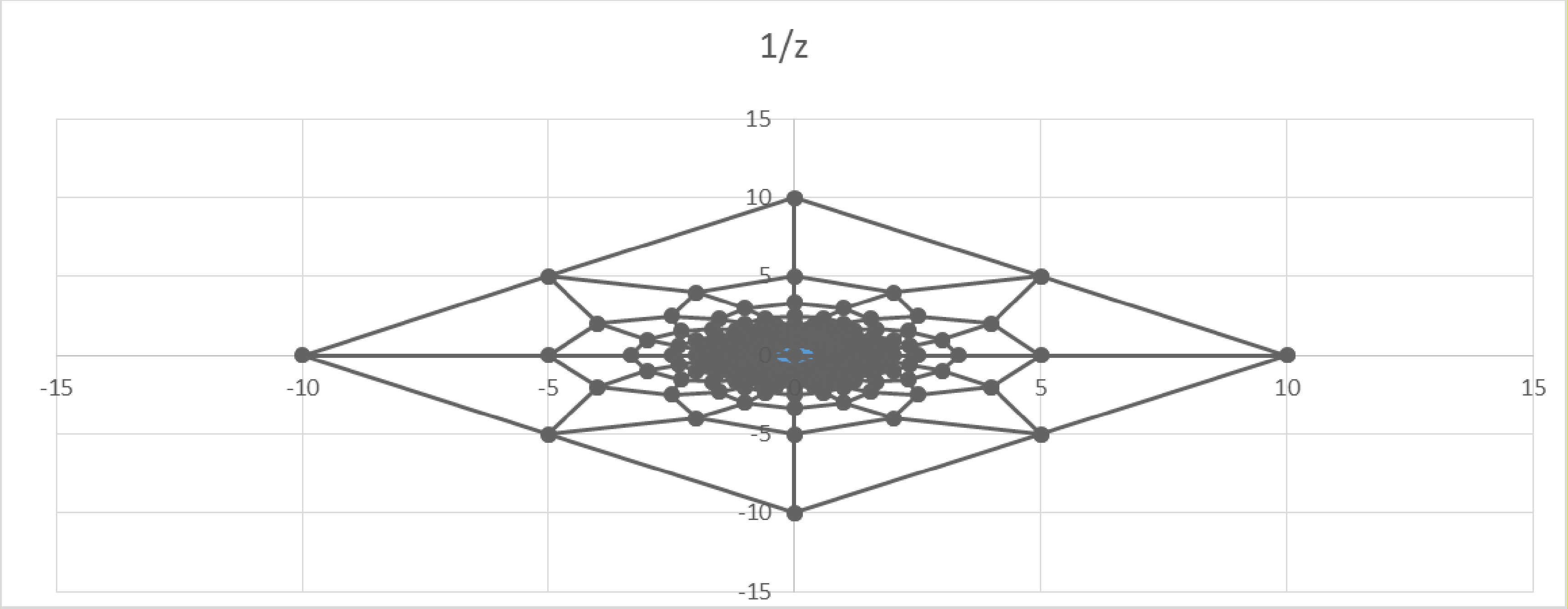
Logarithmic Functions

log(z) Function: w=log(z) :z=x+iy



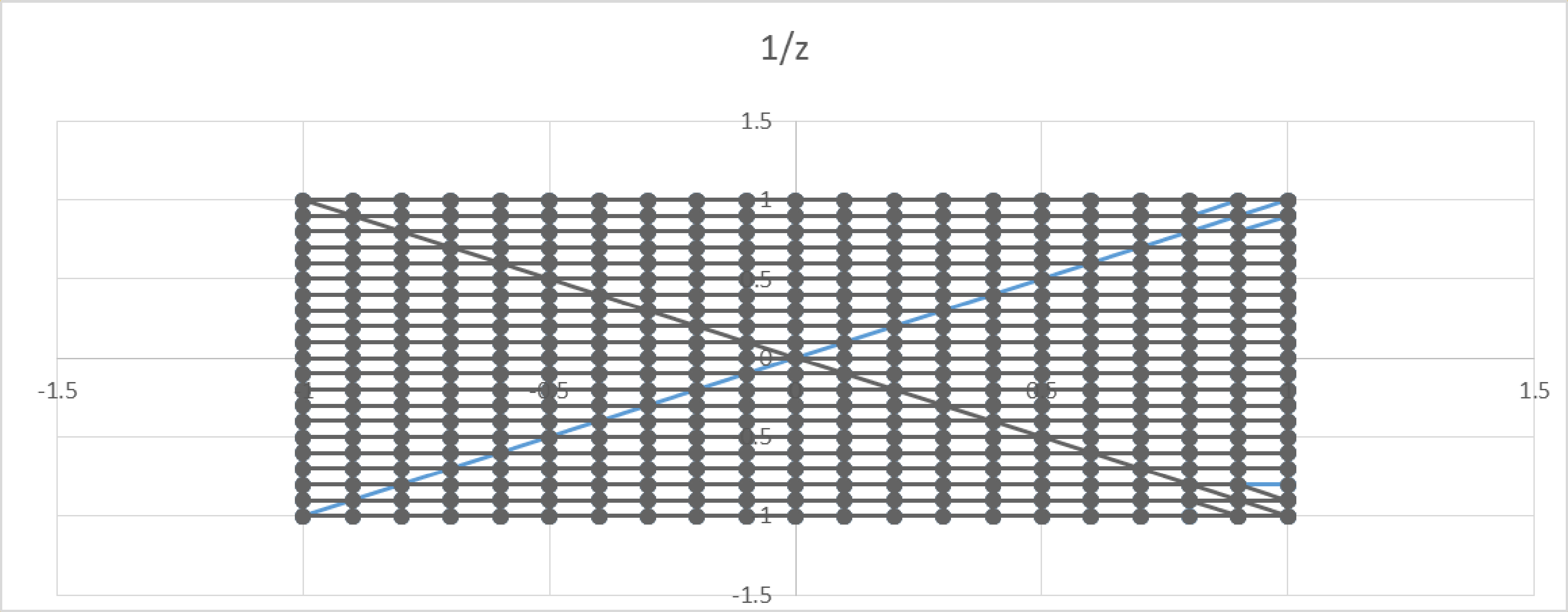
Reciprocal Functions

Reciprocal Function: w=1/z :z=x+iy



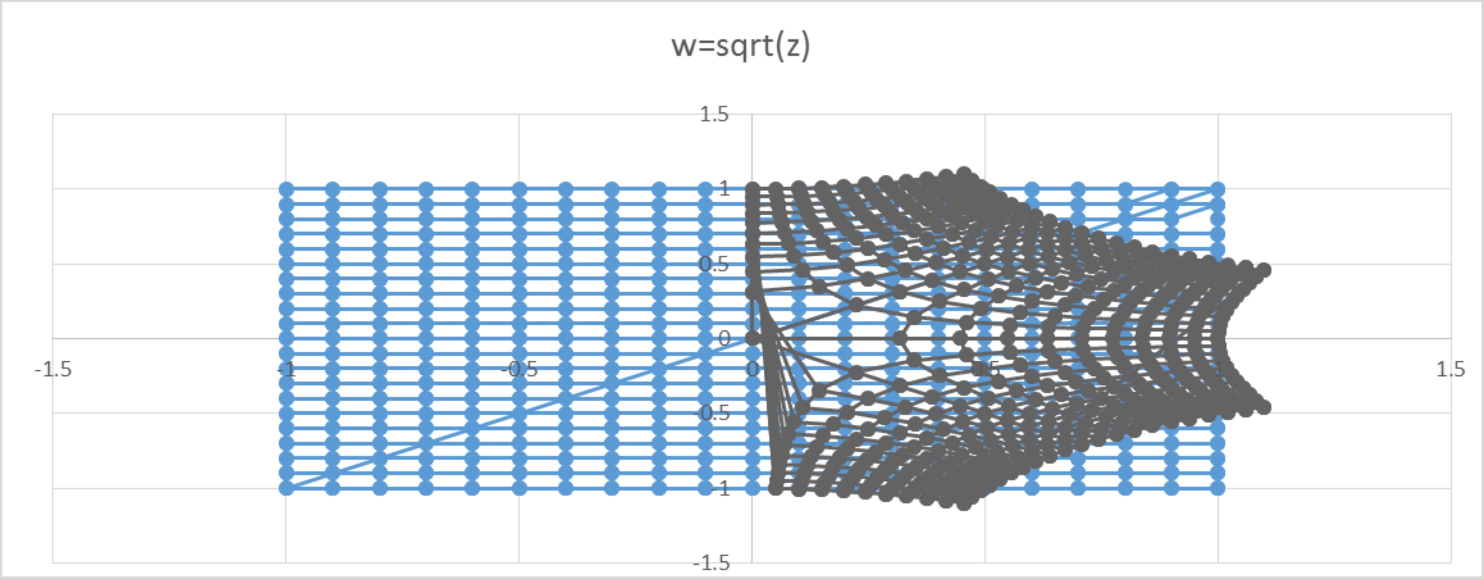
Conjugate Functions

Conjugate Function: w=z- :z=x+iy



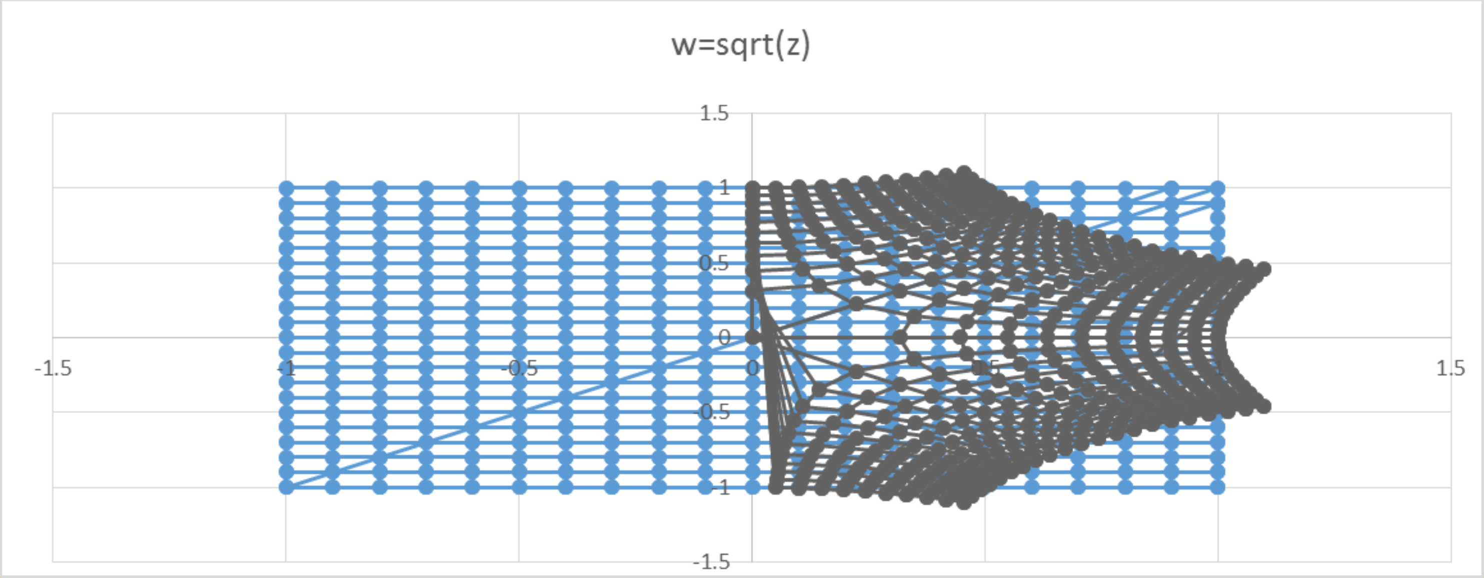
Root Functions

Square Root Function: $w=\sqrt{z}$ $:z=x+iy$



Root Functions

Square Root Function: $w=\sqrt{z}$:$z=x+iy$



Mapping

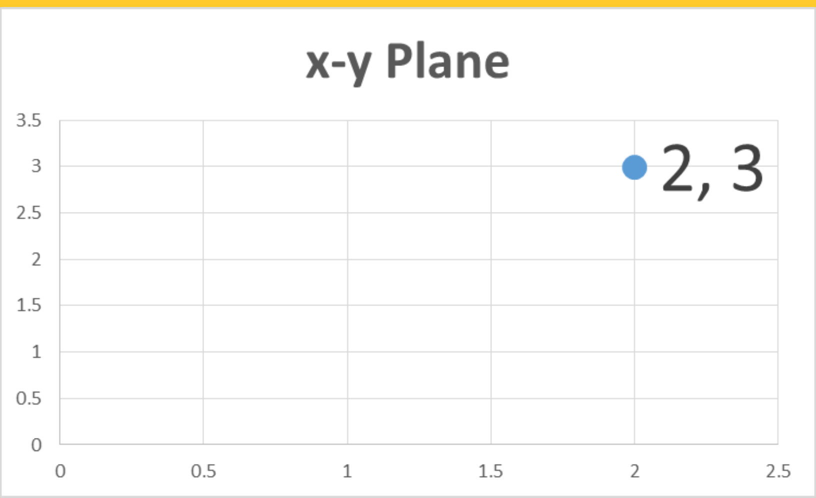
$w=f(z)$

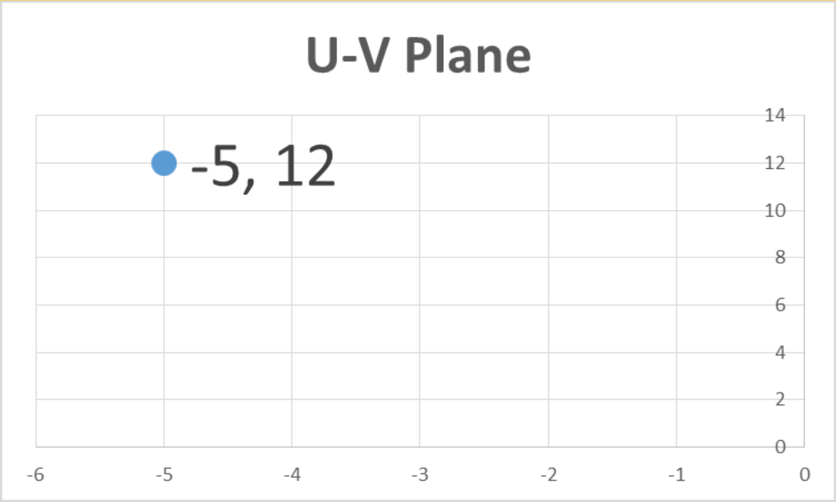
Function is a rule which maps z to w

Complex Function and Mapping

Mapping: For a complex function w=fz, for each point in z-plane there is a corresponding point in w plane. This is called mapping. This geometric approach to complex analysis helps in visualizing the nature of a complex function.

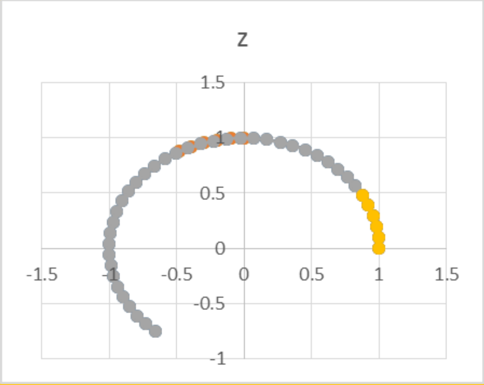
x	y	z	w=z2	u	v
2	3	2+3i	-5+12i	-5	12

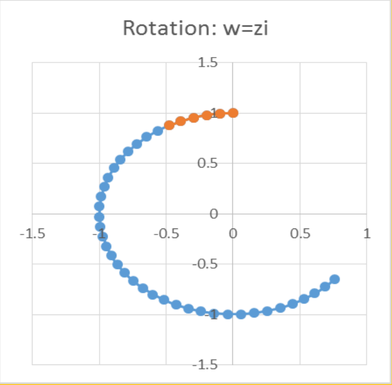


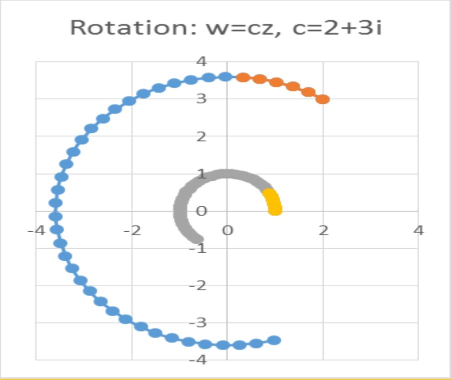


Mapping-Rotation

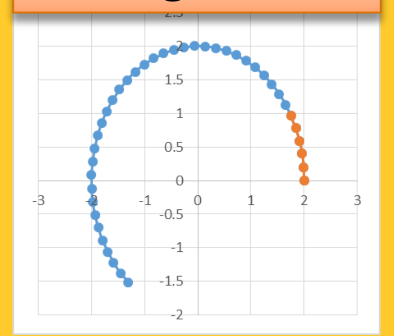
Rotation: Multiplying a complex number by i , rotates the z by 90 degree. Multiplication of a complex number by a complex number, rotates and scales the number. Multiplying a complex number by a scalar, scales the complex number.





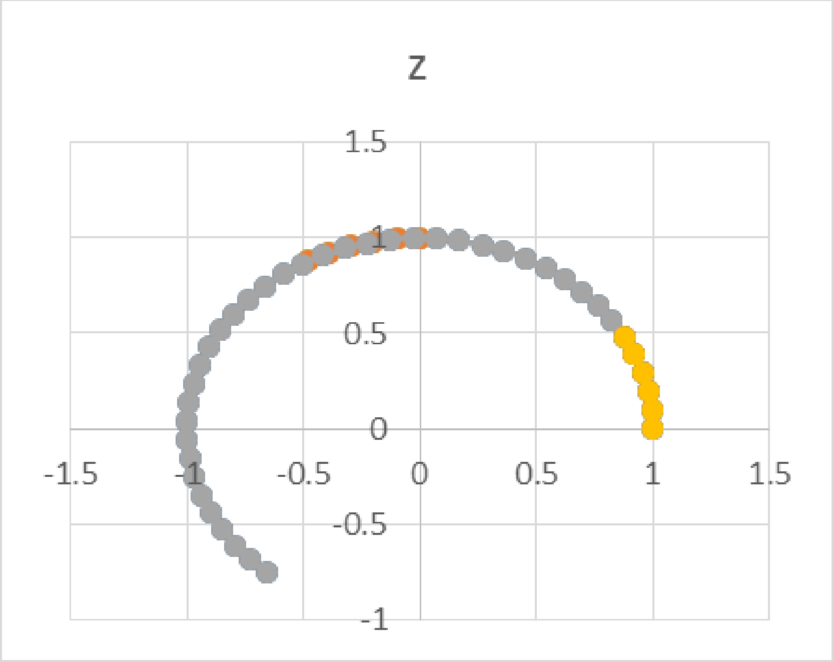


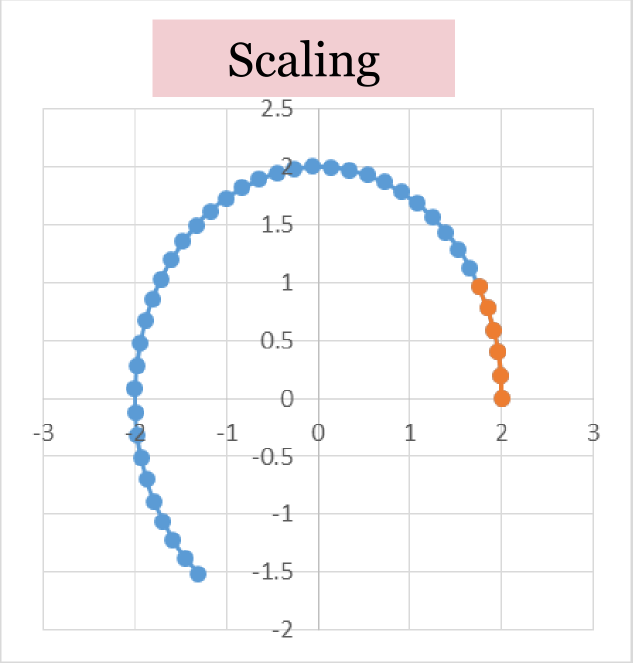
Scaling w=2z



Mapping-Scaling

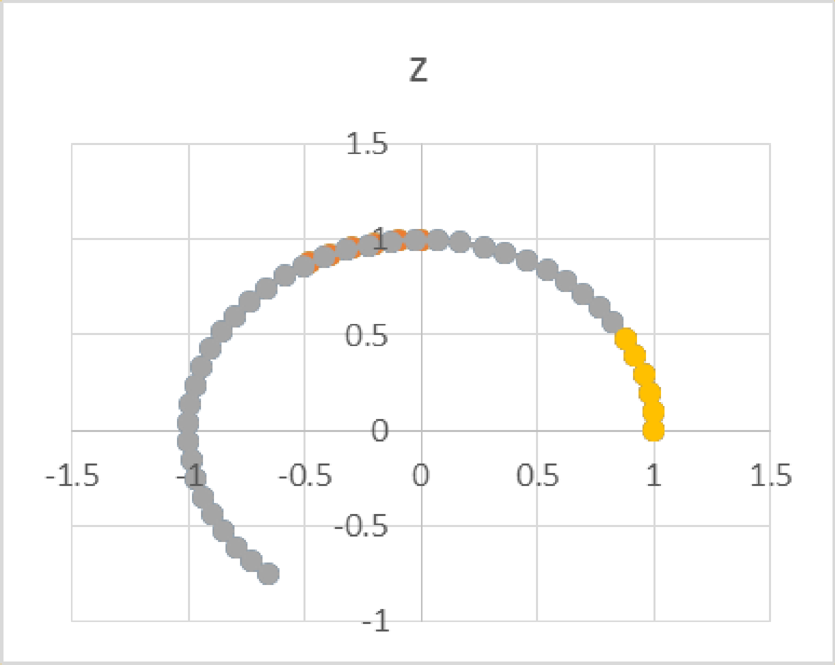
Scaling: Multiplying a complex number by a scalar, scales the complex number.

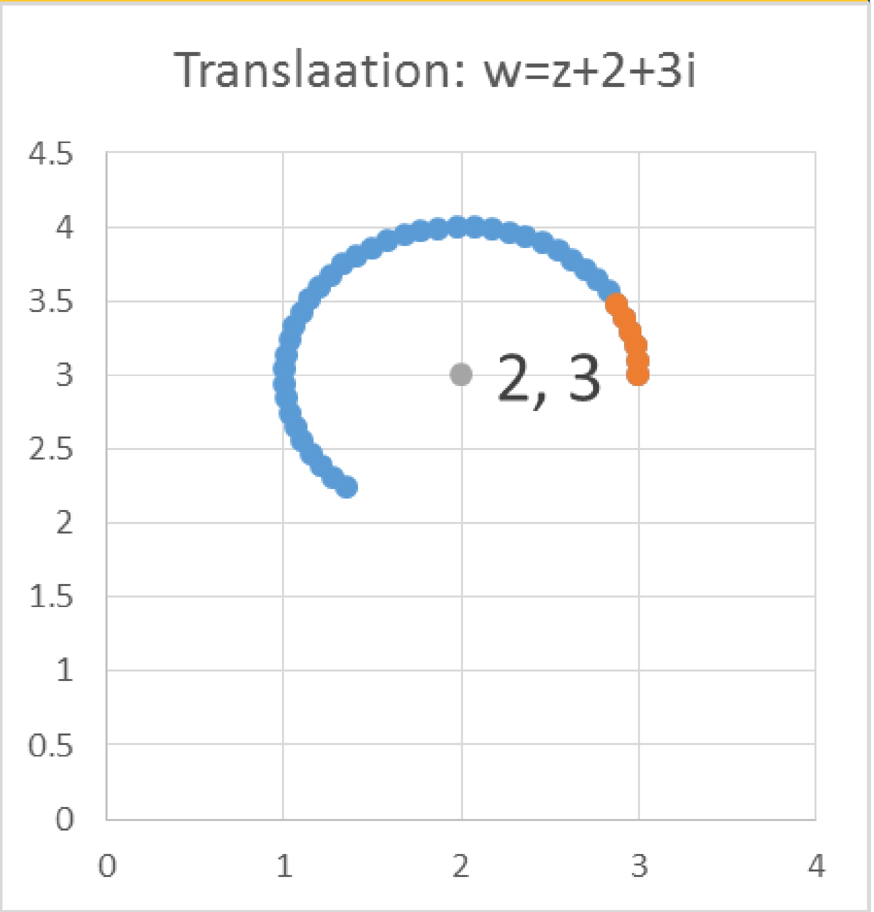




Mapping-Translation

Translation: Adding a constant complex number to a ,complex number, translate the complex number.





Conformal Mapping

- A mapping in the plane is said to be conformal or angle preserving if it preserves the angle between oriented curves in magnitude as well as direction or sense.
- The angle α is the angle between their oriented tangents at the point of intersection.
- A curve in the complex plane can be represented parametrically as-
- zt=xt+iyt
- Example: zt=r cos(t)+i sin(t) represents a curve = circle
- Example: zt=t+it^2 represents a parabola

Differentiation and Conformal Mapping

- The direction of increasing values of t is called positive direction or sense
- The tangent of the given curve is given by dz/dt
- If the analytic function w=f(z) or f(t)=f(z(t)), then the tangent of a point is given by- dw/dt=df/dz*dz/dt

- All these three terms represent complex numbers so the angle of dw/dt=the angle (df/dz)+angle (dz/dt)
- The mapping w=f(z) scales the length by a factor |f'(z)|
- Hence the area is scaled by a factor $|f'(z)|^2$ which can be obtained

by the jacobian d(u,v)/d(x,y)= $\begin{vmatrix} du/dx & du/dy \\ dv/dx & dv/dy \end{vmatrix} = \begin{vmatrix} du/dx & du/dy \\ -du/dy & dv/dx \end{vmatrix} =$

$\left|\left(\frac{du}{dx}\right)^2 - (du/dy)2\right| = \left|\left(\frac{du}{dx}\right) - i(du/dy)\right|^2 = |f'(z)|^2$

Differentiation and Conformal Mapping

- The point where $f'(z) = 0$ is called critical points

Mobius Transformations

1. w=$\frac{az+b}{cz+d}$

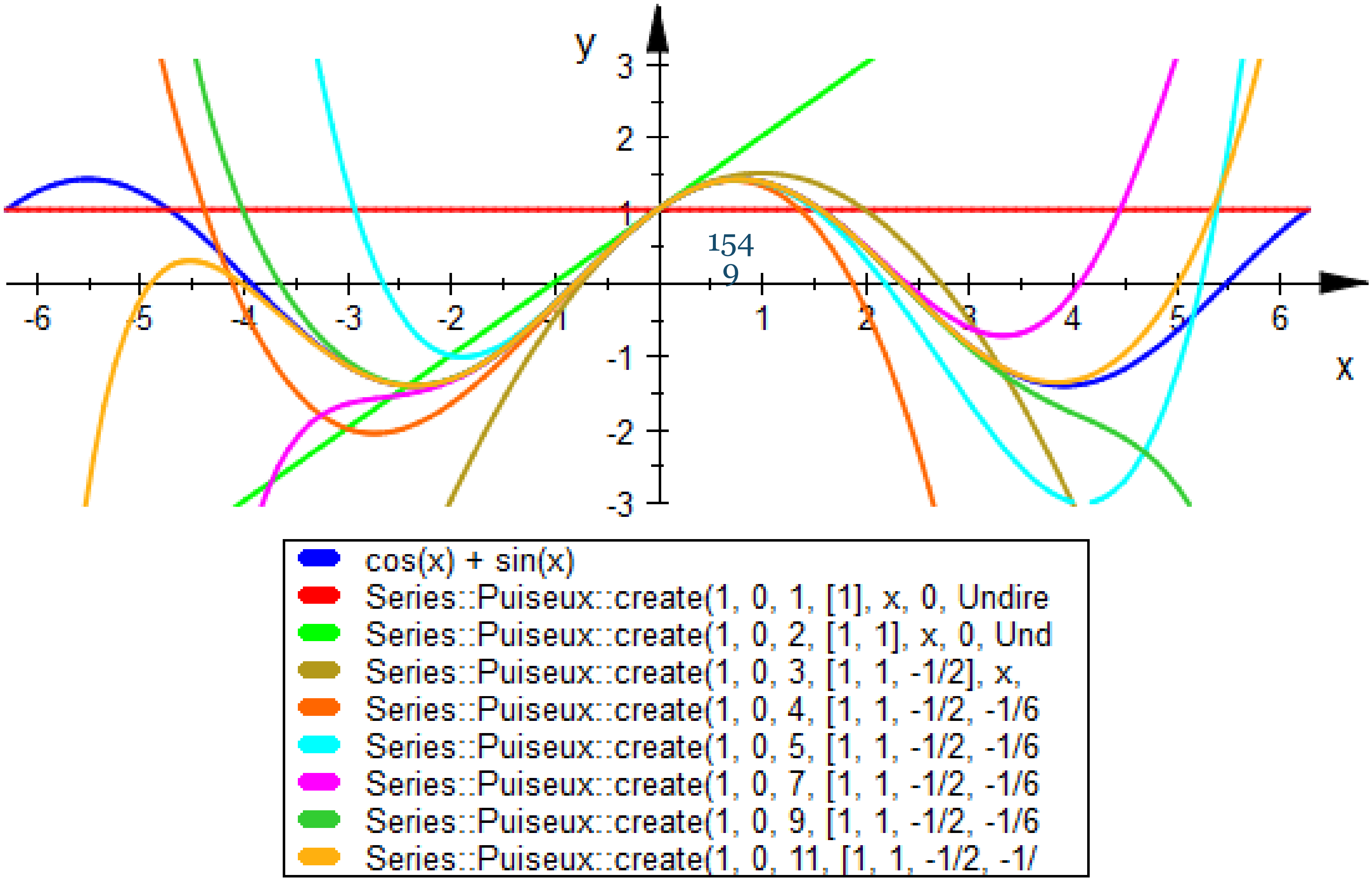
Mobius Transformations

1. w=$\frac{az+b}{cz+d}$

Riemann Surface

•

Sequence, Series


y
3
2
1
-1
-2
-3
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
x
cos(x) + sin(x)
Series::Puiseux::create(1, 0, 1, [1], x, 0, Undire
Series::Puiseux::create(1, 0, 2, [1, 1], x, 0, Und
Series::Puiseux::create(1, 0, 3, [1, 1, -1/2], x,
Series::Puiseux::create(1, 0, 4, [1, 1, -1/2, -1/6
Series::Puiseux::create(1, 0, 5, [1, 1, -1/2, -1/6
Series::Puiseux::create(1, 0, 7, [1, 1, -1/2, -1/6
Series::Puiseux::create(1, 0, 9, [1, 1, -1/2, -1/6
Series::Puiseux::create(1, 0, 11, [1, 1, -1/2, -1/

Series Expansion of Sin(x), Cos(x), Exp(x)

The formula for power series expansion of exponential of x generated from power series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

The Power Series is given below:

$$y = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \;\ldots..$$

If the coefficients are appropriately chosen to diminish, successive terms gets smaller and smaller and series approaches a limiting value.

Series Expansion of Sin(x), Cos(x), Exp(x)

The co-efficients are chosen by differentiating the series and equating to 0.

The Power Series is given below:

$$y = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \dots\dots$$

Putting x=0, we get, a_0=y

Differentiating, and equating to 0, we get, a_1=y'(0)

Differentiating and equating to 0, we get, a_2=y''(0)/2.1

Differentiating and equating to 0, we get, a_3=y'''(0)/2.3

General term of co-efficient becomes, a_n=y^n(0)/fact(n)

Series Expansion of Exp(x)

General term of co-efficient becomes,

$a_n = y^n(0) / \text{fact}(n)$

The Power Series is given below:

$$y = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 \ldots\ldots$$

The Power Series now transformed like :

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \ldots\ldots$$

Original Function was, y=exp(x)

Putting, y(0)=1 and other coefficients, we get,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

Series Expansion of Sin(x)

y=sin(x), x=0, sin(0)=0, y(0)=0

y'=cos(x), x=0, cos(x)=1, y'(0)=1

y''=-sin(x), x=0, -sin(0)=0,y''=0

y'''=-cos(x), x=0, -cos(0)=-1, y'''=-1

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \dots$$

Original Function, y=sin(x) becomes,

$y=\sin(x)=x-x^3/3!+x^5/5!-x^7/7!\dots\dots$

Series Expansion of Cos(x)

y=cos(x), x=0, cos(0)=1, y(0)=1

y'=-sin(x), x=0, -sin(0)=0, y'(0)=0

y''=-cos(x), x=0, -cos(0)=-1,y''=-1

y'''=sin(x), x=0, sin(0)=0, y'''=0

$$y = y(0) + y'(0)x^1 + y''(0)x^2/2! + y'''x^3/3! \ldots..$$

Original Function, y=cos(x) becomes,

$y=\cos(x)=1-x^2/2!+x^4/4!-x^6/6!\ldots\ldots\ldots$

Exp(ix)=sin(ix)+cos(ix)

$y=\sin(x)=x-x^3/3!+x^5/5!-x^7/7!$……. And

$y=\cos(x)=1-x^2/2!+x^4/4!-x^6/6!$………

$y=\sin(ix)=ix-(ix)^3/3!+(ix)^5/5!-(ix)^7/7!$….

$y=\cos(x)=1-(ix)^2/2!+(ix)^4/4!-(ix)^6/6!$……

Exp(ix)=sin(ix)+cos(ix)

$y=\sin(ix)=ix-(ix)^3/3!+(ix)^5/5!-(ix)^7/7!....$

$y=\sin(ix)=i(x-i^2x^3/3!+i^4x^5/5!-i^6x^7/7!.....$

$y=\sin(ix)=i(x+x^3/3!+x^5/5!+x^7/7!.....)$

$y=\cos(ix)=1-(ix)^2/2!+(ix)^4/4!-(ix)^6/6!......$

$y=\cos(ix)=1+(x)^2/2!+(x)^4/4!+(x)^6/6!.......$

$$e^{ix}=1+\frac{ix}{1!}+\frac{i^2x^2}{2!}+\frac{i^3x^3}{3!}+\cdots,$$

$$-\infty<x<\infty$$

$$e^{ix}=cos(x)+i*sin(x)$$

Relation between Trigonometry, Exponential and imaginary numbers

```
exp=taylor(exp(x))
%x^6/720+x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

sine=taylor(sin(x),7)
%x^5/120 - x^3/6 + x

cos=taylor(cos(x),7)
%- x^6/720 + x^4/24 - x^2/2 + 1
```

exp(i*x)=cos(i*x)+sin(i*x)=cos(x)+isin(x)

exp(i*pi)=cos(i*pi)+sin(i*pi) = cos(pi)+isin(pi)=-1
(From Taylor expansion, putting x=ix, i^2=1)

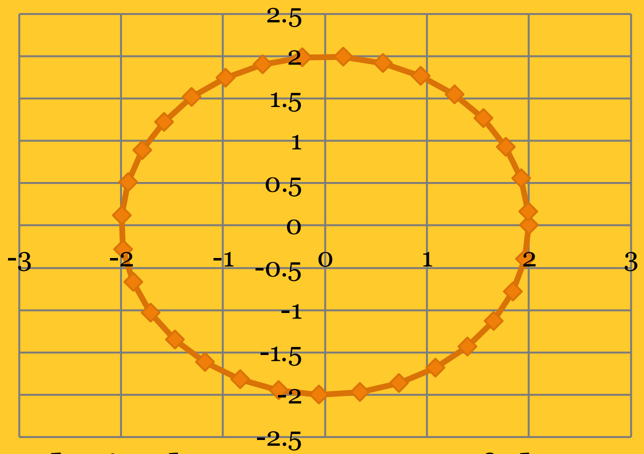
Note: Exp(i*pi)= cos(pi)+isin(pi) links five most important symbols in mathematics

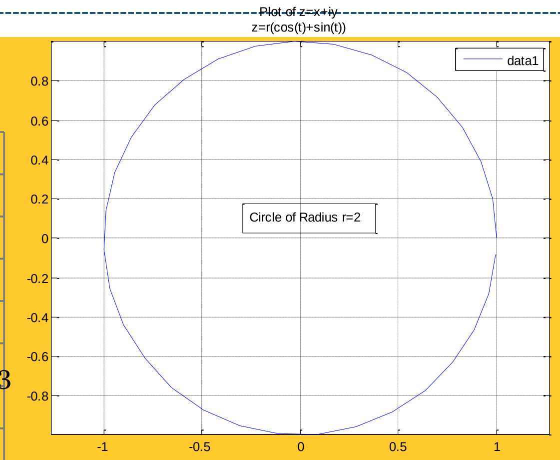
Relation between Complex number and Trigonometry

z=x+iy
x=rcos(t)
y=rsin(t)

z=r*cos(t)+i*r*sin(t)
z=r*(cos(t)+i*sin(t))
z=r*exp(i*t)

Circle generated from z=r*exp(i*t)





Note: r is the modulus and t is the argument of the complex number (angle with x-axis)

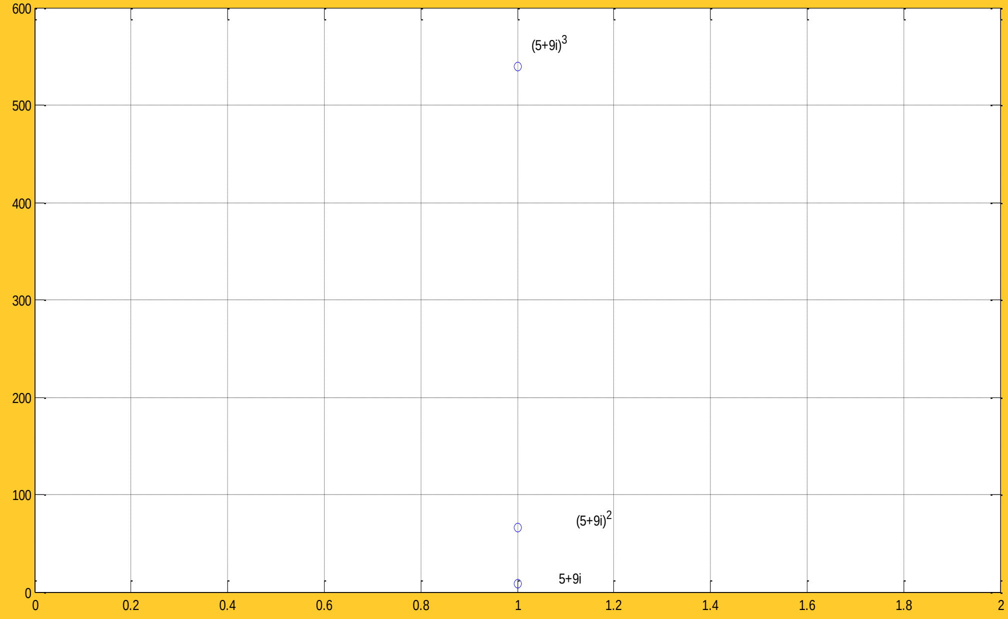
Note: In mathematics, an **argument** of a function is a specific input in the function, also known as an independent variable. In logic and philosophy, an **argument** is an attempt to persuade someone of something.

Operations on Complex numbers

z=x+iy
x=rcos(t)
y=rsin(t)

z=r*cos(t)+i*r*sin(t)
z=r*(cos(t)+i*sin(t))
z=r*exp(i*t)

z=5+9i
z^2=(5+9i)^2



Euler's Formula
Factorial Permutation & Combination

>**Factorial** (n!) Definition: Factorial(**n**) is the product of all positive integers less than **n.**

n!=n*(n-1)*(n-2)*(n-3)-------3*2*1

>**Counting**: If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of occurrence of the event in the given order is m*n.

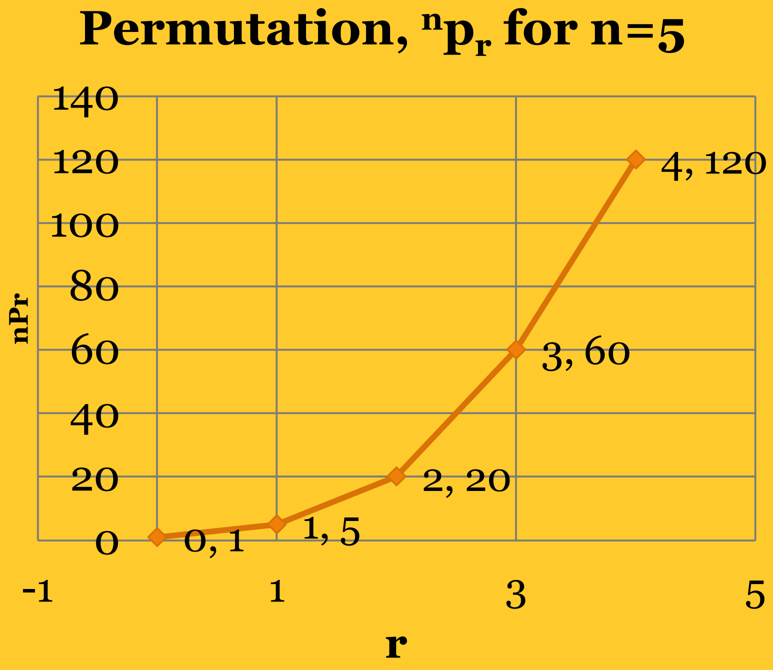
>**Permutation:** Definition: Counting the number of ways in which some or all objects can **be arranged** at a time.

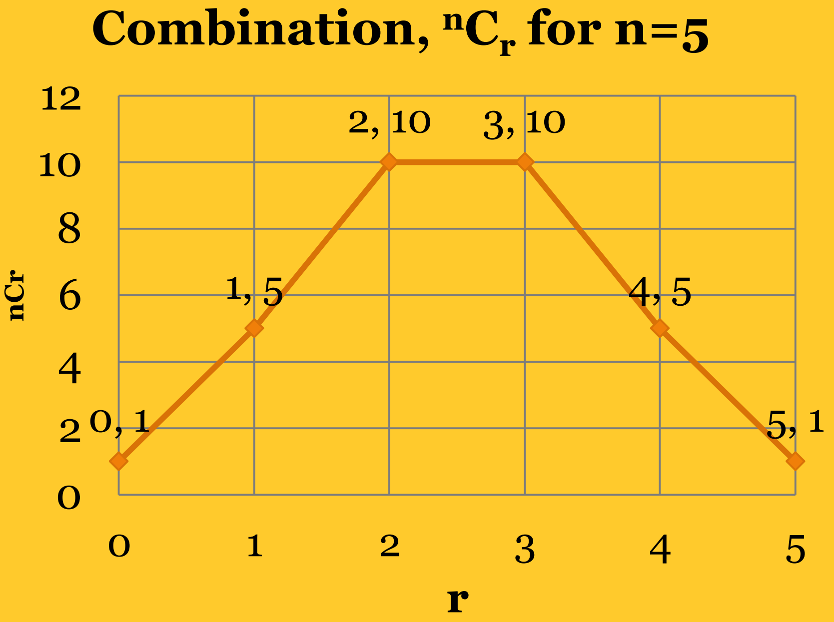
>**Permutation**, $^{n}P_{r}$ **=Factorial(n)/Factorial(n-r)**

>**Combination:** Definition: Counting number of ways in which fixed number of objects (r) can **be chosen** from (n) objects,

Combination, $^{n}C_{r}$ = **Factorial(n)/Factorial(n-r)*(Factorial(r)**

Graphs of $^{n}p_{r}$ and $^{n}C_{r}$





Binomial Theorem

>Definition: Binomial theorem deals with the algebraic expression generated by the expansion of powers(n) to the binomial (a+b).

>The power (n) of the binomial can be 0, 1, 2, 3,……………

Case-1: Power(n=0): (a+b)^0 =1

Case-2: Power(n=1): (a+b)^1 =1, 1

Case-3: Power(n=2): (a+b)^2 =1, 2, 1

Case-4: Power(n=3): (a+b)^3 =1, 3, 3, 1

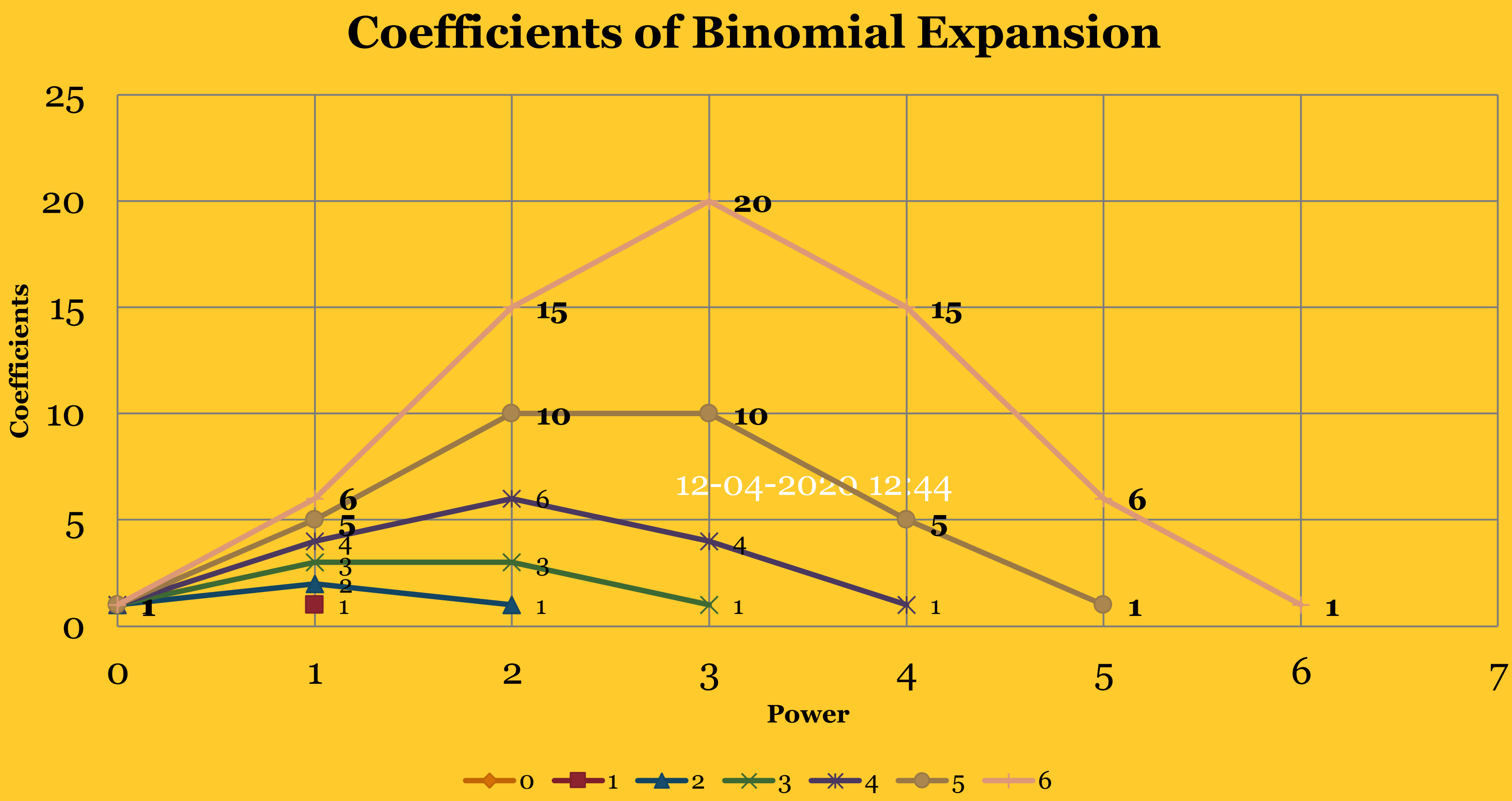
Case-5: Power(n=4): (a+b)^4 =1, 4, 6, 4,1

Case-6: Power(n=5): (a+b)^5 =1, 5, 10, 10, 5, 1

Case-7: Power(n=n): (a+b)^n = $^{n}c_{0}$, $^{n}c_{1}$, $^{n}c_{2}$, $^{n}c_{3}$, $^{n}c_{4}$…. $^{n}c_{(n-2)}$ $^{n}c_{(n-1)}$ $^{n}c_{n}$

(Only coefficients of the expansion considered)

Coefficients of Binomial Expansion



Coefficients of Binomial Expansion

n=	7	10
r	nCr	nCr
0	1	1
1	7	10
2	21	45
3	35	120
4	35	210
5	21	252
6	7	210
7	1	120
8	#NUM!	45
9	#NUM!	10
10	#NUM!	1

>It is seen from the expansion that the coefficients of each term formed a sequence and the sum of the sequence forms a finite series.

>The total no of terms is one more than index.

>Power of 1 goes on decreasing and power of x goes on increasing.

>Sum of indices of 1 and x in each term is same and equal to n.

>Pascal's triangle can be used for finding coefficients of a higher power of binomial expansion

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} 1^k x^{n-k}$$

POWER SERIES

Power Series: Power series

From Wikipedia, the free encyclopedia

In mathematics, a **power series** (in one variable) is an infinite series of the form

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c)^1 + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

Where a_n represents the coefficient of the *n*th term, *c* is a constant, and *x* varies around *c*(for this reason one sometimes speaks of the series as being *centered* at *c*). This series usually arises as the Taylor series of some known function.

In many situations *c* is equal to zero, for instance when considering a Maclaurin series. In such cases, the power series takes the simpler form

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots .$$

>Many functions as well as differential equations can be expanded as an infinite power series.

>The advantage of expanding the functions in power series is that the differentiation and integration becomes easier.

➢DE's also become easier to solve.

➢The power series is like a polynomial without an upper limiting power.

➢**A function can be expanded in power series if it can be differentiated infinitely.**

EXPANSION OF FUNCTION TO POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

For expansion of function to a power series, it is required to find the coefficients of the terms of the series.
In binomial series, the coefficients are generated from the index(n).
In power series, this is done by putting x=0 and sequentially find the values of y by successive differentiation.

Step-1: Put x=0 in the infinite power series. This gives y(0)=a0 or a0=y(0) at x=0.

Step-2: Differentiate the series to get,
$f'(y)=a1+2a^2x+3a^3x^2+4a^4x^3+$........
Put x=0 to get a1=y'(0)
Step-3: Follow step-2 repeatedly.
The pattern is $a_n=y^n(0)/n!$

EXPANSION OF EXPONENTIAL FUNCTION TO POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

Given function: y=exp(x)

Step-1: Put x=0 in y=exp(x). This gives a0=y(0) = 1 at x=0.

Step-2: Differentiate the series to get, y’=f‘(y)=a1+2a^{2}x+3a^{3}x^2+4a^{4}x^3+......

Put x=0 to get a1=y’(0)=1

Step-3: Follow step-2 repeatedly to get a2=1/2.1, a3=1/3.2.1, a4=1/4.3.2.1 etc

The pattern is a_n=y^n(0)/n!
exp(x)=1+x+x^2/2!+x^3/3!+…………………

Source: Computer Graphics Through Key Mathematics-Huw Jones

Power Series

A function can be expanded in power series if it can be differentiated infinitely.

y=exp(x), y'=exp(x), t"=exp(x)……

y=sin(x), y'=-cos(x), y"=-sin(x)…….

y=cos(x),y'=sin(x), y"=-cos(x)……

As these functions are differentiable infinitely it can be expanded as a power series.

Relation between Trigonometry, Exponential and imaginary numbers

exp(x) =taylor(exp(x))

x^6/720+x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

sin(x)=taylor(sin(x),7)

x^5/120 - x^3/6 + x

cos(x)=taylor(cos(x),7)

-x^6/720 + x^4/24 - x^2/2 + 1

exp(i*x)= (i*x)^5/120 + (i*x)^4/24 + (i*x)^3/6 + (i*x)^2/2 + i*x + 1

exp(i*x)=(x^5*i)/120 + x^4/24 - (x^3*i)/6 - x^2/2 + x*i + 1

exp(i*x)= + x^4/24 - x^2/2 + 1 + (x^5*i)/120 - (x^3*i)/6 + x*i

exp(i*x)= + x^4/24 - x^2/2 + 1 + i{(x^5)/120 - (x^3)/6 + x}

exp(i*x)= cos(x)+ i*sin(x)

exp(i*pi)= cos(pi)+isin(pi)=-1

(AS cos(pi)=-1 and sin(pi)=0)

Note: exp(i*pi)= cos(pi)+isin(pi) =-1

links five most important symbols in mathematics

Euler's Formula
Express sin(x) in terms of exp(x)

We know:
exp(i*x)=cos(x)+i sin(x)
exp(i*-x)=cos(x)-i sin(x)

exp(i*x)+ exp(i*-x)=cos(x)+sin(i*x)+cos(x)-i sin(x)
exp(i*x)+ exp(i*-x)= 2cos(x)
cos(x) = (exp(i*x)+ exp(i*-x))/2

exp(i*x)- exp(i*-x)=(cos(x)+i sin(x))- (cos(x)-i sin(x))
exp(i*x)- exp(i*-x)=2 i sin(x)
sin(x)=(exp(i*x)- exp(i*-x))/2 i

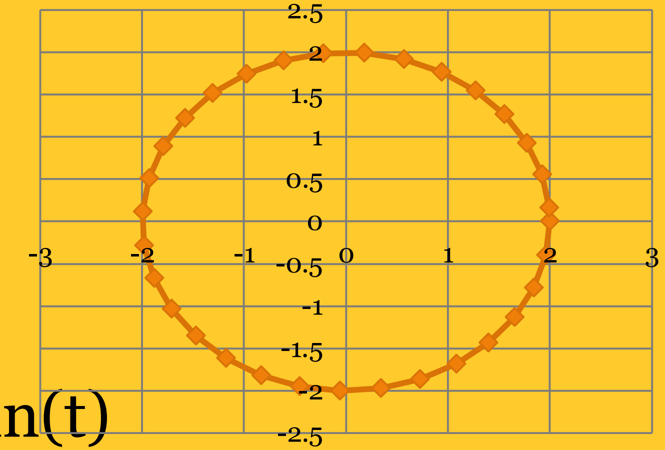
Note: exp(i*pi)= cos(pi)+isin(pi) =-1
links five most important symbols in mathematics

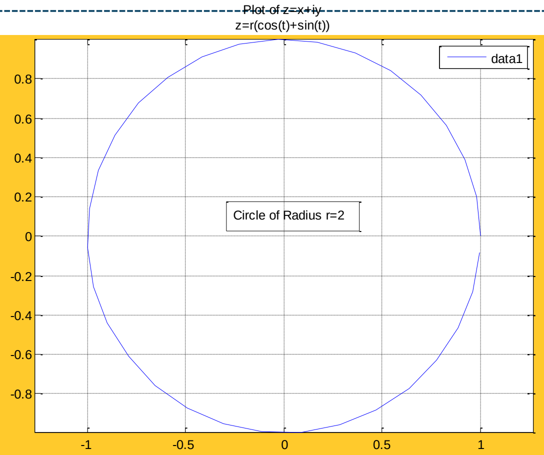
De Moivre's Theorem

$$(\cos(x) + i\sin(x))^n = \cos(nx) + i\sin(nx)$$

Polar form of Complex Number
Relation between Complex number and Trigonometry

Circle generated from z=r*exp(i*t)





z=x+iy
x=rcos(t)
y=rsin(t)

z=r*cos(t)+i*r*sin(t)
z=r*(cos(t)+i*sin(t))
z=r*exp(i*t)

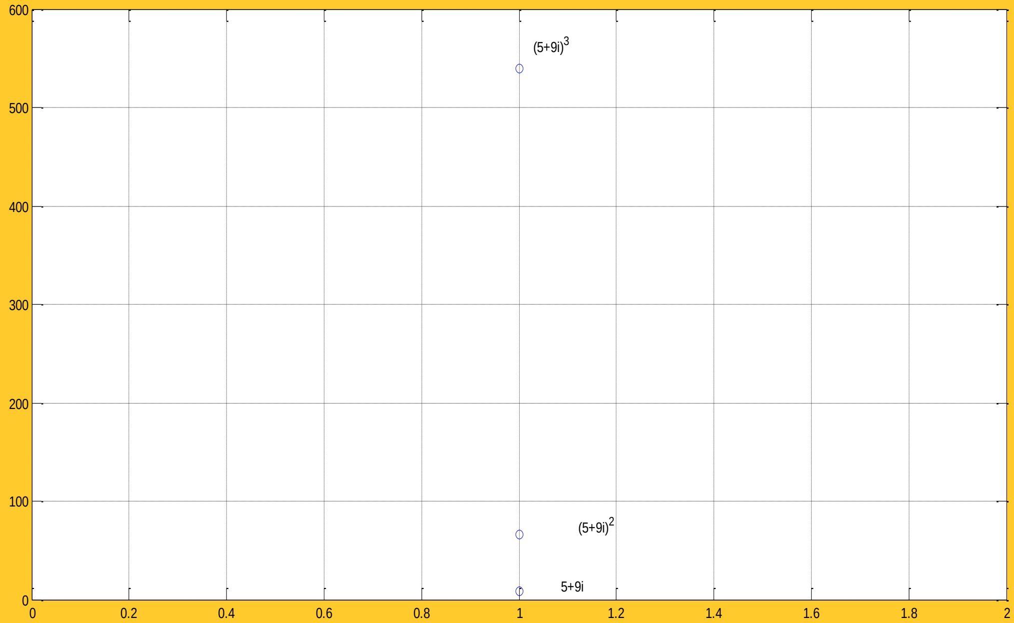
Note: r is the modulus and t is the argument of the complex number (angle with x-axis)

Operations on Complex numbers

z=x+iy
x=rcos(t)
y=rsin(t)

z=r*cos(t)+i*r*sin(t)
z=r*(cos(t)+i*sin(t))
z=r*exp(i*t)

z=5+9i
z^2=(5+9i)^2

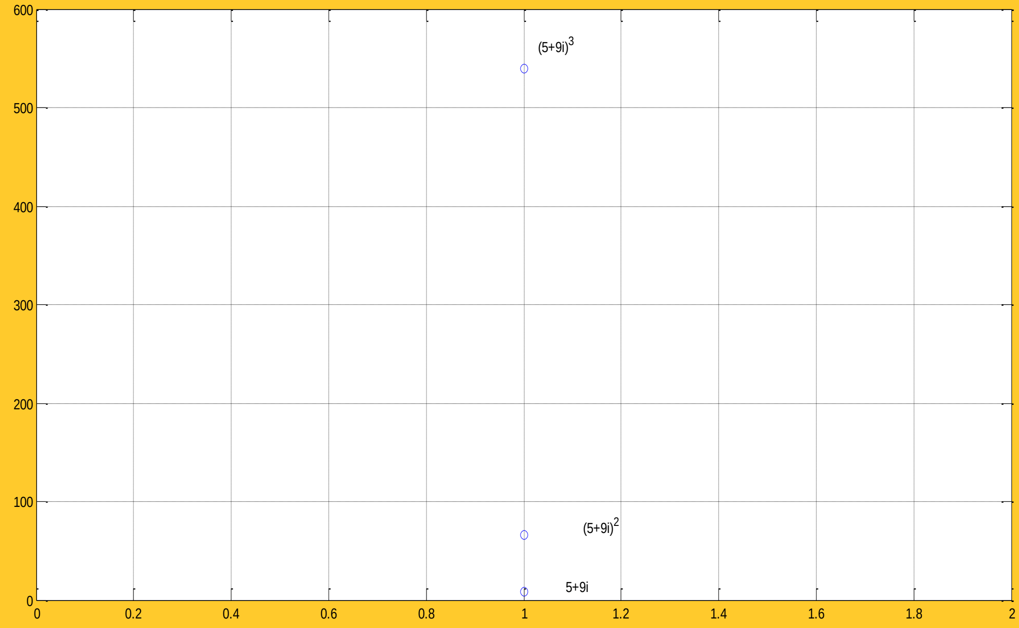


Operations on Complex numbers

z=x+iy
x=rcos(t)
y=rsin(t)

z=r*cos(t)+i*r*sin(t)
z=r*(cos(t)+i*sin(t))
z=r*exp(i*t)

z=5+9i
z^2=(5+9i)^2



Eigenvalues and Eigenvectors

DR R C SHAH

Case-1: Non-symmetric, Distinct Eigen Vector Example 12.1

- Find Eigenvalue and Eigenvector of:

$$A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$$

- Characteristic equation:

$$\begin{vmatrix} 5-\lambda & 3 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(3-\lambda) - 3 = 0$$

$$\Rightarrow \lambda^2 - 8\lambda + 12 = 0$$

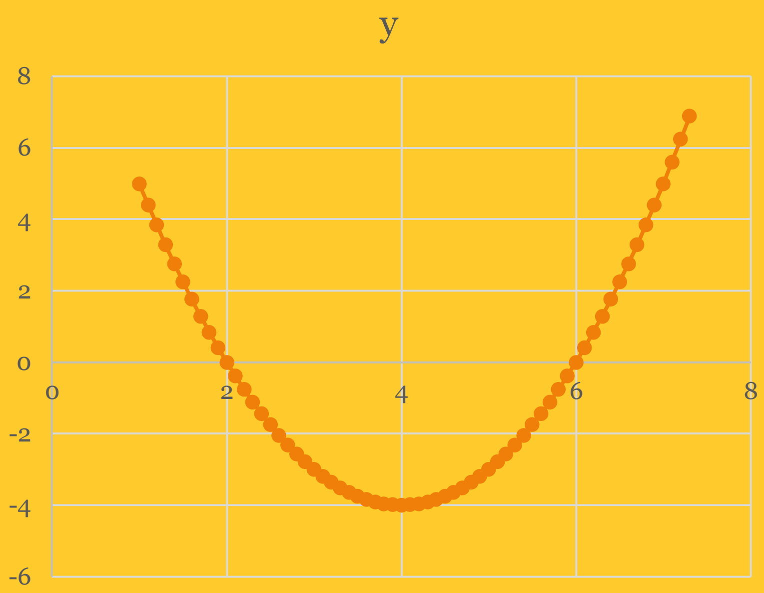
$$\Rightarrow \lambda_1 = 6, \lambda_2 = 2$$

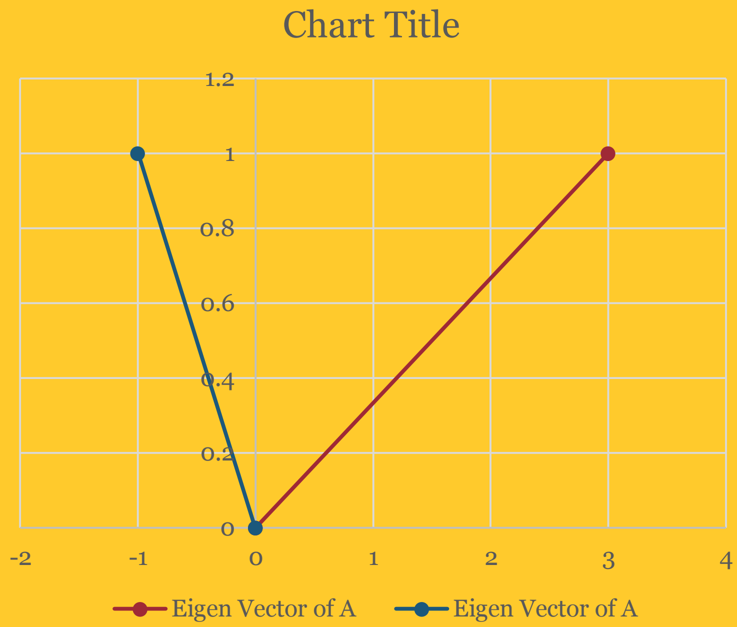
Example 12.1

- Eigenvectors: $v_1 = (3, 1)$; $v_2 = (-1, 1)$
- Observations:
  - Tr(A) = Sum of Diagonal Elements=5 + 3 = 8 = Sum of Eigenvalues = 6 + 2
  - Det(A) = Determinant of A=(5X3-3X1)= 12 = Product of Eigenvalues = 6 x 2
- Reason:
  - For any quadratic equation $lx^2 + mx + n = 0$,
    - the sum of roots is $-m/l$
    - Product of the roots is c/a
- For any 2x2 square matrix, the characteristic equation will be:

$$\begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0$$

Characteristic Equation, Eigen Value, Eigen Vector





Example 12.1

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (a-\lambda)(d-\lambda) - bc = 0$$

$$\Rightarrow \lambda^2 - (a+d)\lambda + ad - bc = 0$$

So, l = 1, m = -(a+d), and n = ad – bc

- The sum of roots = a+d = trace of the matrix
- The product of the roots = ad – bc = determinant of the matrix
- The roots of this equation are the eigenvalues.
- Similar relations can be established for higher order matrices and their eigenvalues

Example 12.2

- Find Eigenvalue and Eigenvector of A^T where,

$$A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$$

- Characteristic equation:

$$\begin{vmatrix} 5-\lambda & 1 \\ 3 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(3-\lambda) - 3 = 0$$

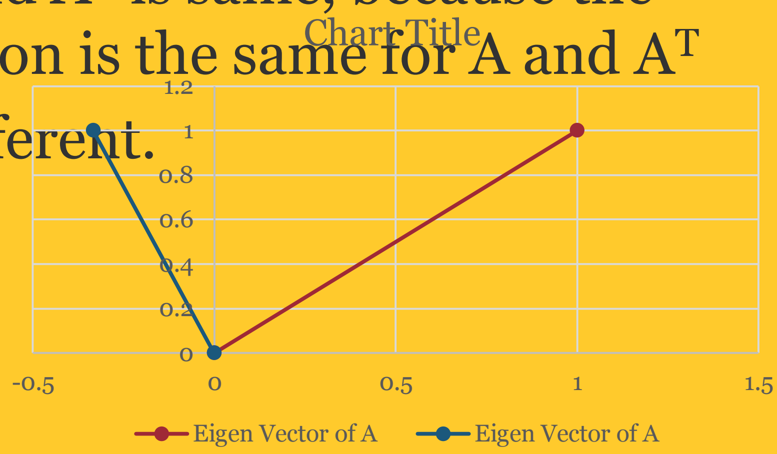
$$\Rightarrow \lambda^2 - 8\lambda + 12 = 0$$

$$\Rightarrow \lambda_1 = 6, \lambda_2 = 2$$

Eigen Values same as matrix A=(6,2)

Example 12.2

- Eigenvectors: v_1 = ()
- Observations:
  - Eigenvalues for A and A^T is same, because the characteristic equation is the same for A and A^T
  - Eigenvectors are different.



Example 12.3

- Find Eigenvalue and Eigenvector of A^{-1} where,

$$A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$$

- Eigenvalue:

$$\Rightarrow \lambda_1 = \frac{1}{6}, \lambda_2 = \frac{1}{2}$$

- Eigenvector: $v_1 = (3, 1)$; $v_2 = (-1, 1)$
- Observations:
  - The eigenvalues of A^{-1} are the reciprocal of eigenvalues of A
  - The eigenvectors of A and A^{-1} are the same.

Example 12.3

- For any 2x2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Its inverse will be:

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

The characteristic equation will be:

$$\begin{vmatrix} \frac{d}{ad-bc} - \lambda & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} - \lambda \end{vmatrix} = 0$$

Example 12.3

$$\Rightarrow \begin{vmatrix} \frac{d}{ad-bc} - \lambda & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - \left(\frac{d}{ad-bc} + \frac{a}{ad-bc}\right)\lambda + \frac{ad}{(ad-bc)^2} - \frac{bc}{(ad-bc)^2} = 0$$

$$\Rightarrow \lambda^2 - \left(\frac{d}{ad-bc} + \frac{a}{ad-bc}\right)\lambda + \frac{1}{ad-bc} = 0$$

Example 12.3

- Let the eigenvalues for A be λ_1 and λ_2 and those of A^{-1} be λ_1' and λ_2'

$$\lambda_1' + \lambda_2' = \frac{a+d}{ad-bc} = \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\lambda_1' \lambda_2' = \frac{1}{ad-bc} = \frac{1}{\lambda_1 \lambda_2}$$

Thus, the eigenvalues of A^{-1} are the reciprocal of the eigenvalues of A

Example 12.4

- Find the eigenvalues and eigenvectors of A^2

$$A^2 = \begin{bmatrix} 28 & 24 \\ 8 & 12 \end{bmatrix}$$

- Characteristic equation:

$$\begin{vmatrix} 28-\lambda & 24 \\ 8 & 12-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - 40\lambda + 144 = 0$$

$$\Rightarrow \lambda_1 = 36;\ \lambda_2 = 4$$

- Eigenvectors: $v_1 = (3, 1)$; $v_2 = (-1, 1)$
- Observation:
  - Eigenvalues of A^2 are the square of eigenvalues of A
  - Eigenvectors of A and A^2 are the same

Example 12.5

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

- Characteristic equation:

$$\begin{vmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{vmatrix} = 0$$

- Eigenvalues: $\lambda_1 = 1$; $\lambda_2 = 2$; $\lambda_3 = 5$
- Eigenvectors: (2, 1, 4); (1, 1, 2); (0, 1, 1)
- Observation: If all eigenvalues are non zero, matrix is non singular

Example 12.6

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 0$; $\lambda_2 = 1$; $\lambda_3 = 1$
- Eigenvectors: (2, -1, 2); (1, -2, 3); (2, -2, 3)
- Observation:
  - If one of the eigenvalues is zero, then the matrix is singular and determinant = 0
  - There can be different eigenvectors corresponding to repeated eigenvalues so that they are linearly independent

Example 12.7

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 1$; $\lambda_2 = 1$; $\lambda_3 = 1$
- Eigenvectors: (1, 1, 1); (1, 1, 1); (1, 1, 1)
- Observation:
  - Here A is non symmetric and all the eigenvalues are repeated. Here it is not possible to find eigenvectors corresponding to the repeated eigenvalues.

Example 12.8

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 0$; $\lambda_2 = 3$; $\lambda_3 = 15$
- Eigenvectors: (1, 2, 2); (2, 1, -2); (2, -2, 1)
- Observation:
  - $x_1.x_2 = x_2.x_3 = x_3.x_1 = 0 \Rightarrow x_1, x_2, x_3$ are orthogonal

Example 12.9

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 1$; $\lambda_2 = 3$; $\lambda_3 = 3$
- Eigenvectors: (-1, 0, 1); (1, 1, 1); (1, -2, 1)
- Observation:
  - Here we chose $x_2 = -2k_2$ for finding x_3 because of making vectors x_1, x_2, x_3 orthogonal to each other.

Example 12.10

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 2$; $\lambda_2 = -1$; $\lambda_3 = -1$
- Eigenvectors: (1, 1, 1); (-1, 1, 0); (1, 1, -2)
- Observation:
  - It is not possible to find out orthogonal eigenvectors for a symmetric matrix having repeated eigenvalues

Example 12.11

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} -3 & -7 & 5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 1$; $\lambda_2 = 1$; $\lambda_3 = 1$
- Eigenvectors: (-3, 1, 1) for $\lambda_1 = 1$

Example 12.12

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 3; \lambda_2 = 2; \lambda_3 = 2$
- Eigenvectors: (-1, -1, 2); (-5, -2, 5); (-5, -2, 5)

Example 12.13

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} -420 & \frac{1}{2} & 576 \\ 0 & 0 & 0.6 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

- A is an upper triangular matrix. So the diagonal elements are the eigenvalues
- The eigenvalues are : $\lambda_1 = -420$; $\lambda_2 = 0$; $\lambda_3 = 3^{1/2}$
- Product of Eigenvalues = |A| = 0
- Hence A is not invertible

Example 12.14

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = +i$; $\lambda_2 = -I$
- Eigenvectors: (-*i*, 1); (*i*, 1)

Example 12.15

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = 1$; $\lambda_2 = 2$; $\lambda_3 = 2$
- Eigenvectors: (-2, 1, 1); (-1, 0, 1); (0, 1, 0)
- Observation:
  - x_2 and x_3 are linearly independent since their scalar product is zero.

Example 12.16

- Find the Eigenvalues and Eigenvectors of:

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

- The eigenvalues are : $\lambda_1 = -1; \lambda_2 = 3$

Example 12.17

- Show that if 0 < θ < π, then

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Has no real eigenvalues and hence no eigenvectors

- Characteristic equation:

$$\lambda^2 - 2\lambda\cos\theta + 1 = 0$$

- Now,

$$\Delta = b^2 - 4ac = 4(\cos\theta)^2 - 4 = -4(\sin\theta)^2 < 0$$

Hence the given matrix has no real eigenvalues and consequently no eigenvectors.

Example 12.18

- Find the eigenvalues of A^9 for:

$$A = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & 5 & 3 & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

- Eigenvalues of A: 1, 0.5, 0, 2
- Eigenvalues of $A^9 = 1^9, 0.5^9, 0^9, 2^9$

Example 12.18

- If A is an invertible matrix, $\lambda = 0$ cannot be an eigenvalue of A

Example 12.20

- Matrix A and A^{-1} have the same Eigenvalue

Cayley – Hamilton Theorem

- This theorem states that every square matrix A satisfies its own characteristic equation:
- If the characteristic equation is:

$$\lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \cdots\cdots + a_1\lambda + a_0 = 0$$

Then:

$$A^n + a_{n-1}A^{n-1} + a_{n-2}A^{n-2} + \cdots\cdots + a_1A + a_0 = 0$$

Example 12.24

- Verify Cayley – Hamilton theorem and find A^4

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$|A - \lambda I| = \lambda^3 - 3\lambda^2 - 7\lambda - 11 = 0$$

To prove: $A^3 - 3A^2 - 7A - 11I = 0$

$$A^2 = \begin{bmatrix} 8 & 8 & 5 \\ 8 & 7 & 8 \\ 13 & 8 & 8 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 42 & 31 & 29 \\ 45 & 39 & 31 \\ 53 & 35 & 42 \end{bmatrix}$$

Example 12.24

$$\text{LHS} = A^3 - 3A^2 - 7A - 11I$$

$$= \begin{bmatrix} 42 & 31 & 29 \\ 45 & 39 & 31 \\ 53 & 35 & 42 \end{bmatrix} - 3\begin{bmatrix} 8 & 8 & 5 \\ 8 & 7 & 8 \\ 13 & 8 & 8 \end{bmatrix} - 7\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} - 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 42 - 24 - 7 - 11 & 31 - 24 - 7 & 29 - 15 - 14 \\ 45 - 24 - 21 & 39 - 21 - 7 - 11 & 31 - 24 - 7 \\ 53 - 39 - 14 & 35 - 24 - 21 & 42 - 24 - 7 - 11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Example 12.24

$$A^4 = A.A^3 = A.(3A^2 + 7A + 11)$$

$$=> A^4 = 3A^3 + 7A^2 + 11A$$

$$\Rightarrow A^4 = 3\begin{bmatrix} 42 & 31 & 29 \\ 45 & 39 & 31 \\ 53 & 35 & 42 \end{bmatrix} + 7\begin{bmatrix} 8 & 8 & 5 \\ 8 & 7 & 8 \\ 13 & 8 & 8 \end{bmatrix} + 11\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 193 & 160 & 144 \\ 224 & 177 & 160 \\ 272 & 224 & 193 \end{bmatrix}$$

Regression Analysis/ Curve Fitting

Definition:
>Regression analysis or curve fitting is a process of fitting a function to a set of data points.

➢The function can be used as a model of the data.

➢ The functions can be Linear, Polynomial, Power, Exponential, etc.

➢ Many times it is known which function will give good fit and only required to find coefficients.

➢ Many time many functions are drawn to know which fits best.

Interpolation Definition:
Interpolation is the process of estimating values between data points.

Regression Analysis/ Curve Fitting

Polynomial Curve fitting

- Curve Passes through all the points

  Note: Degree of polynomial one less than data points

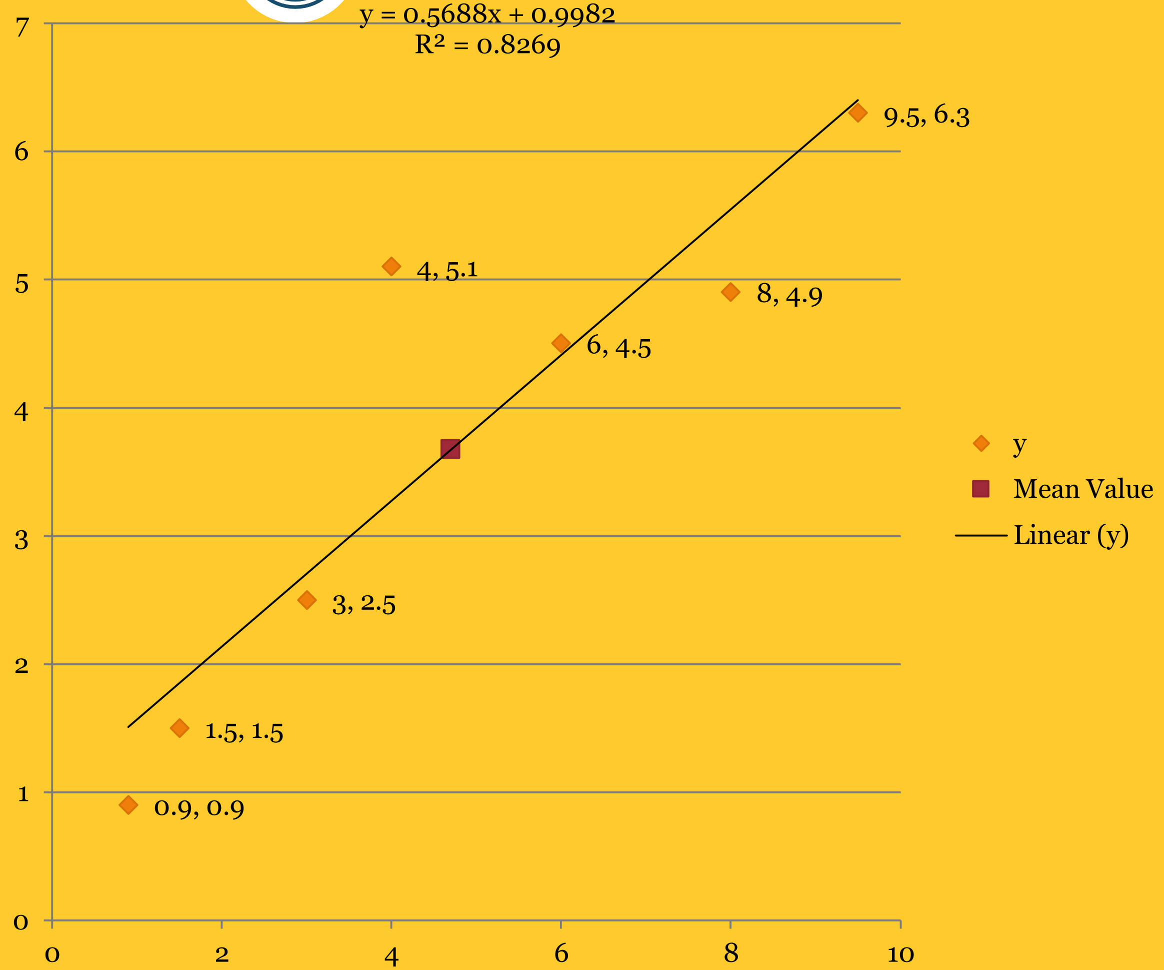
- Polynomial that do not necessarily pass any of the points

Polynomial of first degree
It is required to find m and c

x	y
0.9	0.9
1.5	1.5
3	2.5
4	5.1
6	4.5
8	4.9
9.5	6.3

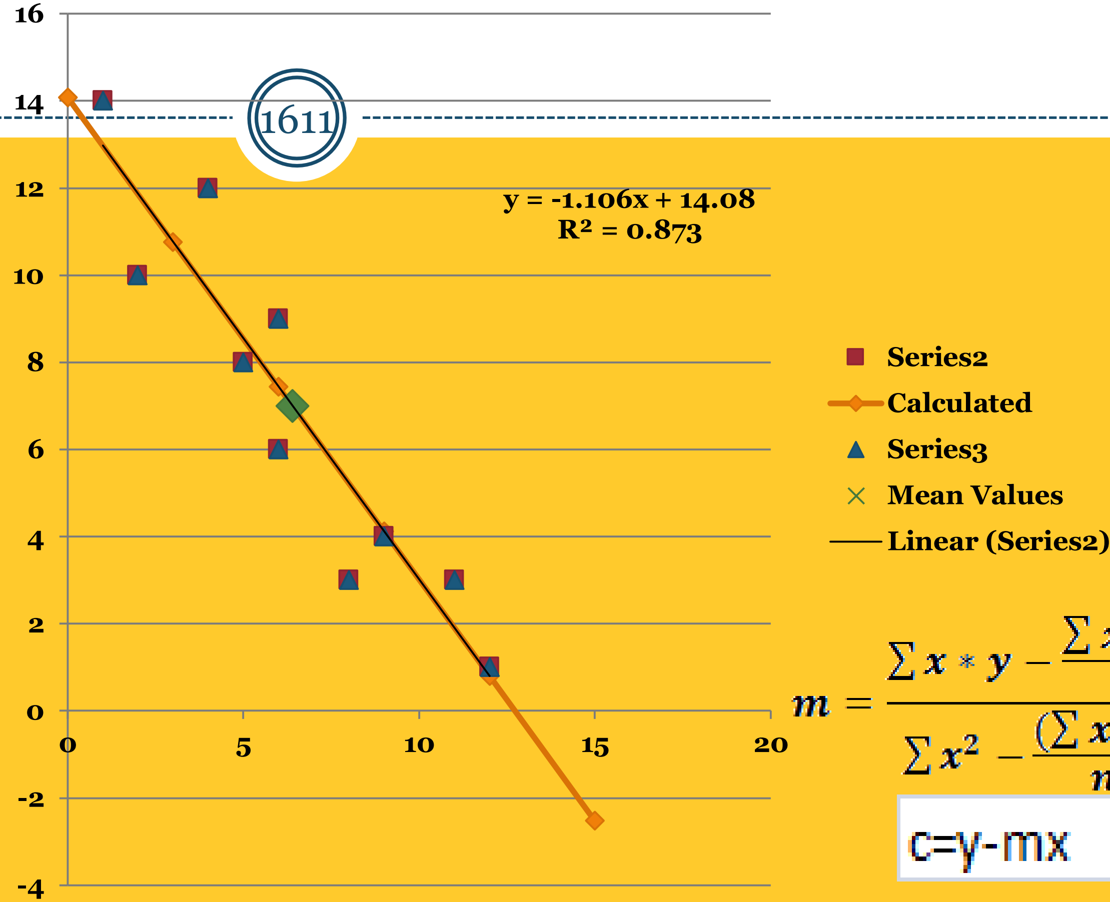
$$m = \frac{\sum x * y - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^\wedge 2}{n}}$$

c=y-mx



Regression Analysis/ Curve Fitting

x	y	x*y	x^2
8	3	24	64
2	10	20	4
11	3	33	121
6	6	36	36
5	8	40	25
4	12	48	16
12	1	12	144
9	4	36	81
6	9	54	36
1	14	14	1
64	70	317	528
4096			
6.4	7		



$$m = \frac{\sum x * y - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

c=y-mx

Polynomial of higher degree/ Excel Option

x	y
0.9	0.9
1.5	1.5
3	2.5
4	5.1
6	4.5
8	4.9
9.5	6.3

$$m = \frac{\sum x * y - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^{\wedge}2}{n}}$$

c=y-mx

Steps for curve fitting in Excel:
Step-1: Plot the data
Step-2:Right click and select the option for Add Trend Line

Select Options from:

1. Exponential
2. Linear
3. Logarithmic
4. Polynomial > Order
5. Power
6. Moving Average

Curve fitting options other than Polynomials

x	y
0.9	0.9
1.5	1.5
3	2.5
4	5.1
6	4.5
8	4.9
9.5	6.3

Different Options for Curve fitting:

1. Exponential, y=b*exp(m*x)
2. Logarithmic, y=m*ln(x)+b
3. Power , y=b*x^m
4. Reciprocal, y=1/(m*x+c)

SETS

1. Sets – Collection of objects of a particular kind.

A = {a,e,i,ou} Roster form (Tabular form)

set element

ε(epsilon), a ε A, a belong to A

Set builder form : A={x:x is an odd natural number}.

Roster form lists all the elements.

Some Examples

Ex-1 Solution set of x^2+x-2=0

(x-1)(x-2)=0, x=1, x=-2

x= {1,-2}

Ex-2 A = {x:x is a positive integer and x^2<40}

= {1,2,3,4,5,6}

Ex-3 A = {1,4,9,16,25,.....}

Set builder form A = {x: x is the square of a natural number}

Ex-4 A = {1/2, 2/3, 3/4, 4/5}

Set builder form A = {x: x/x+1, where x is a natural number and x<6}

Type of Sets

1. Empty sets (ϕ, { })
2. Finite and Infinite sets
3. Equal sets
4. Sub sets A contain B

$$a \varepsilon A \Rightarrow a \varepsilon B$$

4a) Subset of sets of real number

4b) Intervals as subsets of R.

5. Power set {1,2}, {ϕ}, {1}, {2}, {1,2}
6. Universal set

Venn Diagram
(Representation of Sets)

Universal sets – Rectangle
Subsets – Circle

1.10 Operations on sets :

a. Union : A = {2,4,6,8}
B = {6,8,10,12}
A U B = {2,4,6,8,10,12}

Laws : Commutative, Associative, Identity, Idempotent law of U.

b. Intersection : common elements.
$A \cap B = \{6 , 8\}$

c. Difference : A –B = {2 ,4}

1.11 Complement of a set, A’ = U-A

PERMUTATION & COMBINATION

7.2 Fundamental Principle of Counting

Definition : If an event can occur in m different ways, following another event can occur in n different ways then the total no. of occurrence of the event is given by m*n.

A. Event without repetition.

B. Event with repetition.

7.3 Permutation : A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

Let total letter is 6 and we are required to form four letter words. First place can be filled in 6 ways, second place by 5 ways, third place by 4 ways & fourth place by 3 ways. So total no. of ways = 6 * 5 * 4 * 3 = 360.

Place	1	2	3	4	.......r
No	n	(n-1)	(n-2)	(n-3)	n-(r+1)

This is expressed as, $^{n}P_{r} = n!/(n-r)!$

7.3.2 Factorial : $8! = 8*7*6*5*4*3*2*1$

$0! = 1, \quad 1! = 1$

7.3.3 $^{n}P_{r} = n!/(n-r)!$

7.3.4 With repetition, $^{n}P_{r} = n!/P_{1}!\, P_{2}!$

NOTE : In permutation order is important.

7.4 Combination : Let r be the no. of events to be chosen from n no. of events, then the combination is

$^{n}C_{r} = n!/(n-r)!r!$

Relationship Between Permutation and Combination

- $^{n}C_{r} = n!/(n-r)!r!$
- $^{n}P_{r} = n!/(n-r)!$

therefore, $^{n}P_{r} = {}^{n}C_{r} * r!$

PROBABILITY

3.9 Chance and Probability

Event – Possible, Impossible, Probable

In mathematical terms:

- Possible-1
- Impossible – 0
- Probable – $0<p<1$

PROBABILITY

In mathematical terms:

- Possible-1
- Impossible – 0
- Probable – $0<p<1$

- Event: Tossing coin – Probability of T is 0.5
- Event: Throw a die – Probability of getting 5 is 1/6

PROBABILITY

- Chance and Probability
- Random Experiment
- Outcomes finite
- Equally likely outcome
- Linking chances to probability
- Outcomes as Event
- Chance and probability is related to life

PROBABILITY

Uncertainty

- Probably
- Doubt
- Most Probably
- Chances
- 50-50 Chance

PROBABILITY

- Empirical Probability= $\frac{No\ of\ trials\ on\ which\ the\ event\ happened}{Total\ number\ of\ trials}$

- Theoretical Probability =

$$\frac{No\ of\ outcomes\ favourable\ to\ event\ e}{Total\ number\ of\ all\ possible\ outcomes\ of\ the\ experimenttrials}$$

- The sum of the probabilities of all the elementary events of an experiment is one.

Probability

- Probability is a measure of uncertainty of various phenomenon

- Probability = $\frac{No\ of\ outcome\ favourable\ to\ the\ event}{No\ of\ eually\ likely\ outcome}$

- OUTCOME AND SAMPLE SPACE

Probability

- EVENT

- Type of events
- 1) Impossible and sure event
- 2) simple event
- ‘
- Algebra of event
- Complimentary event
- Mutually exclusive event
- Exhaustive event

1. Axonometric approach to probability

Probability

Axonometric approach to probability

1. Probability of an event
2. Probabilities of equally likely outcomes
3. Probability of event A or B
4. Probability of event ‘not A’

Probability

D

PROBABILITY

Probability is the measure of uncertainty.

Probability= No. of outcomes favorable to the event / No. of equally likely outcome

16.2 RANDOM EXPERIMENT –

Condition –

1. There is more than one possible outcome.
2. It is not possible to predict the outcome in advance.

16.2.1 Sample Space – All possible outcome of an experiment :-

(HH HT TH TT)

16.2.2 Sample Point – Each element of a sample space is called a sample point.

EVENT

16.3 Event : An event of an experiment is a subset (ε) of a sample space (S).

16.3.1 Occurrence of an event

16.3.2 Types of events :

a. Impossible or Sure event.

b. Simple event.

c. Compound event.

16.3.3 Algebra of events : As the event is a subset it can be used as analogous set notations like union, intersection, difference, complementary.

Complementary Events

1. Complementary Event : A’ = S – A
2. The event ‘A or B’ => A U B
3. The event ‘A and B’ => A ∩ B
4. The event ‘A but not B’ => A – B

16.3.4 Mutually Exclusive Event

A U B , A ∩ B , A – B are mutually exclusive event.
They cannot occur simultaneously.

16.3.5 Exhaustive Event

Two or more events are called exhaustive if at least one of them necessarily occurs whenever the experiment is performed.

Mutually exclusive & exhaustive events are $\varepsilon i \cap \varepsilon j = \Phi$ for I & j.

16.4 Axiomatic approach to probability (to understand refer to chapter – 14 mathematical reasoning).

NOTE : Probability related

Chapters – 1. Sets (Chapter – 1)

2. Principle of Mathematical Induction (Chapter – 4)

3. Permutation & Combination (Ch – 7)

4. Mathematical Reasoning (Ch – 14)

5. Probability (Ch – 16).

Principle of Mathematical Induction

Statements – 1. Socrates is a man.

2. Man is mortal.

3. Socrates is mortal.

If statement 1 & 2 is true then 3 is true.

Definition : ?

Note : It is felt that we should read chapter 14 on Mathematical Reasoning before chapter 4.

Mathematical Reasoning

All the human (living organs) has the ability to reason and power of reasoning varies from man to man.

14.2 Statements : A sentence is called a mathematically acceptable statement if it is either true or false but not both.

Example – Two and two is five.

There are 35 days in a month.

14.3 New statement from old

a. Negation of a statement.

b. Compound statement

And   or – Inclusive

Exclusive

14.4 Special words / Phrases in a statement

1.
   a. And
   b. Or - Inclusive
      Exclusive

2. Quantifiers
   For -> there exists

14.5 Implications

1. If -> then
2. Only if
3. If and only if

14.5.1 Contrapositive and Converse

Validating Statement

1. Validating by rule.
2. Validating by contradiction.

A 1.4 : Kind of Statement

1. Theorem
2. Conjecture
3. Axioms

1. Theorem is a statement whose truth has been established.
2. A Conjecture is a statement which we believe is true based on our understanding & experience.

Statistics

Statistics deals with group of numbers called data

Statistics-First Step

Statistics
Data Collection

Statistics
Data Analysis

Statistics-Objective

Statistics is a branch of mathematics dealing with the collection, analysis, interpretation, presentation, and organization of data

Statistics-Organizing Data

1. **Recording Data**
2. **Organizing Data**
3. **Pictograph**
4. **Drawing a pictograph**
5. **Bar Graph**
6. **Drawing a Bar Graph**

Statistics-Representative Values

1. **Central Tendency**
2. **Dispersion**

Statistics-Central Tendency

1. **Average**
2. **or Arithmetic Mean**
3. **Mode**
4. **Median**

Statistics-Dispersion

1. Range

Statistics

1. **Collection of Data**
2. **Organization of Data**
3. **Arithmetic Mean**
4. **Range**
5. **Mode**
6. **Median**
7. **Use of Bar Graph**

Statistics

Graphical Representation of Data

1. **Pictograph**
2. **Bar Graph**

Organizing Data

1. **Grouping of data**
2. **Frequency**

Circle Graph or Pie Chart

Drawing Pie Charts

Statistics

Graphical Representation of Data

1. **Bar Graphs**
2. **Histogram**
3. **Frequency Polygon**

Statistics

Measure of Central Tendency

1. **Mean**
2. **Median**
3. **Mode**

Statistics

Measure of Central Tendency

1. **Mean of Grouped data**
2. **Class Mark**
3. **Deviation**
4. **Mode of Grouped Data**
5. **Median of Grouped Data**

6. **3 Median=Mode+2 Mean**

Statistics

Measure of Central Tendency
Mean.
Median
Mode

Statistics

Measure of Dispersion

Range

Mean Deviation

Mean Deviation for Grouped Data

1. **Discrete Frequency Distribution**
   i. **Mean Deviation about mean**
   ii. **Mean Deviation about Median**
2. **Continuous Frequency Distribution**
   i. **Mean Deviation about mean**
   ii. **Mean Deviation about Median**

Limitation of mean deviation

Statistics

Variance and Standard Deviation

1. **Standard Deviation**
2. **Standard deviation of a discrete frequency distribution**
3. **Standard deviation of a continuous frequency distribution**

Short cut method to find variance and standard deviation

Statistics

mm

Analysis of Frequency Distribution

1. **Comparison of two frequency distribution with same mean**

AVEDEV Returns the average of the absolute deviations of data points from their mean
AVERAGE Returns the average of its arguments
AVERAGEA Returns the average of its arguments, including numbers, text, and logical values
AVERAGEIF Returns the average (arithmetic mean) of all the cells in a range that meet a given criteria
AVERAGEIFS Returns the average (arithmetic mean) of all cells that meet multiple criteria.
BETADIST Returns the beta cumulative distribution function
BETAINV Returns the inverse of the cumulative distribution function for a specified beta distribution

BINOMDIST Returns the individual term binomial distribution probability
CHIDIST Returns the one-tailed probability of the chi-squared distribution
CHIINV Returns the inverse of the one-tailed probability of the chi-squared distribution CHITEST Returns the test for independence CONFIDENCE Returns the confidence interval for a population mean

CORREL Returns the correlation coefficient between two data sets COUNT Counts how many numbers are in the list of arguments COUNTA Counts how many values are in the list of arguments COUNTBLANK Counts the number of blank cells within a range COUNTIF Counts the number of cells within a range that meet the given criteria COUNTIFS Counts the number of cells within a range that meet multiple criteria COVAR Returns covariance, the average of the products of paired deviations CRITBINOM Returns the smallest value for which the cumulative binomial distribution is less than or equal to a criterion value DEVSQ Returns the sum of squares of deviations EXPONDIST Returns the exponential

FDIST Returns the F probability distribution FINV Returns the inverse of the F probability distribution FISHER Returns the Fisher transformation FISHERINV Returns the inverse of the Fisher transformation FORECAST Returns a value along a linear trend FREQUENCY Returns a frequency distribution as a vertical array FTEST Returns the result of an F-test GAMMADIST Returns the gamma distribution GAMMAINV Returns the inverse of the gamma cumulative distribution GAMMALN Returns the natural logarithm of the gamma function, Γ(x) GEOMEAN Returns the geometric mean GROWTH Returns values along an exponential trend HARMEAN

FIST Returns the F probability distribution FINV Returns the inverse of the F probability distribution FISHER Returns the Fisher transformation FISHERINV Returns the inverse of the Fisher transformation FORECAST Returns a value along a linear trend FREQUENCY Returns a frequency distribution as a vertical array FTEST Returns the result of an F-test GAMMADIST Returns the gamma distribution GAMMAINV Returns the inverse of the gamma cumulative distribution GAMMALN Returns the natural logarithm of the gamma function, Γ(x) GEOMEAN Returns the geometric mean

GROWTH Returns values along an exponential trend HARMEAN Returns the harmonic mean HYPGEOMDIST Returns the hypergeometric distribution INTERCEPT Returns the intercept of the linear regression line KURT Returns the kurtosis of a data set LARGE Returns the k-th largest value in a data set LINEST Returns the parameters of a linear trend LOGEST Returns the parameters of an exponential trend LOGINV Returns the inverse of the lognormal distribution LOGNORMDIST Returns the cumulative lognormal distribution MAX Returns the maximum value in a list of arguments MAXA Returns the maximum value in a list of arguments, including numbers, text, and logical values MEDIAN Returns the median of the given numbers MIN Returns the minimum value in a list of arguments MINA

Returns the smallest value in a list of arguments, including numbers, text, and logical values MODE Returns the most common value in a data set NEGBINOMDIST Returns the negative binomial distribution NORMDIST Returns the normal cumulative distribution NORMINV Returns the inverse of the normal cumulative distribution NORMSDIST Returns the standard normal cumulative distribution NORMSINV Returns the inverse of the standard normal cumulative distribution PEARSON Returns the Pearson product moment correlation coefficient

PERCENTILE Returns the k-th percentile of values in a range PERCENTRANK Returns the percentage rank of a value in a data set PERMUT Returns the number of permutations for a given number of objects POISSON Returns the Poisson distribution PROB Returns the probability that values in a range are between two limits QUARTILE Returns the quartile of a data set RANK Returns the rank of a number in a list of numbers RSQ Returns the square of the Pearson product moment correlation coefficient

SKEW Returns the skewness of a distribution SLOPE Returns the slope of the linear regression line SMALL Returns the k-th smallest value in a data set STANDARDIZE Returns a normalized value STDEV Estimates standard deviation based on a sample STDEVA Estimates standard deviation based on a sample, including numbers, text, and logical values STDEVP Calculates standard deviation based on the entire population STDEVPA Calculates standard deviation based on the entire population, including numbers, text, and logical values STEYX Returns the standard error of the predicted y-value for each x in the regression TDIST Returns the Student's t-distribution TINV Returns the inverse of the Student's t-distribution TREND Returns values along a linear trend TRIMMEAN Returns the mean of the interior of a data set TTEST Returns the probability associated with a Student's t-test VAR Estimates variance based on a sample VARA Estimates variance based on a sample, including numbers, text, and logical values VARP Calculates variance based on the entire population VARPA Calculates variance based on the entire population, including numbers, text, and logical values WEIBULL Returns the Weibull distribution ZTEST Returns the one-tailed probability-value of a z-test

Resources and Reference

- Mathematical Elements for Computer Graphics (2nd Edition) *by David F. Rogers and J. Alan Adams*
- *Mathematics for computer graphics by John Vince*
- *Computer Graphics through key mathematics by Huw Jones*
- *Getting started with Matlab-Rudra Pratap*
- *Getting started with Mupad-M Majewski*
- *Linear Algebra and its Applications – Gilbert Strang*
- *An introduction to mathematical Techniques for Economic Analusis – Jayadeb Sarkhel, Anindita Bhukta*

- NCERT Books
- Microsoft Math Software
- Microsoft Excel
- www.ocw.mit.edu
- www.wikipedia.com

Contact Us…

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Thank You!!!

Q&A!!!

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Last Updated 29th May 2016
Last updated – 17 – MAY – 2014
Revision – 18-MAY-2014
Total Slides-184

Topics covered:

1. General concepts
2. Matrix Transformation
3. Linear Algebra
4. Vector
5. Calculus
6. Differential Calculus
7. Integral Calculus
8. Differential Equations
9. Set Page-19

Last updated – 18 – MAY – 2014

Topics to be covered

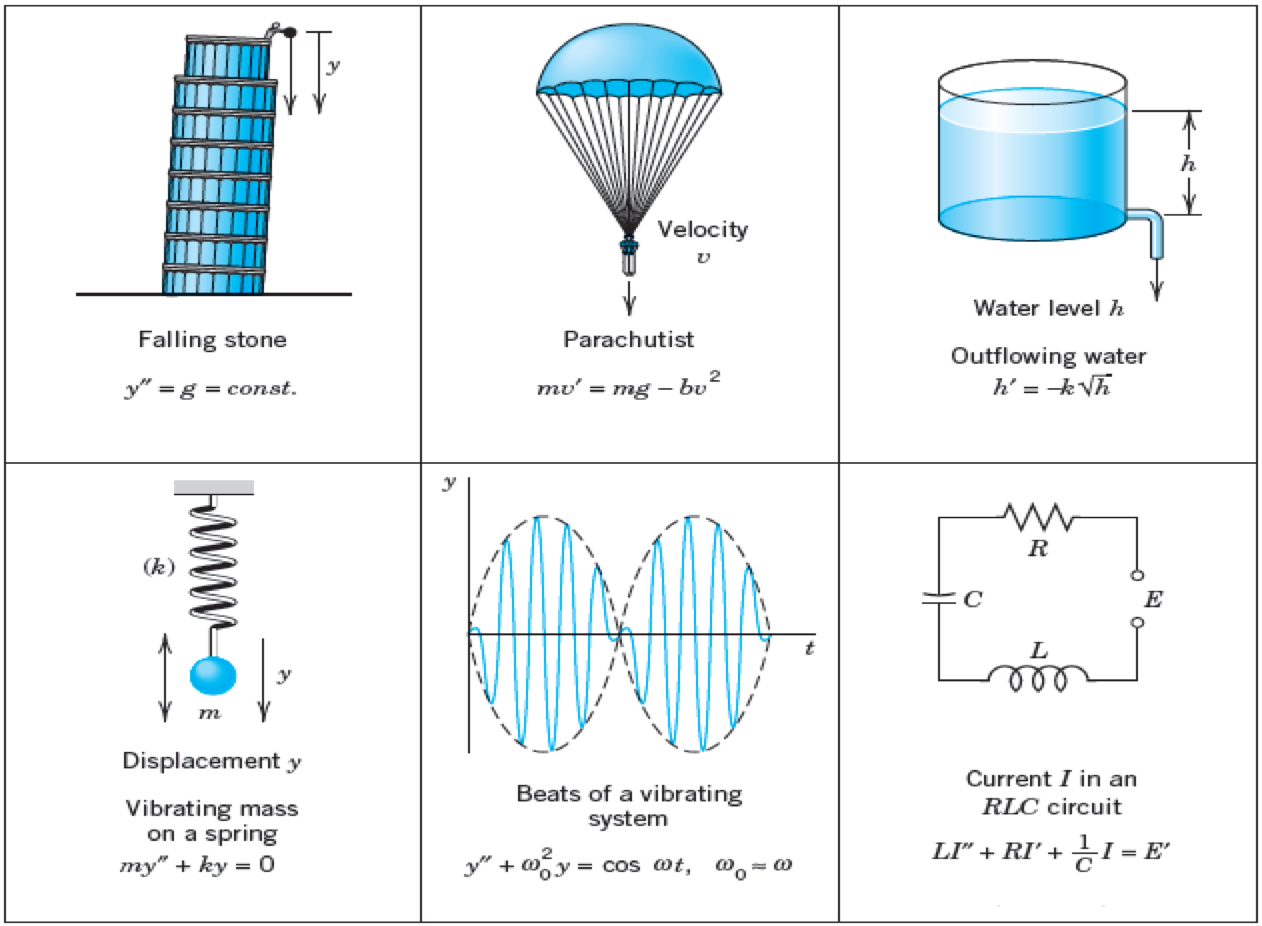
1. ~~Series~~
2. ~~Complex Number~~
3. ~~Trigonometry~~
4. Interpolation
5. ~~Conic Sections~~
6. Multivariate
7. Construction of Surfaces/ Solids

Note: Last working slide 263 complex conjugate 19may2014 @0045hr.

INDEX

SLIDE NO.	TOPIC
86	TRANSLATION/HOMOGENEOUS COORDINATE
90/91	TRANSLATION AND PROJECTION
96	3 DIMENSIONAL TRANSFORMATION
113	LINEAR ALGEBRA
125	LINEAR ALGEBRA
126	EIGEN VALUE AND EIGEN VECTOR
129	VECTOR
146	CALCULUS
162	DIFFERENTIATION
184	INTEGRATION

Where do they appear?



Note: Last working slide 263 complex conjugate 19may2014 @0045hr.

19may2014: Included Set theory at slide 19-

To do: Insert Function graphs at slide 160 from class XI book

Permutation and combination: 238

25MAY2014 0100 HRS. Updated Slide 276

06June2014: 1934hr added slides 315 to 329 for logics

06june2014 total slide 336

Comments of Stanford University Math Video Lecture

Xin Li
6 months ago

he is making sample things difficult!
Reply

·

2
Krishnan Sundaar Wednesday Meets you
5 months ago

he's actually doing the opposite...............he's pretty much spoon feeding you any idea you will ever need to have. without doing this, he's wasting your time........"teaching" steps is a waste of everyone's time
Reply

Lecture by Professor Brad Osgood for the Electrical Engineering course, The Fourier Transforms and its Applications (EE 261). Professor Osgood's lecture addresses the question- How can we use such simple functions, sin(t) and cos(t) to model such periodic phenomenon? He takes the students through the first steps in analyzing general periodic phenomenon.

https://www.youtube.com/watch?v=1rqJl7Rs6ps

Functions of Several Variable

- $z=f(x,y)$
- Here the value of z changes as a result of any change in either x or y or both.
- But it does not give any information about the rate of change of z in respect to any change in the value or values of the independent variables x and y.

Functions of Several Variable

- Case-1: When x and y is not related. If x is changed it will effect z but not y.
- Partial Derivative, dz/dx, y constant
- Partial Derivative, dz/dy, x constant
- Total Differential, dz/dx*dz/dy, x, y both varies

Case-2: When x and y are related. If x is changed, it will effect z in two ways – z is affected directly as x changed and affected indirectly by the change in x via the change of y.

Total Derivative- The rate of change of z due to change in x is measured by total derivative.

Linear Time Invariant

Matlab Command: ltiview

```
n1=[0 1 3]
d1=[2 4 1]
s1=tf(n1,d1)
n2=[3 2]
d2=[6 4 5]
s2=tf(n2,d2)
```

LTI

- ltiview

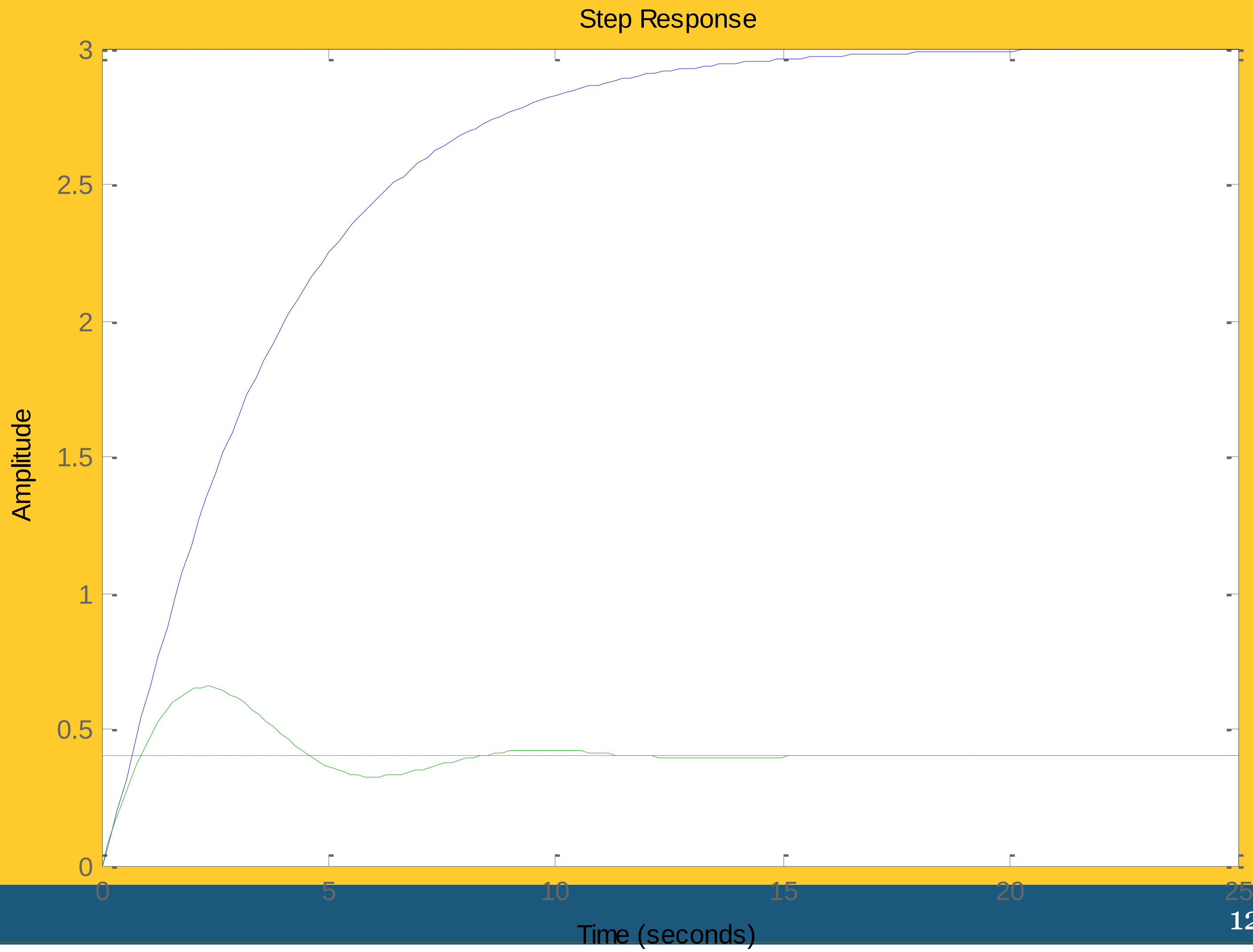
Import s1 s2 n2=[3 2]

d2=[6 4 5]

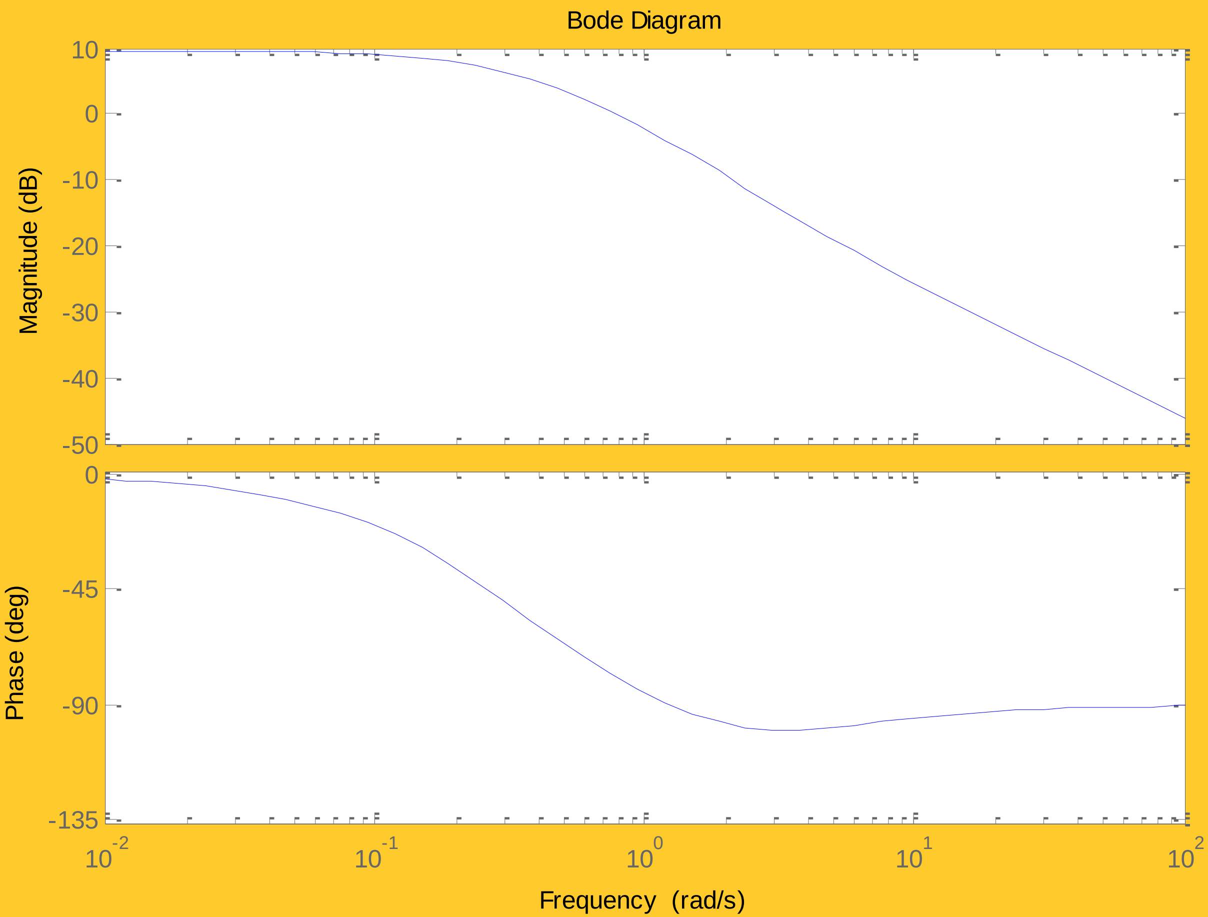
s2=tf[n2,d2]

ltiview(‘bode’,s1)

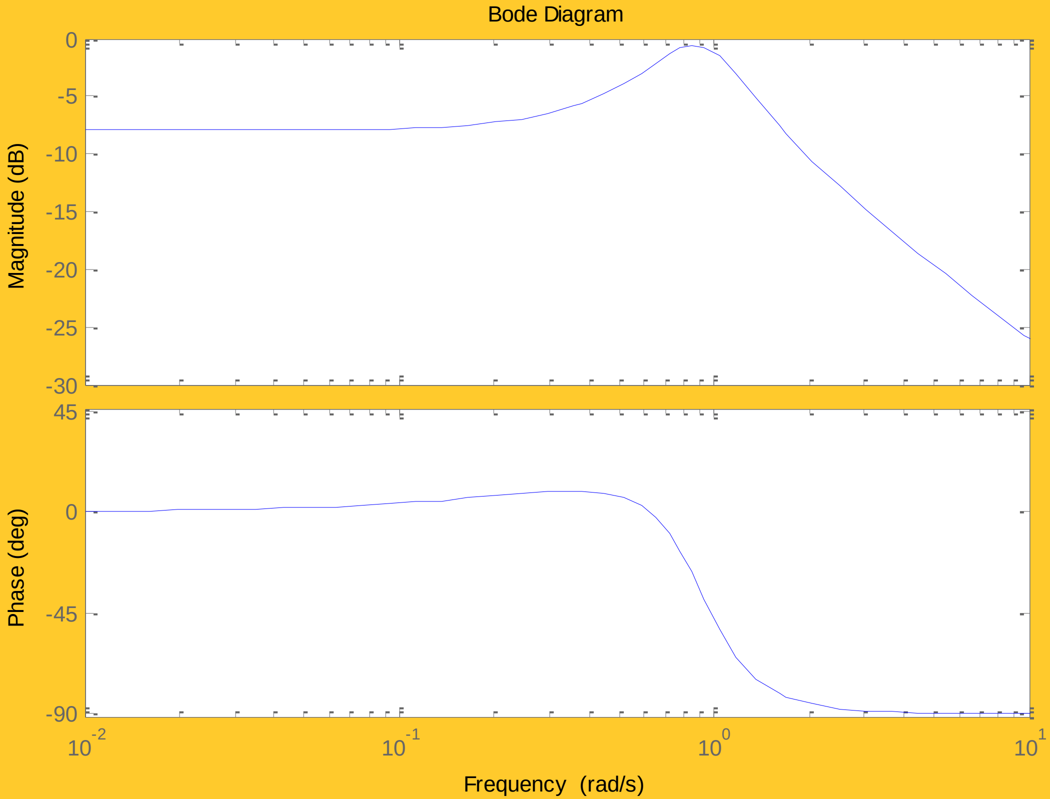
Imported Graph s1,s2



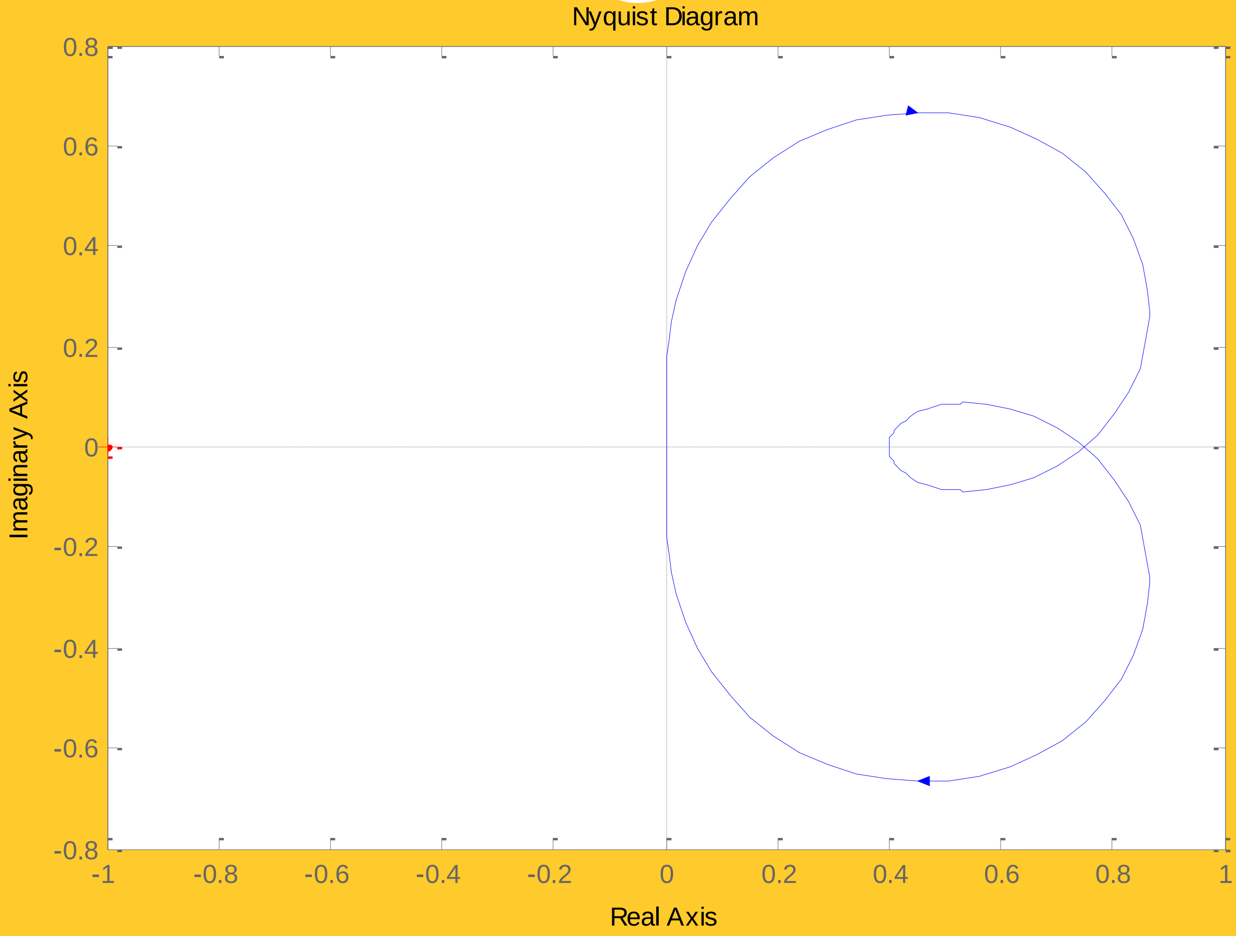
Bode Graph s1



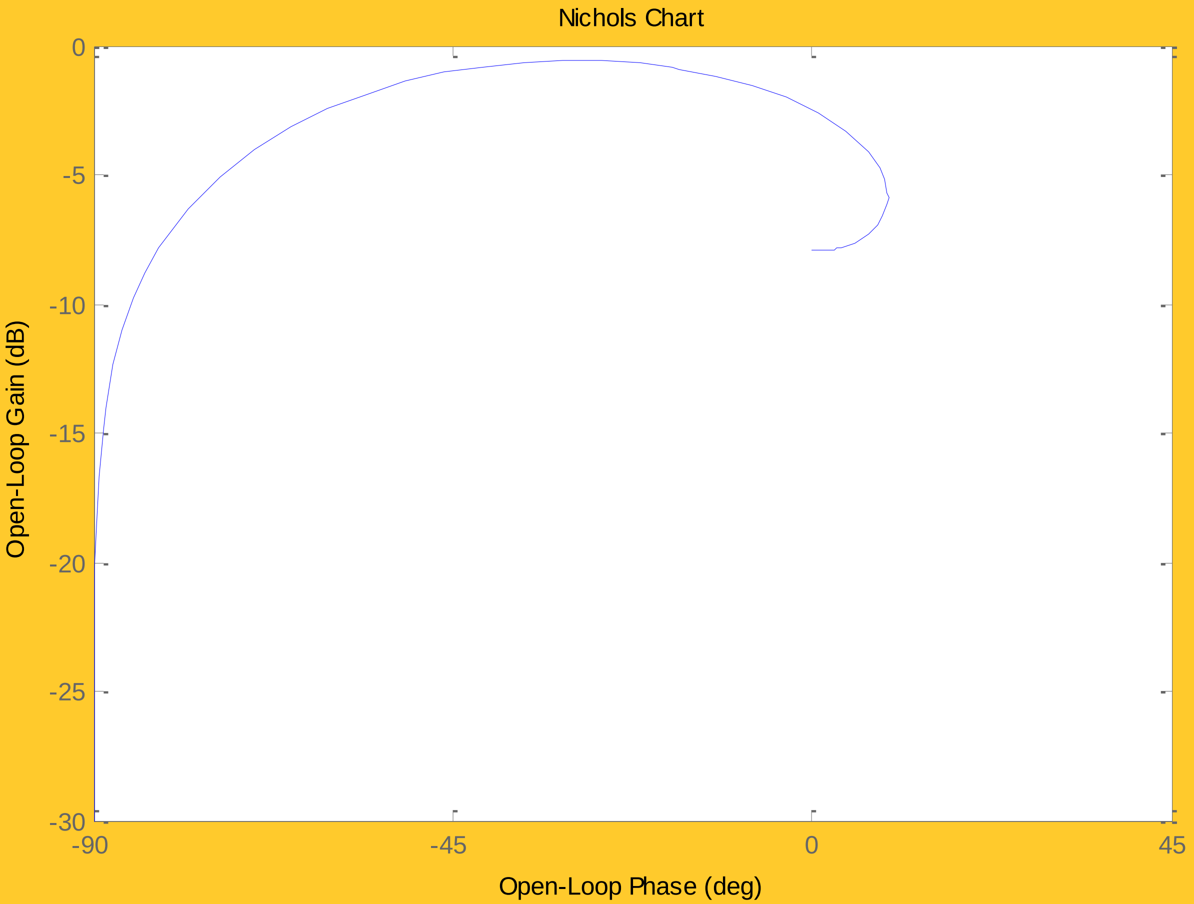
Bode Graph S2



Nyquist diagram



Nuchols Chart



Calculus of Polynomial

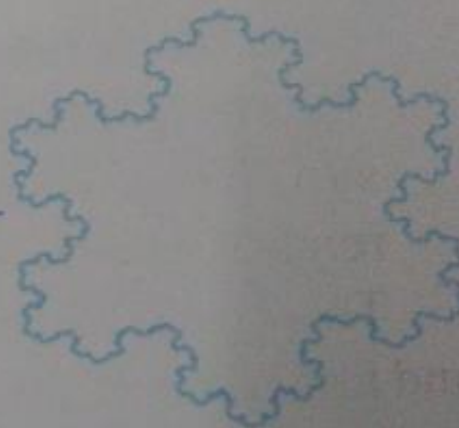
- dydx=polyder(p1)
- dydx = 24 24 4 3
- Dydxp1*p2=polyder(p1,p2)
- Dydxp1p2 = 72 230 224 96 58 31
- [n,d]=polyder(p1,p2)
- [n,d]=polyder(p1,p2)
- n = 24 106 128 52 -12 -19
- d = 4 20 33 20 4

Fractal

When We Encounter Fractal First?

At which class, we first read about Fractal?

- Observe this beautiful figure.
  It is a symmetric pattern known as Koch's Snowflake. (If you have access to a computer, browse through the topic "Fractals" and find more such beauties!).
  Find the lines of symmetry in this figure.



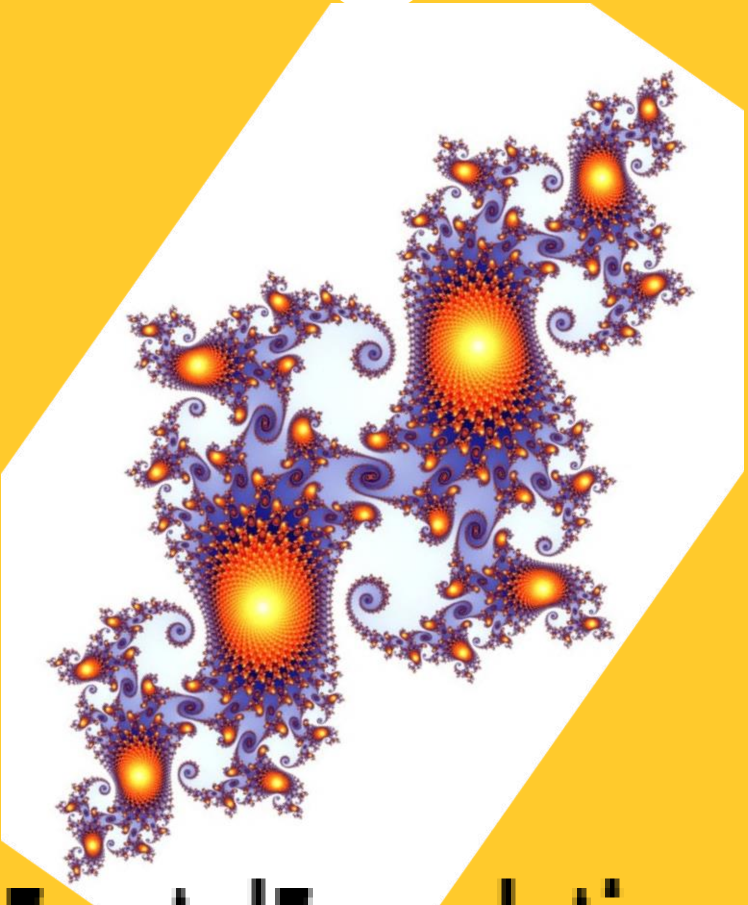
What is Fractal? (from internet)

- A curve or geometrical figure, each part of which has the same statistical character as the whole. They are useful in modelling structures (such as snowflakes) in which similar patterns recur at progressively smaller scales, and in describing **partly random** or **chaotic phenomena** such as crystal growth and galaxy formation.

Fractal - Wiki

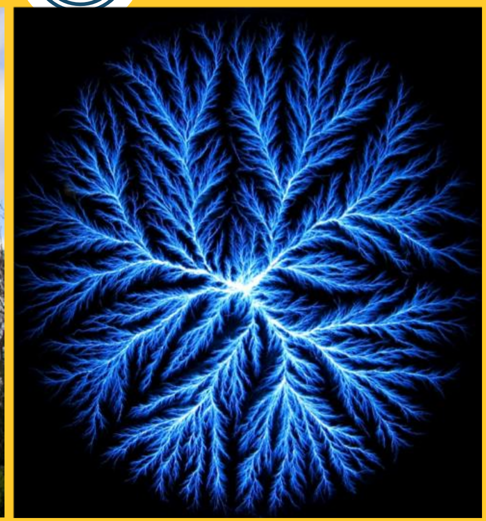
- A **fractal** is a natural phenomenon or a mathematical set that exhibits a repeating pattern that displays at every scale. It is also known as expanding symmetry or evolving symmetry. If the replication is exactly the same at every scale, it is called a self-similar pattern.

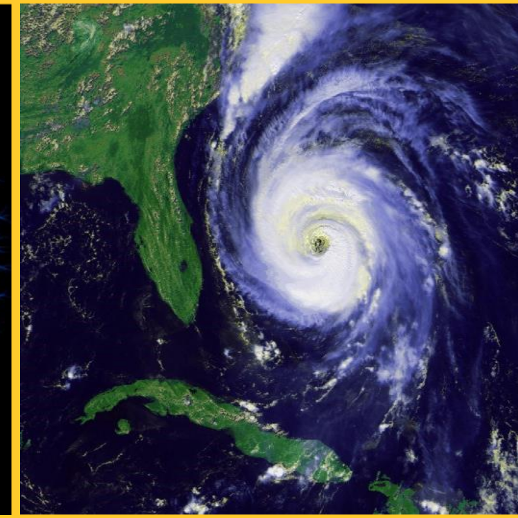
Fractals Are $MART: Science, Math & Art!

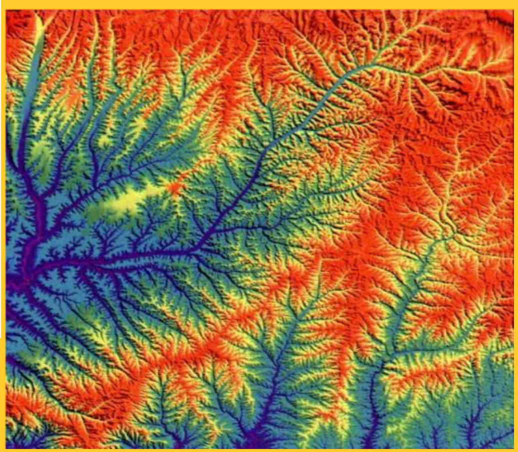


www.FractalFoundation.org

Example of Fractals







Fractal Definition

- A fractal is a never ending pattern that repeats itself at different scales. This property is called "Self-Similarity."
- Fractals are extremely complex, sometimes infinitely complex - meaning you can zoom in and find the same shapes forever.
- **Amazingly, fractals are extremely simple to make**.
- A fractal is made by repeating a simple process again and again.

Fractal – Huw Jones

Definition:

- A fractal is by definition a set for which the Housdorff Besicovitch dimension strictly exceeds the topological dimension.

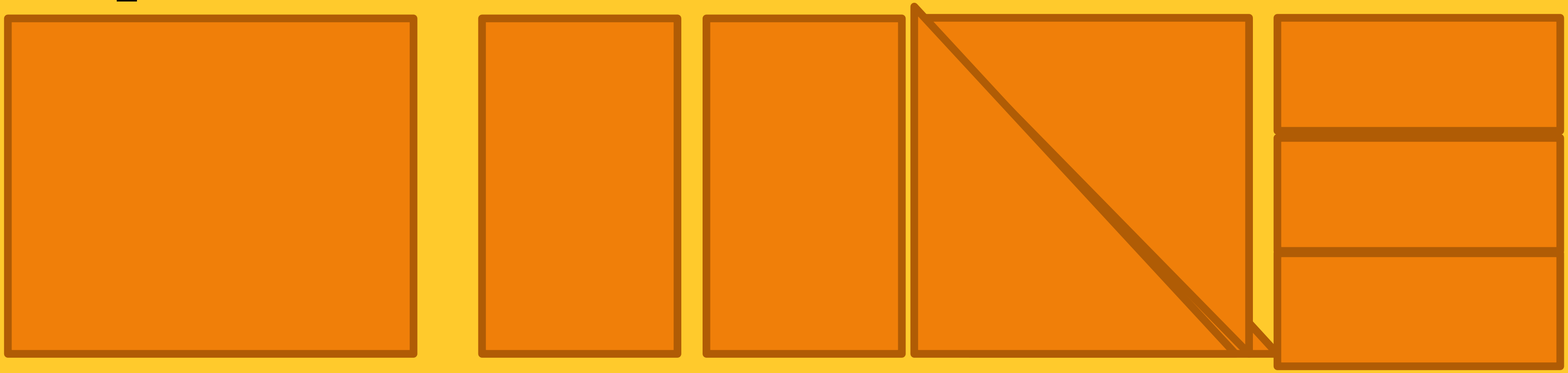
Properties of fractal :

- **Self similarity:** Any sub set of the object is, in some sense, a copy of the whole,
- **Infinitesimal sub divisibility:** There is no apparent change in the amount of detail observed at different levels of magnification.

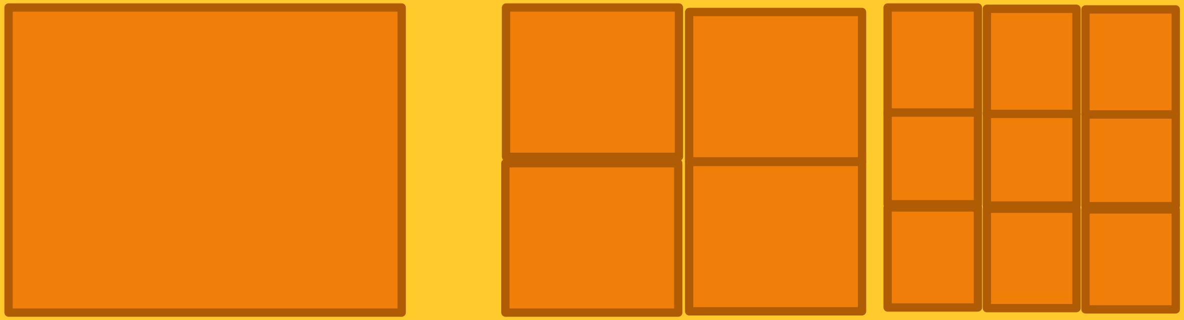
Understanding Concept of Fractal

- Infinitesimal Divisibility and Self Similarity

Square **Divided but not Self Similar**



Divided and Self Similar



Viewing Fractal

Fractivity

Explore Fractals with XaoS

http://fractalfoundation.org/resources/fractal-software/

XaoS

XaoS can create many different fractal types, which can be accessed by using the number keys:

Keys 1 to 5 are Mandelbrot sets with various powers. The “normal” X^2 Mandelbrot set is on key 1. (Hitting “1” is a good way to reset yourself if you get lost!)

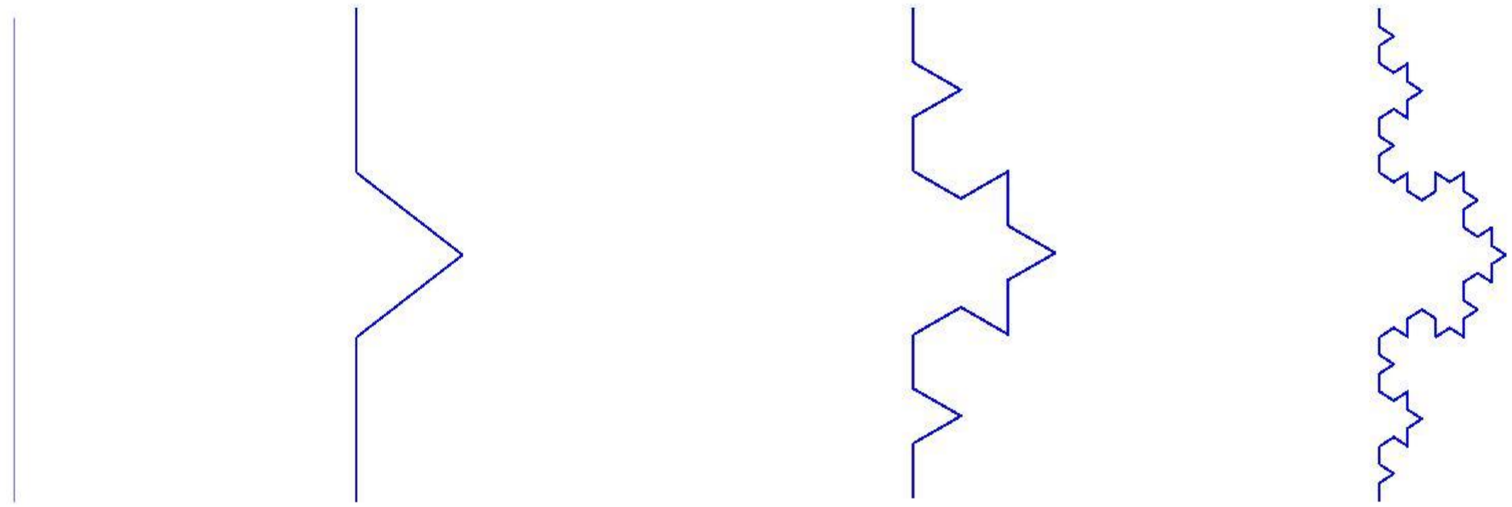
Key 6 is a Newton fractal, exponent 3, illustrating Newton’s method for finding roots to 3’d order polynomial equations.
Key 7 is the Newton fractal for exponent 4.
Key 8,9, and 0 are Barnsley fractals.
Key A - N are several other fascinating fractal formulas

The Koch curve (1904) and Cantor set :

- Koch curve is generated by recursively replacing line segments by 'poly lines' consisting of four line segments each 1/3rd of the original length. The first 4 stages of the process are shown below.

```
lsys:=plot::Lsys(PI/3,
"F","F"="F-F++F-F",
Generations=3):
plot(lsys)
```

Koch Curve – 1st 4 Generations



Koch Curve

- Limitless repetition of the process generates a true fractal form. The curve contains four exact copies itself, each at one third of the original scale, so at each stage of generation every line is replaced by four thirds of its original length.

- If the original line length has length one, the total length after n stages is $(4/3)^n$. It becomes infinitely large, it is said to approach infinity as 'n' approaches infinity.

- Thus, the pure fractal object has infinite length- there is not enough ink in the universe to draw it properly.

Koch Curve - Length

- This means that every sub copy is also infinite length - 1/3 of infinity must still be infinity.

- That goes for sub-sub copies and so on. Any two points in Koch curve is separated by an infinite distance .

Koch Curve - Area

- Now consider the area between the center and the original defining line.
- At the first stage a single triangle is added and we suppose its area is A. This is a isosceles triangle with sides equal to 1/3 and A can be found as $\sqrt{3}/36$, but the actual amount is irrelevant to the argument that follows.
- At each successive stage , four times as new triangles are added, each one ninth the area of the previous stage. When shape lengths are multiplies by 1/3, areas are multiplied by $(1/3)^2$ or 1/9 , a consequence of area being two dimensional. So the added area at any stage is 4/9 that added at the previous stage.

Koch Curve - Area

- The total area can be expressed as the sum of the series-

  S=A + (4 /9) A + (4/9) (4/9) A+ (4/9) (4/9) (4/9) A ....
- $S=A + (4/9)^1 A + (4/9)^2 A + (4/9)^3 A$ ....
- Here, each term of the series becomes smaller by a fraction of 4/9. The series like this with constant multiplies is called geometric series. As 4/9 is less than 1, the series converses. Multiplying both side by (4/9), we get,

  $(4/9) S = 4/9 A + (4/9)^2 A + (4/9)^3 A + - - -$
- Subtracting, this equation with previous equation get,
- S- 4/9 S = A or or 5/9 S = A or S = 9A/5 or Total Area=1.8 A

  As we know, A = √ 3/36.
- S = 9/5* √ 3/36 or S= √ 3/20.
- Here, the S is finite, ie, the area is finite Hence, we have an infinite length curve with a finite area. The area can be painted easily but detailed curve cannot be drawn

Fractal, Turtle Graphics, Lsys

- Fractal are complex geometry and requires specialized tool to draw it.
- Mupad provides excellent and simple tool to draw the fractals. These tools are Turtle Graphics and Lsys.
- Before exploring further, let's try to understand the process of iteration and recursion and how turtle graphics and Lsys works.

Recursion and Iteration

- Recursion and iteration both repeatedly executes the set of instructions.
- Recursion is when a statement in a function calls itself repeatedly.
- The iteration is when a loop repeatedly executes till the controlling condition becomes false.
- The primary difference between recursion and iteration is that a **recursion** is a process, always applied to a function. The **iteration** is applied to the set of instructions which we want to get repeatedly executed.

LOGO/ TURTLE GRAPHICS, L-Sys

- Both classes do not use coordinate geometry
- Using them, many geometric pattern can be produced based on line segment
- New turtle always faces upward, to the top edge of the computer screen
- Logo was first introduced by Jim Muller.
- With few simple commands, fascinating geometric objects can be created.

Logo and Turtle Graphics

- In European countries, Logo is the very first programing language used for teaching computing to students in primary schools and sometimes even in kindergarten.

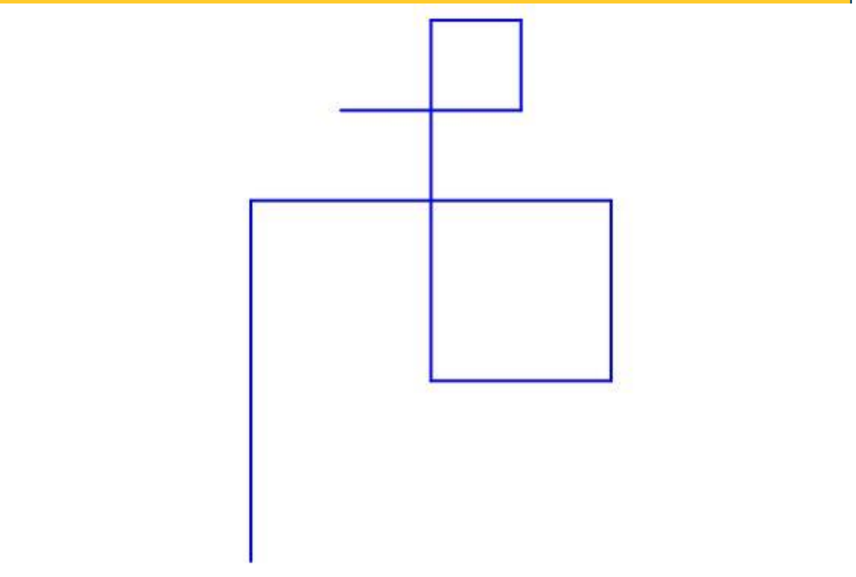
- With simple commands to Turtle like forward, left, right we can create many interesting patterns.

TURTLE COMMANDS

Left(angle)	• Turn Left
Right(angle)	• Turn right
Forward(length)	• Draw a line
PenUp ()	• Stop Drawing
PenDown()	• Start Drawing
Push()	• Save
Pop()	• Go back last saved Position

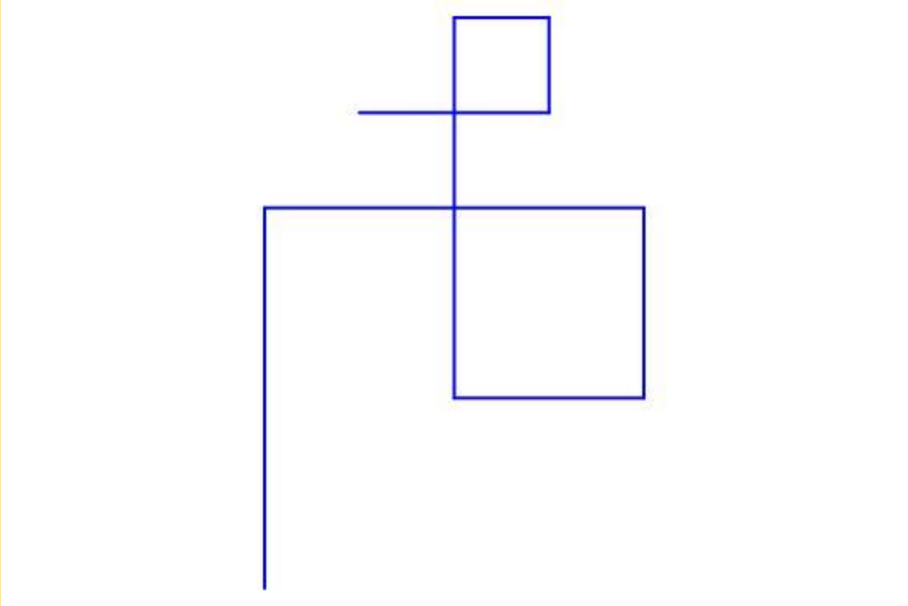
Example of Turtle Commands in Mupad

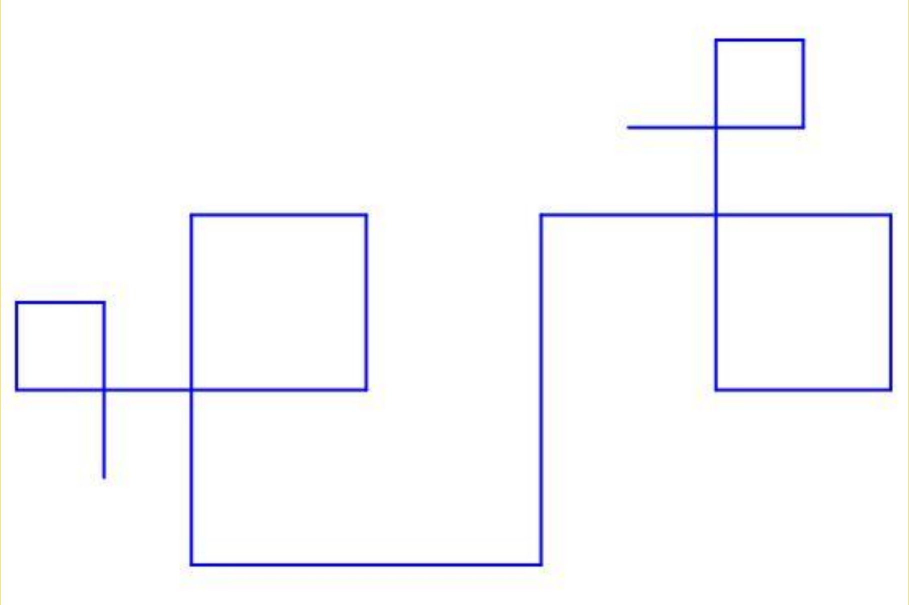

```
t:=plot::Turtle():t::forward(100):
t::right(PI/2):t::forward(100):
t::right(PI/2):t::forward(50):
t::right(PI/2):t::forward(50):
t::right(PI/2):t::forward(100):
t::right(PI/2):t::forward(25):
t::right(PI/2):t::forward(25):
t::right(PI/2):t::forward(50):
plot(t)
```



Rotate Turtle Graphics

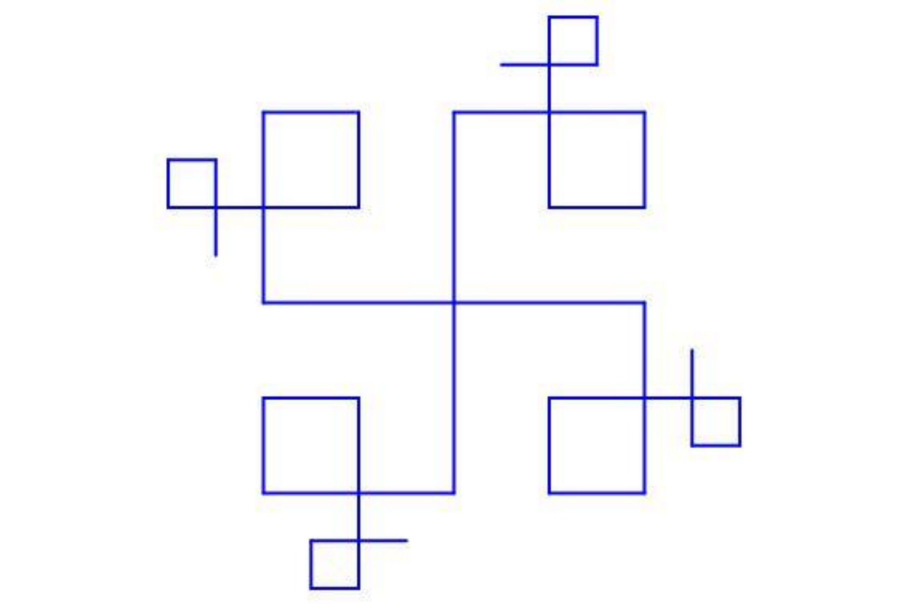
- t2:=plot::Rotate2d(PI/2,[0,0],t)





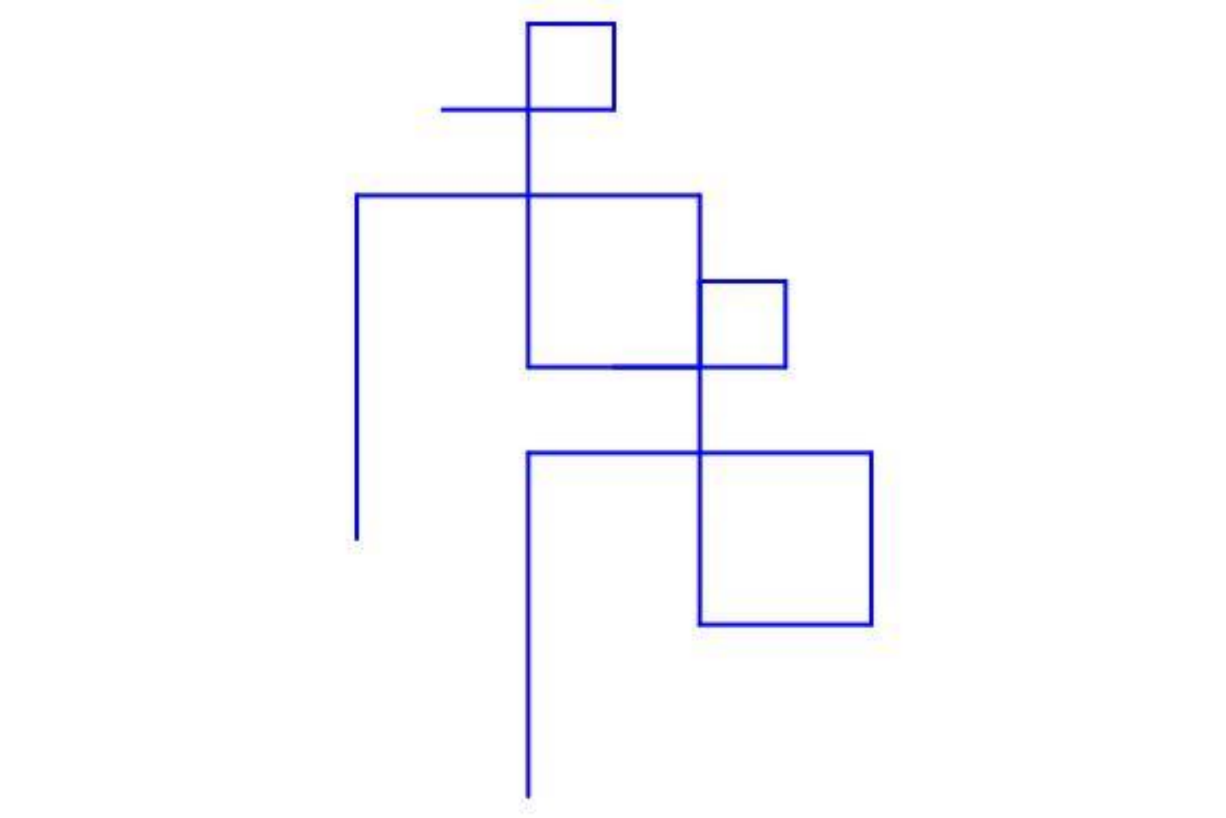
Create a Pattern

- T2:= Plot :: rotate 2d (PI/2), (0,0), t):
- T3:= Plot:: rotate 2d (PI,(0,0),t)
- T4:= Plot:: rotate 2d (3 *PI/2,(0,0),t)
- Plot (t, t2,td3,t4)



Translate Turtle

- a1:=plot::Translate2d([50,-75],t)



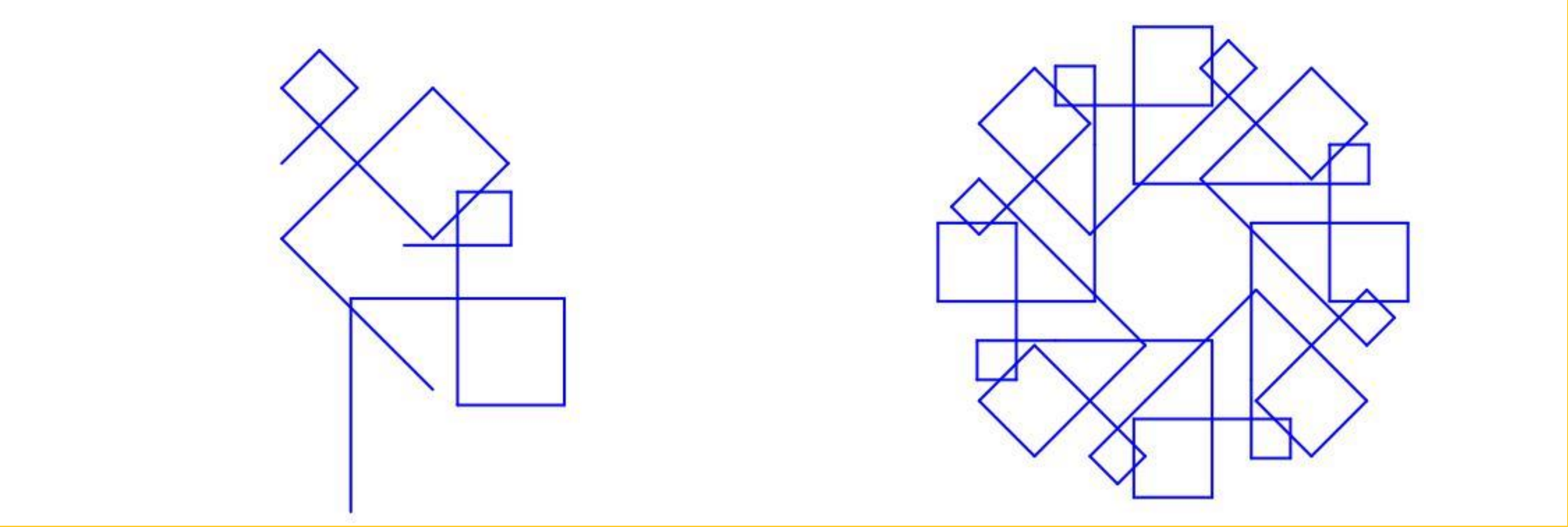
Turtle

a2:=plot::Rotate2d(PI/4,[0,0],a1)

a3:=plot::Rotate2d(2*PI/4,[0,0],a1)

a6:=plot::Rotate2d(5*PI/4,[0,0],a1):

a7:=plot::Rotate2d(6*PI/4,[0,0],a1):



Lindenmayer systems: L-systems

- It is seen that with Turtle graphics very good pattern can be developed but programming is little bit cumbersome.
- Lindenmayer systems is an comparatively easy way to create fractals.
- It was developed in 1960 by the Biologist Dr Aristid Lindenmayer to describe the branching structure of plants and similar objects.
- Structures are generated by repeatedly applying replacement rules or productions to the elements of a defined alphabet, starting with an initial word or axiom.

Lindenmayer systems: L-systems

- Let we have a line of length one. We divide it in three equal parts, attach an equilateral triangle to its central part, pointing to the right with a side length of 1/3 of the segment. Finally, we will remove the base of the triangle. This will create 2^{nd} stage, now by applying the same process, to the segments of the resulting figure, we get subsequent stages. This process can be continued as many times as possible. This can be done very easily by the Mupad turtle graphics.

L-sys

- This rules, applied simultaneously to all characters, generate relatively complicated words after few iterations of the process.
- The characters of the alphabet are generally interpreted as geometric features.
- These are usually related to turtle graphics commands used in the language LOGO.
- In this, the turtle is controlled by simple commands to move around the screen.

Characters of L-sys

- Typical characters of Lsys and their turtle interpretations are-
  1. F – Forward drawing step
  2. f - Forward one step without drawing
  3. + - A left turn at an given angle
  4. - - A right turn at given angle
  5. [- Initiation of branching
  6.] – Termination of branching

Example of Lsys Command

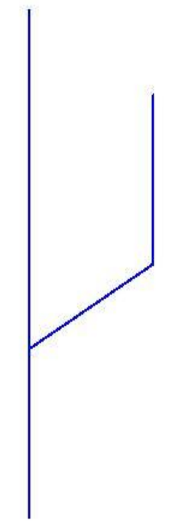
- Forward-Branch-Right-Forward-Left-Forward-Terminate-Forward-Forward

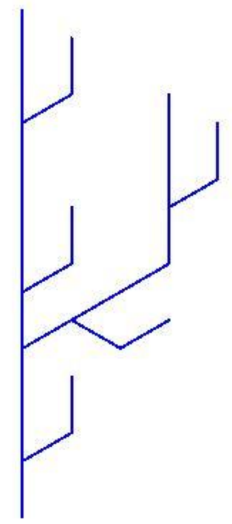
- F ->F[-F+F]FF

- lsys:=plot::Lsys(PI/3, "F", "F"="F[-F+F]FF", Generations=0):
  plot(lsys)

Example of 4 Generations of branching

F ->F[-F+F]FF





Generation = 0: F
Generation = 1: F[-F+F]FF
Generations=2: F[-F+F]FF [-F[-F+F]FF +F[-F+F]FF] F[-F+F]FF F[-F+F]FF
1F-5F-25F-125F-625F.......

L-sys: Stages for the development:

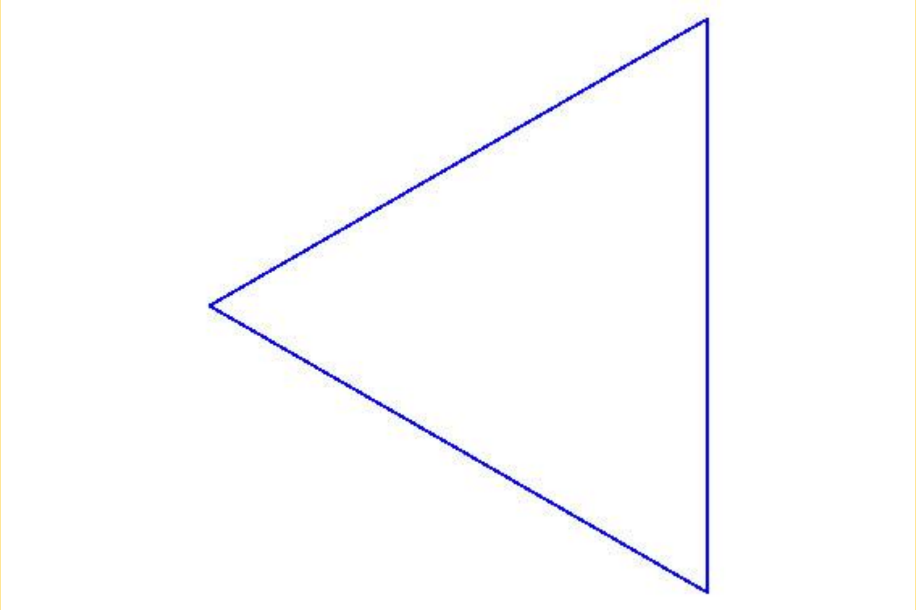
- Seed: Seed is the staring figure .
- Iteration rule: the rule for creating new figures called iteration rule.
- Orbit: the sequence of figures obtained will be called an orbit.
- Fractal: the final result is called a fractal.
- Generations: obtained figures in each sequence are called generations.

Lsys

- The Lsystems are implemented with the use of turtle graphics and we must supply the turtle with the information about the seed, the iteration rule and generations that should be produced.
- Code for 4th generation Koch curve.
- Kochcurve:= plot: : L sys (// stars a new L = Sys
- PI/3, // turtle always turn PI/3
- " F ++ F ++ F" // this is seed
- "F"="F-F++F-F",// Iteration Rule
- Generations = 4// number of generations,:
- Plot (Kochcurve) // now plot the path

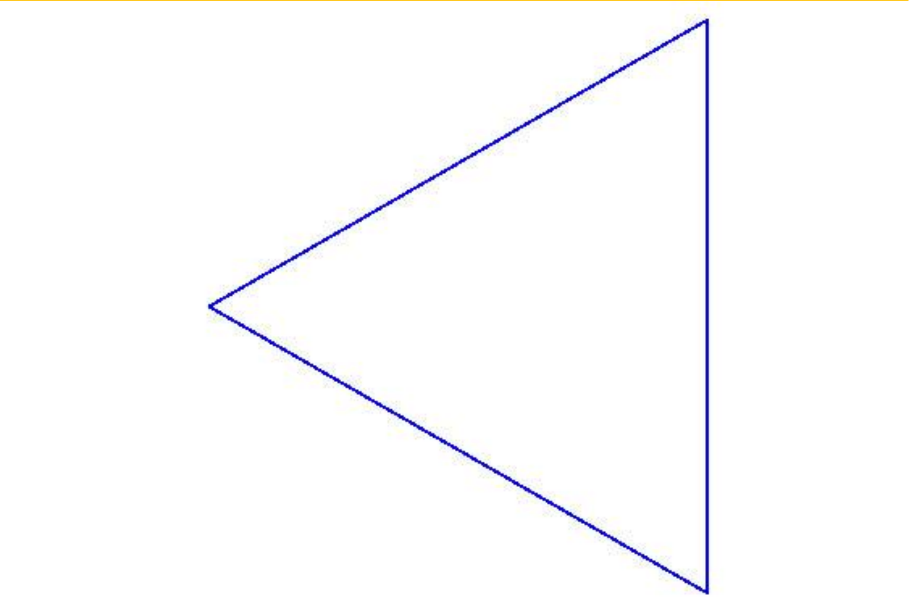
Fractal

- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=0)
- plot(kochcurve)

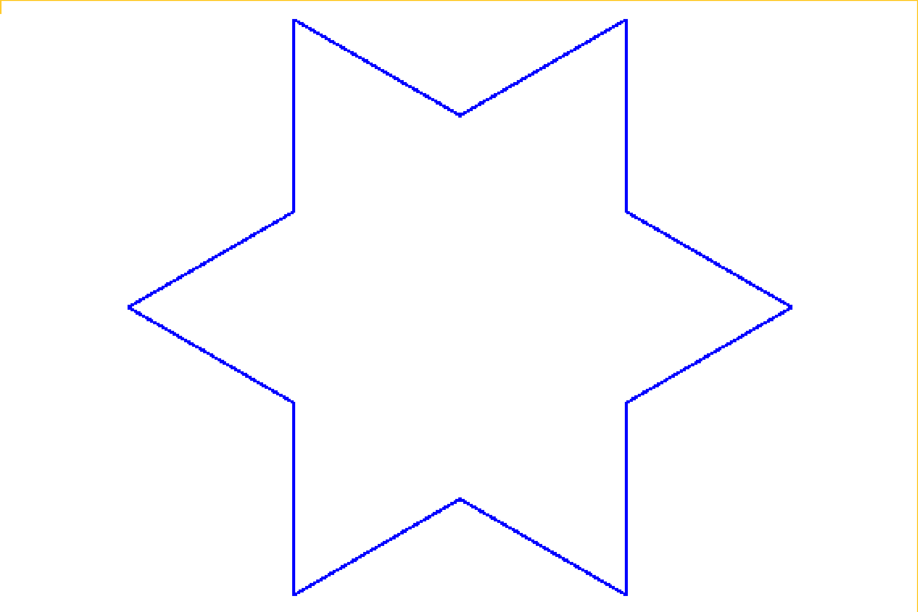


Fractal

- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=1)
- plot(kochcurve)

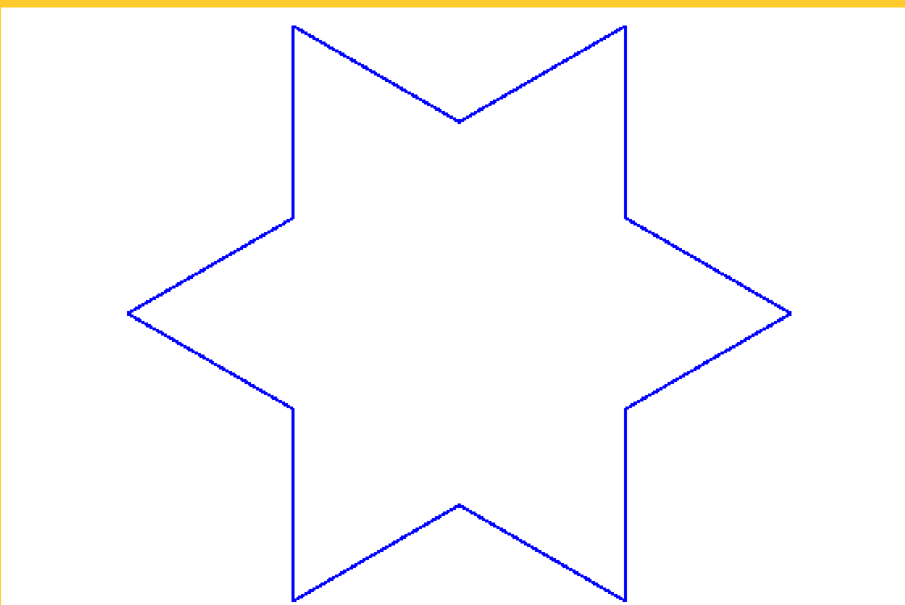


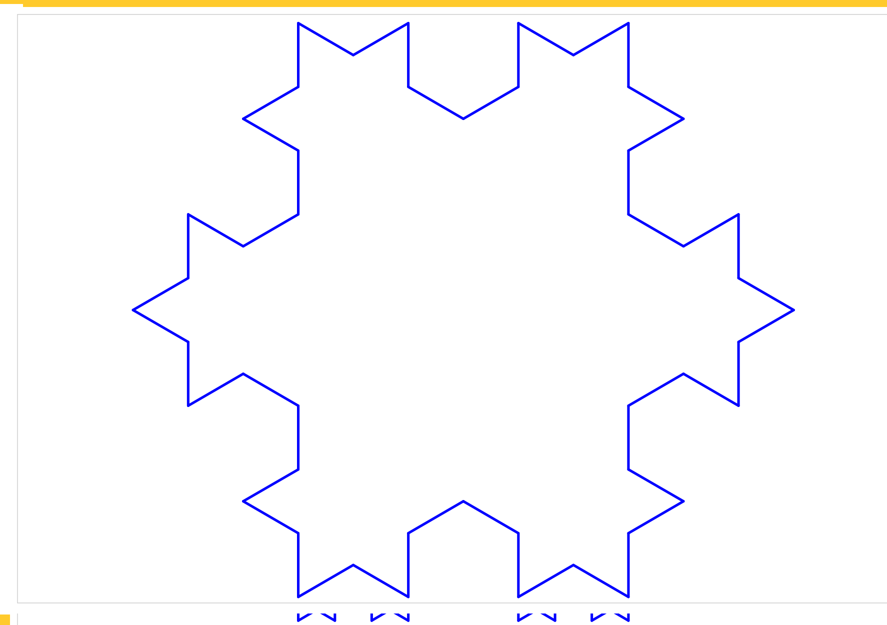
Generations-1



Fractal

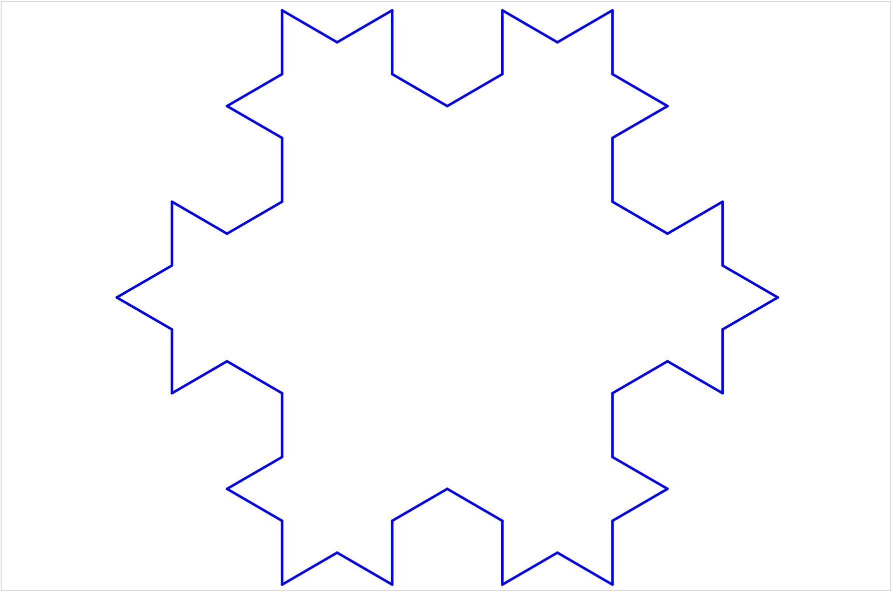
- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=2)
- plot(kochcurve)

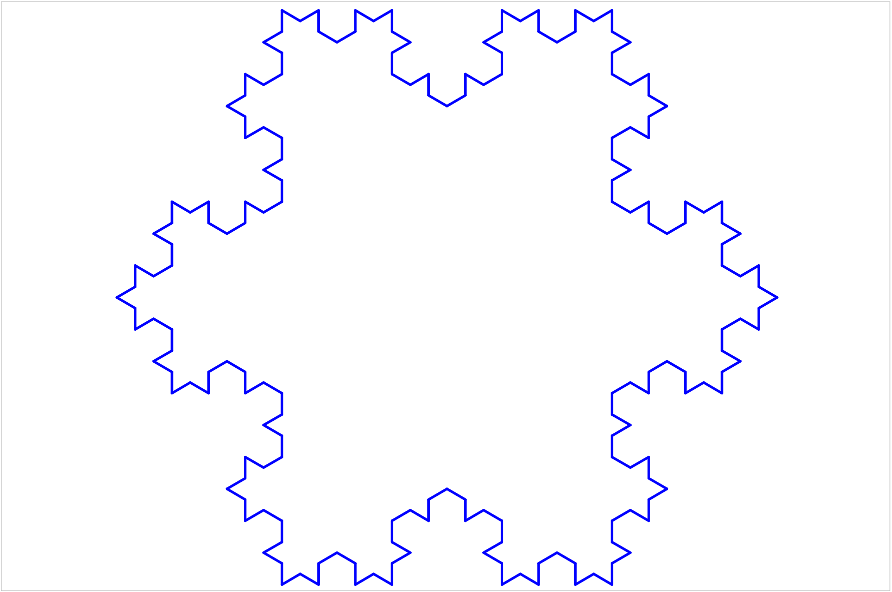




Fractal

- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=3)
- plot(kochcurve)





The meaning of the symbol used for L–sys:

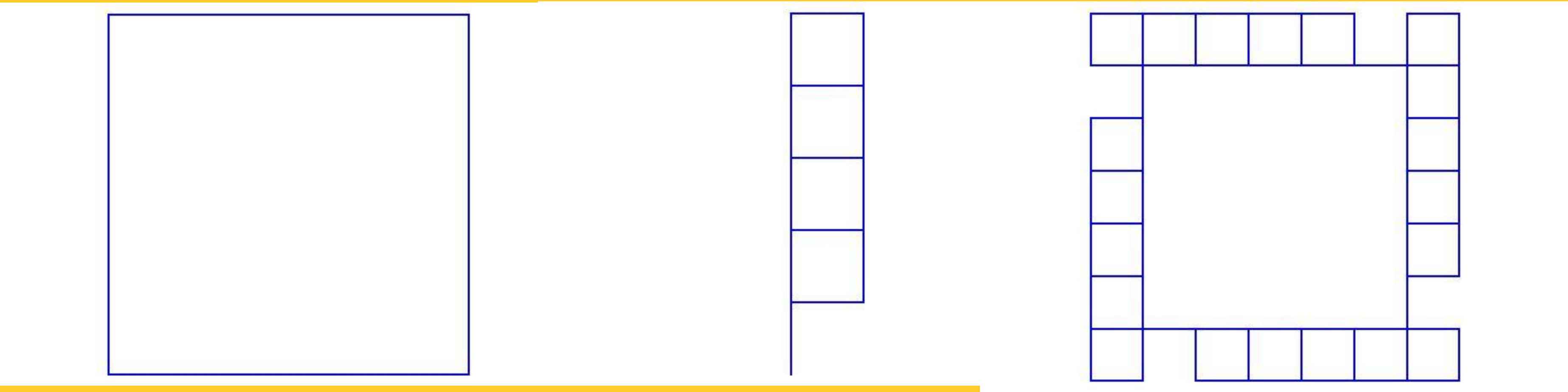
- <u>Command:</u>
- Koch:= plot : : L sys

(P1/3, "F++ F++F", "F" = "F-F++F- F", Generations=4)
Angle Seed Iteration Rule No of generations

1. F means a single segment of length- 1
2. + means turn left ,
3. – means turn right,
4. "F"= Iteration Rule
5. "F – F++F-F" -> Each segment F is replaced by the turtle path F-F++F-F
6. Generations=5 Defines no of iterations
7. f means go forward without drawing a line,
8. [save the current position (branching symbol
9.] go back to the last saved position (Branching symbol)

Example-2

- [Koch1:= plot :: Lsys (PI/2) “F-F-F-F”,
- “F”=” FF –F-F-F”, Generations =2):
- plot (Koch1)



Cantor Set

- Cantor set is another set of fractals
- The Cantor set is created by repeatedly taking out the middle third of all the line segment involved.
- All the points are distinct
- This is impossible to show in an image.
- All points are fused after fifth subdivision

Length of the Cantor Set

- Let the length of the original line be one unit.
- One third of the length is taken out in first stage
- Two lengths of one ninth taken out in second stage
- Four lengths of one twenty seventh taken out in third stage
- And the process goes on…

Length of the Cantor Set

- Total length eliminated is-

- $L=(1/3)+2(1/3)^2+2^2(1/3)^3+2^3(1/3)^4+.....$
- $L=(1/3)\{(1+2(1/3)+2^2(1/3)^2+2^3(1/3)^3+......$
- $L=(1/3)\{1+(2/3)+(2/3)^2+(2/3)^3+(2/3)^4......$
- This is a geometric series with successive terms multiplied by 2/3 and 2/3 is less than one
- Multiplying both side by 2/3, we get-
- $(2/3)L=(1/3)\{(2/3)+(2/3)^2+(2/3)^3+(2/3)^4......$

Length of the Cantor Set

- $(2/3)L=(1/3)\{(2/3)+(2/3)^2+(2/3)^3+(2/3)^4......$
- Previous equation-
  $L=(1/3)\{1+(2/3)+(2/3)^2+(2/3)^3+(2/3)^4......$
- To eliminate the tail (remaining term), we subtract – the two equations to get.

  $L-(2/3)L=(1/3)$

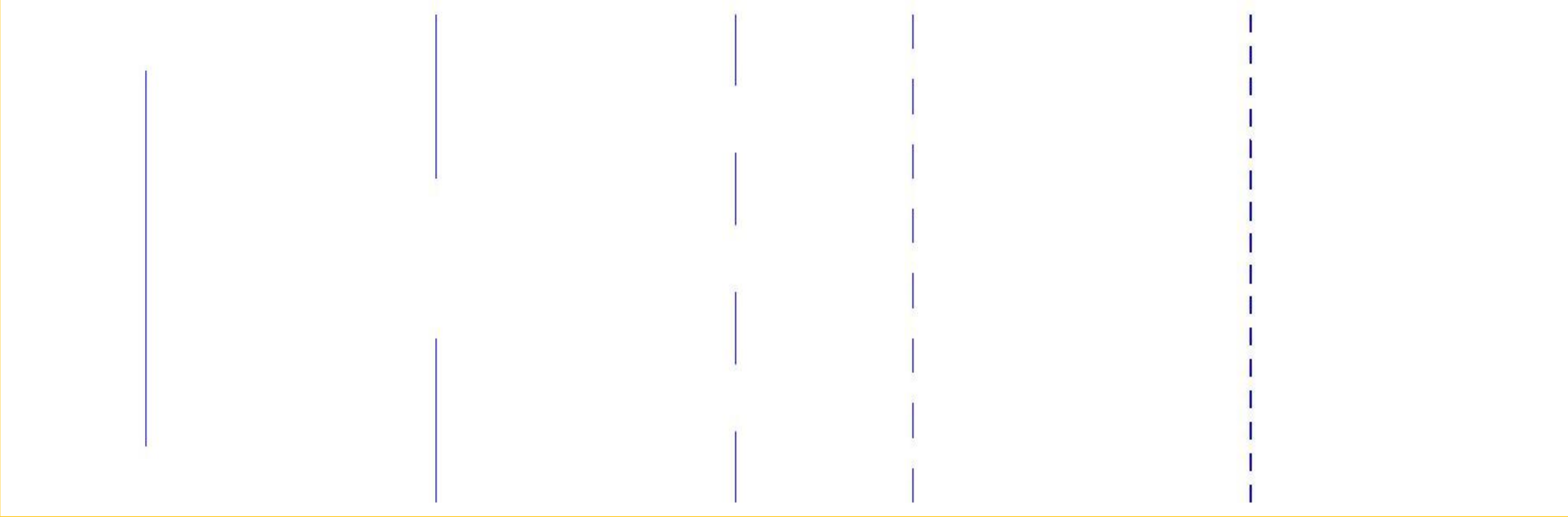
  Or $1/3\ L=1/3$

  Or $L=1$

  - Whole the length has been extracted stills leaves the points in the line.
  - The process creates a pattern with two similar copies at $1/3^{rd}$ Scale

Creating Cantor Sets

- Mupad Lsys gives very good way to create Cantor Curve using Lsys:
- [cantor0:=plot::Lsys(0,"FfF","F"="FfF",Generations =0): plot(cantor0)



The Sierpinski Triangle or Gasket

- A sierpinsky triangle is formed by recursively extracting triangular forms from within an original triangle

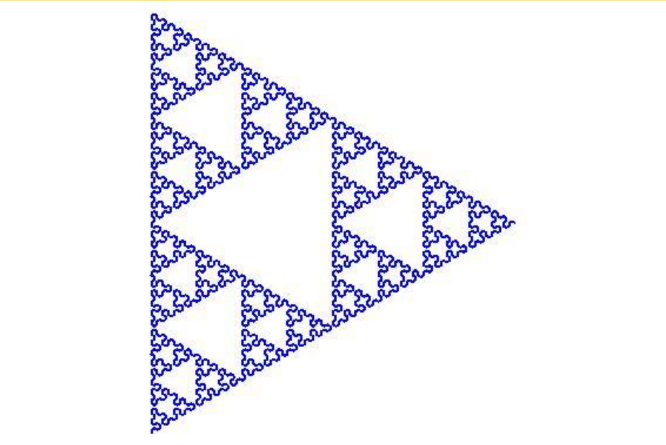
- At each stage three times as many triangles are extracted.
- Each being a quarter of the area of those used in the previous stage.

The Sierpinski Triangle or Gasket

- The total area extracted is equal to the area of the original triangle still there remains many points.
- The resulting fractal object is contains three copies of itself at half linear scale.

Construction of The Sierpinski Triangle

- The total area extracted is equal to the area of the original triangle still there remains many points.
- The resulting fractal object contains three copies of itself at half linear scale. Lsys can be used to draw the curve
- l := plot::Lsys(PI/3, "R", "L" = "R+L+R", "R" = "L-R-L",
  "L" = Line, "R" = Line,
  Generations = 7):
  plot(l)



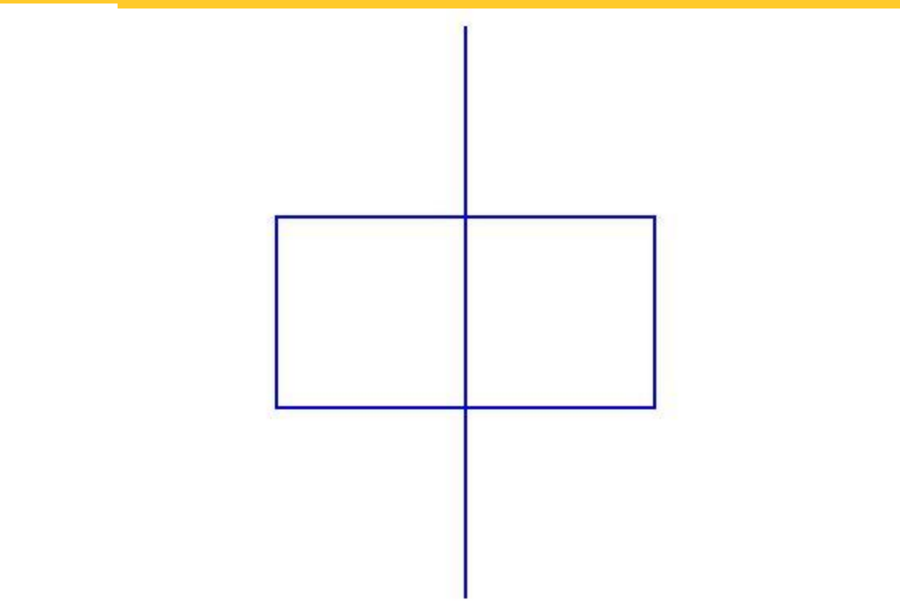
Peano Curve

- This fractal was created by Giuseppe Peano in 1890
- Peano Curves are called "Space Filling Curve"
- The peano curve visits every point within a two dimensional region

- Here also Lsys can be used to draw Peano Curve.
- The first three stage and fifth stage are shown below:

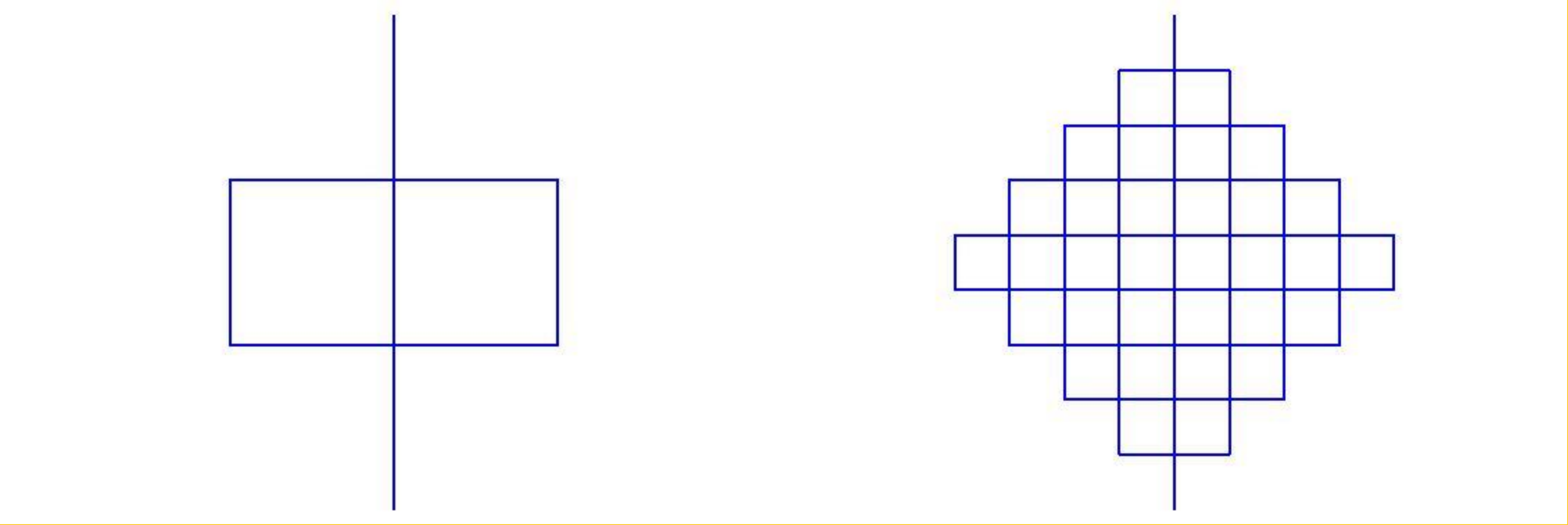
Peano Curve – Generations 0/1

- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"), peano::Generations := 0: plot(peano0)



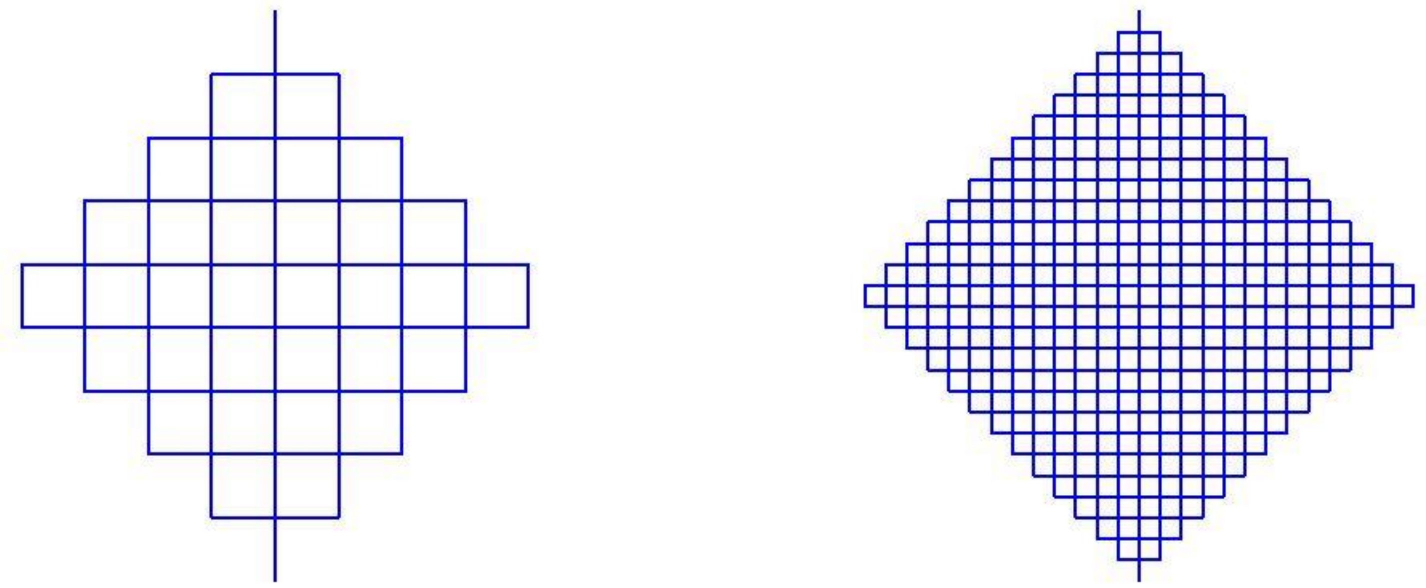
Peano Curve – Generation – 1 / 2

- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"),
  peano::Generations := 0:
  plot(peano0)



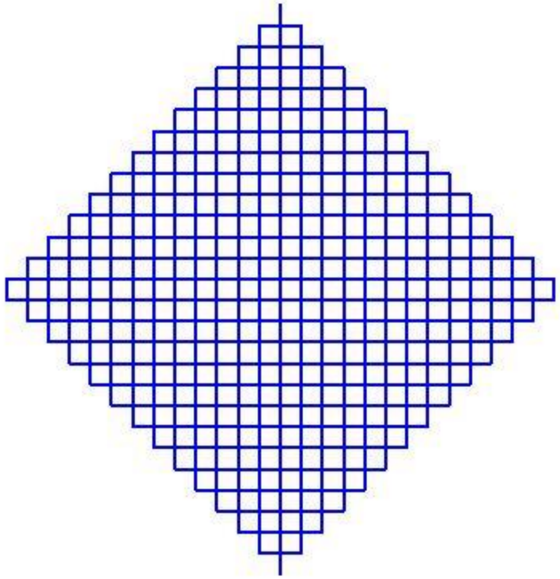
Peano Curve– Generation – 2/3

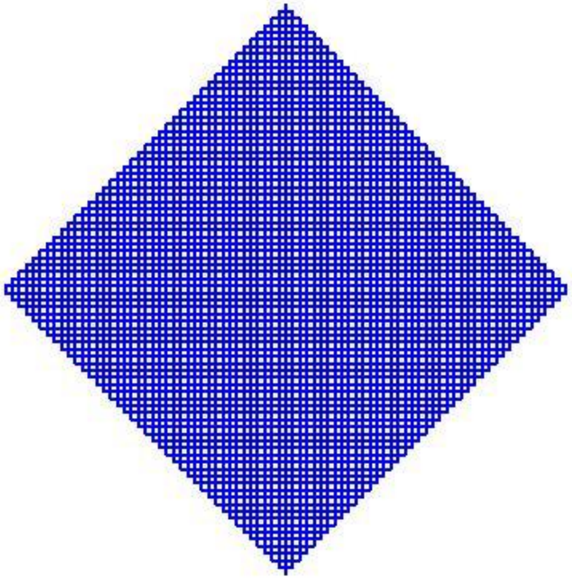
- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"), peano::Generations := 0: plot(peano0)



Peano Curve– Generation – 3/4

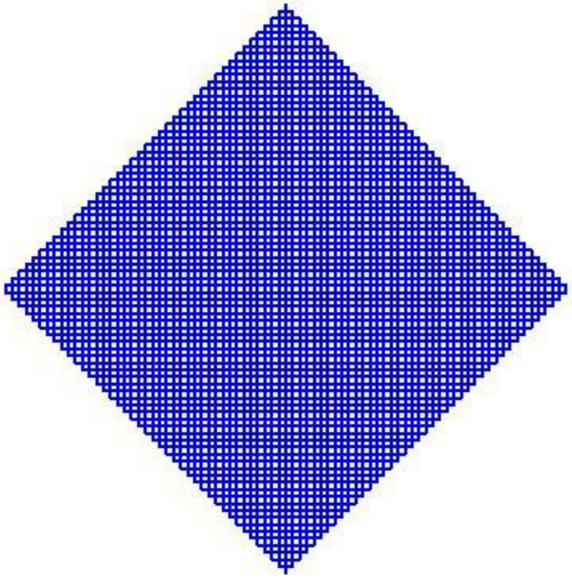
- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"),
  peano::Generations := 0:
  plot(peano0)

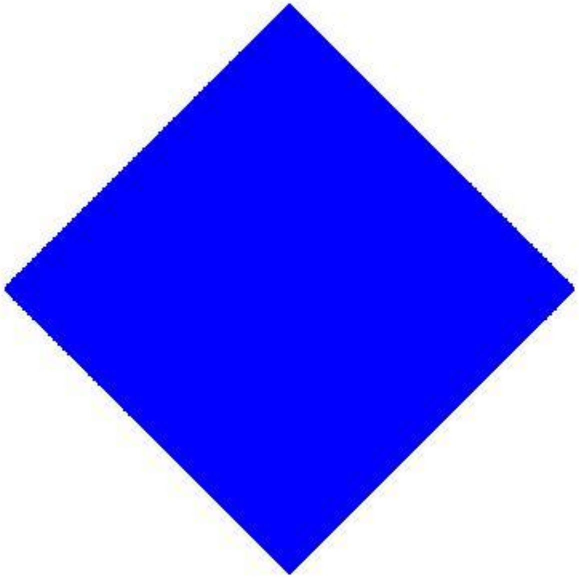




Peano Curve– Generation – 4/5

- peano0 := plot::Lsys(PI/2, "F",
 "F" = "F+F-F-F-F+F+F+F-F"),
peano::Generations := 0:
plot(peano0)





Fractals and it's Dimensions

- When the space filling curve repeated several times, it fills the region.
- In it's construction, a replacement method is used.
- Every time, each line recursively replaced by nine other line segments at one third scale.
- This generates an object that contains nine copies of itself at one third scale.

Fractals and it’s Dimensions

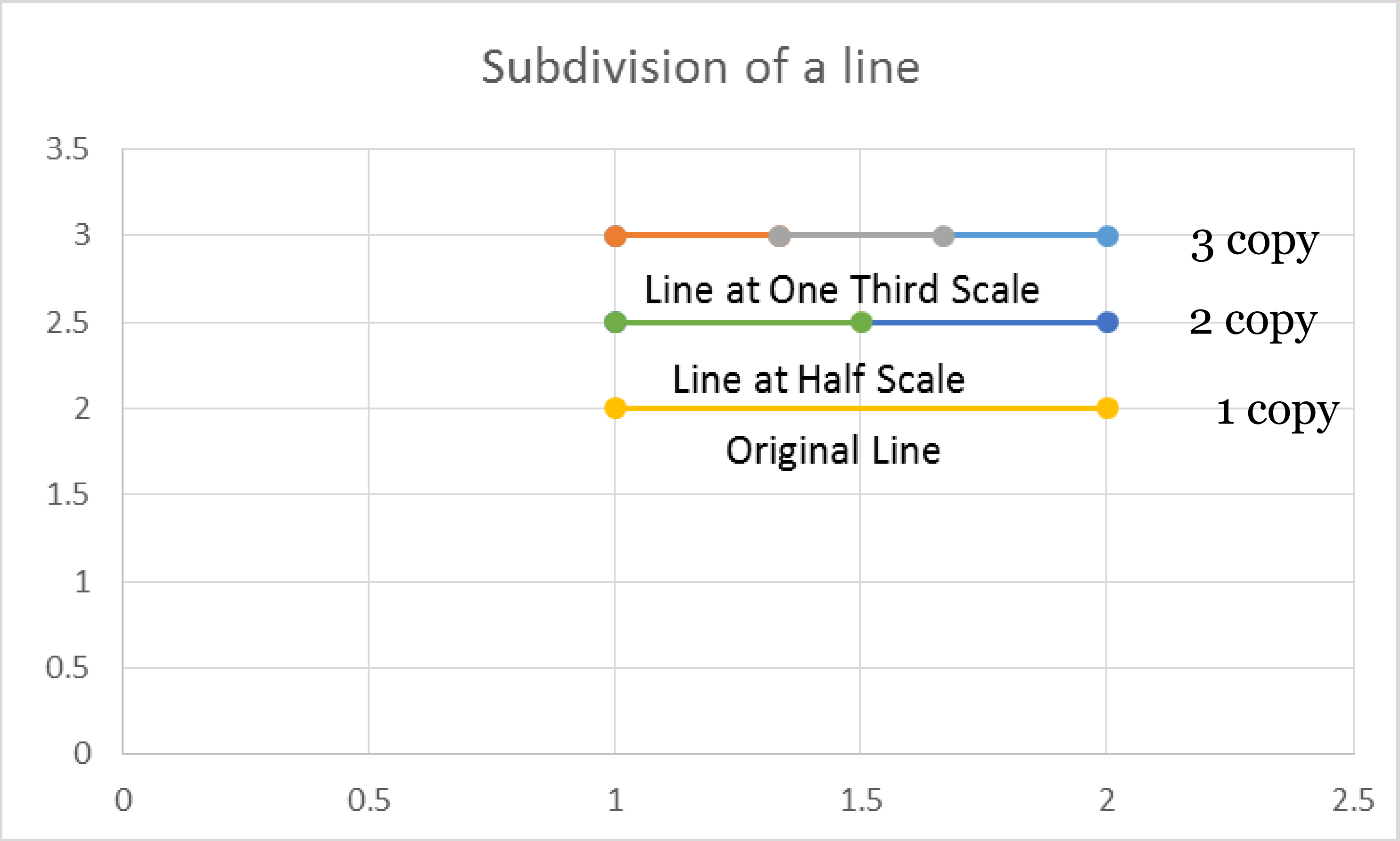
- We started with one dimensional object, whose length approaches infinity as the process continues, and end with an object that fills a region of two dimensional space.

- Now the question comes – What is the dimension of these objects?

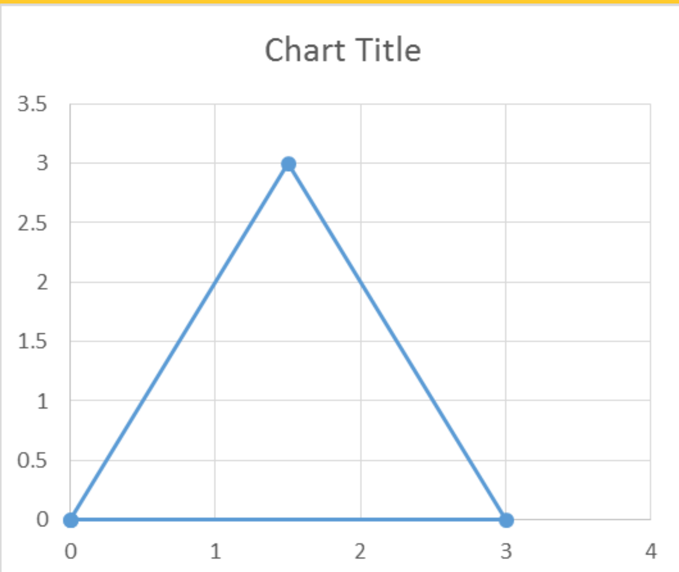
What is Dimension?

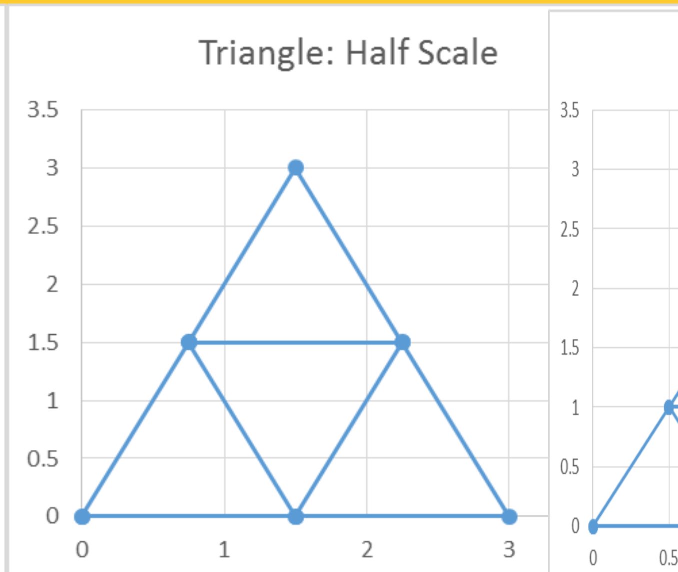
- Point – 0 Dimension
- Line – 1 Dimension – ½ scale – 2 copy, 1/3rd scale – 3 copy
- Triangle – 2 Dimension -1/2 scale, 4 copy, 1/3rd scale – 9 copy
- Square – 2 Dimension – ½ scale, 4 copy, 1/3rd scale – 9 copy
- Cube – 3 Dimension -1/2 scale, 8 copy, 1/3rd scale – 27 copy

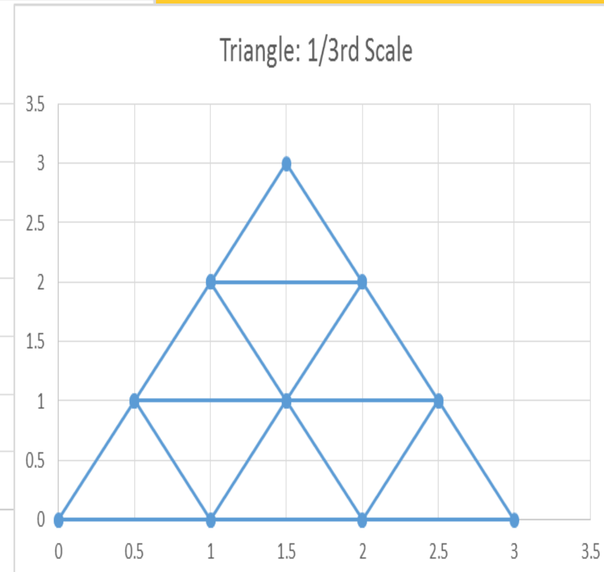
Fractals and it’s Dimensions



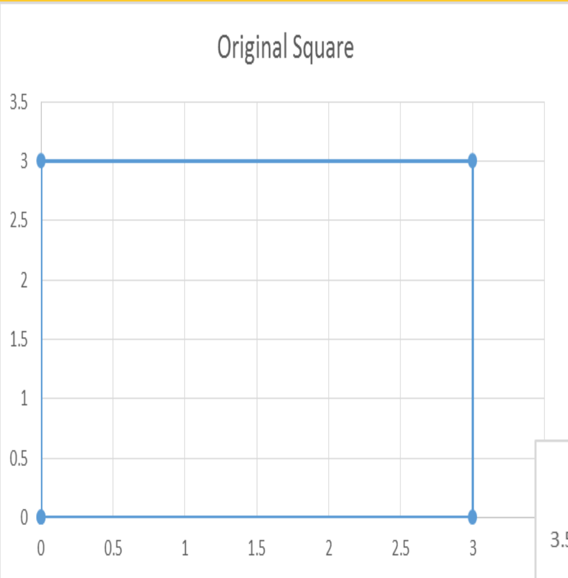
Fractals and it's Dimensions

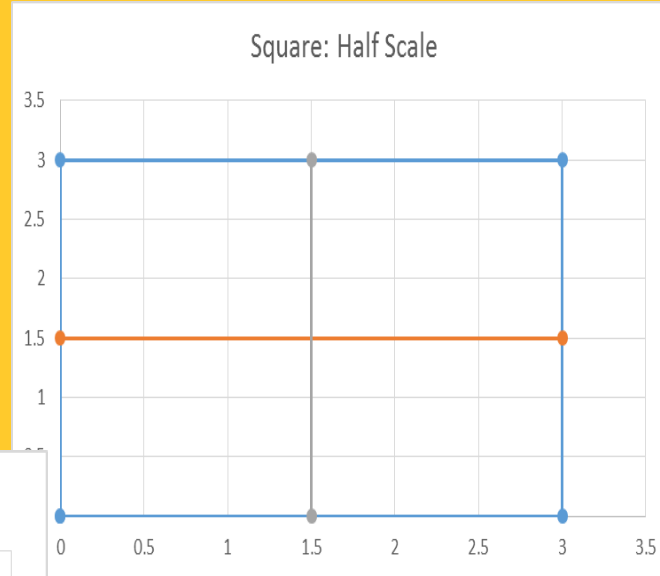


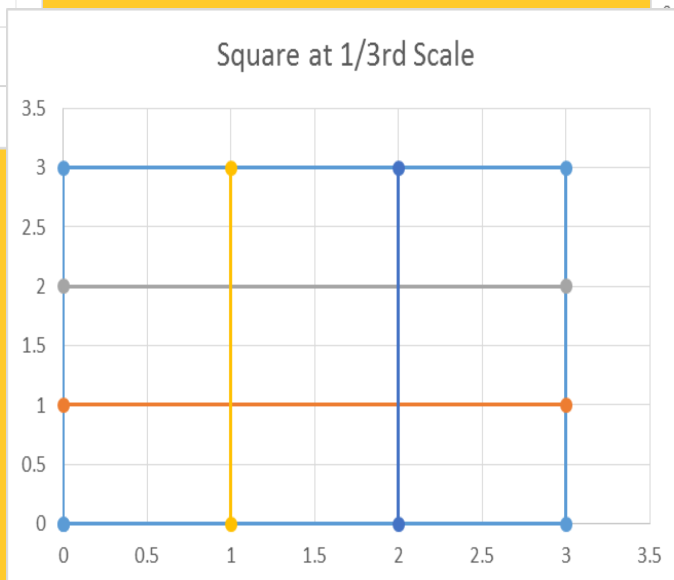




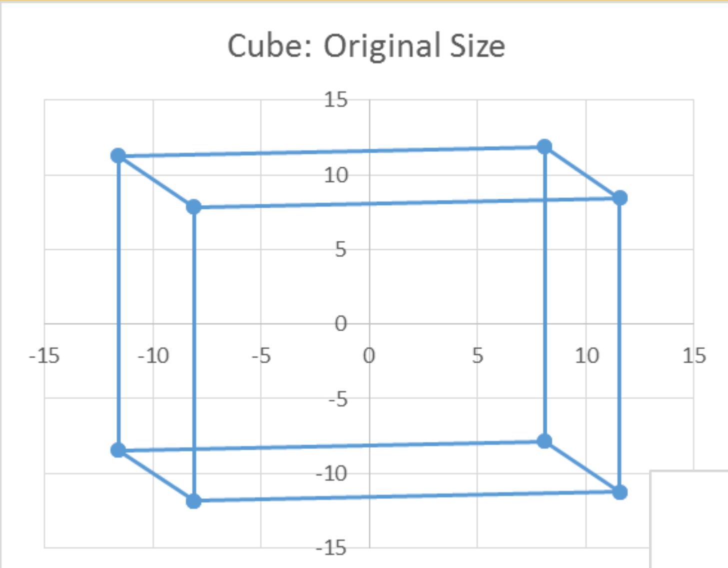
Fractals and it's Dimensions

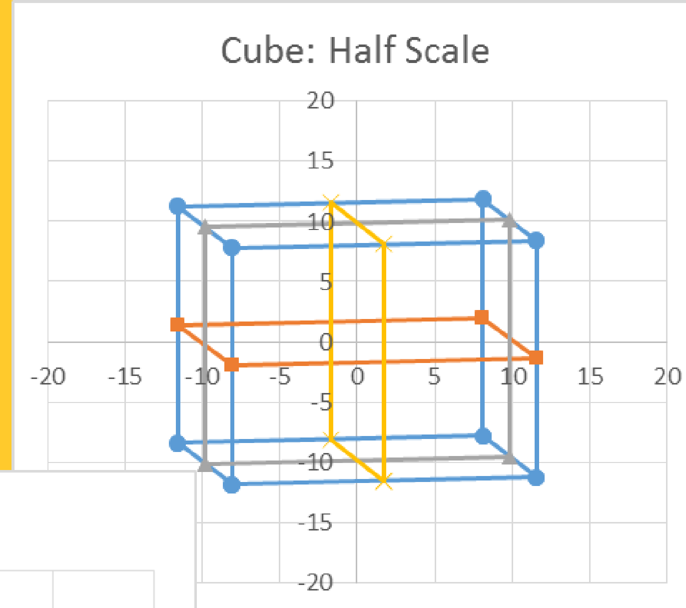


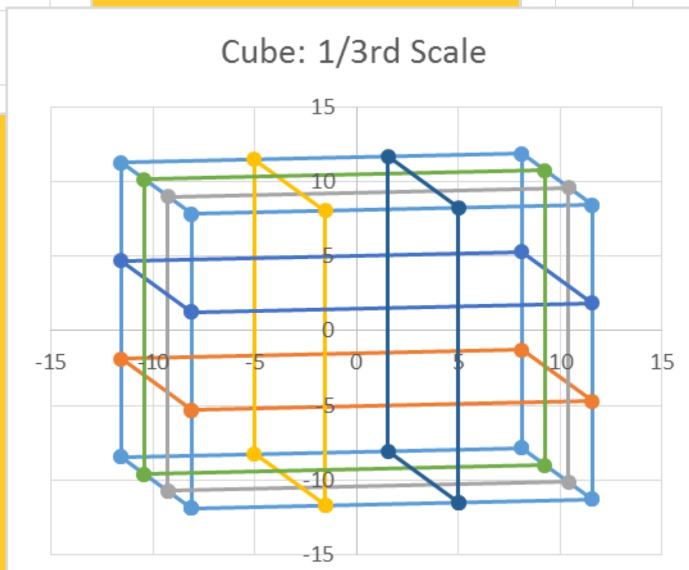




Fractals and it’s Dimensions







Fractals and it's Dimensions

- Line is an one dimensional object
- Line contains 2 copies at half scale
- It contains 3 copies at one third Scale
- It contains 4 copies at one forth scale

Triangle and it's Dimensions

- Triangle is a two dimensional object

- Triangle contains four copies at half scale
- It contains nine copies at one third scale

Square and it’s Dimensions

- Square is a two dimensional object
- Square contains four copies at half scale
- It contains nine copies at one third scale

Cube and it's Dimensions

- Cube is a three dimensional object
- Cube contains eight copies at half scale
- It contains 27 copies at one third scale

Fractals and it's Dimensions

- It is seen that the number of copies, N, the scale factor, f, and the dimension, D, of the object are related.
- The relation among the three parameters are given by-
- $N=(1/f)^D$
- Taking log in both side, we get
- $\log(N)=D*\log(1/f)$
- $D=\log(N)/\log(1/f)$

Fractal Dimension

Sl No	Fractal	N	F=1/f	D=Log(N)/log(F)	
1	Cantor Set	2	3	0.63	
2	Koch Curve	4	3	1.26	
3	Sierpinski Gasket	3	2	1.58	
4	Peano Curve	9	3	2	
5	Sierpinski Tetrahedron	4	2	2	

Fractal Dimension

- Cantor set is more than a mere point, D=0.63
- Koch set is more than a curve, D=1.26
- Sierpinski set is less than a triangle, D=1.58
- Peano curve start as curve fills a plane, D=2

Fractals and Naturally occurring objects

Naturally Occurring objects:

- Clouds
- Coast Line
- Fire
- Terrain
- Mountains
- Forests
- Water Falls
- Waves
- Galaxies

Fractals and Naturally occurring objects

- Sections of these objects are not exact copies of the whole but their general features are, on the whole, indistinguishable from the over all form.

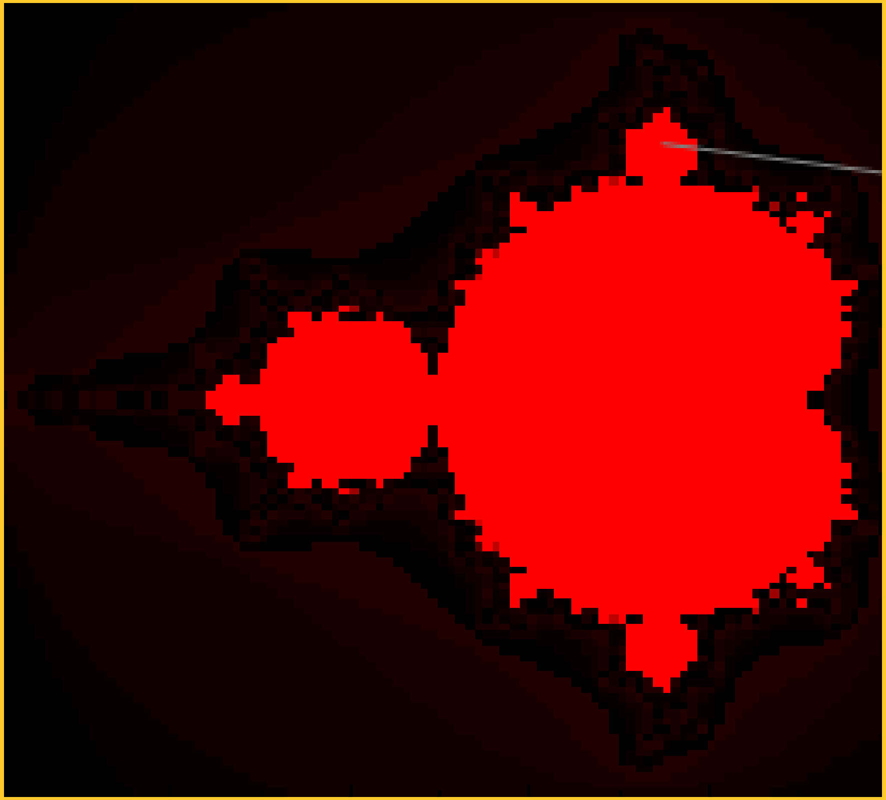
- There is an absence of scale about such fractals
- The fractal property of sub divisibility is hold up because it end up with a single sand grain.
- But if we consider a reasonable lengths, the fractal properties of self similarity and sub divisibility are maintained.

Fractals from Functions

- There are fractals which are created from repeated application of mathematical formulas
- These fractals are Julia set, Mandelbrot set etc.

Julia and Mandelbrot Set

- Here is a picture of Mandelbrot Fractal



Julia and Mandelbrot Set

- These fractals are based in complex plane
- A complex number can be written as z=x+iy
- The main engine is a loop of instructions that takes its starting complex number and applies the arithmetic rules to it.
- For a Mandelbrot set, the rule is
  - z=z^2+c
- Here z begins with zero and c is a complex number corresponds to the point to be tested

Julia and Mandelbrot Set

- The loop continues like –
  - Take 0, multiply it by it self and add the starting number, c
  - Take the result as starting number, multiply it by itself and add the starting number, c.
  - Continue the cycle
- To break the loop, the loop needs to watch the running total.
- If the total heads to infinity, moving further and further from the center, the original point does not belong to the set.

Julia and Mandelbrot Set

- If the running total becomes greater than 2 or smaller than -2, it is surely heading off to infinity.

- If the program repeats many times without becoming greater than 2, then the point is part of the set.
- How many times depends on the amount of magnifications. It can be 100, 200 or any number even 1000.

Julia and Mandelbrot Set

- The program must repeat this process for each of thousand of points on a grid, with a scale that can be adjusted for greater magnification.
- Each of the point inside and outside of the set are colored differently.
- The colors reveal the contours of the terrain of the fractal set.

Mathematics of Mandelbrot Set

- Any complex number can be represented as x+i y
- In polar form it is represented as r (cos (t)+i sin (t))
- Any complex number has two properties, magnitude or absolute value or length, r= sqrt(x^2+y^2) and angle or amplitude t = atan(y/x)
- When a complex number is squared, z^2, it's absolute value gets squared and amplitude gets doubled.

Mathematics of Mandelbrot Set

- We know, for a Mandelbrot set, the rule is
    - $z=z^2+c$
- Here z begins with zero and c is a complex number corresponds to the point to be tested.
- As the iteration continues, three cases may arise –
    - The sequence diverge – meaning that the points move further and further from original location
    - It may converge on a single fixed location, or,
    - It may remain in a cycle fairly close to the original point

Three Cases of Iteration Results

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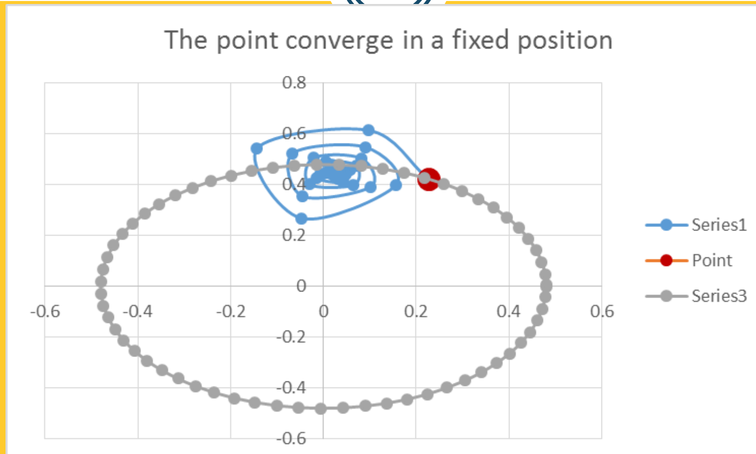

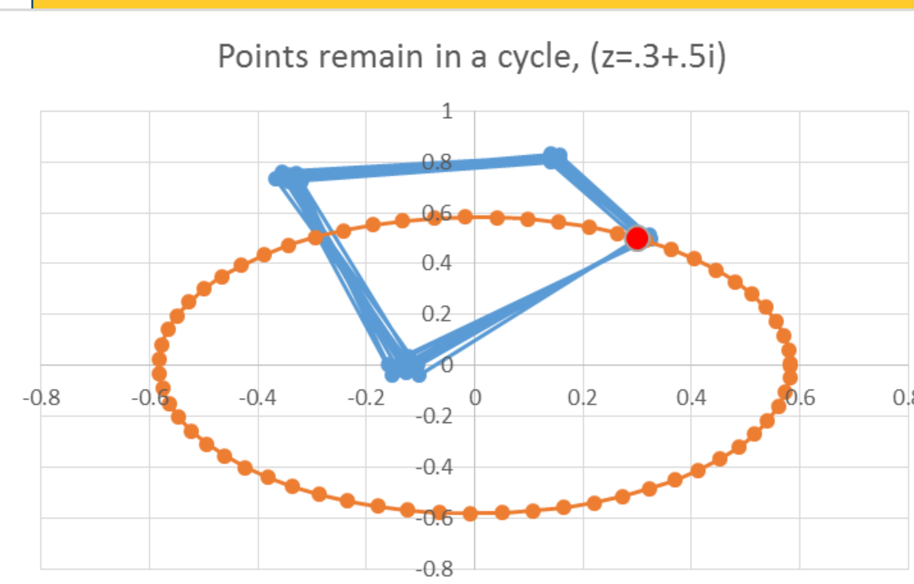

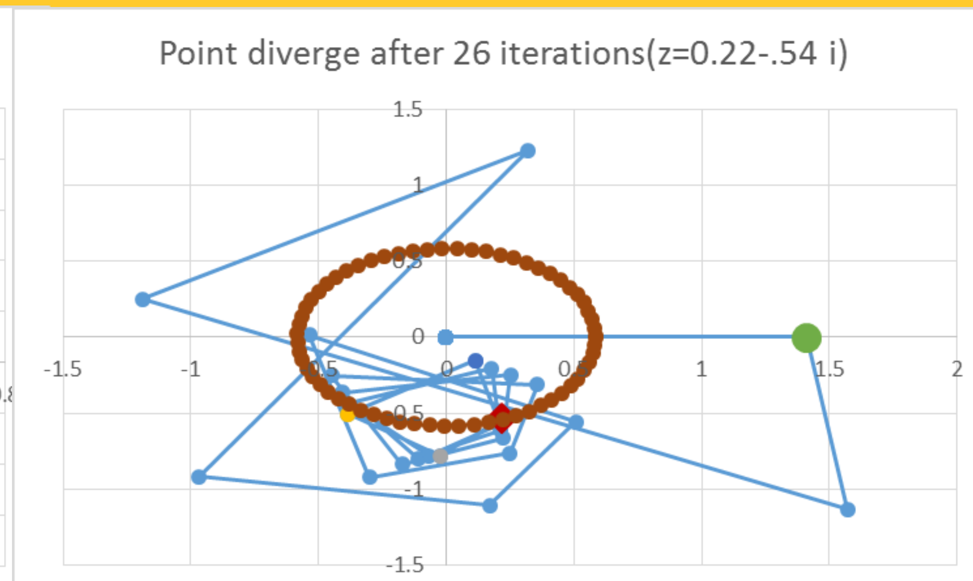

The formation of Mandelbrot Set

Notes for preparing this presentation

1772

14-05-2018

TODAY I AM PREPARING THE PRESENTATION FOR THE TALK AT IIT GANDHINAGAR ON 18-05-2017

17052017 SLIDES 273

IIT Gandhinagar
Indian Institute of Technology Gandhinagar

DASS SRL
DASS Scientific Research Labs Pvt. Ltd.

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Talk on Fractals

By

CHANCHAL DASS, FIE

CHAIRMAN
DASS SCIENTIFIC RESEARCH LABS (P) LTD
EMAIL-CDASS01@GMAIL.COM
MOBILE-9427030155

18TH MAY 2017

IIT Gandhinagar

Indian Institute of Technology Gandhinagar

DASS SRL

DASS Scientific Research Labs Pvt. Ltd.

Welcome Everyone

IIT Gandhinagar

Indian Institute of Technology Gandhinagar

DASS SRL

DASS Scientific Research Labs Pvt. Ltd.

1775

Sincere Thanks To

Prof. (Dr.) Sudhir Jain, Director, IITGN

Dr Indranath Sengupta, Associate Professor, IITGN

CHANCHAL DASS, FIE

The Journey

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Making Mathematics Popular

Main and Single Objective

Make Advanced Mathematical Concepts Simple and Promote These Simplified Mathematical Concepts Globally

Other Objective

Take participants to the

“Path of Self Discovery and Self Learning”

Dass Scientific Research Labs Private Limited

7th June 2012

DASS SRL

DASS Scientific Research Labs Pvt. Ltd.

eInfochips Training and Research Academy
07th December 2013

Crafting Preferred Engineers

National Institute of Oceanography

समुचित ज्ञान समन्वय

Dhirubhai Ambani
Institute of Information and Communication Technology

National Design Business Incubation(NDBI)

NATIONAL DESIGN
BUSINESS INCUBATOR

NATIONAL INSTITUTE OF DESIGN

Silver Oak College of Engineering &Technology

Venus International College of Technology

SAHAJANAND LASER TECHNOLOGY LIMITED

Dr. Arvind Patel Group

Sahajanand LASER TECHNOLOGY LIMITED

SRI PADMAVATI MAHILA VISVAVIDYALAYAM

Imperial College London

Ganpat University

GANPAT
UNIVERSITY
॥ विद्यया समाजोत्कर्षः ॥

गुजरात युनिवर्सिटी

। योगः कर्मसु कौशलम् ।

Shree Swaminarayan Institute Of Technology

Indian Institute of Technology

IIT Gandhinagar

Indian Institute of Technology Gandhinagar

Theme: Math Boliye: Talk on Fractal

- The idea behind the mathematics talk is to inculcate a culture of speaking about mathematics in common parlance.
- India has enormous contribution towards the advancements of Science, Technology, Engineering and Mathematics.
- The innovative thought of promoting mathematics through mass communication with "Math Boliye" Campaign will add another dimension to the advancement of modern civilization.

Started on 7th June 2014 at GTU

Inaugurated By VC-GTU

1796

About The Speaker- Chanchal Dass, FIE

Inventor
- MZWC Technology, Contract Bridge Gaming App, Math Teaching and Demonstration Technology

Qualifications
- AMIE (Mechanical Engineering), PGD 'Operations Research', MBA (Finance)

Experience
- (35 yrs) 8 Yrs in HFCL, 21 Yrs in ONGC, 6 Yrs in Dass OTPL/ Dass SRL

Skill
- Reservoir Engineering, well testing, Reservoir Simulation, EOR, Sick Well Analysis, Work-over job planning, Chemical Flooding (FPR- Sanand, Jhalora), Teaching Mathematics

Awards & Recognition
- World Oil Award, SPE President Award, SPE Regional Service Award, ONGC Director / Regional Director's Award, IIGP-2011, NASSCOM 10000 Startup Initiative, PALF 2015, CII Innovation Award 2015 and many more

Membership
- Fellow of Institution of Engineers, Life Member of Society of Petroleum Engineers, Society of Petroleum Geophysicist, Indian Mathematical Society, American Mathematics Society, ACBL

Publications
- SPE IOGCE, SPEIOGCEC, Forums, ATW, NATC, IPTC as presenter, Session Chair, Committee Member and many more

Countries Visited
- US(2), France(2), Holland, Belgium, Germany, Luxembourg, Switzerland, Cairo, Qatar, Dubai(2), Sri Lanka, Abu Dhabi, Sharjah, China(3), Malaysia(4), Thailand(2), Singapore, Georgia, Unitel Kindom, Azarbijan

First Requirement

INFORMAL

HEALTHY INTERACTION

FREELY SHARING OF VIEWS

NO KNOWLEDGE JUDGEMENT

LANGUAGE POLICY-COMMUNICATION MATTERS

Topics to be discussed

Definition of Fractals, Properties of Fractals, Concept of divisibility, Concept of Self Similarity, Details of Xaos Program, Display of Fractals at Xaos, Koch Curve, Cantor Set, Length of Koch Curve, Area of Koch Curve, Fractal, Turtle graphics, LOGO software, Mupad,Turtle commands, Examples of Turtle commands, Branching of Plants and Lindenmayer System, L-sys in Mupad and its commands, Examples of Lsys, Generation of Koch Curve with Lsys, Creation of Cantor set, Length of Cantor set, Sierpinski Triangle or Gasket, Construction of Sierpinski Triangle, Peano Curve, Fractals and its dimensions, Defining dimension, Fractals and Naturally occurring objects, Julia and Mandelbrot sets, Mathematics of Mandelbrot sets, Formation of Mandelbrot sets, Fractal Fern.

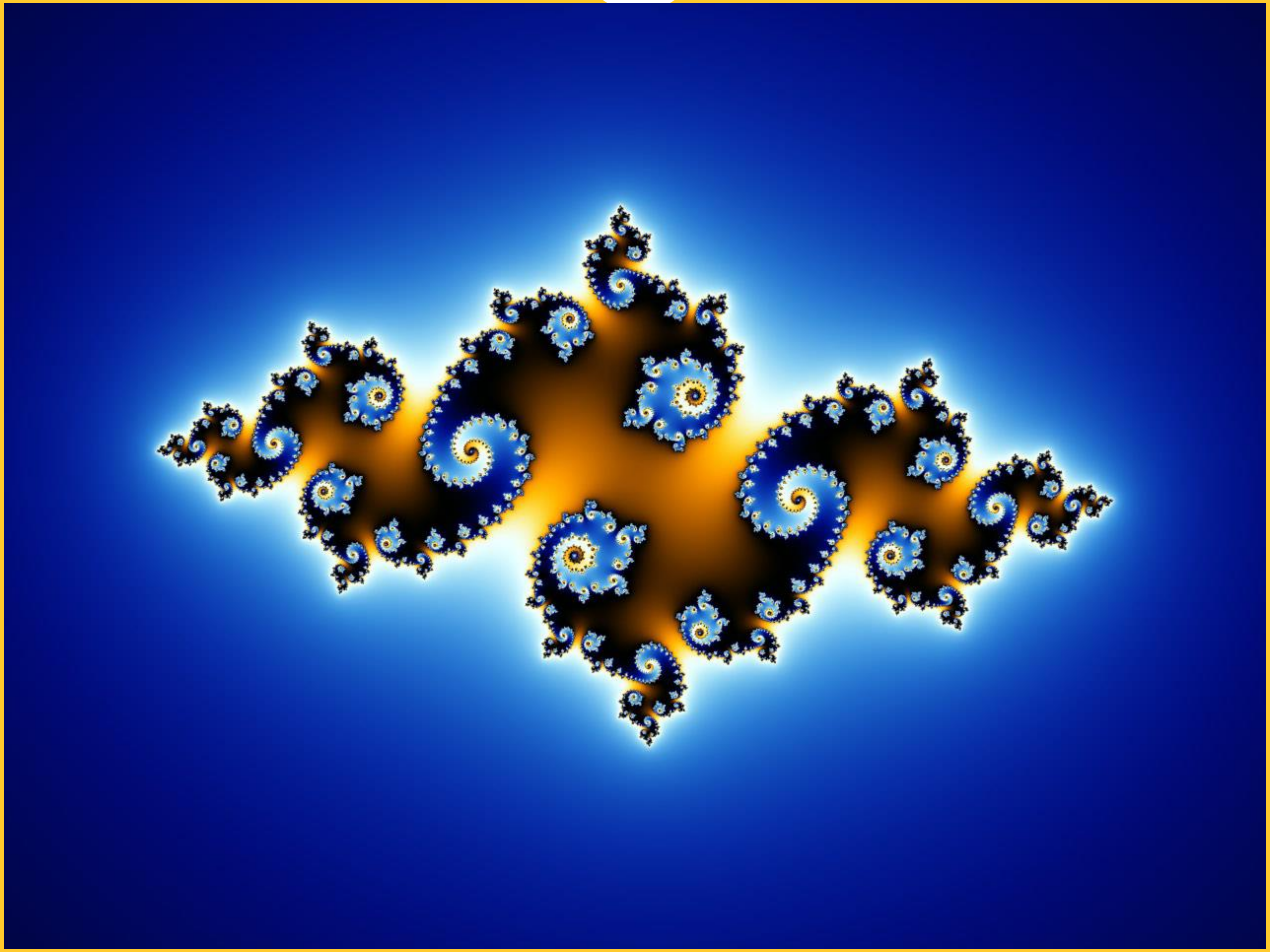
Topics To Be Discussed

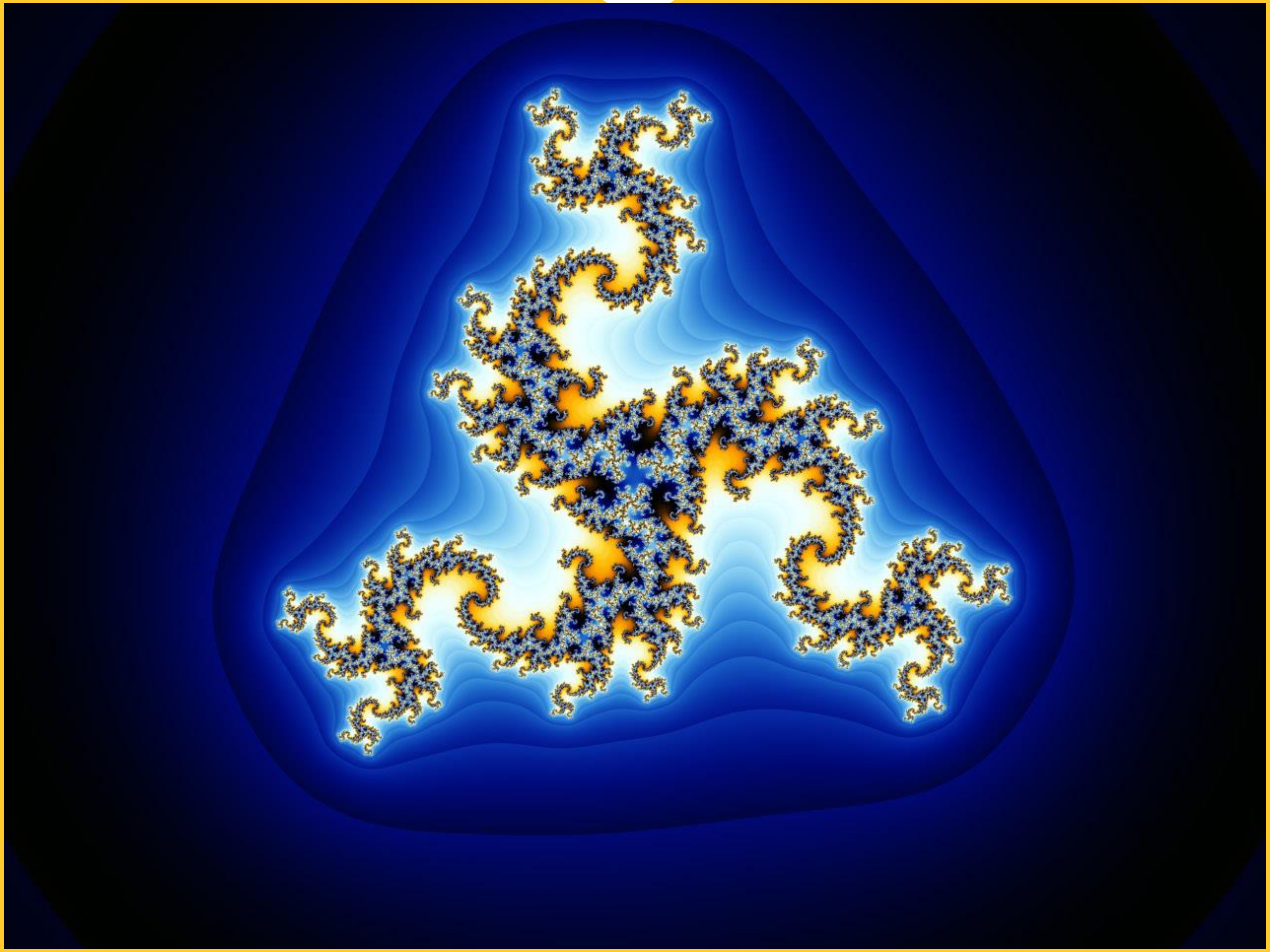
- Definition of Fractals
- Properties of Fractals
- Concept of divisibility, Multiplication, Scaling
- Concept of Self Similarity
- Details of Xaos Program
- Koch Curve
- Cantor Set
- Lindenmayer System
- Sierpinski Triangle
- Peano Curve

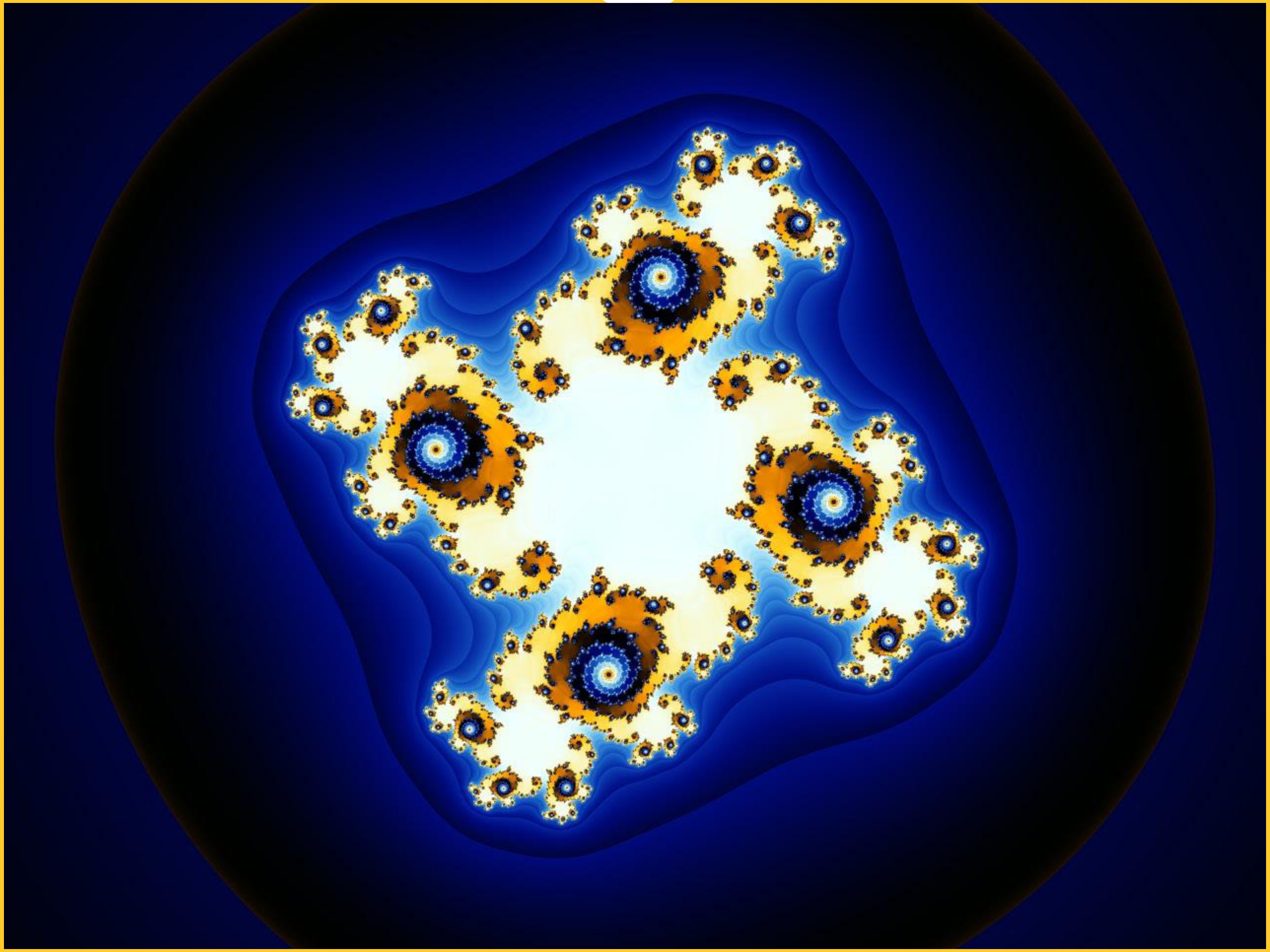
Topics To Be Discussed

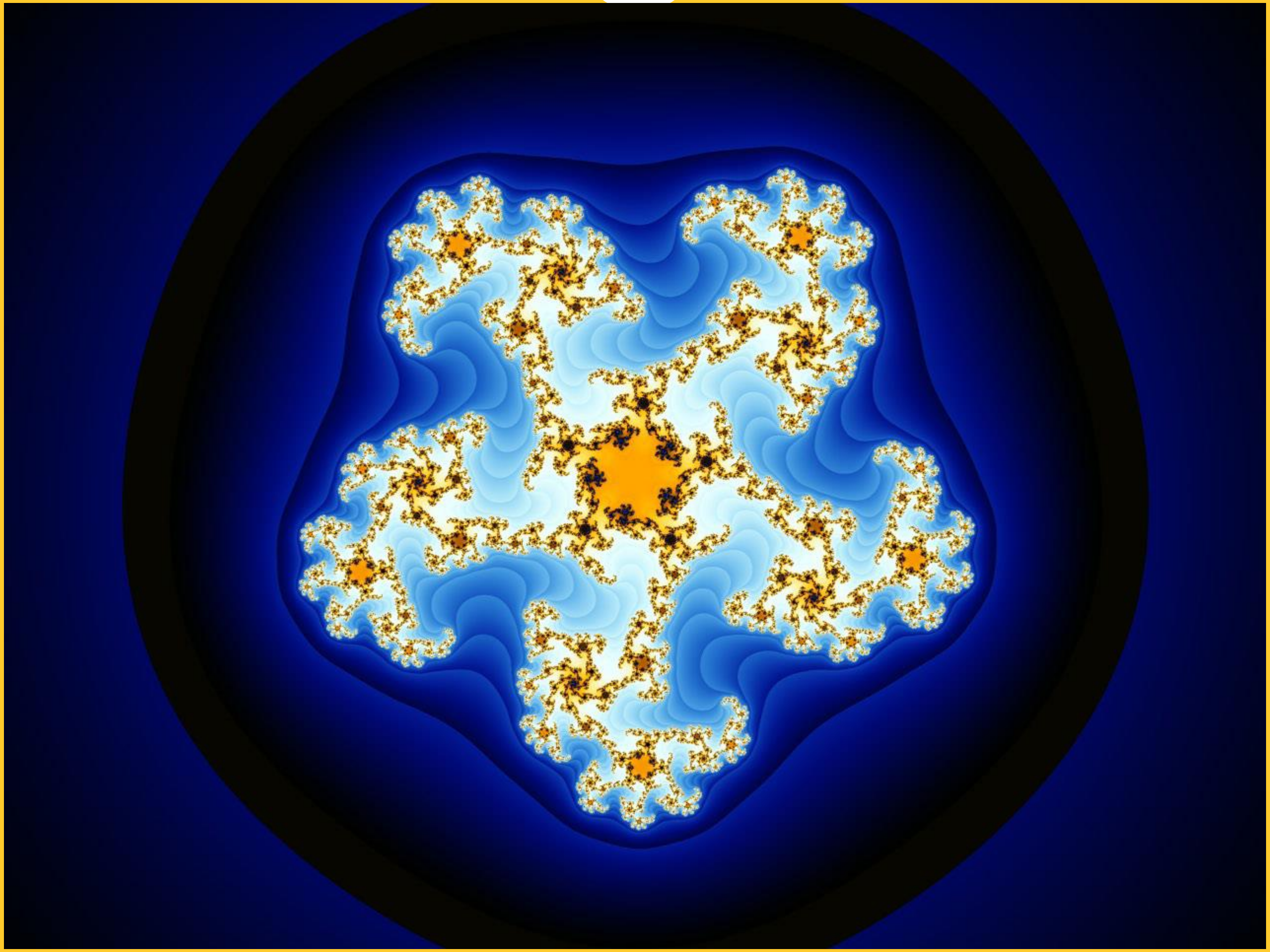
- Fractals and its dimensions
- Fractals and Naturally occurring objects
- Julia and Mandelbrot sets
- Mathematics of Mandelbrot sets
- Fractal Fern
- Introduction to Mupad
- Introduction to LOGO
- References
- QA
- Feedback

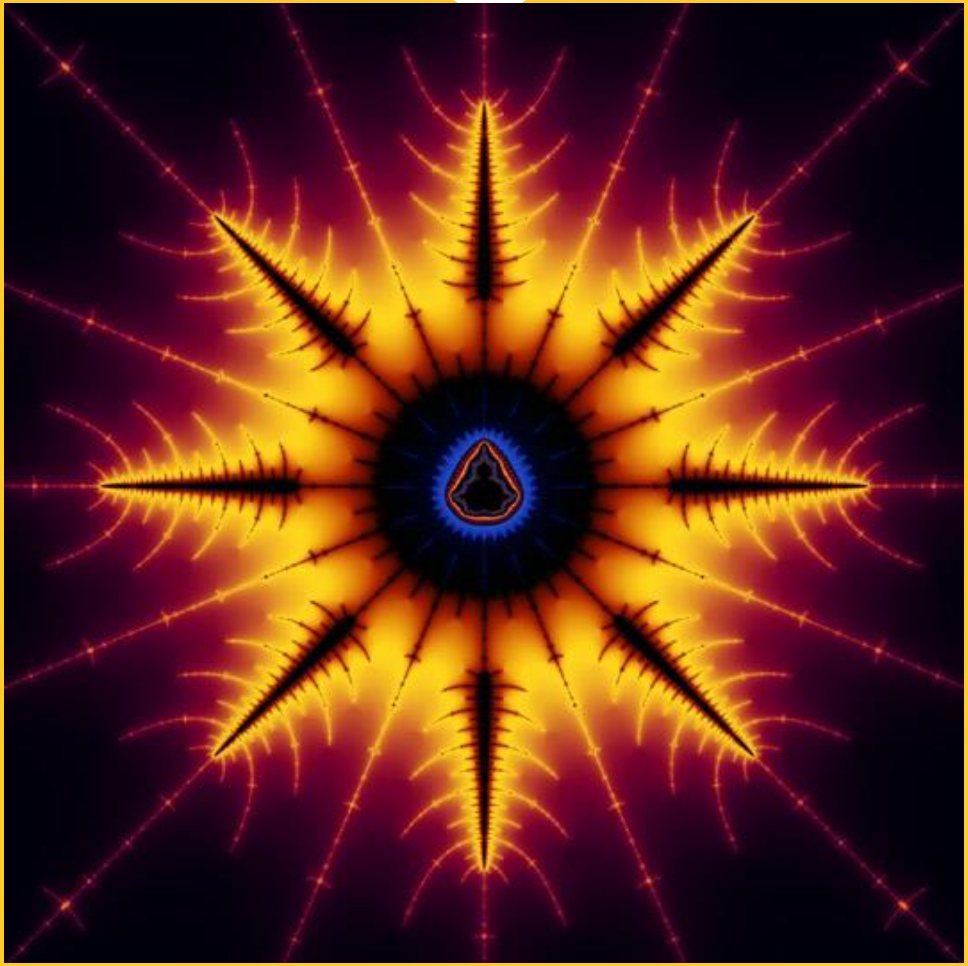
Why This Talk











Benoit B Mandelbrot

Born: 20 November 1924, Warsaw, Poland
Died: 14 October 2010, Cambridge, Massachusetts, United States

Complexity vs Simplicity

- Fractals are beautiful and appears that very complex mathematics involved in creating these fractals.
- Later I have seen that the mathematics involved in creating these fractals are very simple.
- This motivated me to promote mathematics through talks on fractals.

Brain Storming

Math is not Hard

Survey Report (18-05-2017)

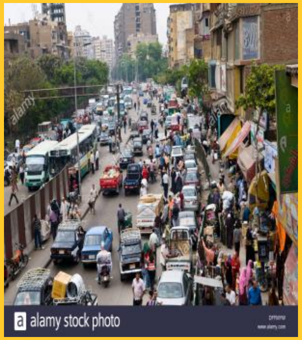
- Total Participant –
- Math is hard –
- Math is not hard -

All are Mathematicians !!!

1812

- **Housewives- Great Mathematicians**

•Maintaining Household Expenditure- Financial Management



•Driving Vehicle
Dynamics, Accident, Time, Sp

•Transaction in market place

Math is everywhere
Animal Kingdom

1813

- Prey and Predator- Crab

- Tiger >> Dear

- Kingfisher >> Fish

- Honey Bee >> Honeycomb

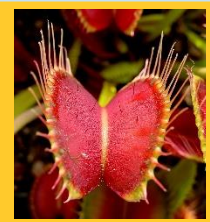
Math is everywhere-Plant Kingdom

•Trees, Leaf, Branches, Flowers – Follows definite Patterns, reasoning, structure, symmetry
(Many guided by Fibonacci Number, Golden Ratios)

Fibonacci was born around 1170.
Michael Maestlin, first to publish a decimal approximation of the golden ratio, in 1597

• Sense of Direction (Sunflower)
• Sense of Season
• Sense of Touch
• Sense of Time (Touch-me-not)





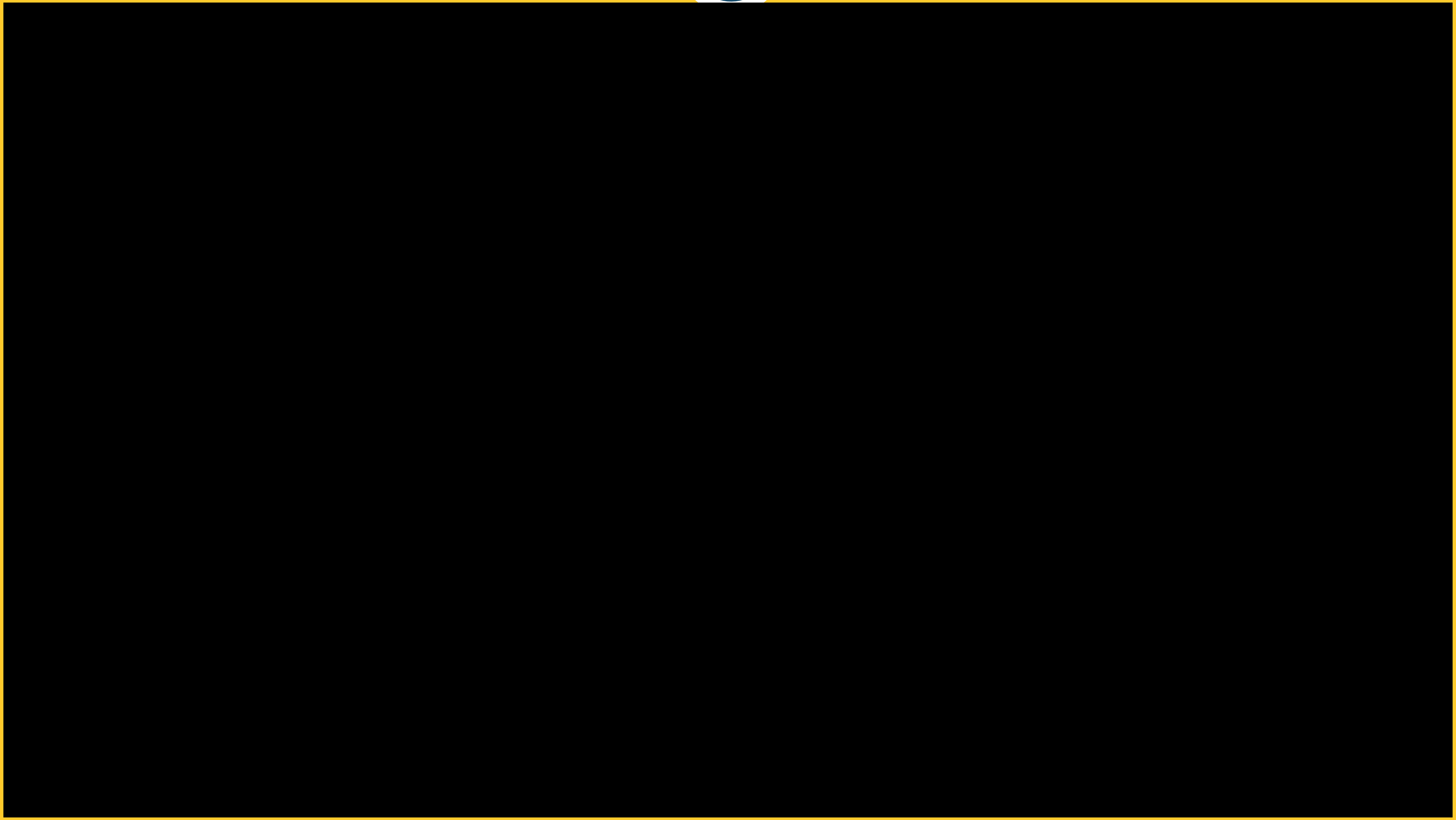

•Fractals
(Cauliflower, Fern)


Fibonacci sequence: Golden Ratio

							H
A	B	C	D	E	F	G	x=sqrt(x+1)
Sl No	Fabonacci Sequence	Golden Ratio=1.618	Square Root	Add One	D+E	Sqrt(F)=1.618	X^2-X-1=0
1	0		100	1	101	10.04988	From Equation
2	1	#DIV/0!	10.04988	1	11.04988	3.324135	1.618034
3	1	1	3.324135	1	4.324135	2.079456	From Series
4	2	2	2.079456	1	3.079456	1.754838	1.618034
5	3	1.5	1.754838	1	2.754838	1.65977	From Golden Ratio
6	5	1.666667	1.65977	1	2.65977	1.63088	1.618034
7	8	1.6	1.63088	1	2.63088	1.621999	From Step function
8	13	1.625	1.621999	1	2.621999	1.619259	1.618182
9	21	1.615385	1.619259	1	2.619259	1.618412	
10	34	1.619048	1.618412	1	2.618412	1.618151	
11	55	1.617647	1.618151	1	2.618151	1.61807	

Golden Ratio and Fractal Geometry

1816



Math is everywhere-Nature



NATURE

•In Nature>> Nothing is Random
(Clouds, Mountains, Rivers, Fire, Coast Line)
>>Many are Fractals

Conclusion

1818

- Math is every where
- Every thing is governed by math
- Math is not hard
- As number system started with natural numbers and embraced different numbers due to different need, Now Euclidian Geometry should embrace Fractal geometry to describe the universe .

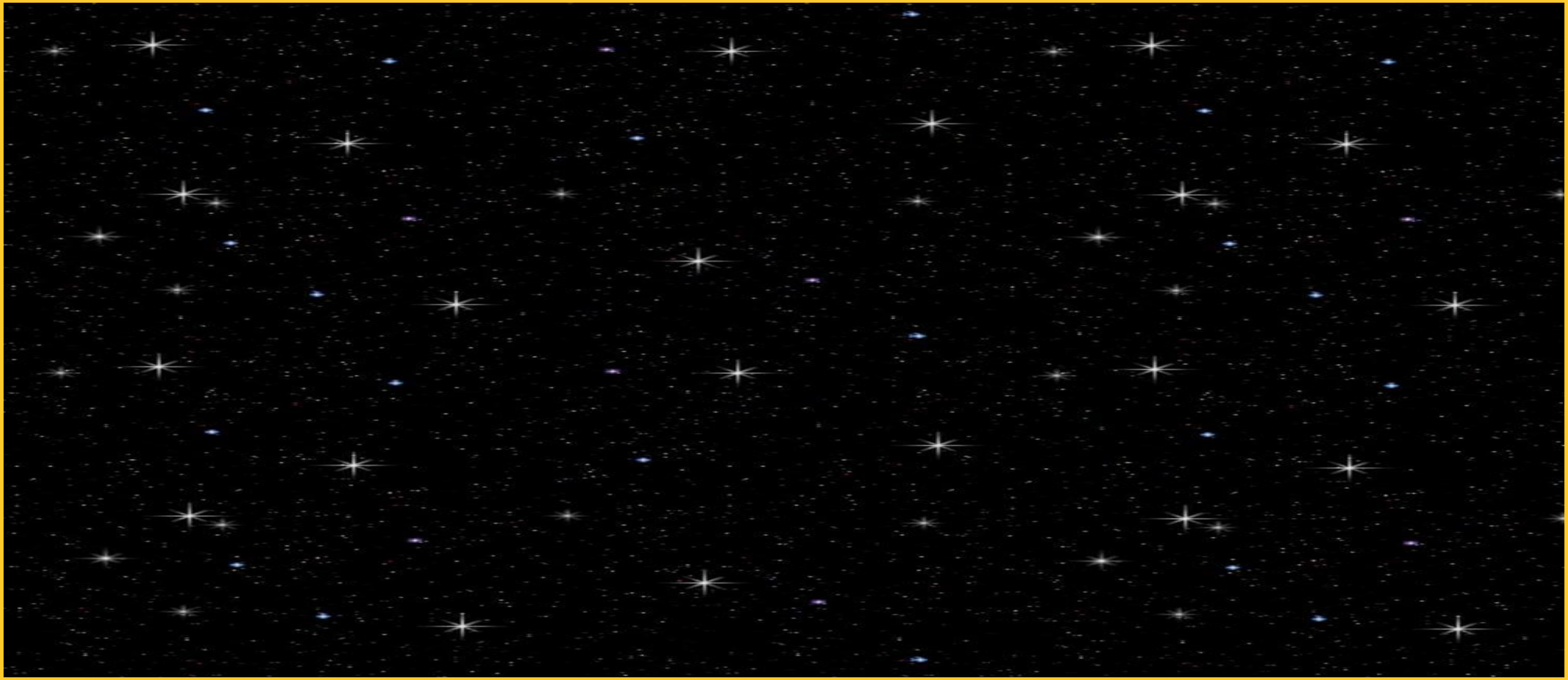
Next Question

1819

- Then why math appears Hard?

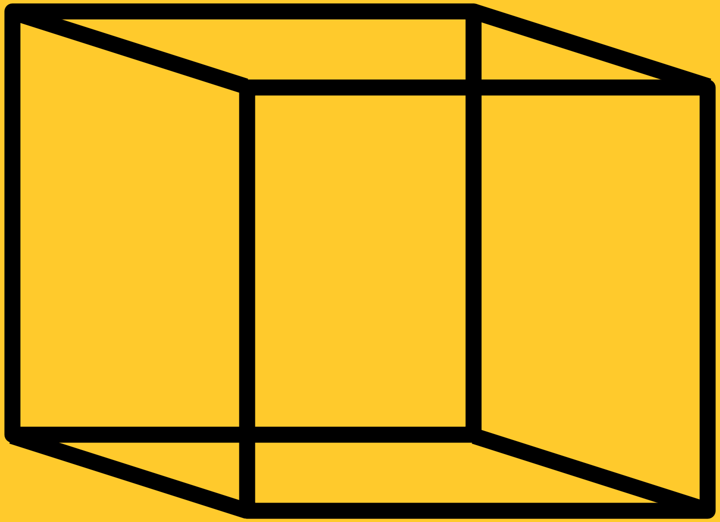
Physical World vs Visual World

If I ask , what is this photograph?
You will answer Night sky, Star. I can not differ.
God has given us an incredible gift – our eyes. It can be tiny but it is so powerful that we can see these stars at an infinite distance.



Physical World vs Visual World

Similarly if I ask you what is this? You will answer cube. I will say, no it is not cube. You will argue. But I will stick in my word.



And the problem lies here.

Physical World vs Visual World

Physical World vs Visual World

- Similarly if I ask you what is this? You will answer, it is railway track. I will say, no it is not a rail track. You will argue. But I will stick in my word.

- And the problem lies here.
- What we see is different from the real world.

Definition of Math

Why do we do math?

Why math is required?
Why do we require mathematics?

- Counting
- Comparing
- Exchanging
- Measuring
- Time Keeping
- Constructing
- Transforming
- Calculating
- Changing
- Identifying
- Characterizing
- Predicting
- Many more

Evolution of Mathematics

- Natural Numbers
- Whole Number
- Integer
- Rational Number
- Irrational Numbers
- Real Numbers
- Imaginary Numbers
- Complex Numbers
- Logarithmic Numbers
- Prime Numbers
- Quaternion and many more

ADDITION
SUBTRACTION
MULTIPLICATION
DIVISION
EXPONENTIATION

ARITHMETICS
GEOMETRY
ALGEBRA
LINEAR ALGEBRA
TRIGONOMETRY
CALCULUS
STATISTICS
MANY OTHERS

Evolution of Mathematics

Numbers	Need	Binary Operations	Branches
Natural Numbers	Counting	Addition +	Arithmetic
Whole Numbers	Comparing	Subtraction -	Algebra
Integer Numbers	Measuring	Multiplication *	Geometry
Rationals	Dividing	Division /	Trigonometry
Irrationals Hippasus Transcendentals	Probability	Exponentiation ^	Calculus
Real s Logarithms(Time)	Calculus		Statistics
Imaginary	Exponentiation		Linear Algebra
Complex	Quantification		Many More
Quaternion	Qualitative		
Vectors	Characterizati on		
Matrices, Sets			12-04-2020 12:47

Basic Geometrical Ideas

Points	Lines	Curves Angle Triangle Quadrilaterals Rhombus Trapezium Pentagon Polygon Circle Ellipse Parabola Hyperbola	Cube Sphere Pyramid Prism Cylinder Ellipsoid Cone Cuboid

One Formula for Universal Shapes

1829

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Different Shapes

From Wiki: Meaning of Polygon: The word "polygon" derives from the Greek adjective πολύς (*polús*) "much", "many" and γωνία (*gōnía*) "corner" or "angle". It has been suggested that γόνυ (*gónu*) "knee" may be the origin of "gon".[1]

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Polygons

1831

- **Triangle**
- **Quadrilateral**
- **Pentagon**
- **Hexagon**
- **Octagon**
- **Nonagon**
- **Decagon**

- **Go on**

Polygons

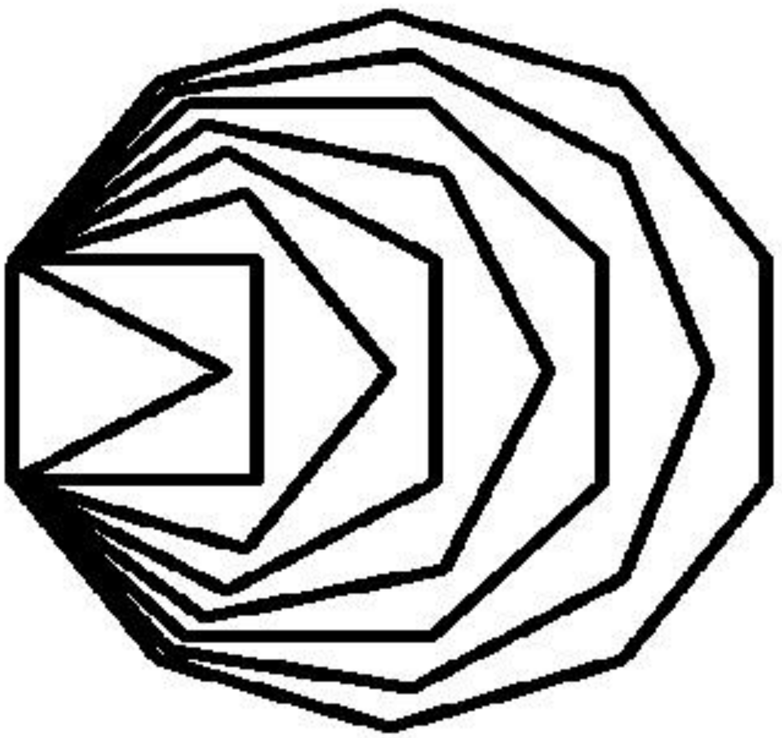
- Zerogon
- Monogon
- Bigon
- Trigon
- Quadrugon
- Pentagon
- Hexagon
- Heptagon
- Octagon
- Go on

Creating Regular Polygons

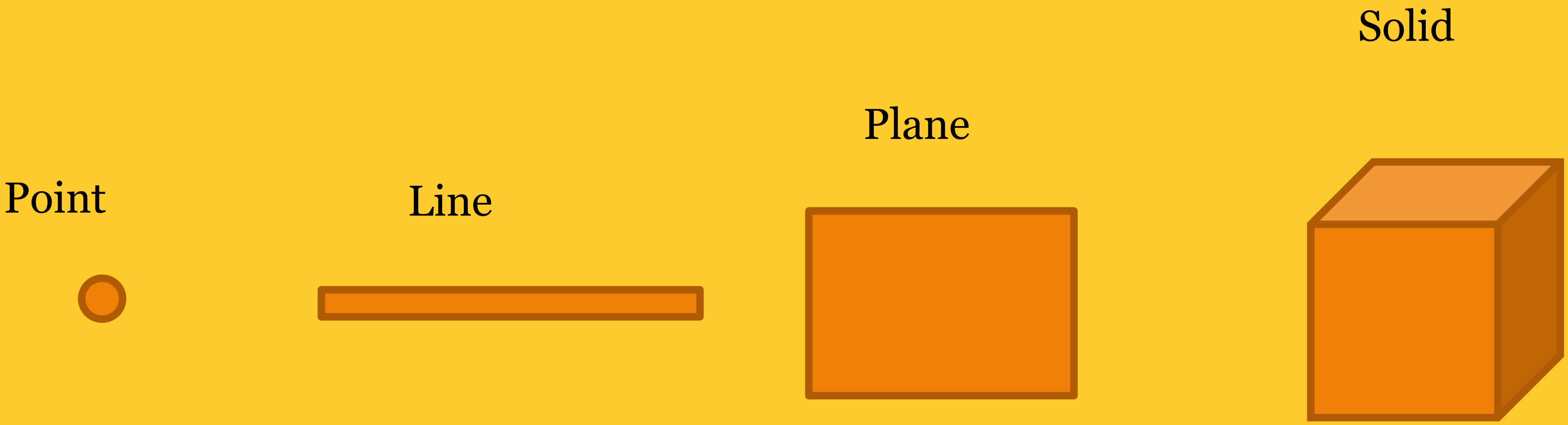
- Regular polygon follows a pattern
- There is a relationship between the no of sides, Angle and Length of the sides of the polygon

```
to polygon :sides :length
repeat :sides  [forward :length rightturn 360/:sides]
end
```

Polygons Created in LOGO



Objects



Dimensions

Polygons are two dimensional objects:

Similarly,

0 Dimension - Point

1 Dimension – Line

2 Dimension – Square

3 Dimension – Cube

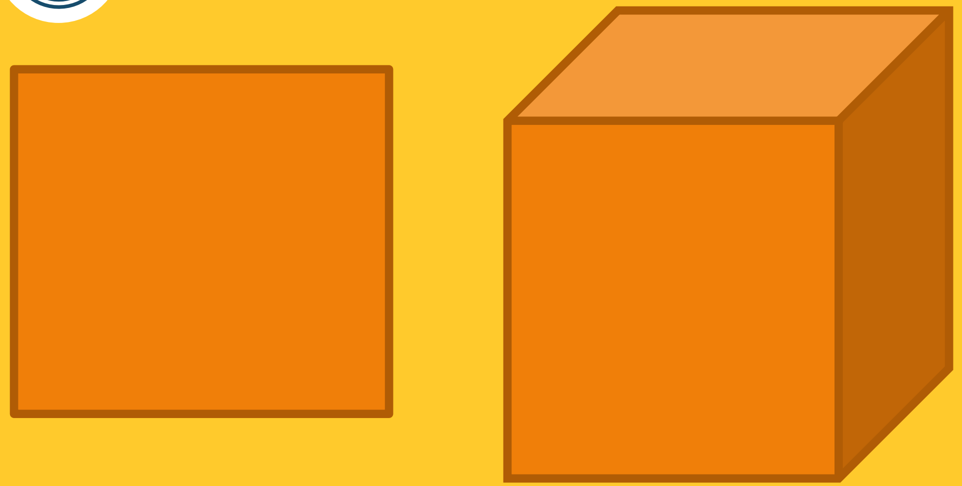
Can there be higher dimensional objects, can there be fractional dimensional objects?

Relevance of Dimension

Can there be higher dimensions than three or can there be fractional dimensions?

This question was irrelevant 500 years ago.

Formation of objects and Dimensions



Drag and Create

1. 0 Dimension - Point
2. 1 Dimension – Line
3. 2 Dimension – Square
4. 3 Dimension – Cube
5. 4 Dimension – Hyper Cube
6. 5 Dimension – Hyper Hyper Cube

Drag and Create: Object and Dimensions

Object	Vertex	Edges	Faces	Solids	Hyper Solid	Dimension
Point	1	0	0	0	0	0
Line	2	1	0	0	0	1
Square	4	4	1	0	0	2
Cube	8	12	6	1	0	3
Hyper Cube	16	32	24	8	1	4
Hyper Hyper Cube	32	80	80	40	10	5

Relationship in 3d Objects: Euler's Formula: Vertex + Face=Edge+2

As dimension can be integers, it can be fractions

Object	Vertex	Edges	Faces	Solids	Hyper Solid	Dimension
Point	1	0	0	0	0	0
Line	2	1	0	0	0	1
Square	4	4	1	0	0	2
Cube	8	12	6	1	0	3
Hyper Cube	16	32	24	8	1	4
Hyper Hyper Cube	32	80	80	40	10	5

And the objects with fractional dimensions are fractals.

Continuity

- Calculus was another topic which is dominating Modern Science. It deals with derivatives of continuous objects.
- But in natures geometry, many are continuous but do not posses any derivatives.
- They are broken, wrinkled, uneven

Fractal

2400 Years Ago

Let no one enter who does not know Geometry

Inscription on Plato's Academy
at Athens (429-347 BC)

32 Years Ago

No one will be considered scientifically literate tomorrow who is not familiar with Fractals – John Archibald Wheeler : **New Scientist** 4 Apr 1985

Basic Geometrical Ideas

- Points
- Lines
- Curves
- Angle
- Triangle
- Quadrilaterals
- Rhombus
- Trapezium
- Pentagon
- Polygon
- Circle
- Ellipse
- Parabola
- Hyperbola

- Cube
- Sphere
- Pyramid
- Prism
- Cylinder
- Ellipsoid
- Cone
- Cuboid

Congruence

Two objects are said to be congruent if the objects are of same size and same shape

Congruence of Angle

- **It can be said that if length of two lines are same, they are congruent.**
- **If two angles have same measure, they are congruent**

Congruence of Triangle

- **Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.**
- **In two congruent triangles, corresponding vertices, angles and sides are equal.**

- **Definition of Similarity: Two figures having same shape and not necessarily same size are called similar figures.**

Observation

- **All congruent figures are similar but all similar figures are not congruent.**

- **Two polygons of same number of sides are congruent, if (i) their corresponding angles are equal and (ii) their corresponding sides are also equal.**
- **Two polygons of same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion)**

Sequence and Series

Very Important branch of Mathematics

Binomial Theorem

Why we study Binomial Theorem????????????????????????

$(a+b)^2=a^2+2ab+b^2$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

>It is easy to calculate. But if we require to calculate $(a+b)^{59}$ or any other power of (a+b), how can we proceed???

>Binomial Theorem helps in these situations.

>Binomial theorem enable us to recognize the pattern hidden behind many mathematical problems.

Series

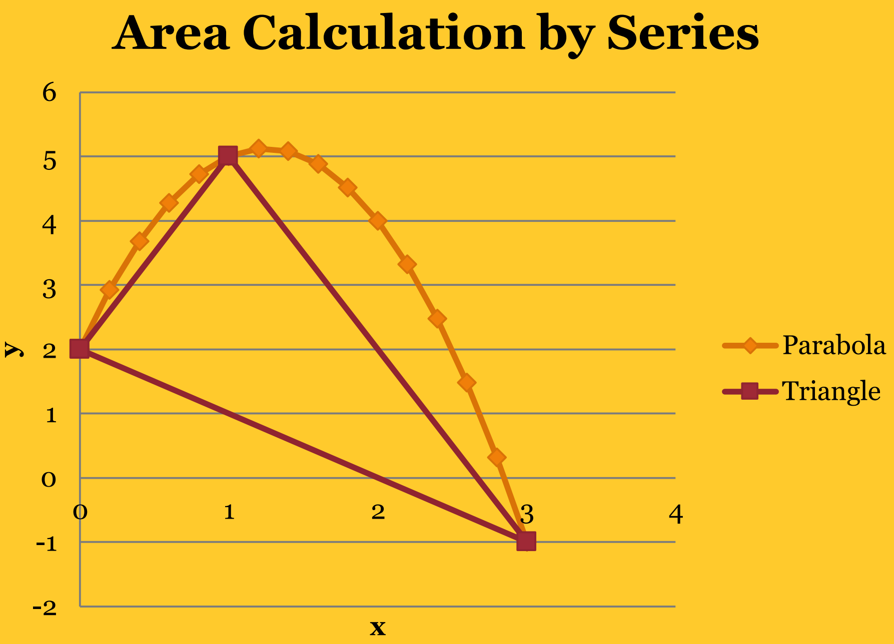
- History: **Archimedes of Syracuse** 287 BC – 212 BC
- In *The Quadrature of the Parabola*, Archimedes proved that the area enclosed by a parabola and a straight line is $4/3$ times the area of a corresponding inscribed triangle . He expressed the solution to the problem as an infinite geometric series with the common ratio $1/4$:
- If the first term in this series is the area of the triangle, then the second is the sum of the areas of two triangles whose bases are the two smaller secant lines, and so on. This proof uses a variation of the series 1/4 + 1/16 + 1/64 + 1/256 + · · · which sums to $1/3$.
- Note: http://en.wikipedia.org/wiki/Archimedes

$\sum_0^n 1/4\hat{}n$

Archimedes Area Calculation

$a_n = 1/4^{(n-1)}$

n	p	a_n	s_n
1	0	1	1
2	1	0.25	1.25
3	2	0.0625	1.3125
4	3	0.015625	1.328125
5	4	0.003906	1.332031
6	5	0.000977	1.333008
7	6	0.000244	1.333252



Binomial Theorem

>Definition: Binomial theorem deals with the algebraic expression generated by the expansion of powers(n) to the binomial (a+b).

>The power (n) of the binomial can be 0, 1, 2, 3,……………

Case-1: Power(n=0): $(a+b)^0$ =1

Case-1: Power(n=1): $(a+b)^1$ =1, 1

Case-1: Power(n=2): $(a+b)^2$ =1, 2, 1

Case-1: Power(n=3): $(a+b)^3$ =1, 3, 3, 1

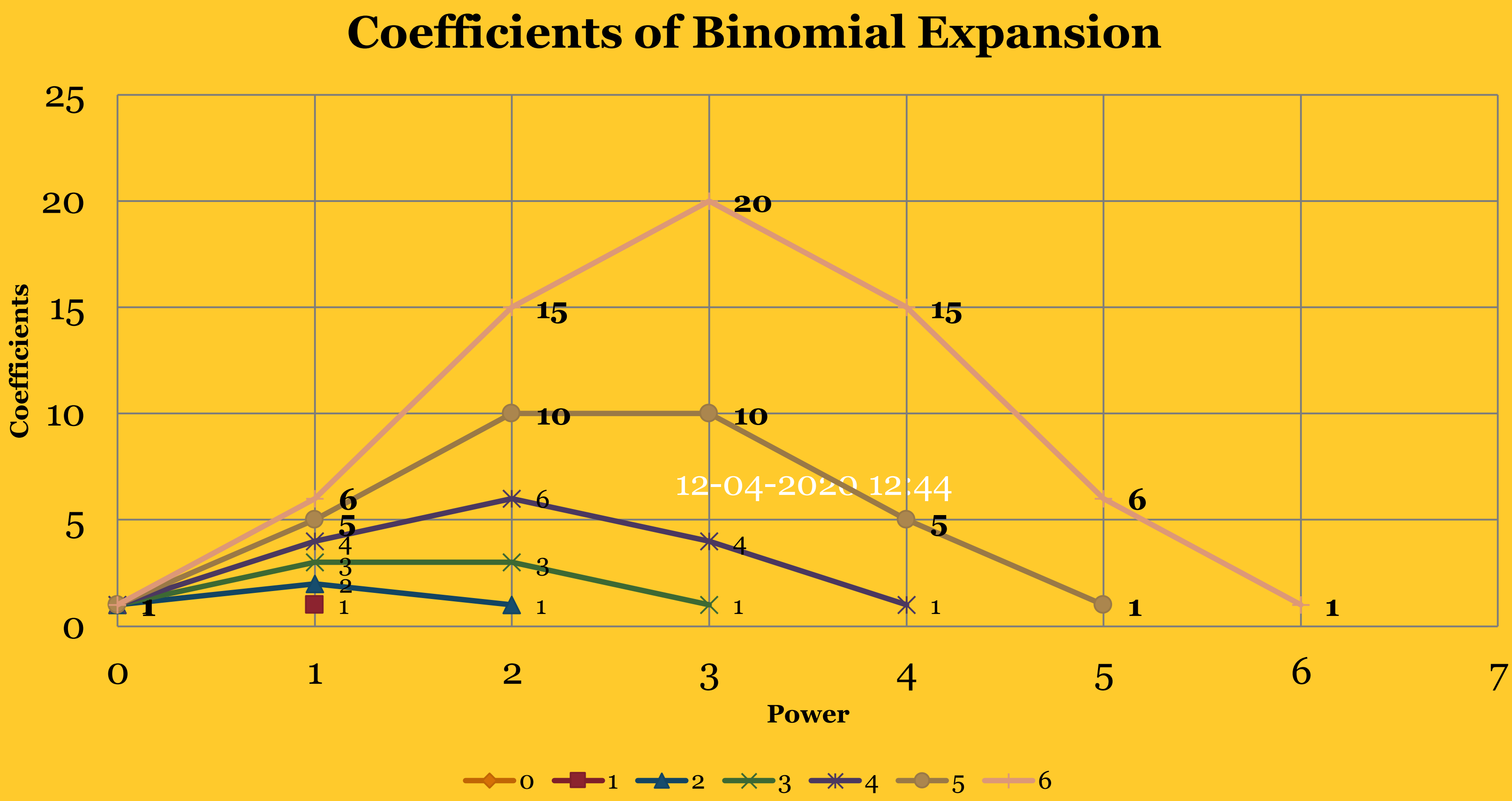
Case-1: Power(n=4): $(a+b)^4$ =1, 4, 6, 4,1

Case-1: Power(n=5): $(a+b)^5$ =1, 5, 10, 10, 5, 1

Case-1: Power(n=n): $(a+b)^n$ = ${}^nc_0, {}^nc_1, {}^nc_2, {}^nc_3, {}^nc_4 \ldots {}^nc_{(n-2)}\ {}^nc_{(n-1)}\ {}^nc_n$

(Only coefficients of the expansion considered)

Coefficients of Binomial Expansion



Sequence, Series, GP

>**Geometric Progression** (GP) is a sequence where each term except the first term bears a constant ratio to the term immediately preceding it.

>Common Difference,

$$r = \frac{an+1}{an}$$

Term	1	2	3	4	5	6	7
Power	0	1	2	3	4	5	6
a_n	2	4	8	16	32	64	128

>

$$an = a * r^{(n-1)}$$

>

$$Sn = \frac{a(r^n - 1)}{(r-1)}$$

Sequence, Series, GP

>**Geometric Mean(GM):** GM is a number, c, between two consecutive terms , a and c, of a GP is given by

$$c = \sqrt{ab}$$

>We can insert any number (n) between two terms. The formula is

$$b = ar^{n+1}$$

$$an = a(\frac{b}{a})\hat{}(n/(n+1)$$

Sequence, Series, AM, GM

>Relation between AM and GM:

AM=(a+b)/2

GM=✓ab

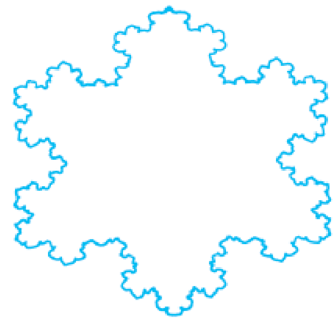
Observation: (AM-GM) is always positive, so AM is always greater than GM

Fractal

When We Encounter Fractal First?

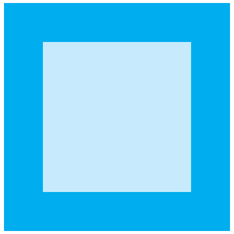
At which class, we first read about Fractal?

- Observe this beautiful figure.
  It is a symmetric pattern known as Koch's
  Snowflake. (If you have access to a computer, browse through the topic "Fractals" and find more such beauties!). Find the lines of symmetry in this figure.



EXERCISE 13.2

1. Find the number of lines of symmety for each of the following shapes :

(a) 

(b)

(c) 

Definition of Fractals

- What is Fractal!

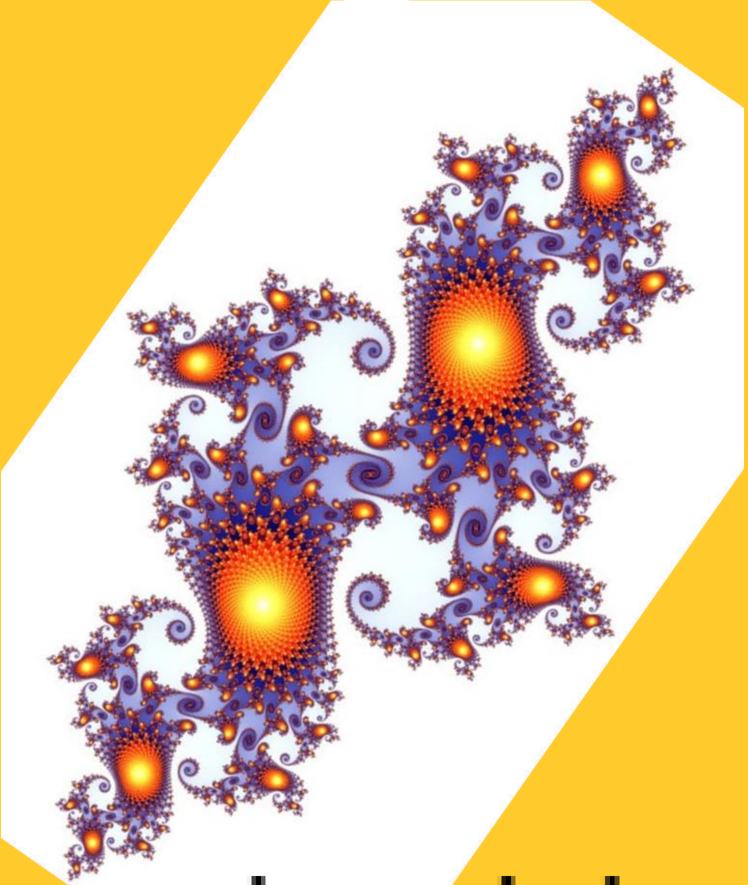
What is Fractal? (from internet)

- A curve or geometrical figure, each part of which has the same statistical character as the whole. They are useful in modelling structures (such as snowflakes) in which similar patterns recur at progressively smaller scales, and in describing **partly random** or **chaotic phenomena** such as crystal growth and galaxy formation.

Fractal - Wiki

- A **fractal** is a natural phenomenon or a mathematical set that exhibits a repeating pattern that displays at every scale. It is also known as expanding symmetry or evolving symmetry. If the replication is exactly the same at every scale, it is called a self-similar pattern.

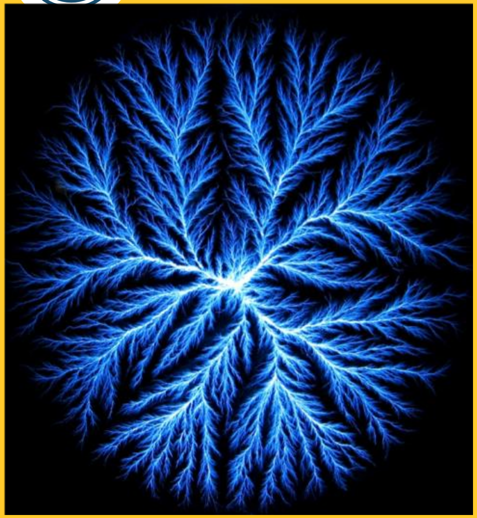
Fractals Are $MART: Science, Math & Art!

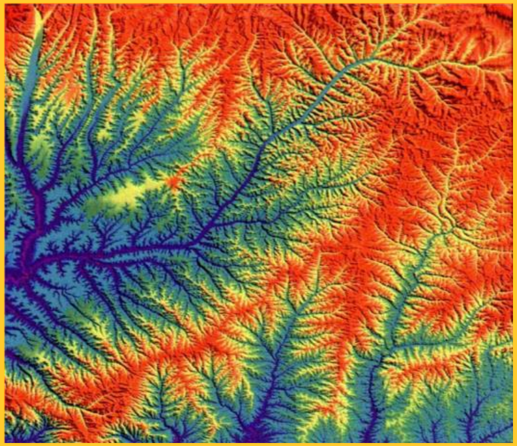


www.FractalFoundation.org

Example of Fractals







Fractal Definition

- A fractal is a never ending pattern that repeats itself at different scales. This property is called "Self-Similarity."
- Fractals are extremely complex, sometimes infinitely complex - meaning you can zoom in and find the same shapes forever.
- **Amazingly, fractals are extremely simple to make**.
- A fractal is made by repeating a simple process again and again.

Fractal – Huw Jones

Definition:

- A fractal is by definition a set for which the Housdorff Besicovitch dimension strictly exceeds the topological dimension.

Properties of fractal :

- **Self similarity:** Any sub set of the object is, in some sense, a copy of the whole,
- **Infinitesimal sub divisibility:** There is no apparent change in the amount of detail observed at different levels of magnification.

Two Main Concepts Governs Fractal Formation

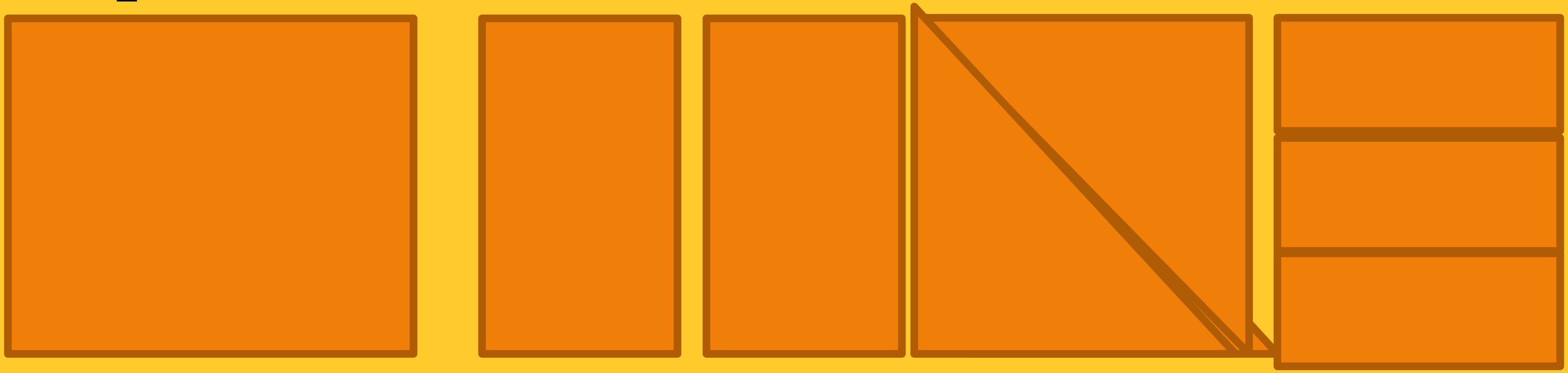
- Concept of infinite divisibility

- Concepts of self similarity

Concepts of Divisibility and Self Similarity

- Infinitesimal Divisibility and Self Similarity

Square



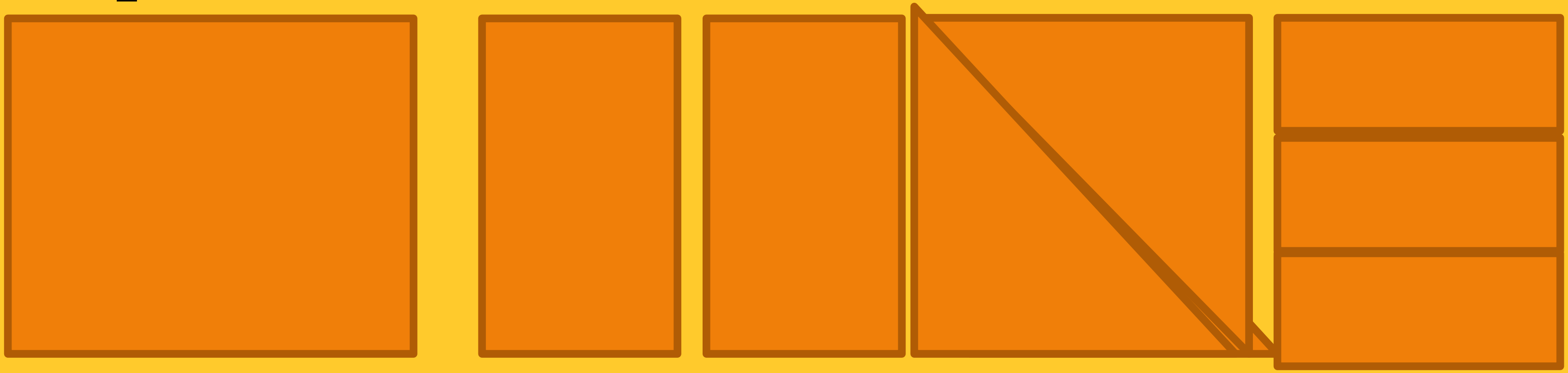
Divided but not Self Similar

Divisibility achieved but Not Self Similarity

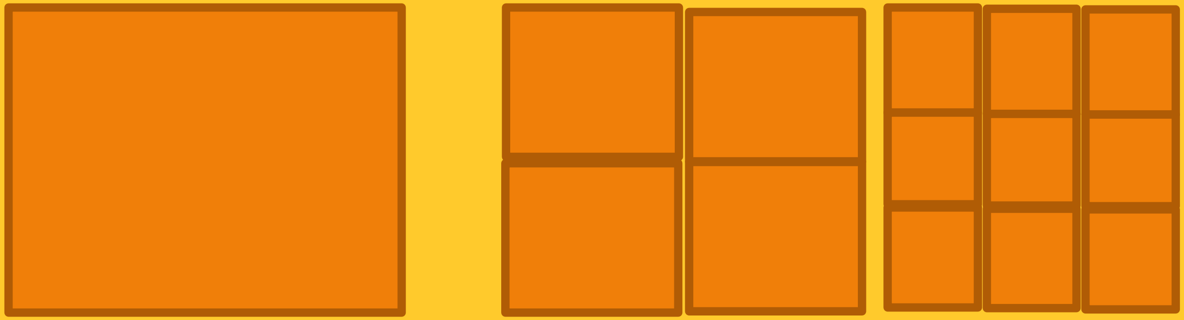
Understanding Concept of Fractal

- Infinitesimal Divisibility and Self Similarity

Square **Divided but not Self Similar**



Divided and Self Similar



Viewing Fractal

Fractivity

Explore Fractals with XaoS

http://fractalfoundation.org/resources/fractal-software/

Fractal And Universe

XaoS

1875

XaoS can create many different fractal types, which can be accessed by using the number keys:

Keys 1 to 5 are Mandelbrot sets with various powers. The “normal” z^2 Mandelbrot set is on key 1. (Hitting “1” is a good way to reset yourself if you get lost!)

Key 6 is a Newton fractal, exponent 3, illustrating Newton’s method for finding roots to 3’d order polynomial equations.
Key 7 is the Newton fractal for exponent 4.
Key 8,9, and 0 are Barnsley fractals.
Key A - N are several other fascinating fractal formulas

Enjoy Xios

Scale

HOW BIG (OR SMALL) ARE FRACTALS ?

Mathematical fractals are infinitely complex. This means we can zoom into them forever, and more detail keeps emerging. To describe the scale of fractals, we must use scientific notation:

Thousand	1,000	10^3
Million	1,000,000	10^6
Billion	1,000,000,000	10^9
Trillion	1,000,000,000,000	10^{12}
Quadrillion	1,000,000,000,000,000	10^{15}

Because of the limits of computer processors, all the fulldome fractal zooms stop at a magnification of 10^{16}. Of course the fractals keep going, but it becomes much slower to compute deeper than that. 10^{16} (or ten quadrillion) is incredibly deep. To put it in

Fractal And Universe

Koch Curve

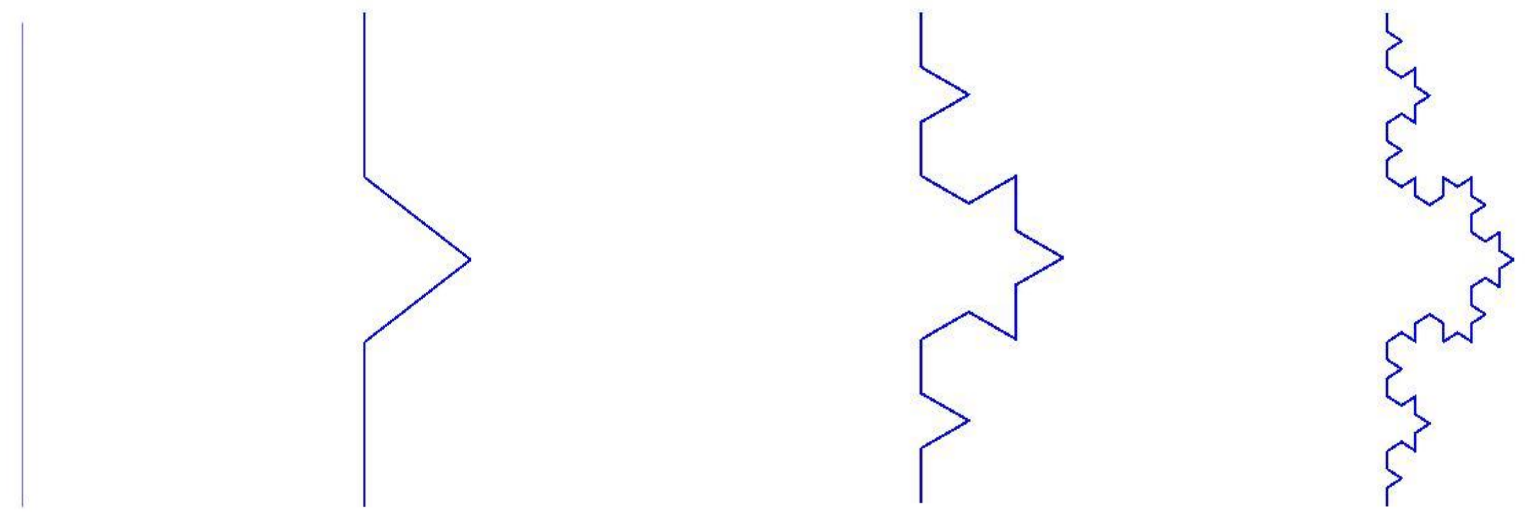
The Koch curve (1904)

- Koch curve is generated by recursively replacing line segments by ‘poly lines’ consisting of four line segments each 1/3rd of the original length. The first 4 stages of the process are shown below.

```
lsys:=plot::Lsys(PI/3,
"F","F"="F-F++F-F",
Generations=3):
plot(lsys)
```

Koch Curve – 1st 4 Generations

1881



- Length Of Koch Curve

Koch Curve

- Limitless repetition of the process generates a true fractal form. The curve contains four exact copies itself, each at one third of the original scale, so at each stage of generation every line is replaced by four thirds of its original length.
- If the original line length has length one, the total length after n stages is $(4/3)^n$. It becomes infinitely large, it is said to approach infinity as ‘n’ approaches infinity.
- Thus, the pure fractal object has infinite length- there is not enough ink in the universe to draw it properly.

Koch Curve - Length

- This means that every sub copy is also infinite length - 1/3 of infinity must still be infinity.

- That goes for sub-sub copies and so on. Any two points in Koch curve is separated by an infinite distance .
- They are continuous but not differentiable

- Area of Koch Curve

Koch Curve - Area

- Now consider the area between the center and the original defining line.
- At the first stage a single triangle is added and we suppose its area is A. This is a isosceles triangle with sides equal to 1/3 and A can be found as $\sqrt{3}/36$, but the actual amount is irrelevant to the argument that follows.
- At each successive stage , four times as new triangles are added, each one ninth the area of the previous stage. When shape lengths are multiplies by 1/3, areas are multiplied by $(1/3)^2$ or 1/9 , a consequence of area being two dimensional. So the added area at any stage is 4/9 that added at the previous stage.

Koch Curve - Area

- The total area can be expressed as the sum of the series-

  $S=A + (4/9) A + (4/9)(4/9) A + (4/9)(4/9)(4/9) A \ldots$
- $S=A + (4/9)^1 A + (4/9)^2 A + (4/9)^3 A \ldots$
- Here, each term of the series becomes smaller by a fraction of 4/9. The series like this with constant multiplies is called geometric series. As 4/9 is less than 1, the series converses. Multiplying both side by (4/9), we get,

  $(4/9) S = 4/9 A + (4/9)^2 A + (4/9)^3 A + - - -$
- Subtracting, this equation with previous equation get,
- $S - 4/9 S = A$ or or $5/9 S = A$ or $S = 9A/5$ or Total Area=1.8 A

  As we know, $A = \sqrt{3}/36$.
- $S = 9/5 * \sqrt{3}/36$ or $S = \sqrt{3}/20$.
- Here, the S is finite, ie, the area is finite Hence, we have an infinite length curve with a finite area. The area can be painted easily but detailed curve cannot be drawn

- Fractal, Turtle Graphics, Lsys

Fractal, Turtle Graphics, Lsys

- Fractal are complex geometry and requires specialized tool to draw it.
- Mupad provides excellent and simple tool to draw the fractals. There are tools like Mupad, LoGOG, Turtle Graphics and Lsys.
- Before exploring further, let’s try to understand the process of iteration and recursion and how turtle graphics and Lsys works.

Recursion and Iteration

- Recursion and iteration both repeatedly executes the set of instructions.
- Recursion is when a statement in a function calls itself repeatedly.
- The iteration is when a loop repeatedly executes till the controlling condition becomes false.
- The primary difference between recursion and iteration is that a **recursion** is a process, always applied to a function. The **iteration** is applied to the set of instructions which we want to get repeatedly executed.

LOGO/ TURTLE GRAPHICS, L-Sys

1891

- Both classes do not use coordinate geometry
- Using them, many geometric pattern can be produced based on line segments
- New turtle always faces upward, to the top edge of the computer screen
- Logo was first introduced by Jim Muller.
- With few simple commands, fascinating geometric objects can be created.

Logo and Turtle Graphics

- In European countries, Logo is the very first programing language used for teaching computing to students in primary schools and sometimes even in kindergarten.

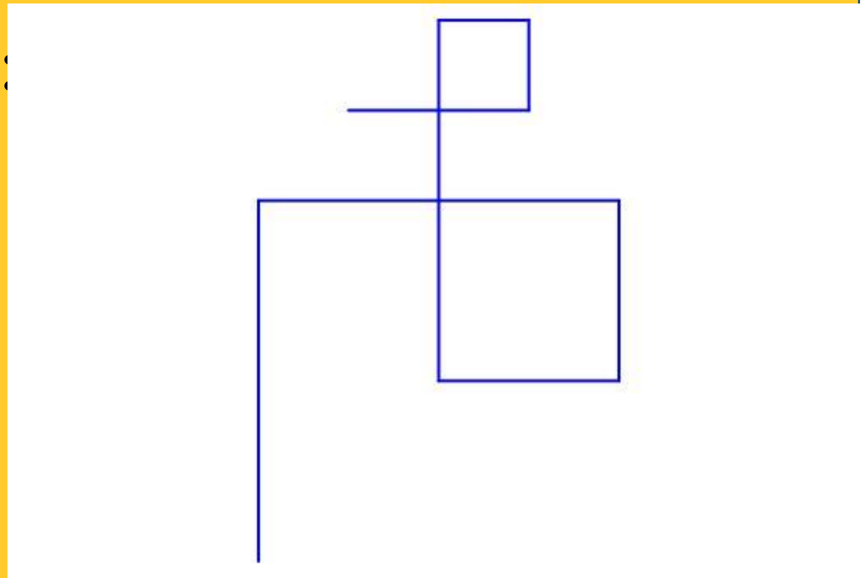
- With simple commands to Turtle like forward, left, right we can create many interesting patterns.

TURTLE COMMANDS

Left(angle)	• Turn Left
Right(angle)	• Turn right
Forward(length)	• Draw a line
PenUp ()	• Stop Drawing
PenDown()	• Start Drawing
Push()	• Save
Pop()	• Go back last saved Position

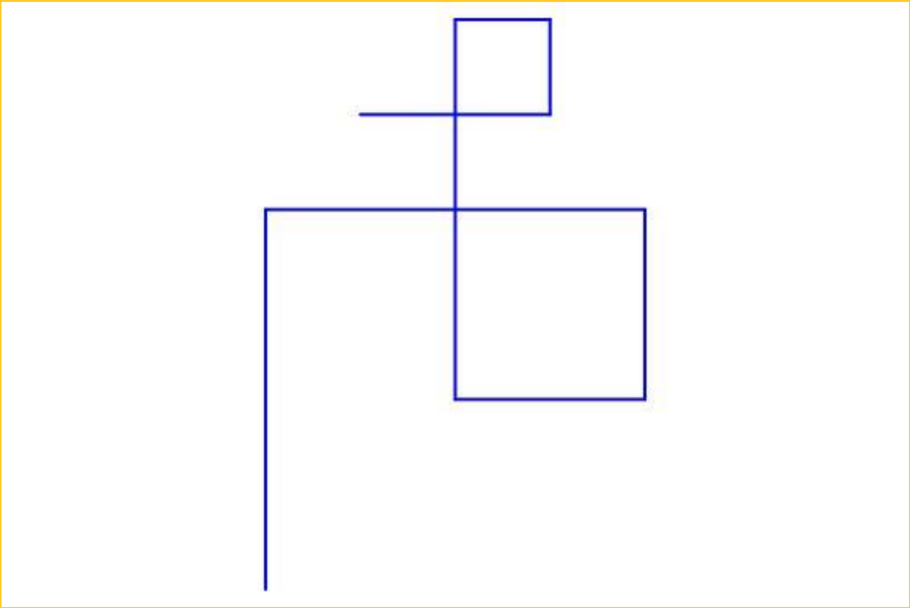
Example of Turtle Commands in Mupad

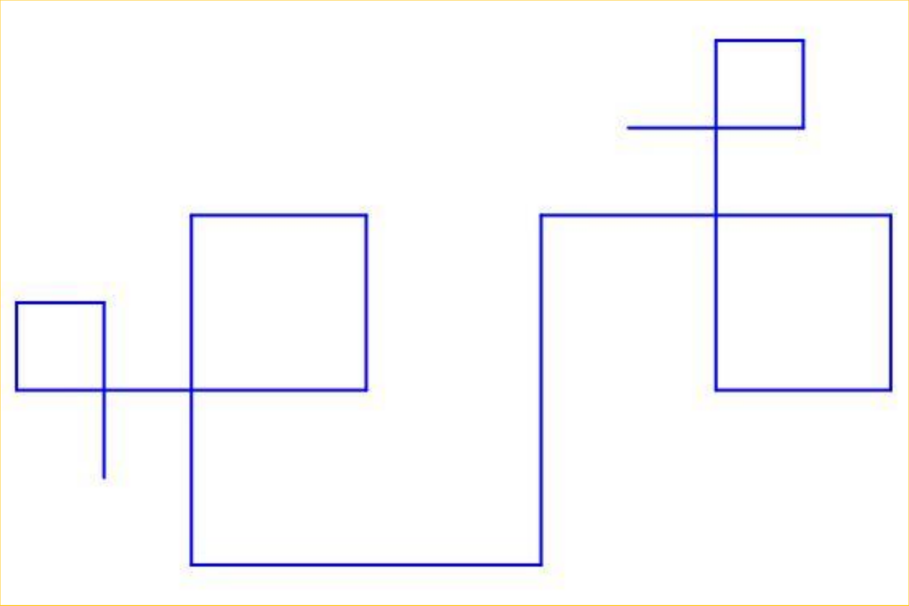

```
t:=plot::Turtle():t::forward(100):
t::right(PI/2):t::forward(100):
t::right(PI/2):t::forward(50):
t::right(PI/2):t::forward(50):
t::right(PI/2):t::forward(100):
t::right(PI/2):t::forward(25):
t::right(PI/2):t::forward(25):
t::right(PI/2):t::forward(50):
plot(t)
```



Rotate Turtle Graphics

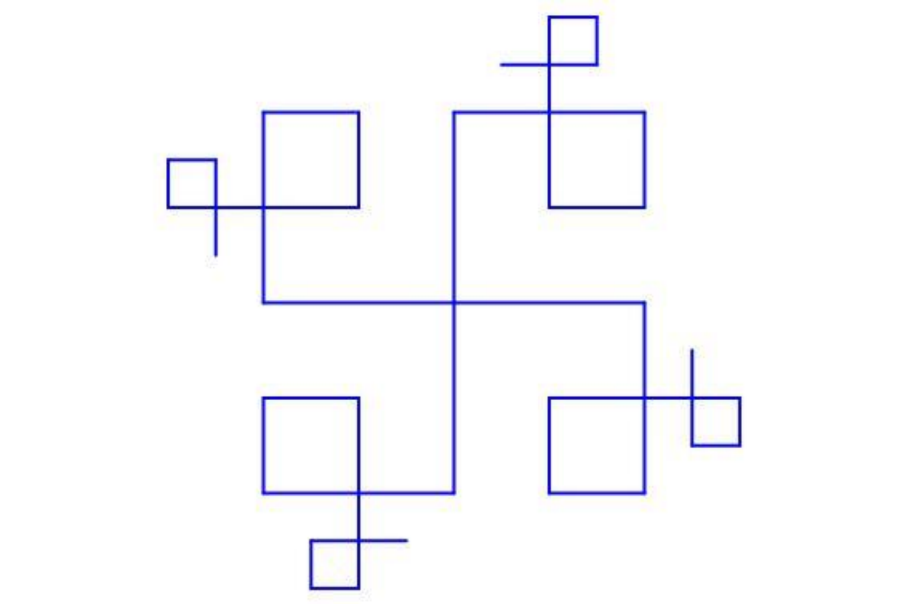
- t2:=plot::Rotate2d(PI/2,[0,0],t)





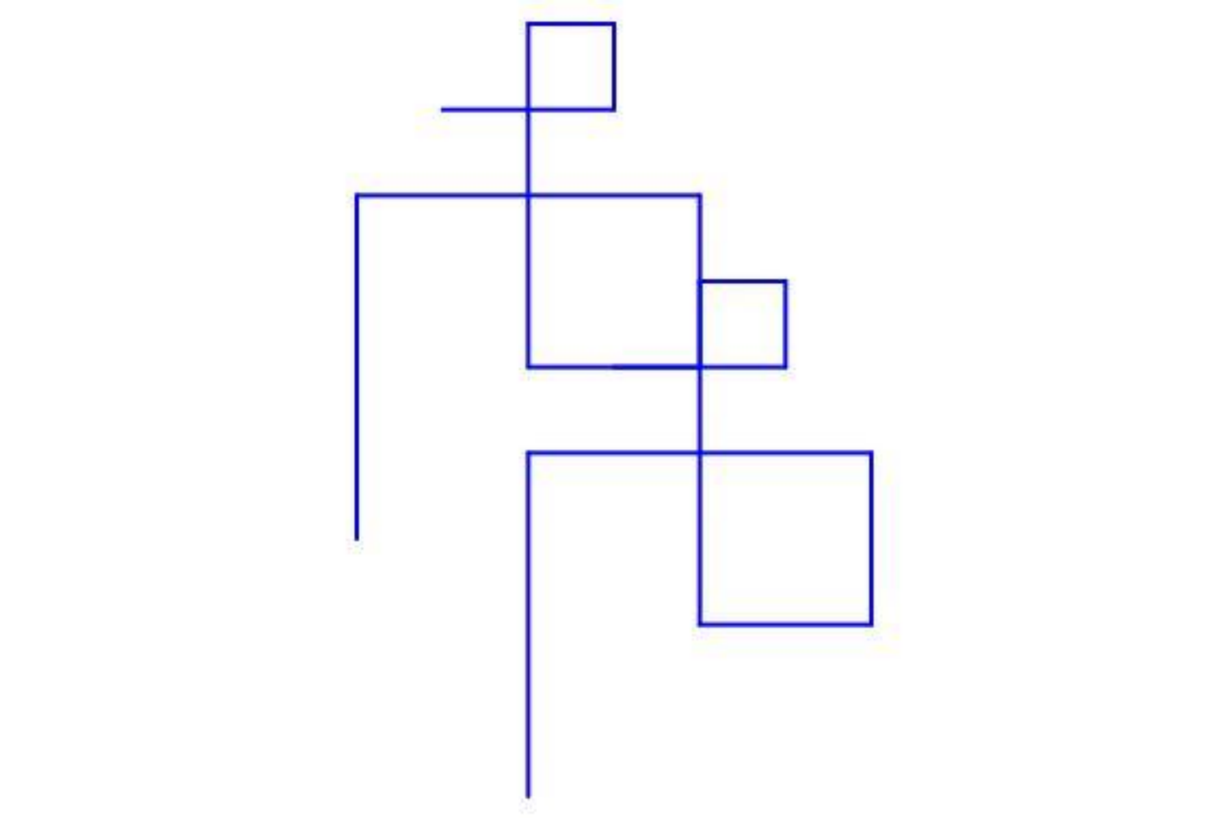
Create a Pattern

- T2:= Plot :: rotate 2d (PI/2), (0,0), t):
- T3:= Plot:: rotate 2d (PI,(0,0),t)
- T4:= Plot:: rotate 2d (3 *PI/2,(0,0),t)
- Plot (t, t2,td3,t4)



Translate Turtle

- a1:=plot::Translate2d([50,-75],t)



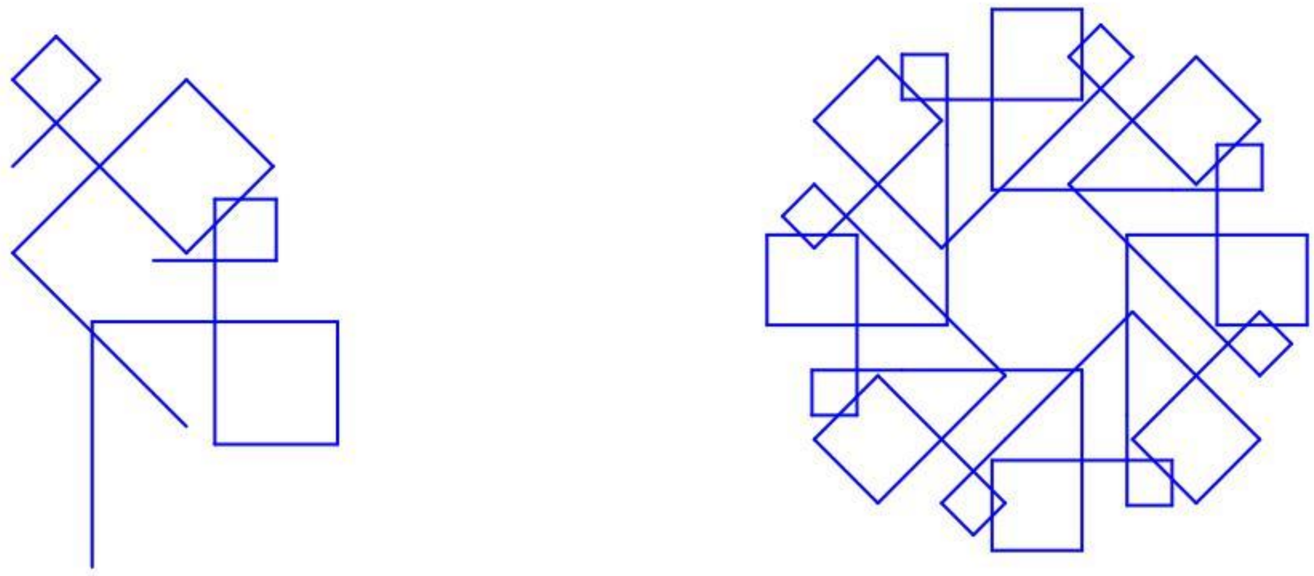
Turtle

a2:=plot::Rotate2d(PI/4,[0,0],a1)

a3:=plot::Rotate2d(2*PI/4,[0,0],a1)

a6:=plot::Rotate2d(5*PI/4,[0,0],a1):

a7:=plot::Rotate2d(6*PI/4,[0,0],a1):



- Lindenmayer systems: L-systems

Lindenmayer systems: L-systems

- It is seen that with Turtle graphics very good pattern can be developed but programming is little bit cumbersome.
- Lindenmayer systems is an comparatively easy way to create fractals.
- It was developed in 1960 by the Biologist Dr Aristid Lindenmayer to describe the branching structure of plants and similar objects.
- Structures are generated by repeatedly applying replacement rules or productions to the elements of a defined alphabet, starting with an initial word or axiom.

Lindenmayer systems: L-systems

- Let we have a line of length one. We divide it in three equal parts, attach an equilateral triangle to its central part, pointing to the right with a side length of 1/3 of the segment. Finally, we will remove the base of the triangle. This will create 2nd stage, now by applying the same process, to the segments of the resulting figure, we get subsequent stages. This process can be continued as many times as possible. This can be done very easily by the Mupad turtle graphics.

L-sys

- This rule, applied simultaneously to all characters, generate relatively complicated words after few iterations of the process.
- The characters of the alphabet are generally interpreted as geometric features.
- These are usually related to turtle graphics commands used in the language LOGO.
- In this, the turtle is controlled by simple commands to move around the screen.

Characters of L-sys

- Typical characters of Lsys and their turtle interpretations are-

1. F – Forward drawing step
2. f - Forward one step without drawing
3. + - A left turn at an given angle
4. - - A right turn at given angle
5. [- Initiation of branching
6.] – Termination of branching

Example of Lsys Command

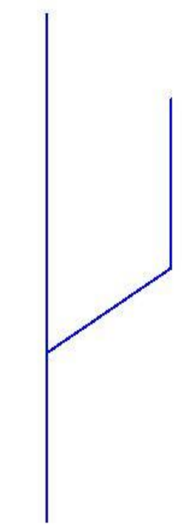
- Forward-Branch-Right-Forward-Left-Forward-Terminate-Forward-Forward

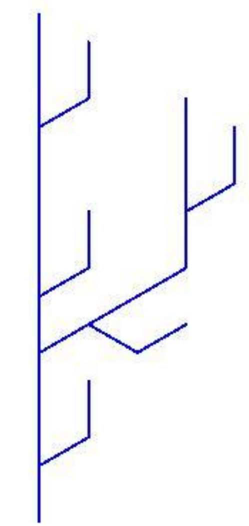
- F ->F[-F+F]FF

- lsys:=plot::Lsys(PI/3, "F", "F"="F[-F+F]FF", Generations=0):
  plot(lsys)

Example of 4 Generations of branching

F ->F[-F+F]FF





Generation = 0: F
Generation = 1: F[-F+F]FF
Generations=2: F[-F+F]FF [-F[-F+F]FF +F[-F+F]FF] F[-F+F]FF F[-F+F]FF
1F-5F-25F-125F-625F…….

L-sys: Stages for the development:

- Seed: Seed is the staring figure .
- Iteration rule: the rule for creating new figures called iteration rule.
- Orbit: the sequence of figures obtained will be called an orbit.
- Fractal: the final result is called a fractal.
- Generations: obtained figures in each sequence are called generations.

Lsys

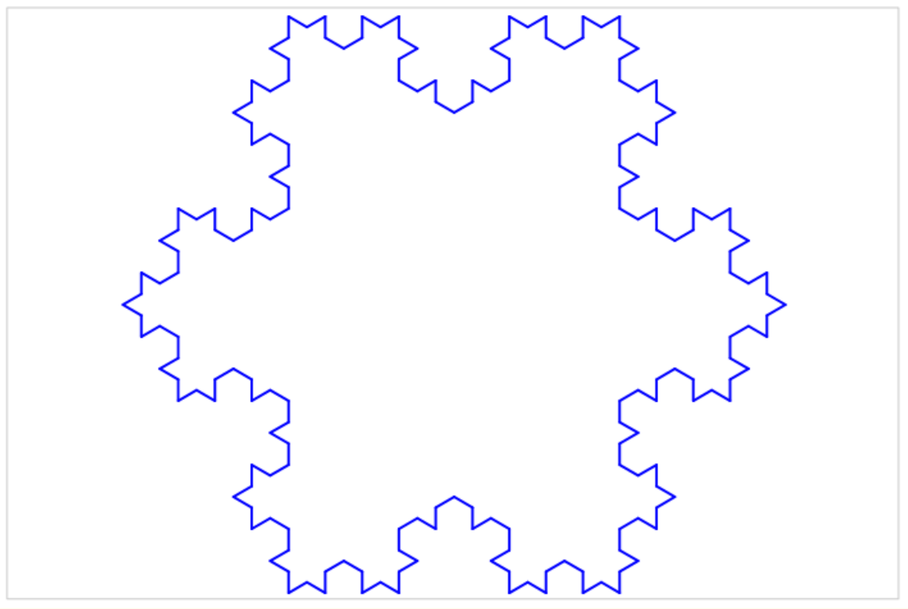
- The Lsystems are implemented with the use of turtle graphics and we must supply the turtle with the information about the seed, the iteration rule and generations that should be produced.
- Code for 4^{th} generation Koch curve.

Kochcurve:= plot: : L sys (// stars a new L = Sys
PI/3, // turtle always turn PI/3
“ F ++ F ++ F” // this is seed
“F”=”F-F++F-F”,// Iteration Rule
Generations = 4// number of generations,:
Plot (Kochcurve) // now plot the path

- Kochcurve:= plot::Lsys(PI/3, "F++F++F","F"="F-F++F-F", Generations = 4):
  plot(Kochcurve)

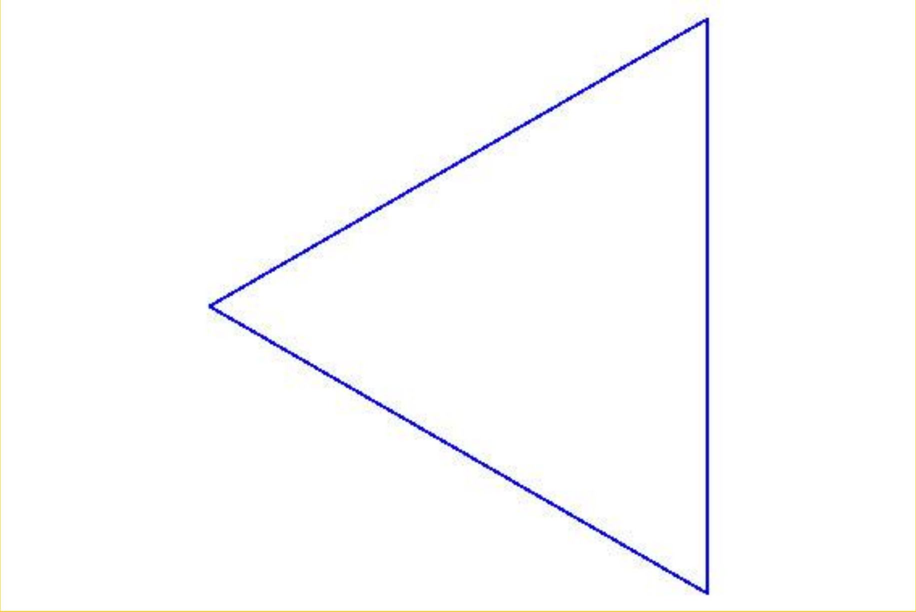
Creating Koch Curve with Lsys

- **Kochcurve:= plot::Lsys(PI/3, "F++F++F","F"="F-F++F-F", Generations = 4):**
  plot(Kochcurve)



Koch Curve

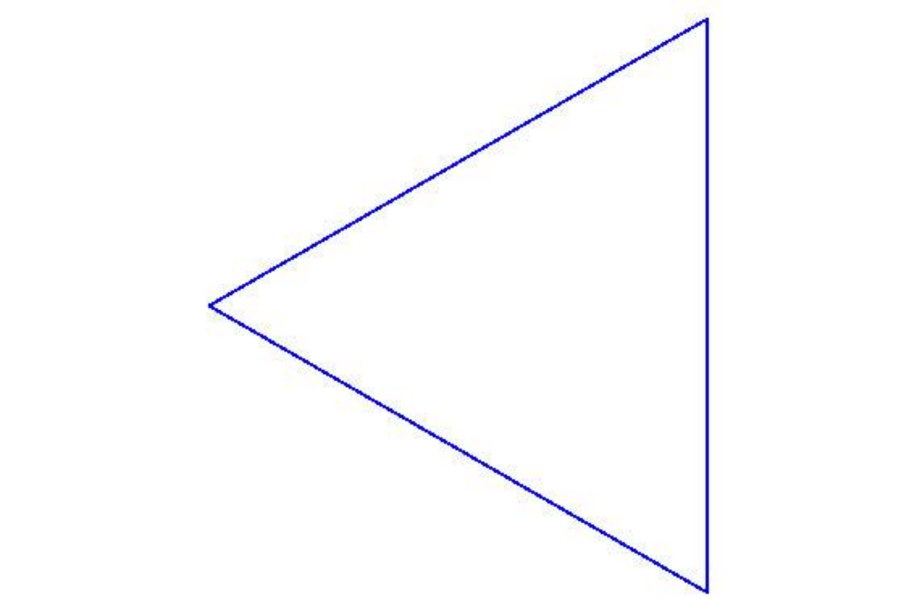
- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=0)
- plot(kochcurve)



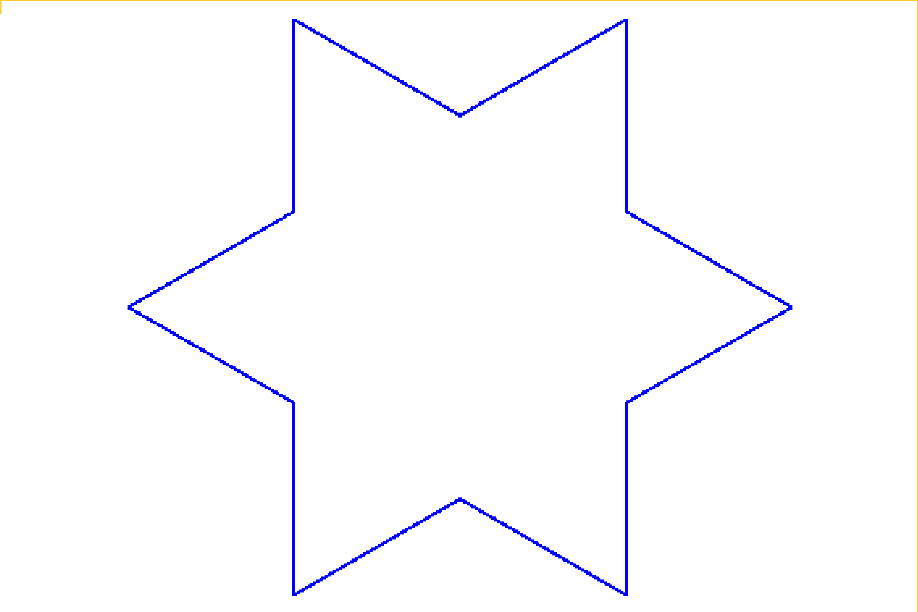
Fractal

1910

- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=1)
- plot(kochcurve)

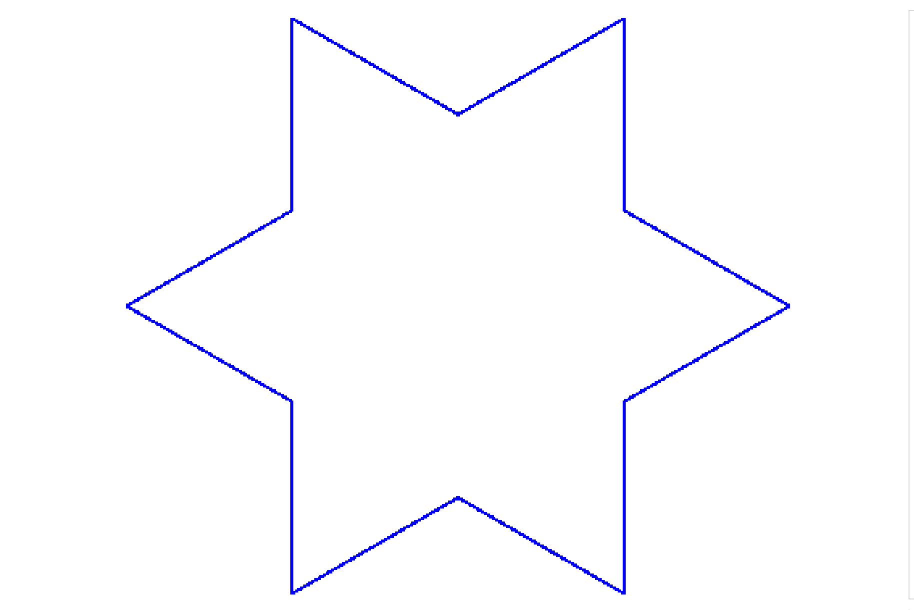


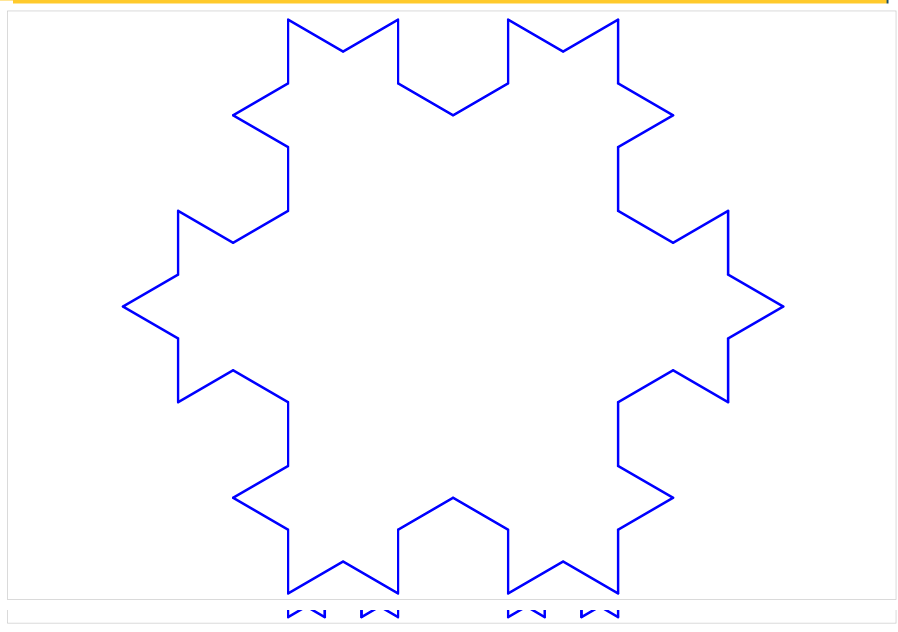
Generations-1



Fractal

- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=2)
- plot(kochcurve)

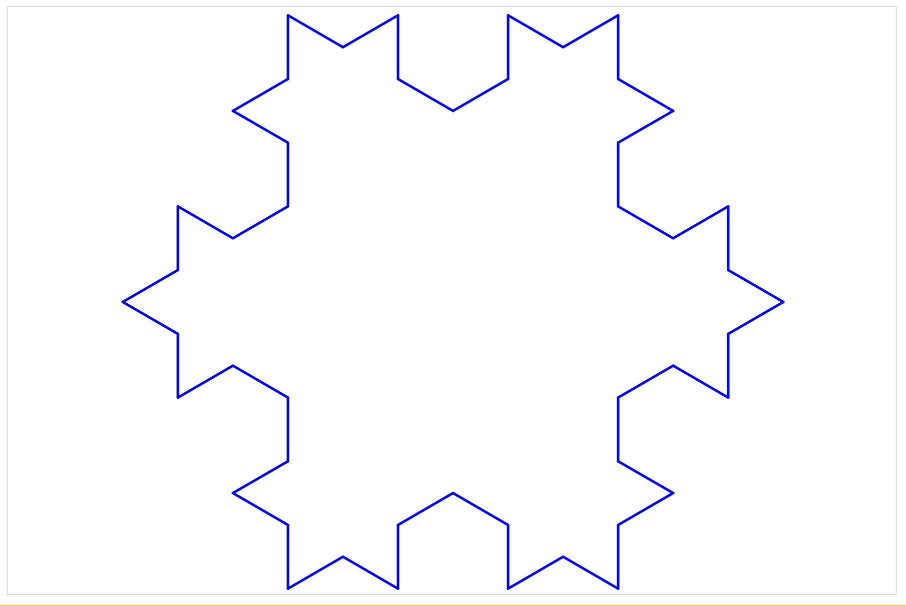


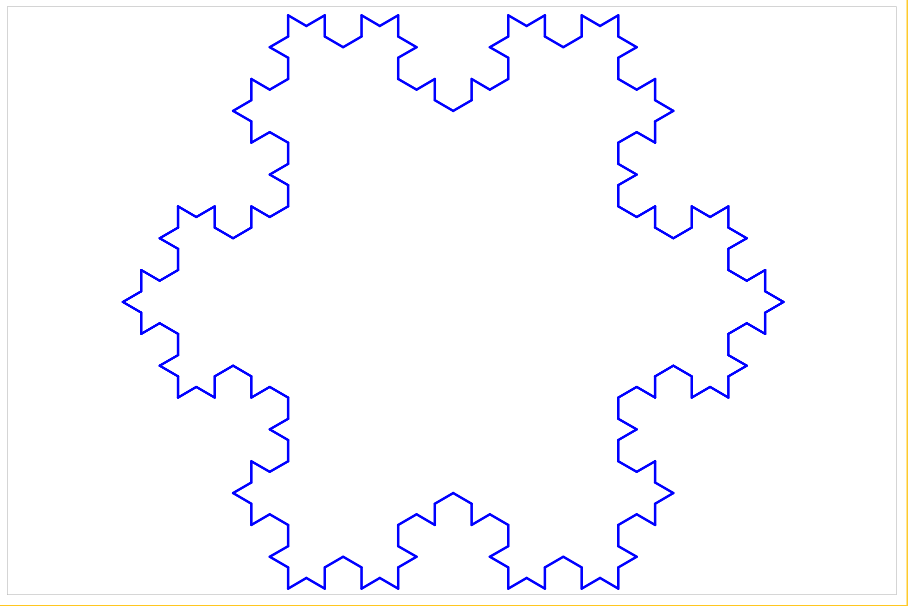


Fractal

1912

- kochcurve:=plot::Lsys(PI/3,"F++F++F","F"="F-F++F-F", Generations=3)
- plot(kochcurve)





The meaning of the symbol used for L–sys:

1913

- <u>Command:</u>
- Koch:= plot : : L sys

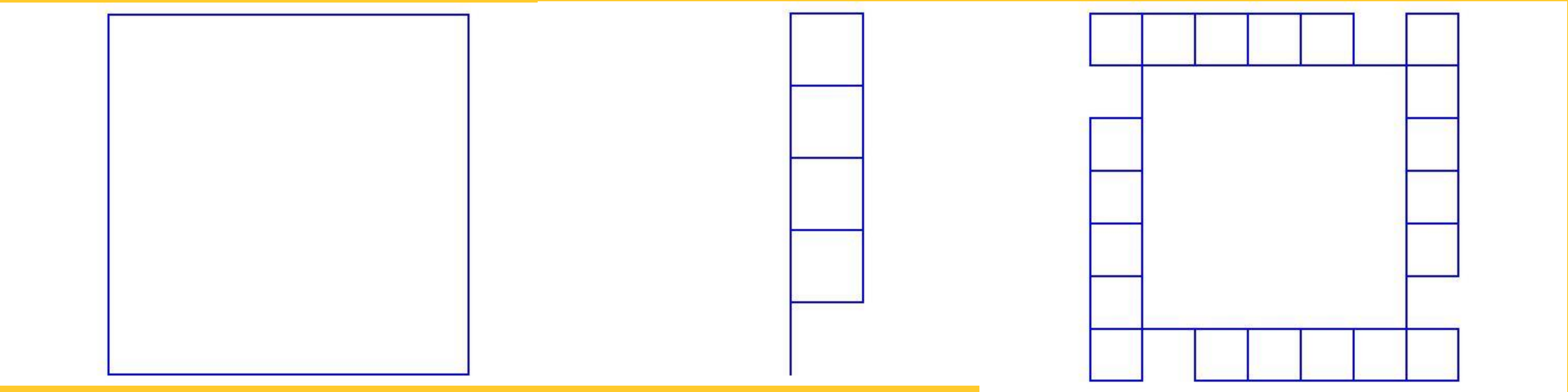
(P1/3, "F++ F++F", "F" = "F-F++F- F", Generations=4)

Angle Seed Iteration Rule No of generations

1. F means a single segment of length- 1
2. + means turn left ,
3. – means turn right,
4. "F"= Iteration Rule
5. "F – F++F-F" -> Each segment F is replaced by the turtle path F-F++F-F
6. Generations=5 Defines no of iterations
7. f means go forward without drawing a line,
8. [save the current position (branching symbol
9.] go back to the last saved position (Branching symbol)

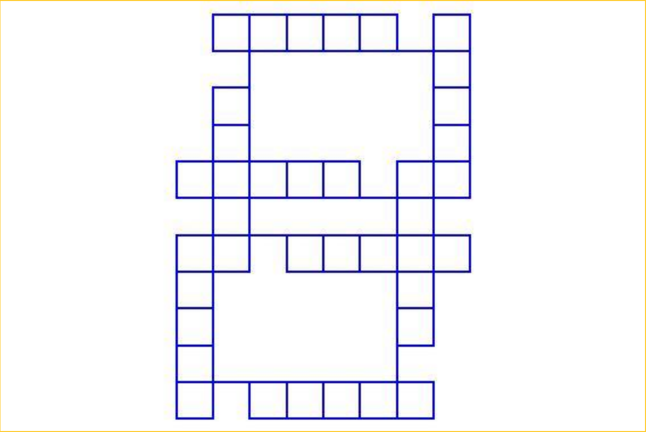
Example-2

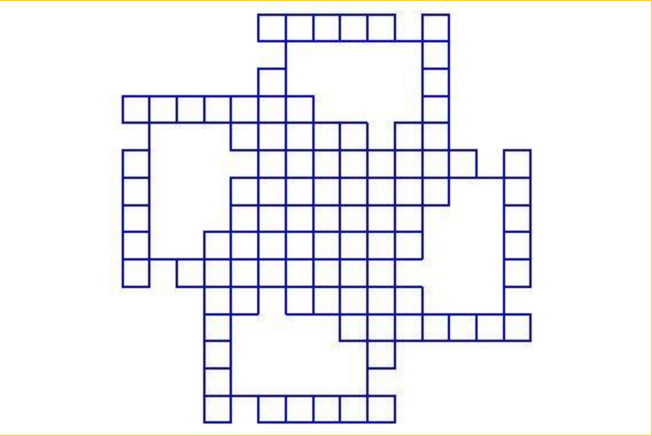
- [Koch1:= plot :: Lsys (PI/2) “F-F-F-F”,
- “F”=” FF –F-F-F”, Generations =2):
- plot (Koch1)

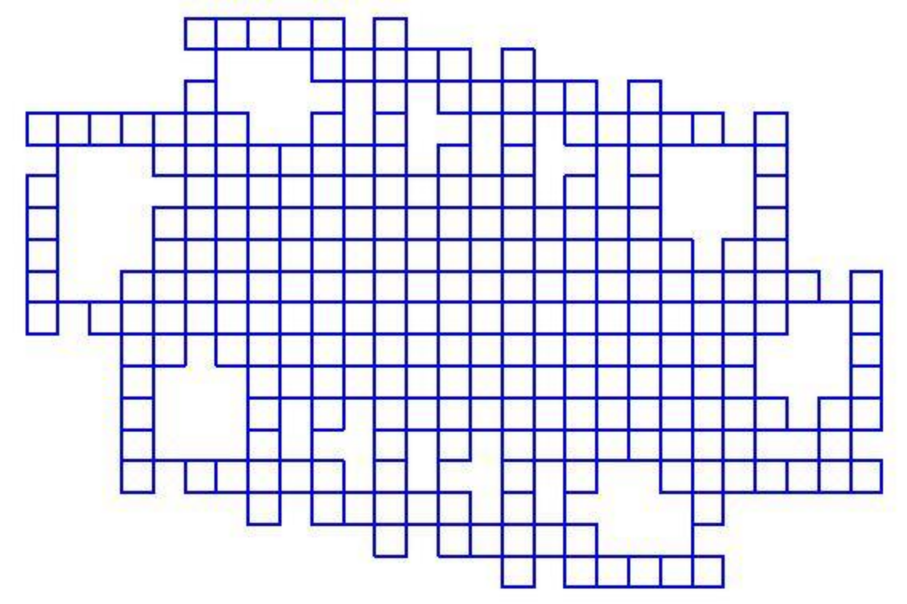


Koch Curve 4th, 5th and 6th Generation

1915







- Cantor Set

Cantor Set

1917

- Cantor set is another set of fractals
- The Cantor set is created by repeatedly taking out the middle third of all the line segment involved.
- All the points are distinct
- This is impossible to show in an image.
- All points are fused after fifth subdivision

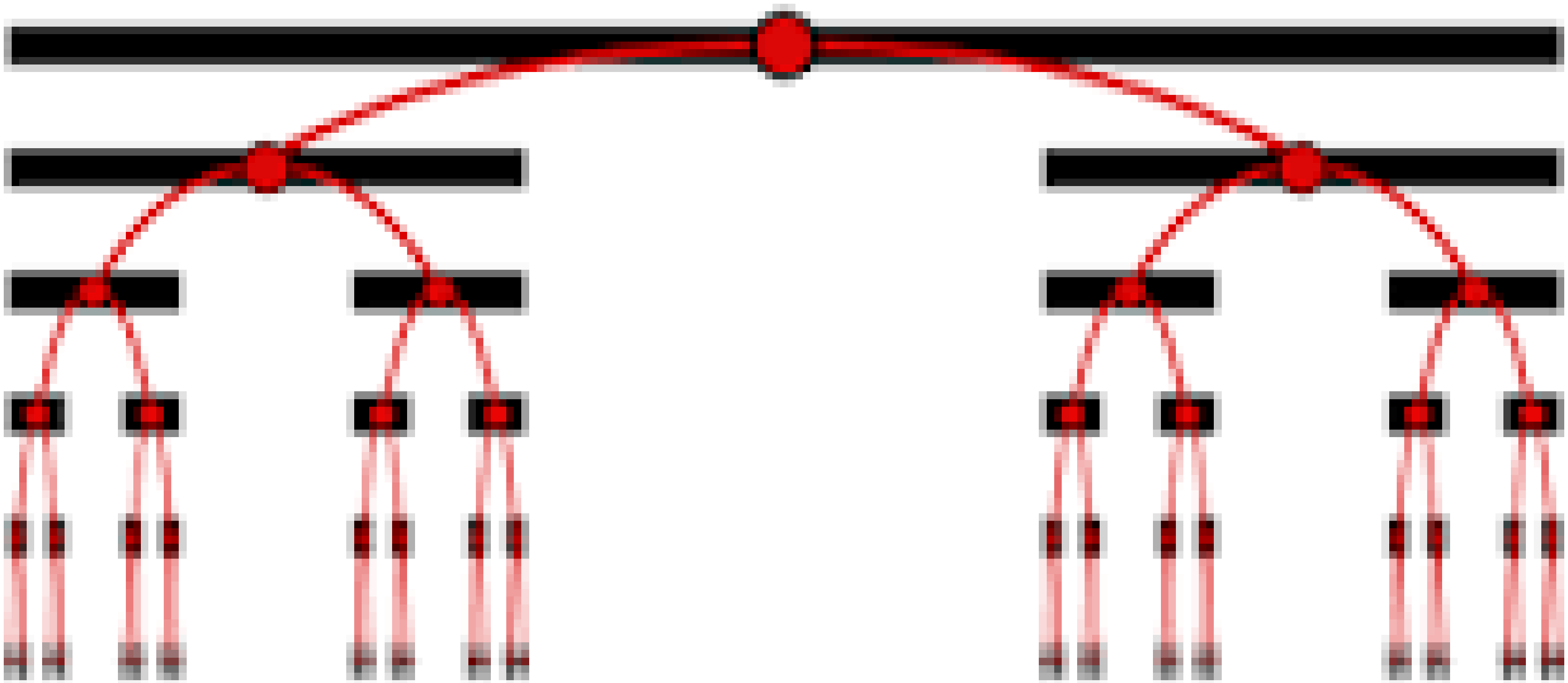
Cantor Set

1918

- In mathematics, the **Cantor set** is a set of points lying on a single line segment that has a number of remarkable and deep properties. It was discovered in 1874 by Henry John Stephen Smith[1][2][3][4] and introduced by German mathematician Georg Cantor in 1883.[5][6]

Cantor Set

1919



Length of the Cantor Set

- Let the length of the original line be one unit. 1
- One third of the length is taken out in first stage 1-1/3
- Two lengths of one ninth taken out in second stage

  2/3-2/9
- Four lengths of one twenty seventh taken out in third stage 4/9-4/27
- And the process goes on…

Length of the Cantor Set

- Total length eliminated is-

- $L=(1/3)+2(1/3)^2+2^2(1/3)^3+2^3(1/3)^4+\ldots..$
- $L=(1/3)\{(1+2(1/3)+2^2(1/3)^2+2^3(1/3)^3+\ldots\ldots$
- $L=(1/3)\{1+(2/3)+(2/3)^2+(2/3)^3+(2/3)^4\ldots\ldots$
- This is a geometric series with successive terms multiplied by 2/3 and 2/3 is less than one
- Multiplying both side by 2/3, we get-
- $(2/3)L=(1/3)\{(2/3)+(2/3)^2+(2/3)^3+(2/3)^4\ldots\ldots$

Length of the Cantor Set

- $(2/3)L=(1/3)\{(2/3)+(2/3)^2+(2/3)^3+(2/3)^4......$
- Previous equation-
  $L=(1/3)\{1+(2/3)+(2/3)^2+(2/3)^3+(2/3)^4......$
- To eliminate the tail (remaining term), we subtract – the two equations to get.

  $L-(2/3)L=(1/3)$

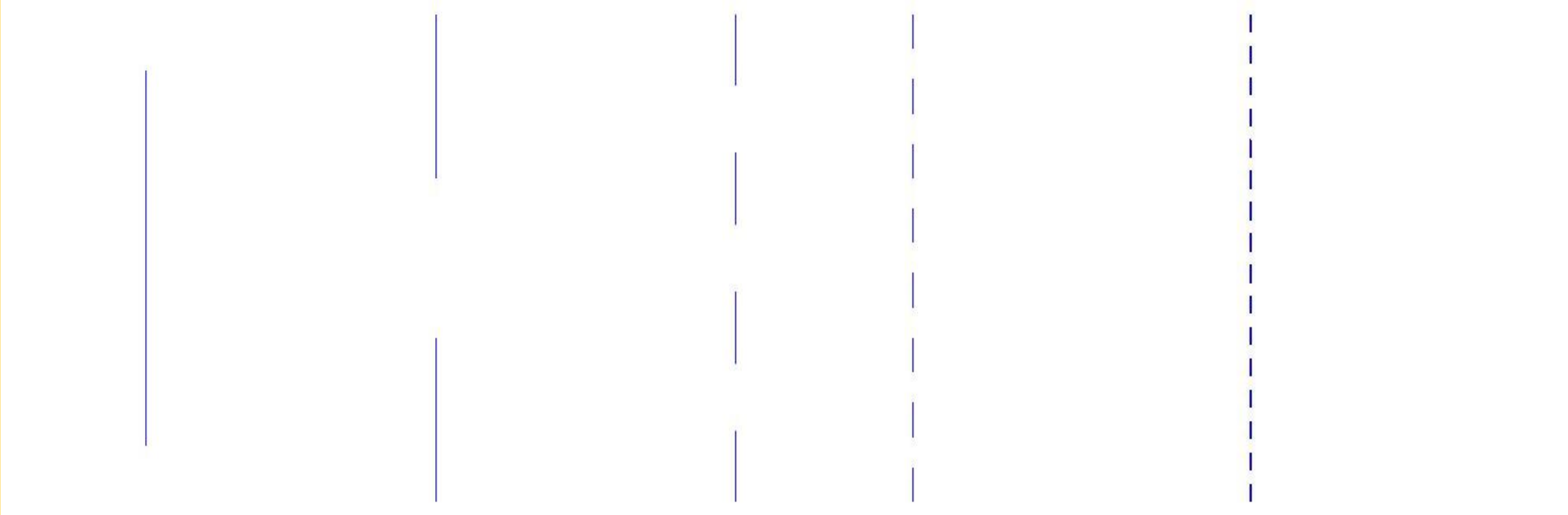
  Or $1/3\ L=1/3$

  Or $L=1$

  - Whole the length has been extracted stills leaves the points in the line.
  - The process creates a pattern with two similar copies at $1/3^{rd}$ Scale

Creating Cantor Sets

- Mupad Lsys gives very good way to create Cantor Curve using Lsys:
- [cantor0:=plot::Lsys(0,"FfF","F"="FfF",Generations =0): plot(cantor0)



- Sierpinski Triangle

The Sierpinski Triangle or Gasket

- A sierpinsky triangle is formed by recursively extracting triangular forms from within an original triangle

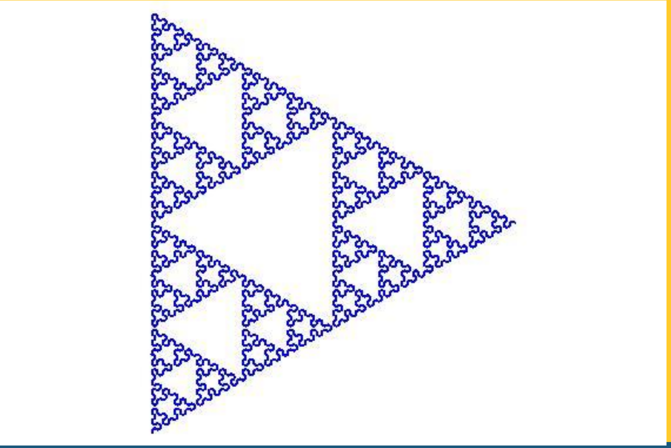
- At each stage three times as many triangles are extracted.
- Each being a quarter of the area of those used in the previous stage.

History of Sierpinski

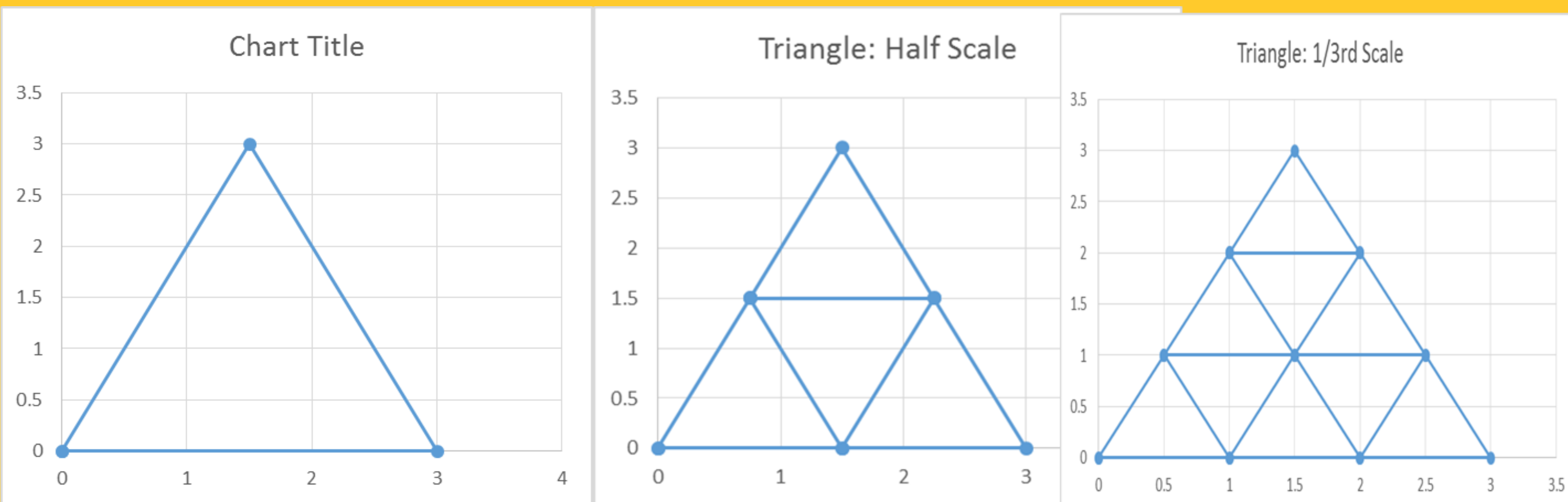
- It is named after the Polish mathematician Wacław Sierpiński, but appeared as a decorative pattern many centuries prior to the work of Sierpiński.[1]

Construction of The Sierpinski Triangle

- The total area extracted is equal to the area of the original triangle still there remains many points.
- The resulting fractal object contains three copies of itself at half linear scale. Lsys can be used to draw the curve
- l := plot::Lsys(PI/3, "R", "L" = "R+L+R", "R" = "L-R-L",
  "L" = Line, "R" = Line,
  Generations = 7):
  plot(l)



Sierpinski Triangle



- Peano Curve

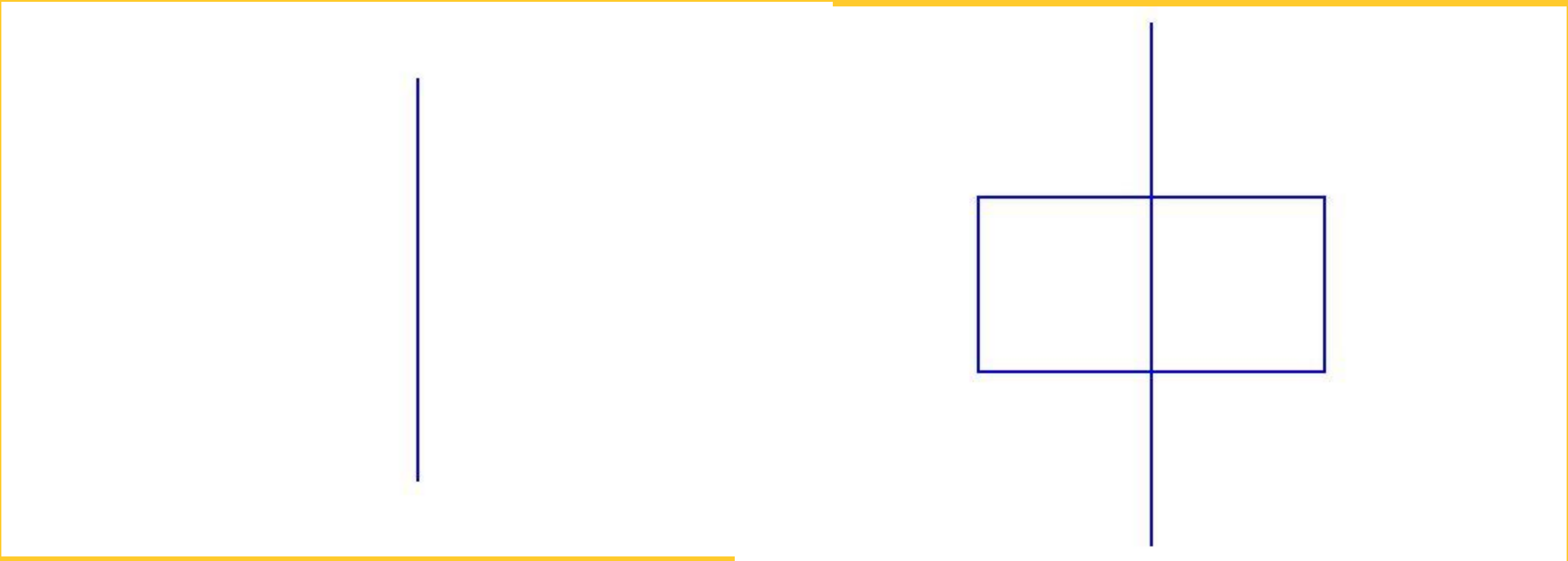
Peano Curve

- This fractal was created by Giuseppe Peano in 1890
- Peano Curves are called “Space Filling Curve”
- The peano curve visits every point within a two dimensional region

- Here also Lsys can be used to draw Peano Curve.
- The first three stage and fifth stage are shown below:

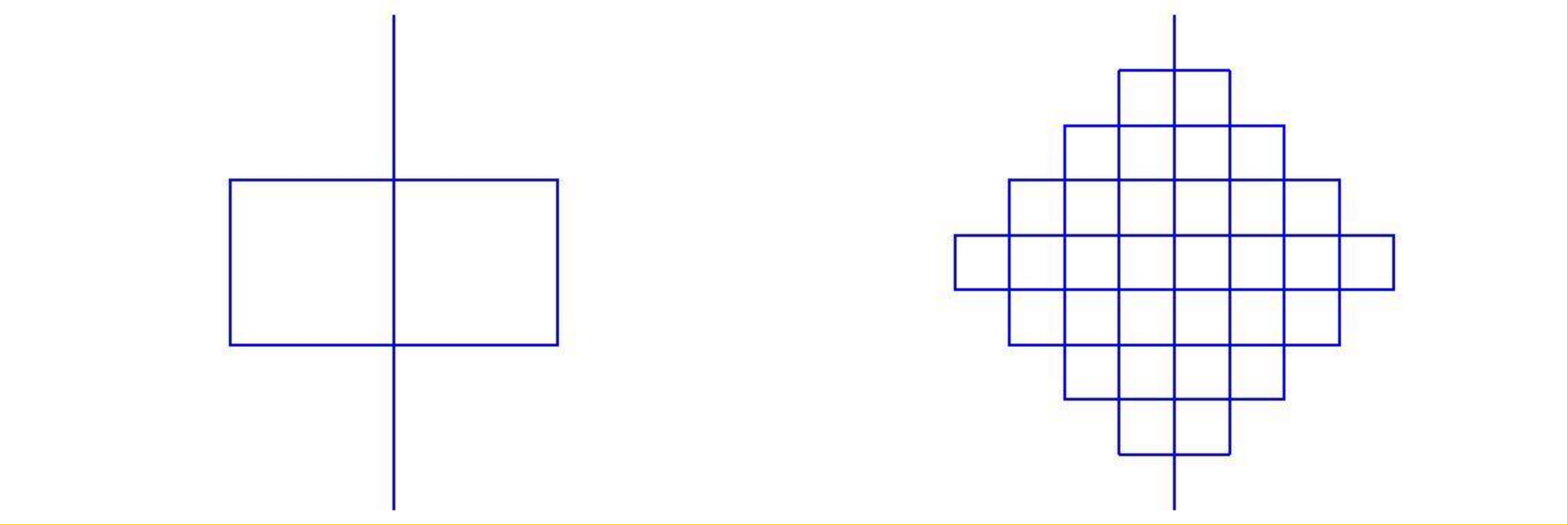
Peano Curve – Generations 0/1

- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"), peano::Generations := 0: plot(peano0)



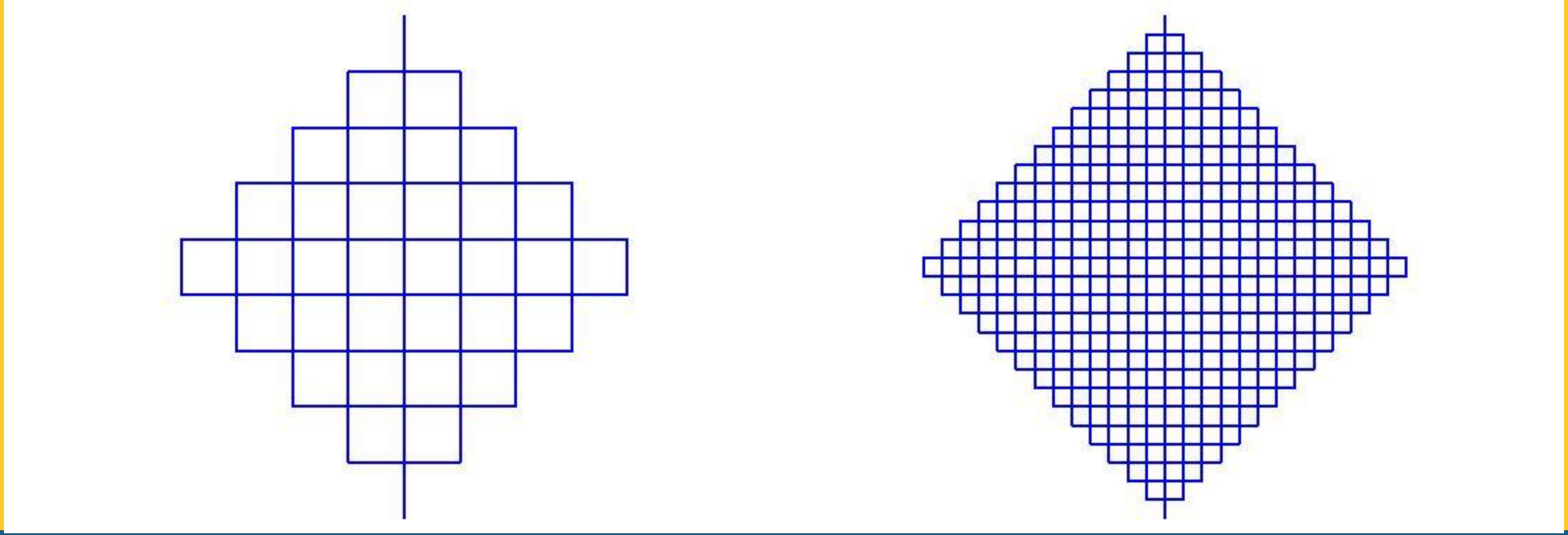
Peano Curve – Generation – 1 / 2

- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"), peano::Generations := 1: plot(peano0)



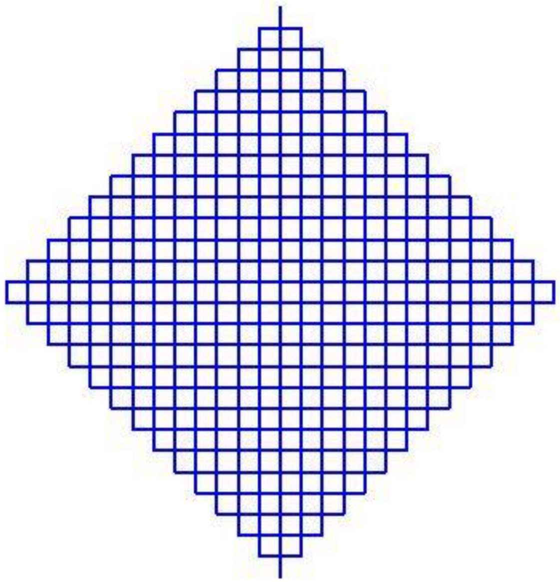
Peano Curve– Generation – 2/3

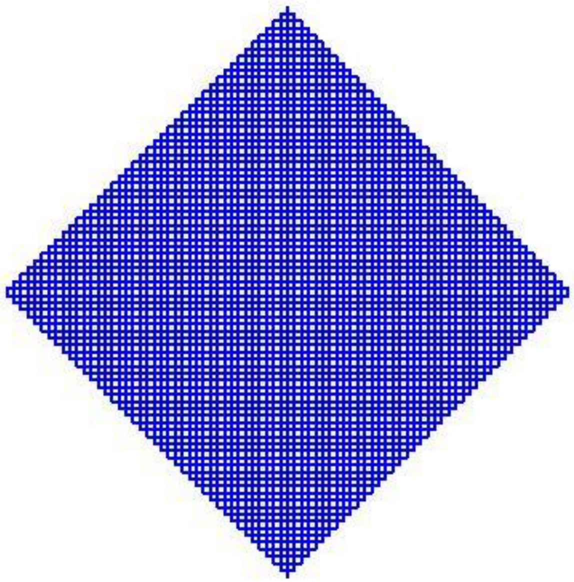
- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"),
  peano::Generations := 2:
  plot(peano0)



Peano Curve– Generation – 3/4

- peano0 := plot::Lsys(PI/2, "F", "F" = "F+F-F-F-F+F+F+F-F"),
  peano::Generations := 3:
  plot(peano0)

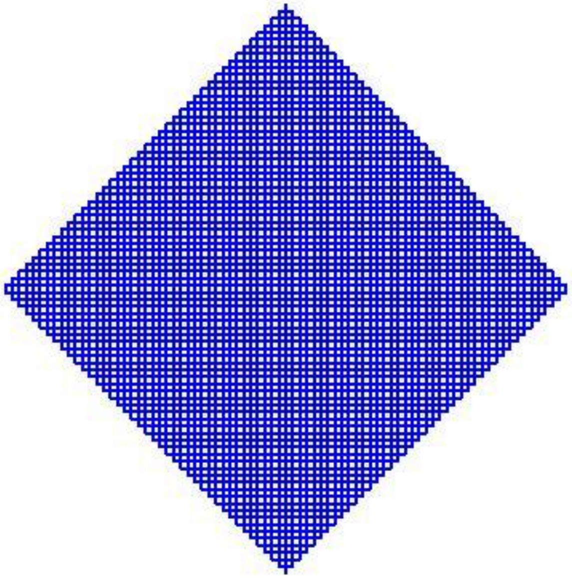


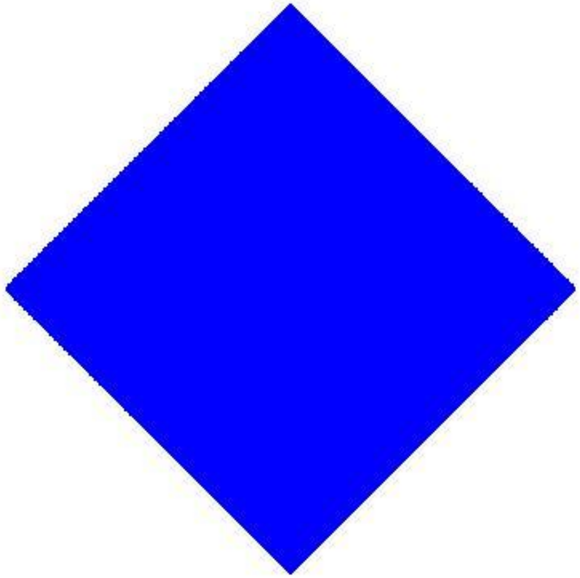


Peano Curve– Generation – 4/5

- peano0 := plot::Lsys(PI/2, "F",

"F" = "F+F-F-F-F+F+F+F-F"),
peano::Generations := 4:
plot(peano0)





Fractals and its Dimension

- Fractal and it's Dimension

Fractals and it's Dimensions

1937

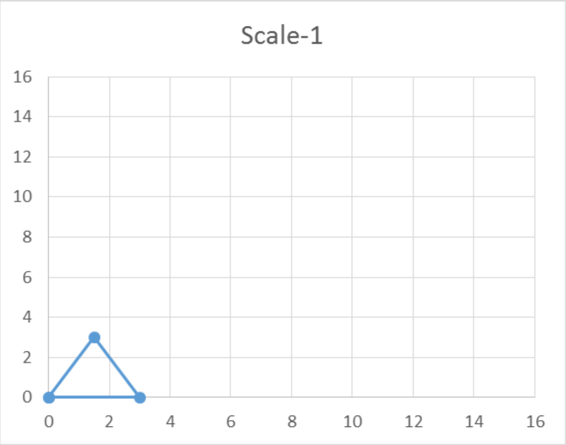
- When the space filling curve repeated several times, it fills the region.
- In it's construction, a replacement method is used.
- Every time, each line recursively replaced by nine other line segments at one third scale.
- This generates an object that contains nine copies of itself at one third scale.

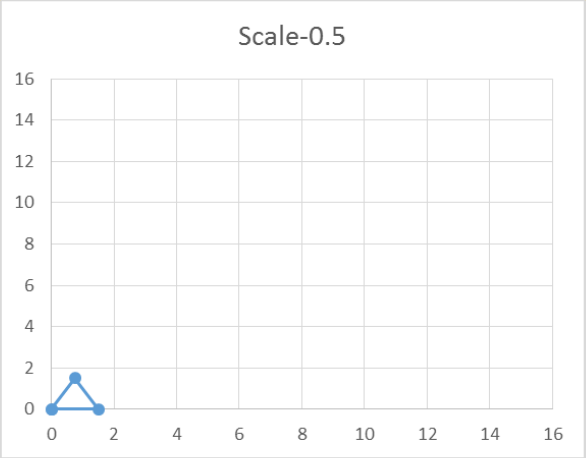
Fractals and it's Dimensions

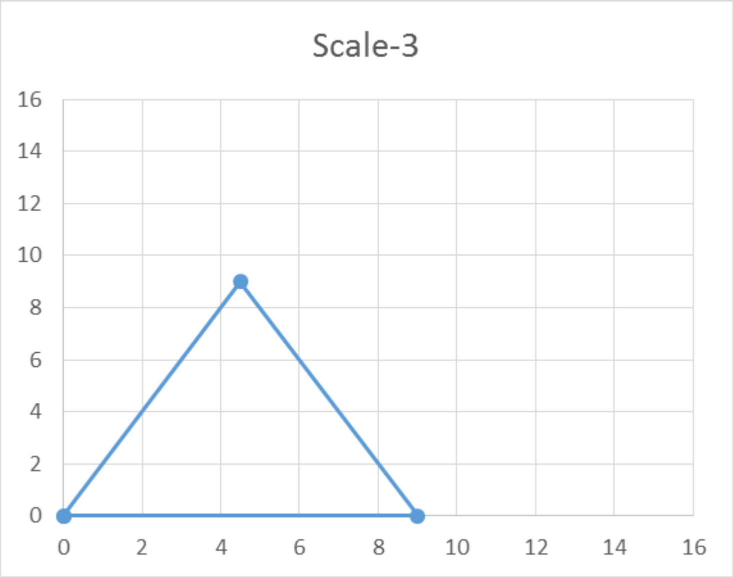
- We started with one dimensional object, whose length approaches infinity as the process continues, and end with an object that fills a region of two dimensional space.

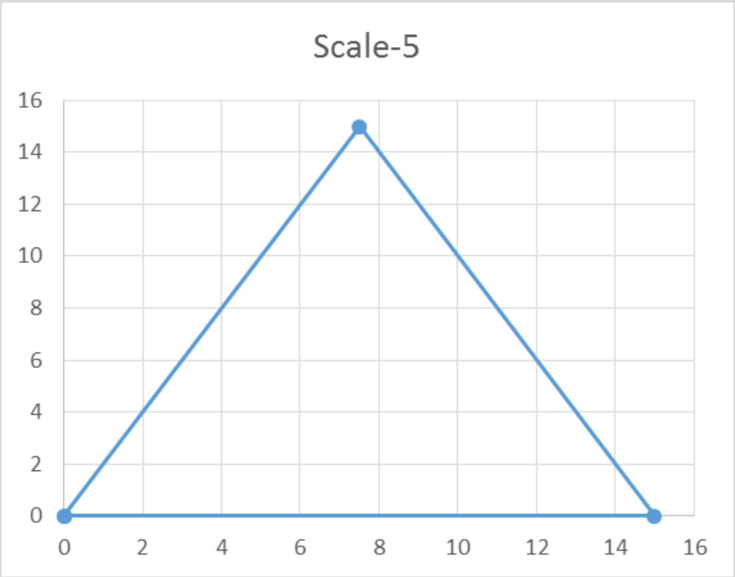
- Now the question comes – What is the dimension of these objects?

Scaling and Multiplication is not same



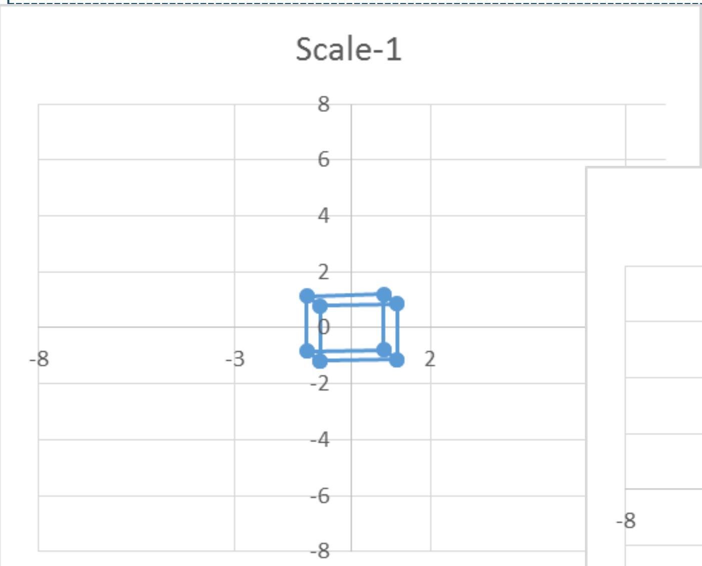


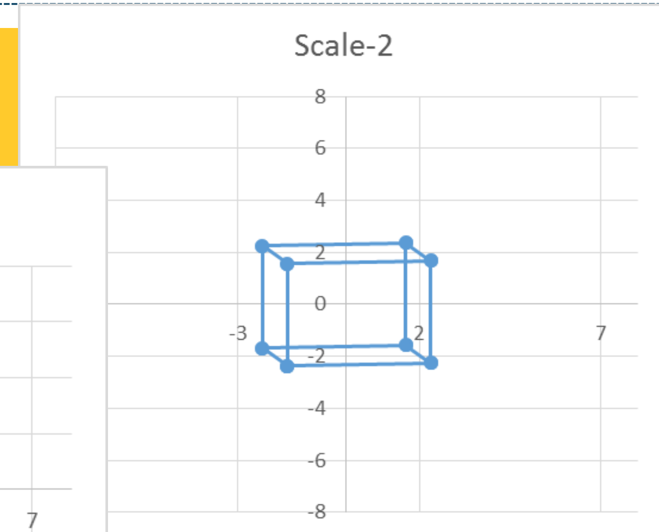


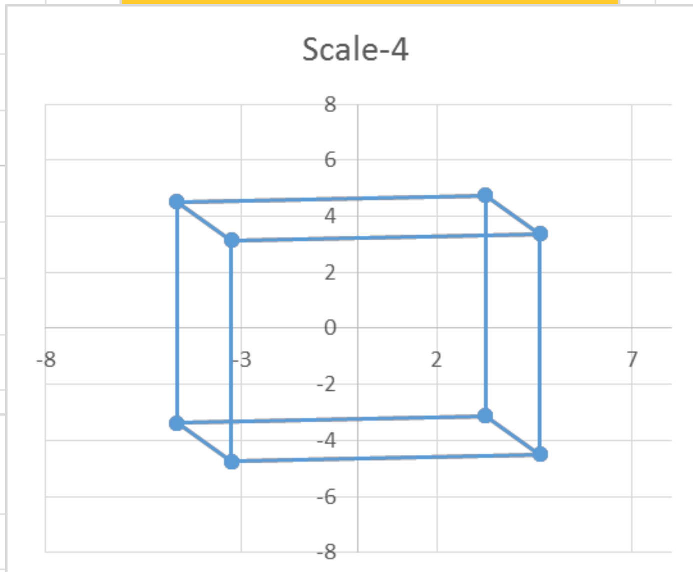


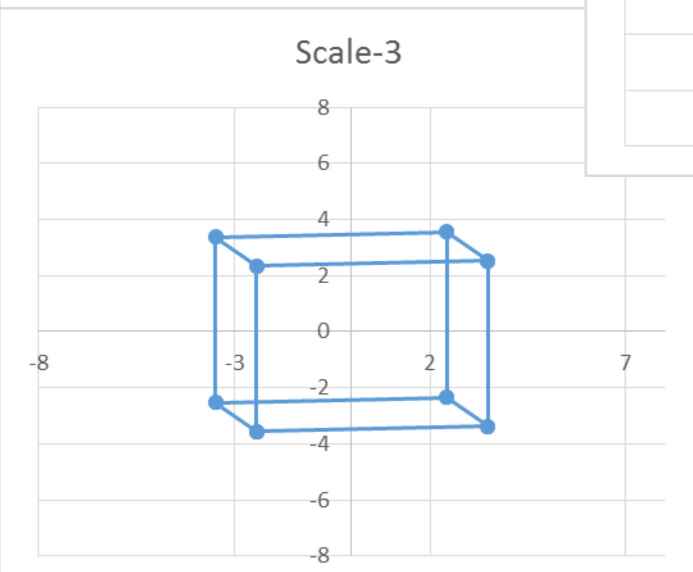
Scale of Cube 1 3 5
Self Similar

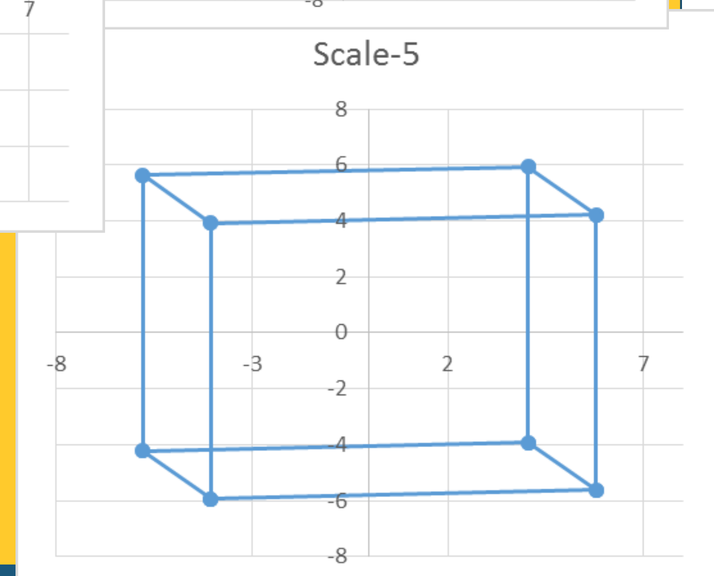
1940







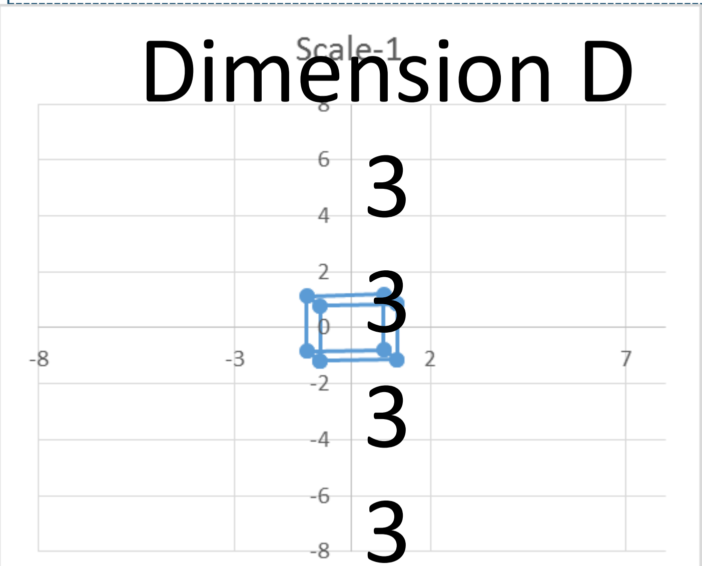


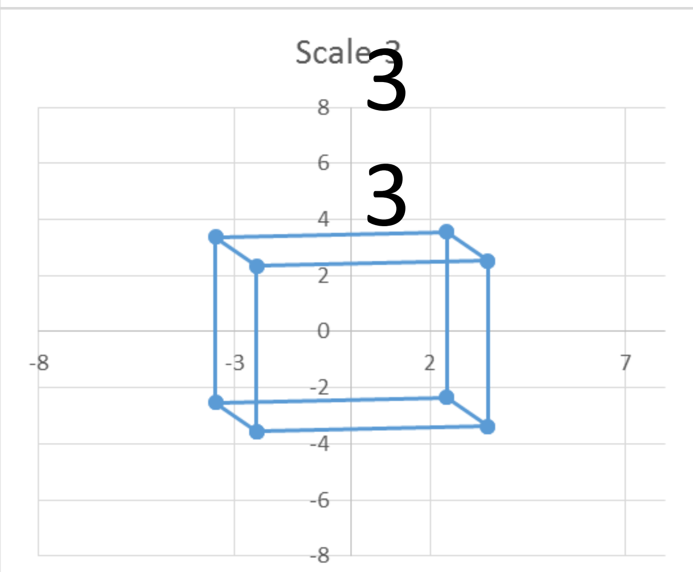


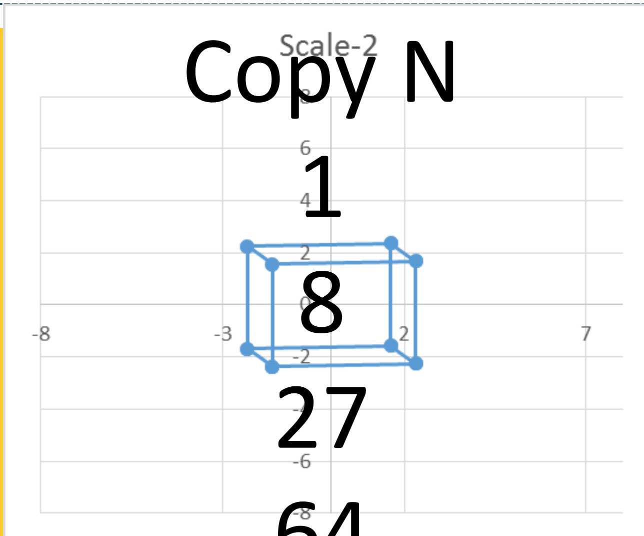
Scale of Cube 1 3 5
Self Similar

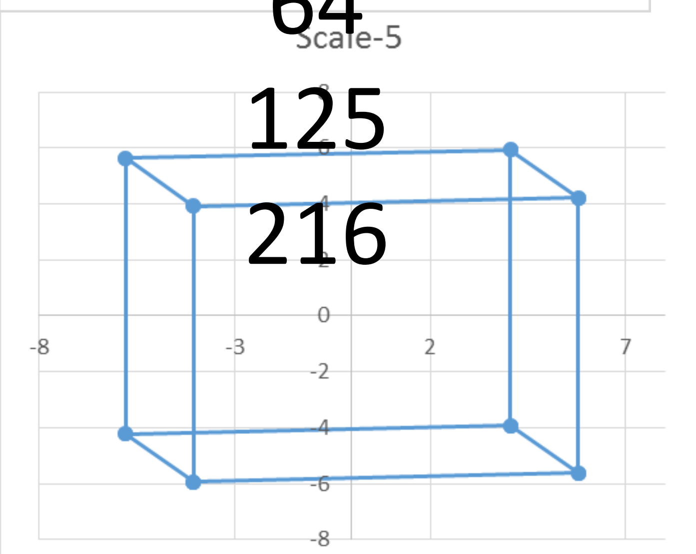
1941

Dimension D	Scale S	Copy N
3	1	1
3	2	8
3	3	27
3	4	64
3	5	125
3	6	216

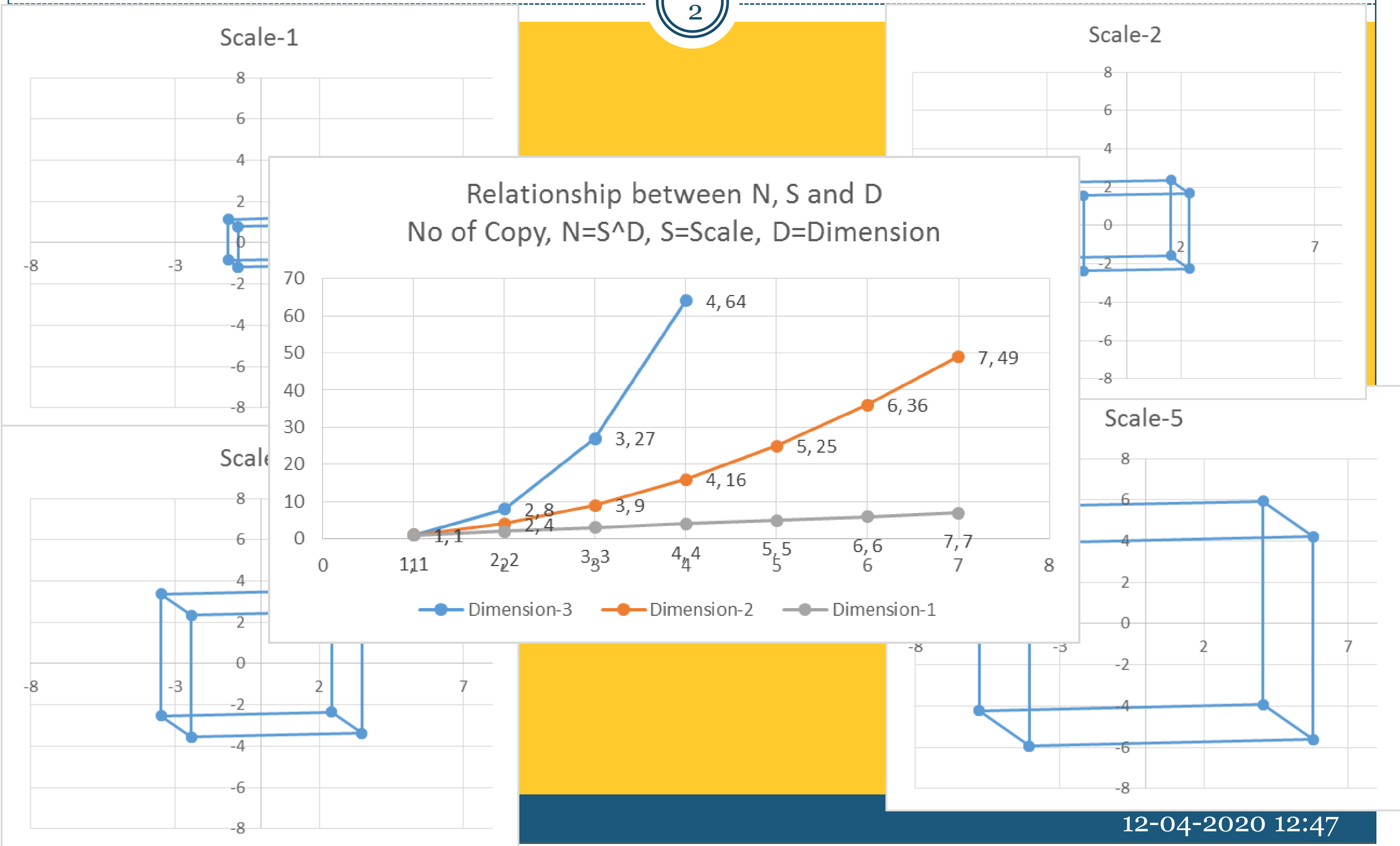




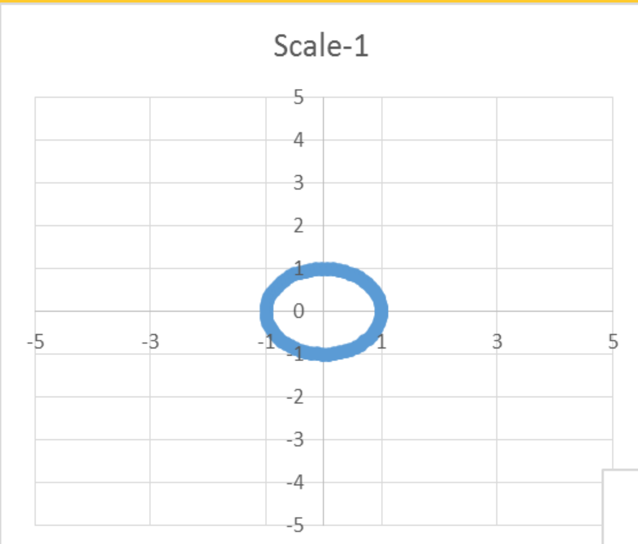


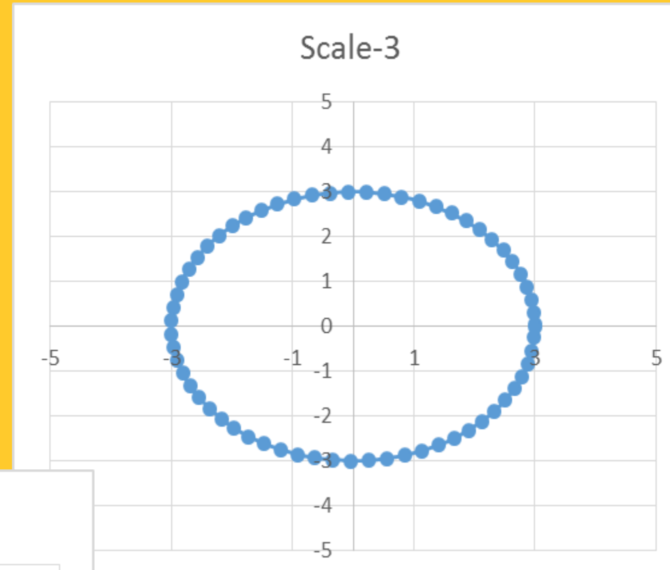


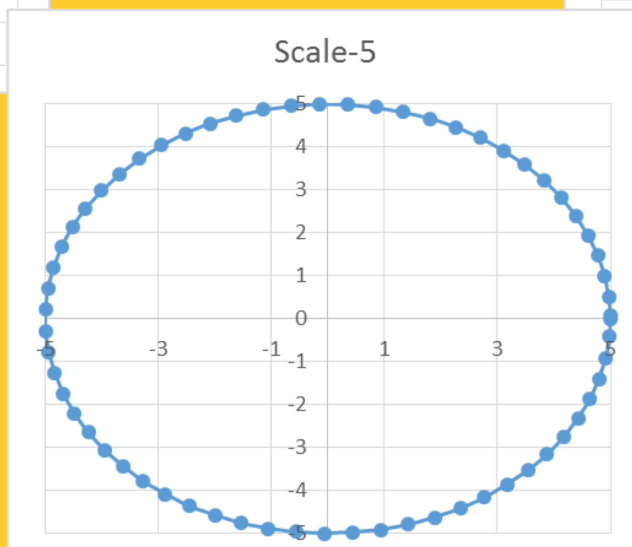
Relationship between N, S and D
No of Copy, N=S^D, S=Scale, D=Dimension



Scaling of Circle
Scaling Rule Follows But Not Self Similar





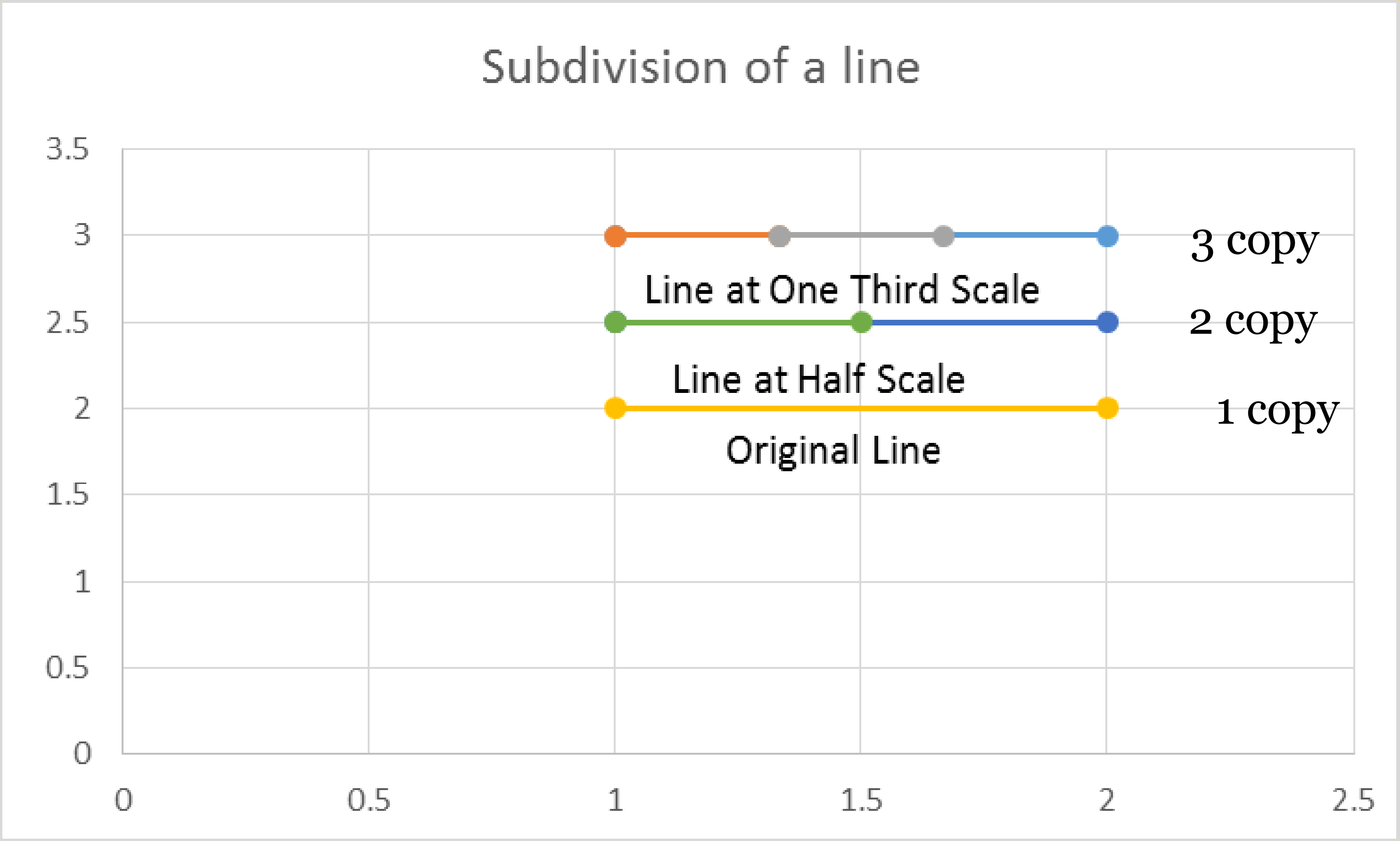


Moving Reverse-What is Dimension?

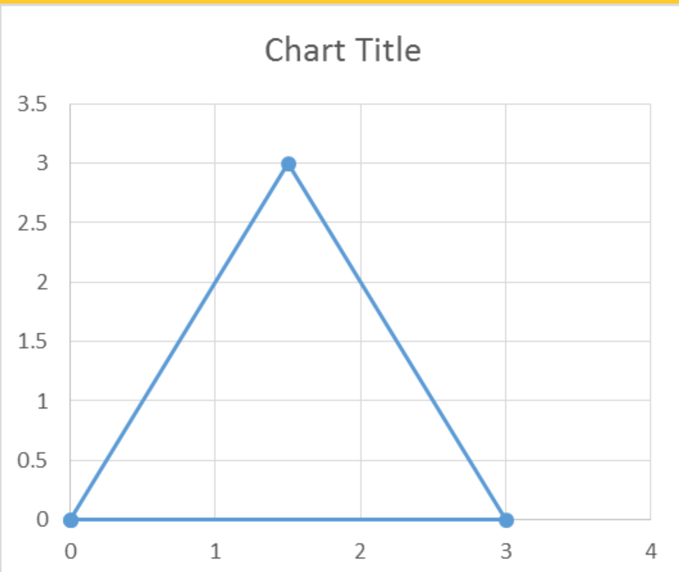
- Point – 0 Dimension
- Line – 1 Dimension – ½ scale – 2 copy, 1/3rd scale – 3 copy
- Triangle – 2 Dimension -1/2 scale, 4 copy, 1/3rd scale – 9 copy
- Square – 2 Dimension – ½ scale, 4 copy, 1/3rd scale – 9 copy
- Cube – 3 Dimension -1/2 scale, 8 copy, 1/3rd scale – 27 copy

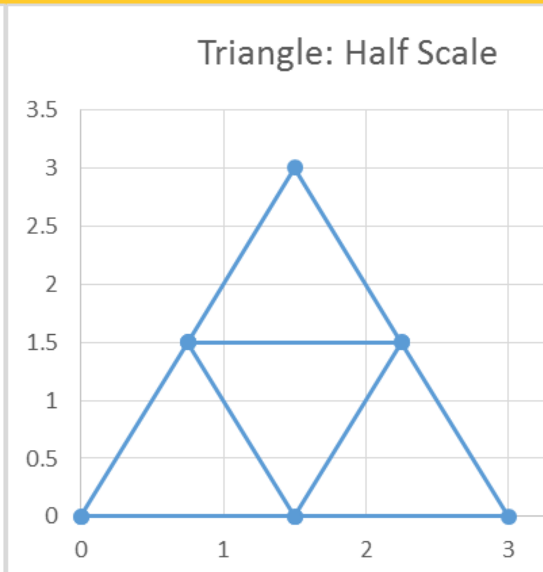
Fractals and it's Dimensions

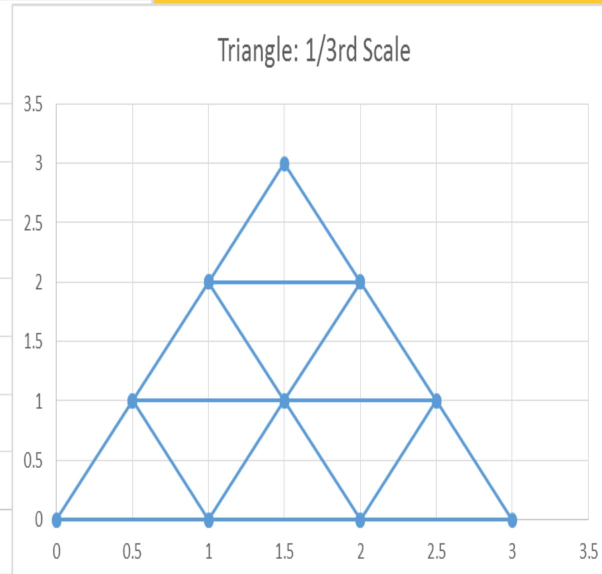
1945



Fractals and it’s Dimensions







N=S^D, S=2, D=2, N=4

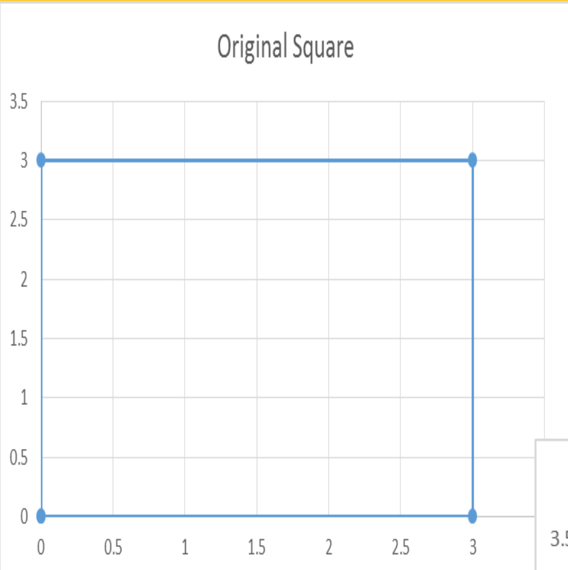
S=3, D=2, N=9

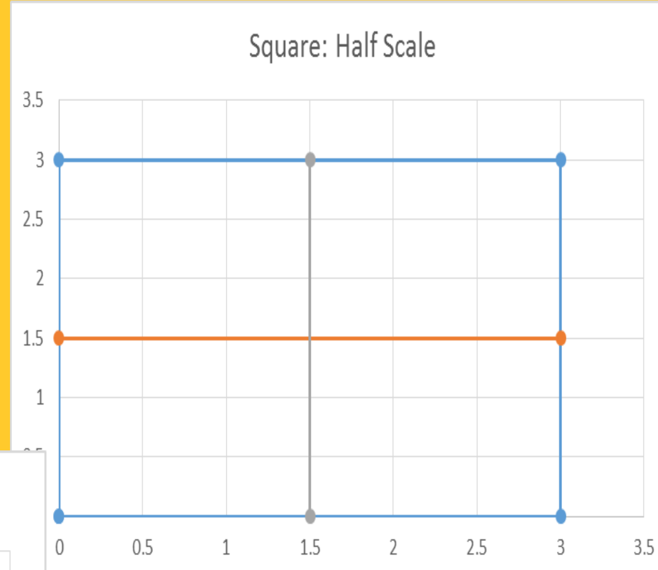
S=4, D=2, N=16

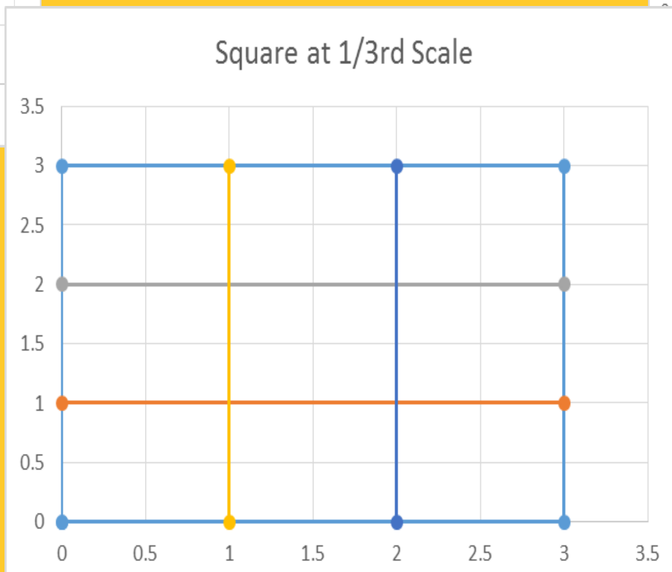
S=5, D=2, N=25

Fractals and it's Dimensions

1947

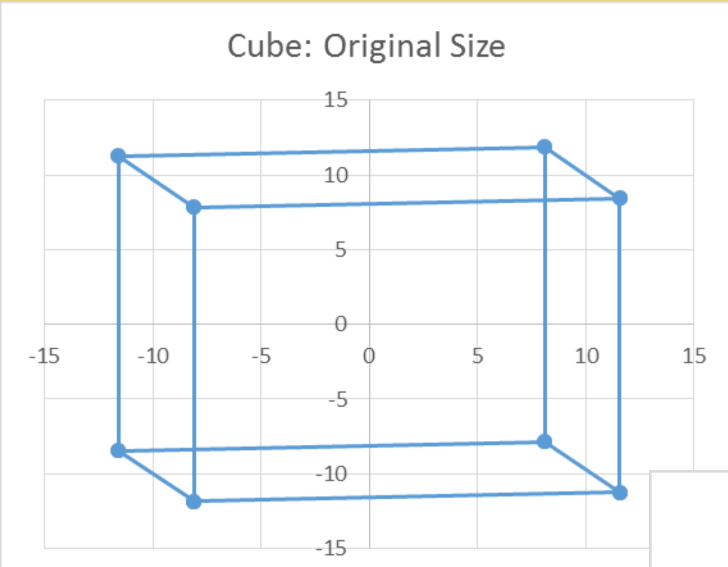


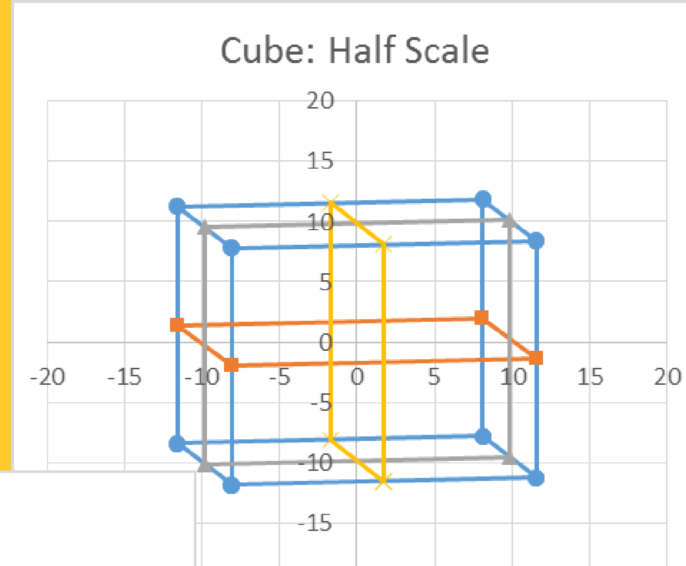




N=S^D, S=3, D=2, N=9

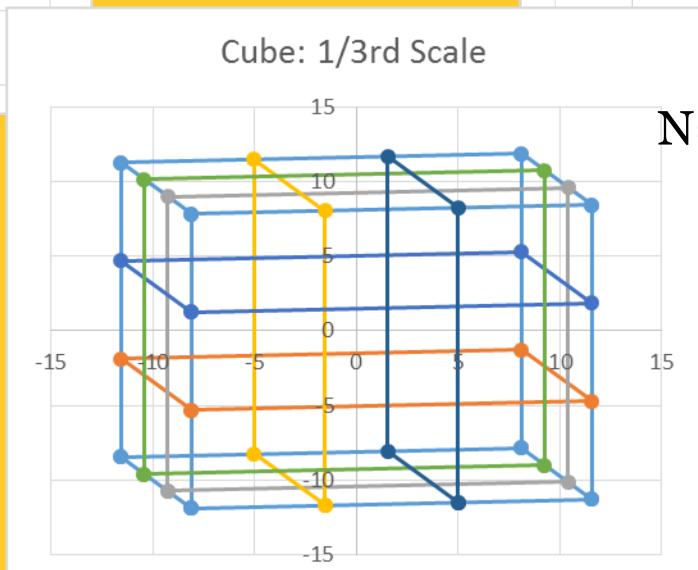
Fractals and it's Dimensions





$N=S^D$, $S=2$, $D=3$, $N=8$

$N=S^D$, $S=1$, $D=3$, $N=1$



$N=S^D$, $S=3$, $D=3$, $N=27$

Fractals and it's Dimensions

- Line is an one dimensional object
- Line contains 2 copies at half scale
- It contains 3 copies at one third Scale
- It contains 4 copies at one forth scale

Triangle and it's Dimensions

- Triangle is a two dimensional object

- Triangle contains four copies at half scale
- It contains nine copies at one third scale

Square and it’s Dimensions

- Square is a two dimensional object
- Square contains four copies at half scale
- It contains nine copies at one third scale

Cube and it's Dimensions

- Cube is a three dimensional object
- Cube contains eight copies at half scale
- It contains 27 copies at one third scale

Fractals and it's Dimensions

- It is seen that the number of copies, N, the scale factor, f, and the dimension, D, of the object are related.
- The relation among the three parameters are given by-
- $N=(1/f)^D$
- Taking log in both side, we get
- $\mathrm{Log}(N)=D*\log(1/f)$
- $D=\log(N)/\log(1/f)$

Fractal Dimension

Sl No	Fractal	N	F=1/f	D=Log(N)/log(F)	
1	Cantor Set	2	3	0.63	
2	Koch Curve	4	3	1.26	
3	Sierpinski Gasket	3	2	1.58	
4	Peano Curve	9	3	2	
5	Sierpinski Tetrahedron	4	2	2	

Fractal Dimension

1955

- Cantor set is more than a mere point, D=0.63
- Koch set is more than a curve, D=1.26
- Sierpinski set is less than a triangle, D=1.58
- Peano curve start as curve fills a plane, D=2

Fractals and Naturally occurring objects

Naturally Occurring objects:

- Clouds
- Coast Line
- Fire
- Terrain
- Mountains
- Forests
- Water Falls
- Waves
- Galaxies

Fractals and Naturally occurring objects

- Sections of these objects are not exact copies of the whole but their general features are, on the whole, indistinguishable from the over all form.

- There is an absence of scale about such fractals
- The fractal property of sub divisibility is hold up because it end up with a single sand grain.
- But if we consider a reasonable lengths, the fractal properties of self similarity and sub divisibility are maintained.

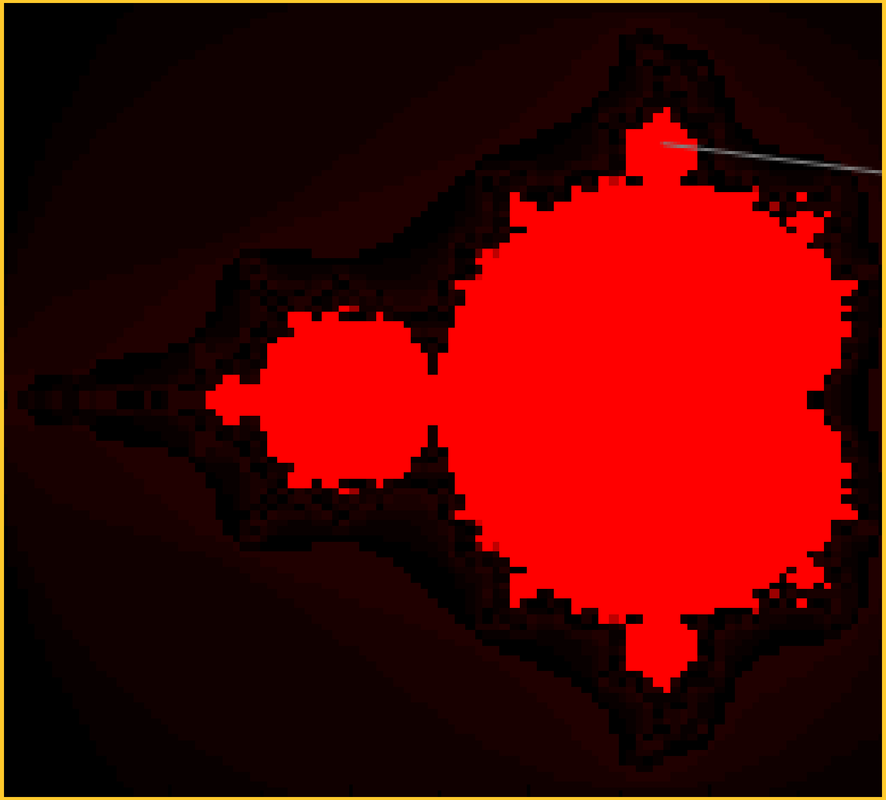
Fractals from Functions

- There are fractals which are created from repeated application of mathematical formulas

- These fractals are Julia set, Mandelbrot set etc.

Julia and Mandelbrot Set

- Here is a picture of Mandelbrot Fractal



Gaston Julia (right), with Gustav Herglotz, comparing dogs	
Born	3 February 1893 Sidi Bel Abbes, French Algeria
Died	19 March 1978 (aged 85) Paris
Nationality	French
Fields	Mathematics
Institutions	University of Paris

Julia and Mandelbrot Set

- These fractals are based in complex plane
- A complex number can be written as z=x+iy
- The main engine is a loop of instructions that takes its starting complex number and applies the arithmetic rules to it.
- For a Mandelbrot set, the rule is
  - $z=z^2+c$
- Here z begins with zero and c is a complex number corresponds to the point to be tested

Julia and Mandelbrot Set

- The loop continues like –
  - Take 0, multiply it by it self and add the starting number, c
  - Take the result as starting number, multiply it by itself and add the starting number, c.
  - Continue the cycle
- To break the loop, the loop needs to watch the running total.
- If the total heads to infinity, moving further and further from the center, the original point does not belong to the set.

Julia and Mandelbrot Set

- If the running total becomes greater than 2 or smaller than -2, it is surely heading off to infinity.

- If the program repeats many times without becoming greater than 2, then the point is part of the set.
- How many times depends on the amount of magnifications. It can be 100, 200 or any number even 1000.

Julia and Mandelbrot Set

- The program must repeat this process for each of thousand of points on a grid, with a scale that can be adjusted for greater magnification.
- Each of the point inside and outside of the set are colored differently.
- The colors reveal the contours of the terrain of the fractal set.

Mathematics of Mandelbrot Set

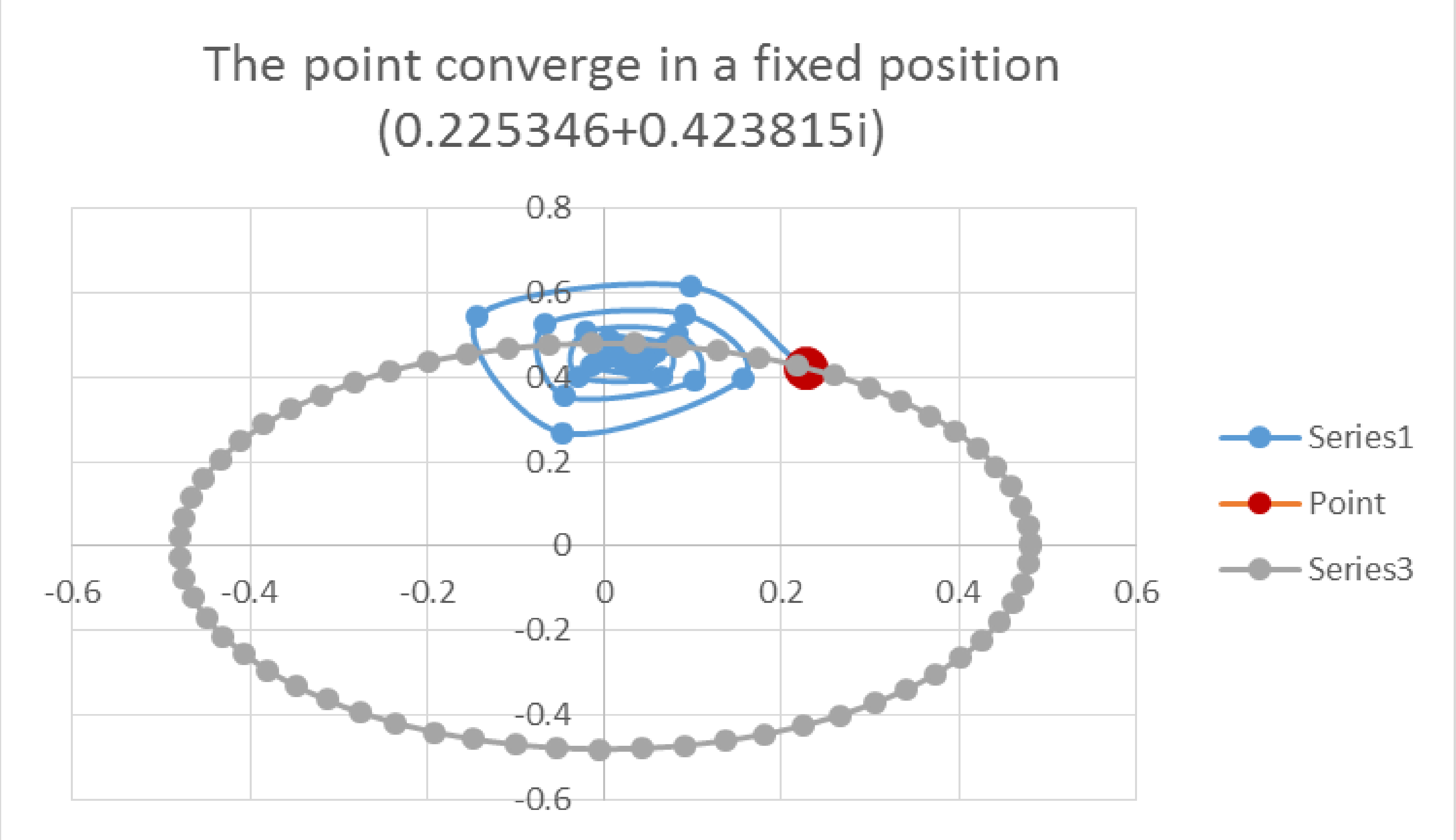
- Any complex number can be represented as x+i y
- In polar form it is represented as r (cos (t)+i sin (t))
- Any complex number has two properties, magnitude or absolute value or length, r= sqrt(x^2+y^2) and angle or amplitude t = atan(y/x)
- When a complex number is squared, z^2, it's absolute value gets squared and amplitude gets doubled.

Mathematics of Mandelbrot Set

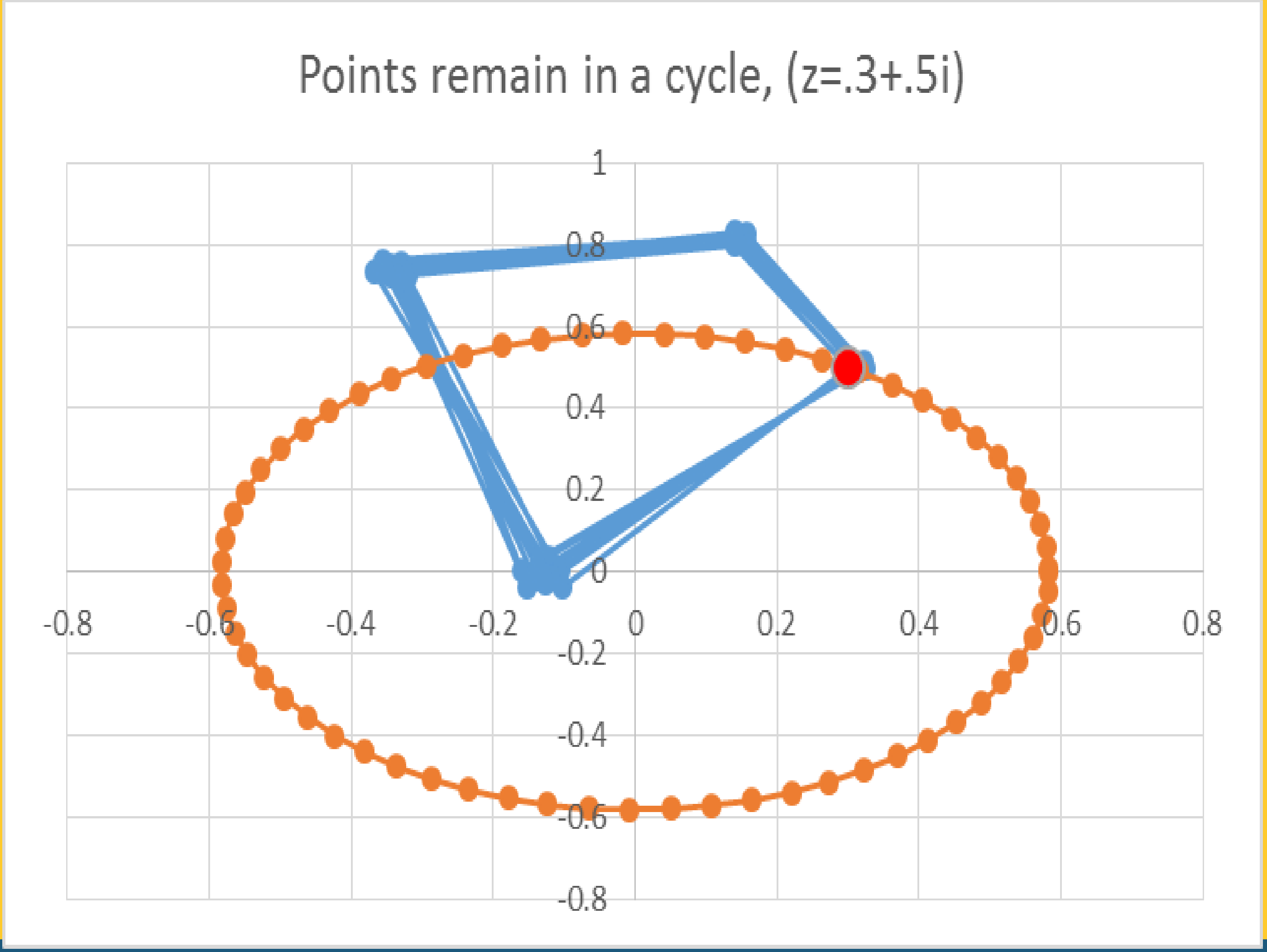
- We know, for a Mandelbrot set, the rule is
  - $z=z^2+c$
- Here z begins with zero and c is a complex number corresponds to the point to be tested.
- As the iteration continues, three cases may arise –
  - The sequence diverge – meaning that the points move further and further from original location
  - It may converge on a single fixed location, or,
  - It may remain in a cycle fairly close to the original point

Three Cases of Iteration Results

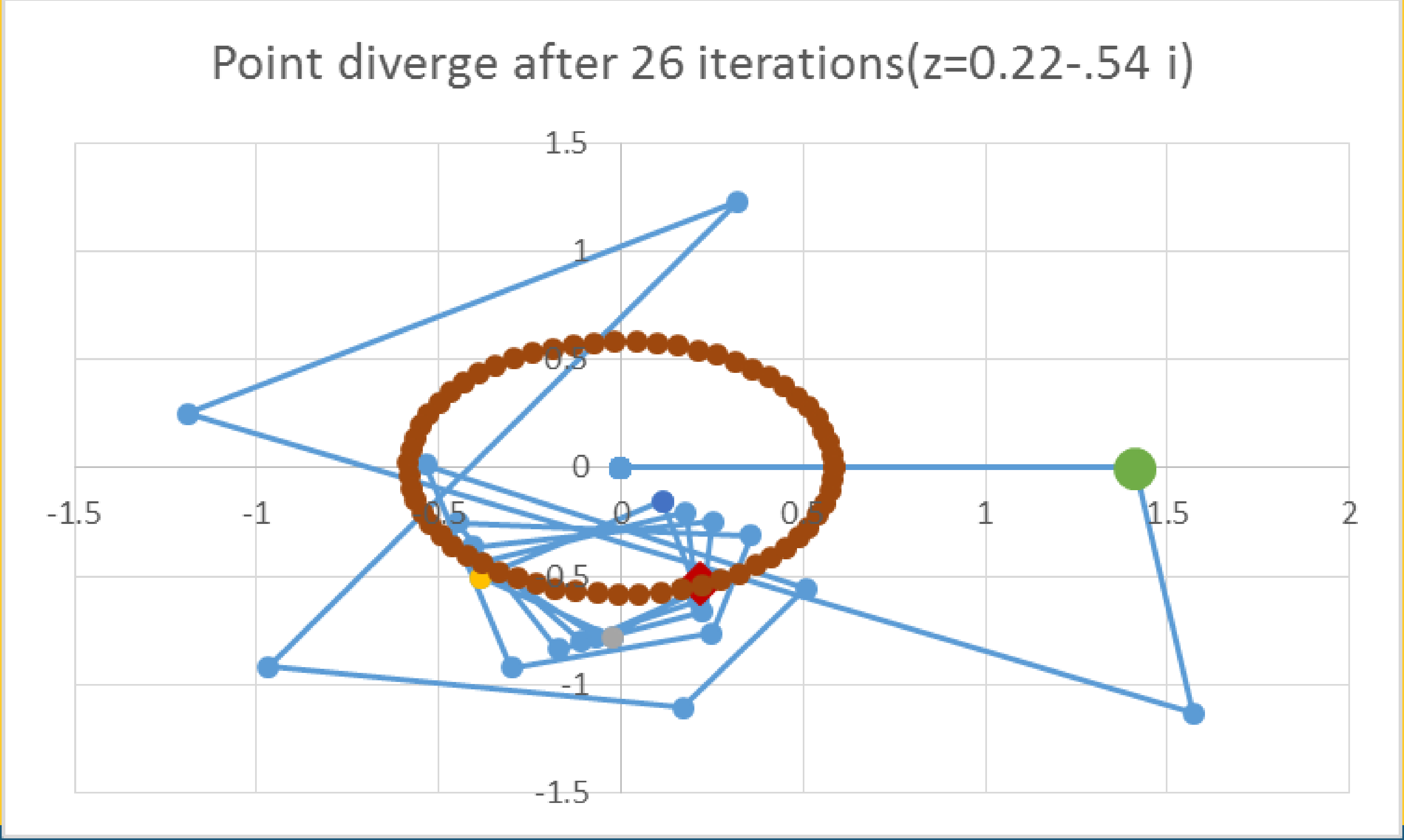
1967



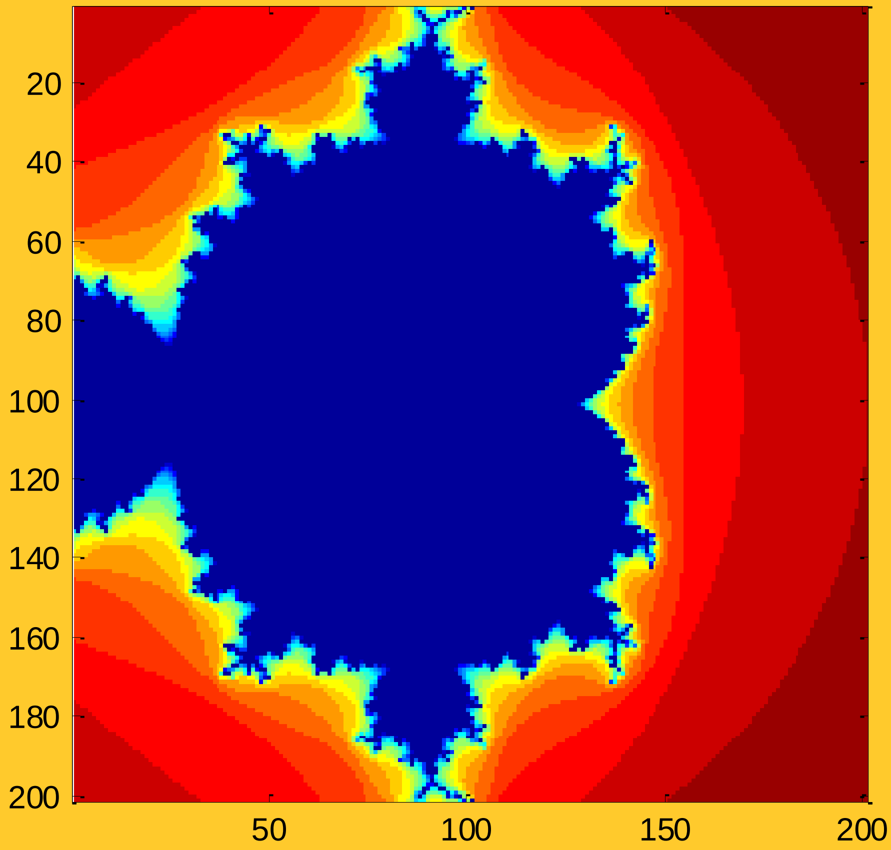
Three Cases of Iteration Results



Three Cases of Iteration Results



The formation of Mandelbrot Set



Program for Mandelbrot Set

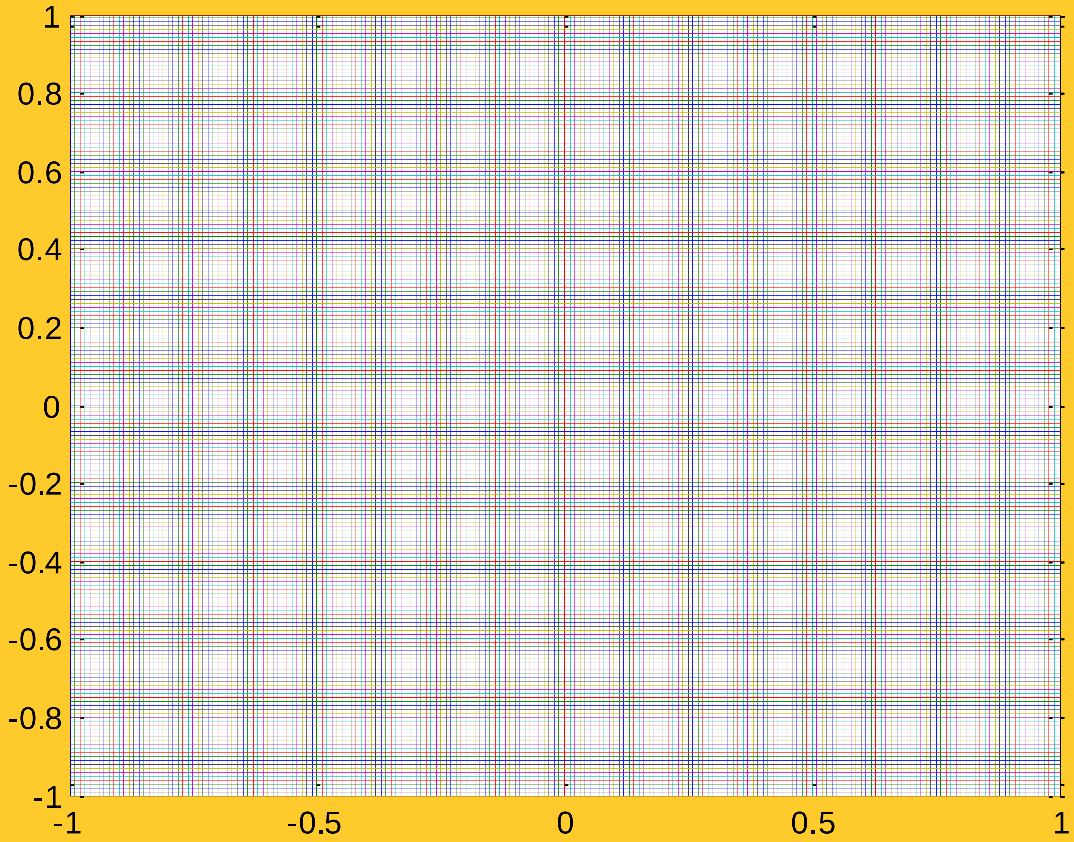
1971

Important Parameters	Grid Size, Depth

X=-1:.01:1
Y=X'
[x,y]=meshgrid(X,Y)
figure
plot(x,y,y,x)
grid
z0=x+i*y
n=length(X)
z=zeros(n,n)
c=zeros(n,n)
depth=20
for k=1:depth
   z=z.^2+z0
   c(abs(z)<2)=k
end
	figure
	c
	
	image(c)
	axis image
	colormap(flipud(jet(depth)))
	figure
	plot(real(z),imag(z))
	%axis([-10 10 -10 10])
	figure
	surf(c)

Grid-0.01

1972

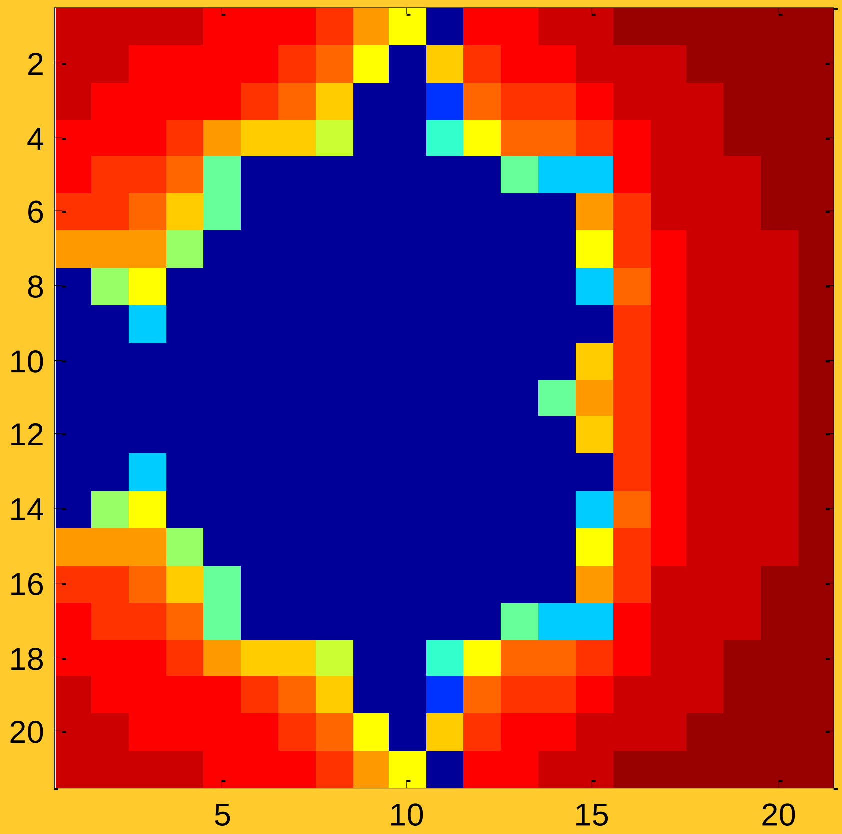


Grid – 0.1

1973

Mandelbrot Set

1974

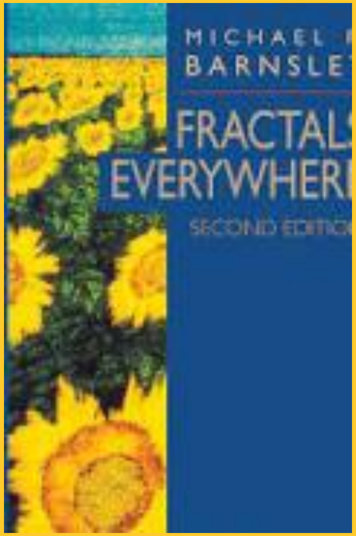


Fractal Fern

1975

Michael Fielding Barnsley is a British mathematician, researcher and an entrepreneur who has worked on fractal compression; he holds several patents on the technology. **Born**: 1946, Folkestone, United Kingdom

Fractal Fern:
They generate and plot a potentially infinite sequence of random, but carefully choreographed, points in the plane.



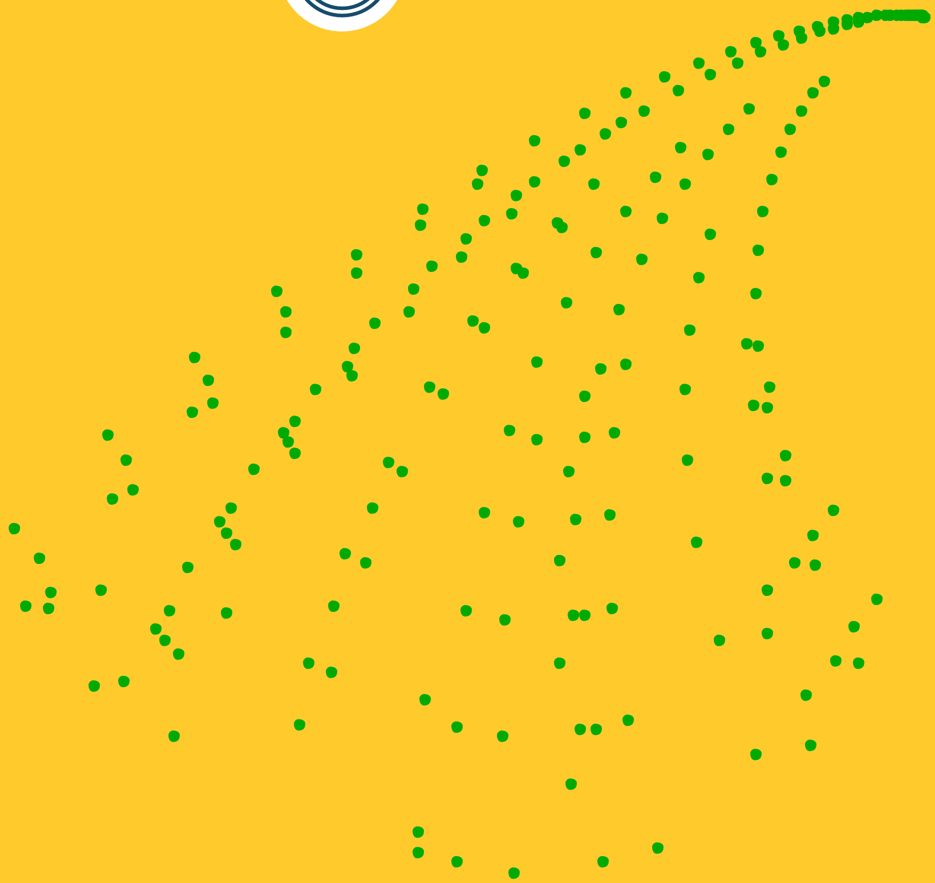
Fractal Fern

x = [.5; .5]; p = 0.8500 0.9200 0.9900 1.0000 A1 = 0.8500 0.0400 -0.0400 0.8500 b1 = 0 1.6000 A2 = 0.2000 -0.2600 0.2300 0.2200 b2 = 0 1.6000 A3 = -0.1500 0.2800 0.2600 0.2400 b3 = 0 0.4400 A4 = 0 0 0 0.1600	x = 0.4450 2.0050 cnt = 2 x = 0.4584 3.2864 cnt = 3 x = 0.5211 4.3751 cnt = 4 x = 0.6180 5.2980 cnt = 5 x = 0.7372 6.0786 cnt = 6 x = 0.8698 6.7373 cnt = 7 x = 1.0088 7.2919 cnt = 8 x = 1.1492 7.7578 cnt = 9 x = 1.2871 8.1482 cnt = 10 x = 1.4200 8.4745

Fractal Fern Program

```
x = [.5; .5];
p  = [ .85  .92  .99  1.00]
A1 = [ .85  .04; -.04  .85]
b1 = [0; 1.6]
A2 = [ .20 -.26;  .23  .22]
b2 = [0; 1.6]
A3 = [-.15  .28;  .26  .24]
b3 = [0; .44]
A4 = [  0    0 ;   0   .16]
cnt = 1
tic
while ~get(stop,'value')
  r = rand;
  if r < p(1)
    x = A1*x + b1
  elseif r < p(2)
    x = A2*x + b2
  elseif r < p(3)
    x = A3*x + b3
  else
    x = A4*x
  end
plot(x(1),x(2),'.','markersize',4,'color',da
rkgreen)
  drawnow
  cnt = cnt + 1
end
t = toc;
s = sprintf('%8.0f points in %6.3f
seconds',cnt,t);
text(-1.5,-0.5,s,'fontweight','bold');
set(stop,'style','pushbutton','string','clos
e','callback','close(gcf)')
```

Points after few iterations

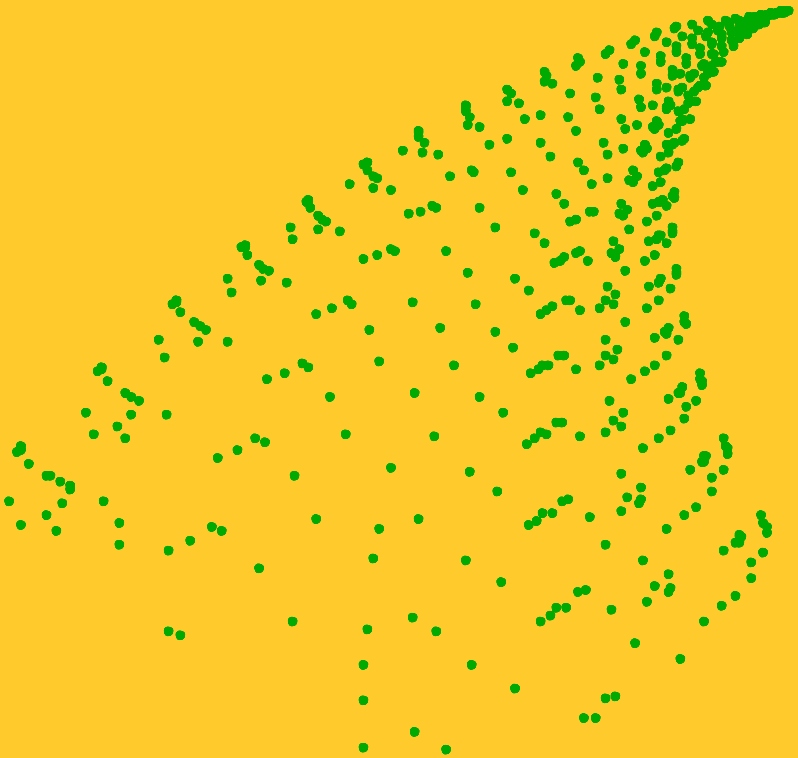


close

202 points in 2.616 seconds

Points after few iterations

1979



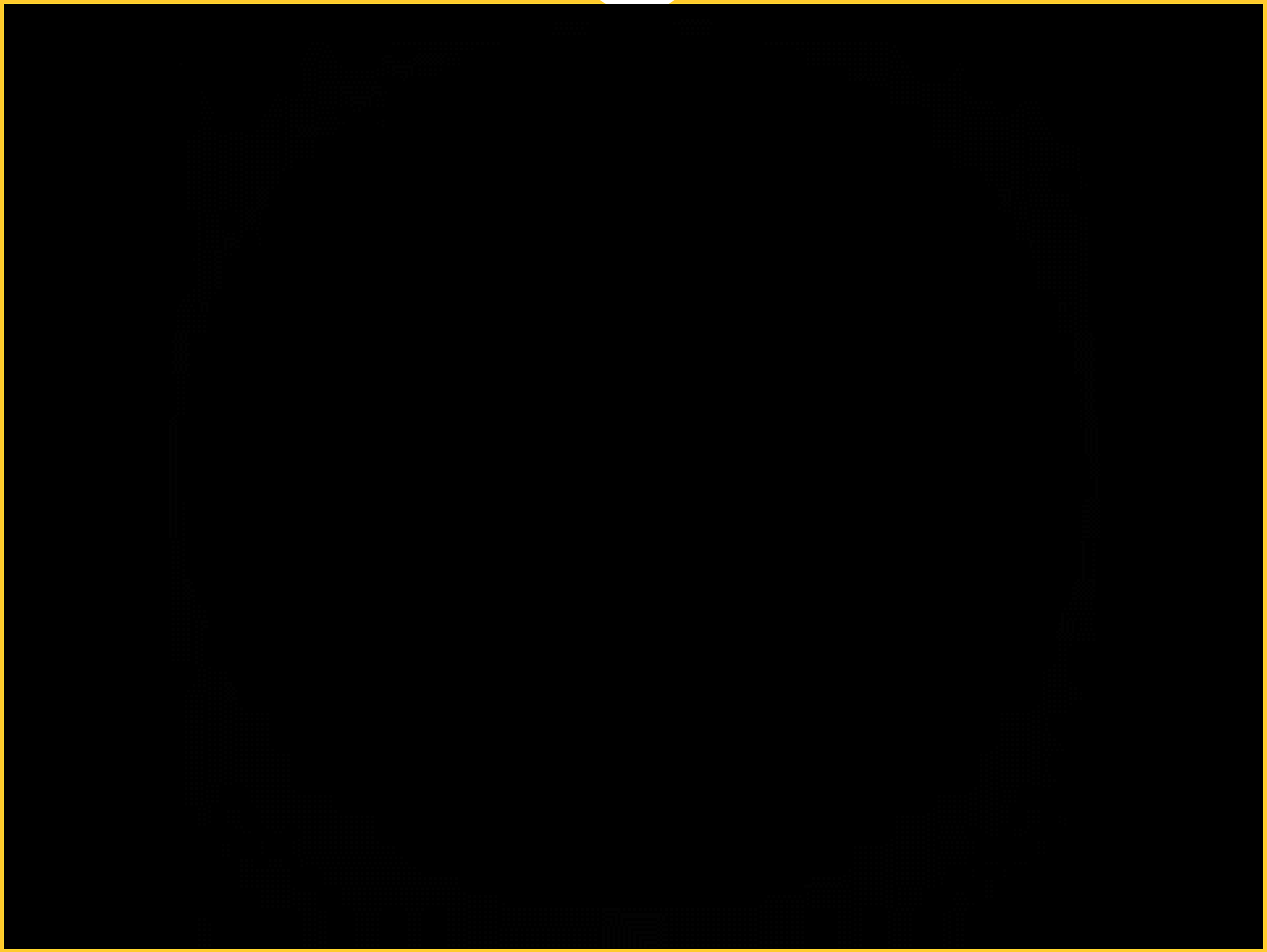
548 points in 15.950 seconds

close

Points after few iterations

Peacock Fractal

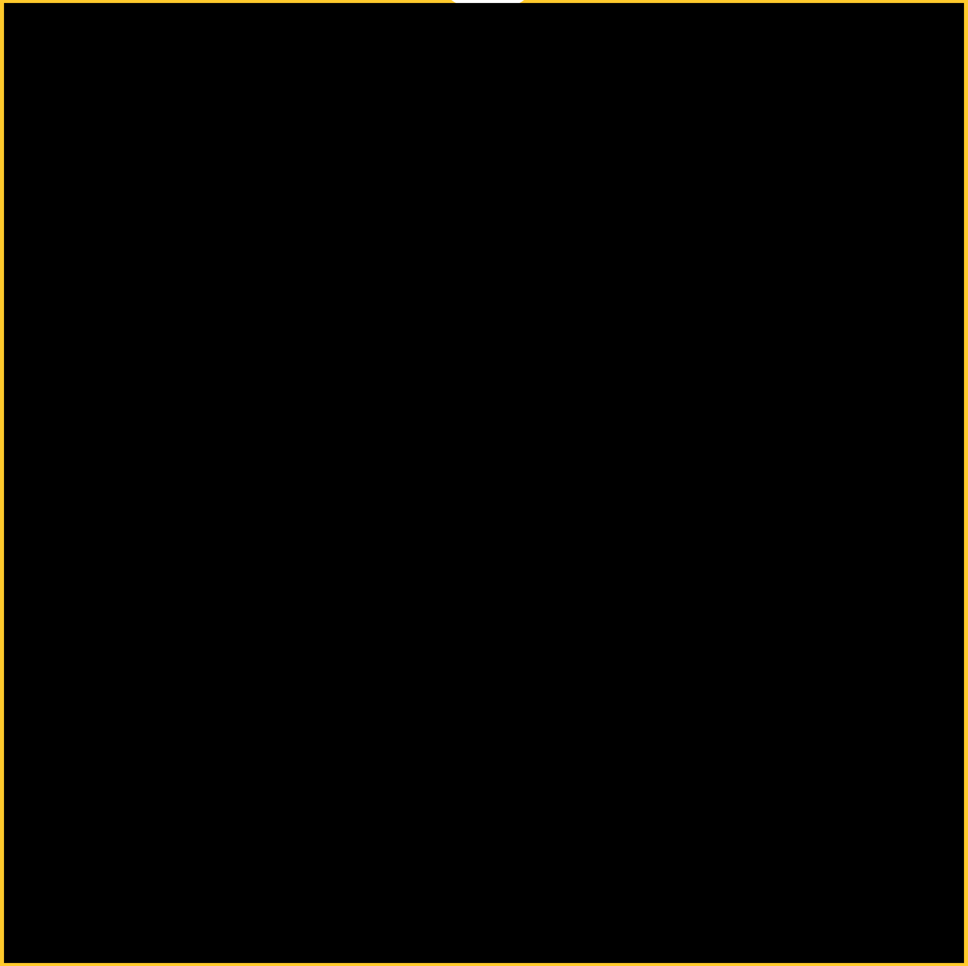
Ahonia Fractal



Butterfly Meltdown

1983

Geometrica Fractals



Juliamorph Fractal

1985

Neuturaleza Fractal

Morphalingus Fractal

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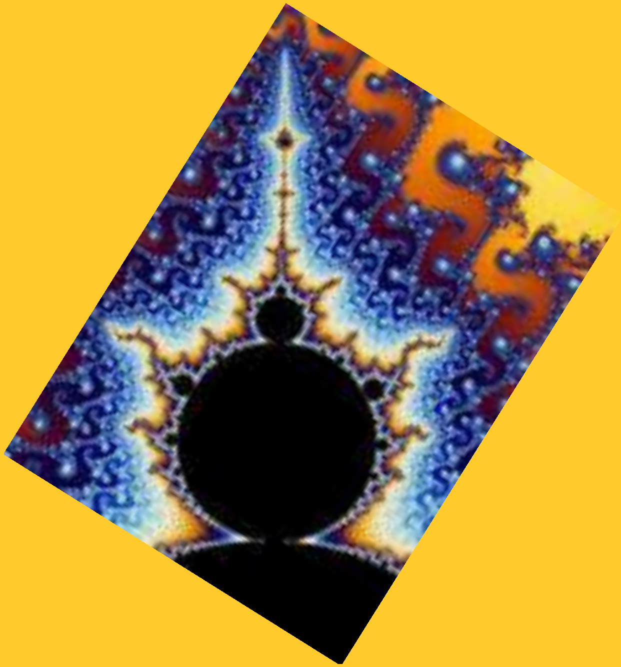
- Rogers D.F., Adams J.A., Mathematical Elements for Computer Graphics. ISBN-13: 978-0-07-048677-5, Published by Tata McGraw Hill

Q&A!!!

Feedback For Improvement

Thank You!!!



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Mupad

- What is it:
- Mupad is a Computer Algebra System.

- What it does?
- It helps in solving mathematical problems.
- Features:
- It can perform symbolic operations that is it can perform operations on expressions containing symbols
- It can carry out operations on numbers with high precision

History of Mupad

- Started in 1989 by Prof Benno Fuchssteiner from Paderborn University
- Students involved:
- Waldemar Wiwianka
- Oliver Kluge
- Karsten Morisse
- Gudrun Oevel
- In 1977 Commercialization started with SciFace GmBH
- Integrated with Matlab

Starting Mupad

Mupad Regions

- There are three regions: Input, Output and text
- [is the input region where the commands are to be typed

- Example: [2+3 [Enter]
- Result: 5
- The area where result are displayed are called output region.
- Text region

Mupad as a Calculator

- [23.3/67.4

- [21!
- [sqrt(2)
- [DIGITS:=50
- [sqrt(2)
- Constant always in capital letter: PI, DIGITS, E, CATALAN, EULER
- Always to use Assignment operator   :=

Solving an equation

- [a:=solve(x^4-4*x^2-1=0,x)
- {-(5^(1/2) + 2)^(1/2), -(2 - 5^(1/2))^(1/2), (5^(1/2) + 2)^(1/2), (2 - 5^(1/2))^(1/2)}

$$\left\{\sqrt{\sqrt{5}+2},\ -\sqrt{\sqrt{5}+2},\ \sqrt{2-\sqrt{5}},\ -\sqrt{2-\sqrt{5}}\right\}$$

- float(a)
- {-2.1, 2.1, 0.49*I, -0.49*I}

$$\{-2.1,\ 2.1,\ 0.49\,\mathrm{i},\ -0.49\,\mathrm{i}\}$$

Solving Equation in Mupad

- a:=solve(x^3-4*x^2-1=0,x)

$$\mathrm{RootOf}(z^3 - 4\ z^2 - 1,\ z)$$

- RootOf(z^3 - 4*z^2 - 1, z)

- [float(a)

$$\{4.1,\ -0.03 + 0.5\ i,\ -0.03 - 0.5\ i\}$$

- {4.1, - 0.03 + (- 0.5*I), - 0.03 + 0.5*I}

Why we should use Computer for solving mathematics problems?

- From Mupad, Page-7: For polynomial equations of order higher than 2, Mupad often produces results as RootsOf if their solutions contain radicals. This is because the resulting formula may be very complex and may not fit on the screen. It can be solved by MaxDegree=3
- Solve Procedure can be used to solve linear equations, quadratic equations, system of equations, differential equations, etc.

Solve

- a:=solve({3*x+2*y-1=0,-6*x+4*y-6=0},{x,y})

What is colour??

Vedic Mathematics

MULTIPLICATION

The base method of multiplication

- This method is used to multiply numbers very close to multiples of 10, like 10, 100, 1000, etc
- Single digit multiplication: 8 x 9
- 8 (10-2) and 9 (10-1) are very close to 10. So our base becomes 10
- **STEP 1:** Put the numbers as shown below:

	9	-1
x	8	-2

The base method of multiplication

- **STEP 2:** Add or subtract crosswise, and put the result as shown below:

	9	-1
x	8	-2
	7	

- **STEP 3:** Multiply vertically the remainders on the right hand side, and put it as shown below:

	9	-1
x	8	-2
	7	2

The base method of multiplication

- So the result obtained is 72.
- **Advantage:** Converted multiplication of large numbers into subtraction and small number multiplication
- **Special Case:** Suppose we are required to multiply 6x7. Here we get 12 on RHS

	6	-4
x	7	-3
	3	$_1$2
	42	

- So we let 2 stay in the unit place. Carry 1 to RHS and add it to 3. So our final answer is 42.

The base method of multiplication

- Two digit multiplication: 99 x 97
- 99 (100-1) and 97 (100-3) are very close to 100. So our base becomes 100
- Follow the same steps, to get the answer 9603.

	99	-01
x	97	-03
	96	03

The base method of multiplication

- Two digit multiplication with carry overs: 89 x 88
- Base is 100
- Follow the same steps, to get the answer 7821

	89	-11
x	88	-12
	77	$_{1}32$
	7832	

The base method of multiplication

- Three digit multiplication with no carry overs: 998 x 997
- Base is 1000
- Follow the same steps, to get the answer 99506

	998	-002
x	997	-003
	995	006

The base method of multiplication

- All from 9 last from 10: 123 x 999
- Base is 1000
- In this case we can see that 123 is far from base, and if we apply base method it will be complex:

	123	-877
x	999	-001
	122	877

- Another example: 1234 x 9999

	1234	- 8766
x	9999	- 0001
	1233	8766

The base method of multiplication

2011

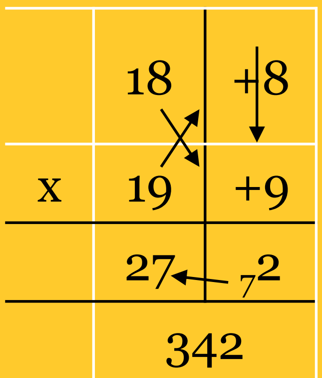
- Above the base: 12 x 13; Both are above 10

	12	+2
x	13	+3
	15	6

- **STEP 1:** Add the excess crosswise
- **STEP 2:** Multiply the right column vertically

The base method of multiplication

- Above the base with carryover: 18 x 19



The base method of multiplication

- Squaring a number with squaring the right column:
- 101 x 101

	101	+01
x	101	+01
	102	01
	10201	

- 102 x 102

	102	+02
x	102	+02
	104	04
	10404	

The base method of multiplication

- Squaring a number with squaring the right column:
- 111 x 111

	111	+11
x	111	+11
	122	$+_1$21
	12321	

- This is a pattern and if you follow the pattern you can do any multiplication

The base method of multiplication

- Above and below the base: 15 x 8

	15	+5
x	8	-2
	13	$-{}_{1}0$
	120	

- 17 x 9

	17	+7
x	9	-1
	16	-7
	160-7	
	153	

The base method of multiplication

- 102 x 99

	102	+02
x	99	-01
	101	-02
	10100-02	
	10098	

The base method of multiplication

- Base other than a power of 10: 44 x 48
- Take base 50 (10x5)
- Since our base is 5 times of 10, we will have to adjust the sum on the RHS. So we will multiply the sum by 5

	44	-6
x	48	-2
	42	$+_{1}2$
42x5 =	210	$+_{1}2$
	2112	

The base method of multiplication

- 49 x 47

	49	-1
x	47	-3
	46	+3
46x5 =	230	+3
	2303	

- 59 x 59: Base is 60

	59	-1
x	59	-1
	58	+1
58x6 =	348	+1
	3481	

The base method of multiplication

- 23 x 23:

	23	+3
x	23	+3
	26	+9
26x2 =	52	+9
	529	

The base method of multiplication

2020

- When the bases are different: 981 x 93
- The bases are 1000 and 100.
- Multiply the smaller number by ratio of the bases.
- Multiply that number (930) with the larger number (981) by the base method

	981	-19
x	930	-70
	911	+$_1$330
	912330	

- Now divide this result by the ratio of the bases to get the final answer: 91233

The base method of multiplication

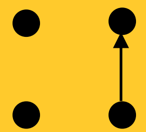
- Example 2: 1006 x 118

	1006	+006
x	1180	+180
	1186	$+_{1}080$
	1187080	
1187080/10	118708	

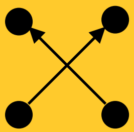
Vertically and crosswise multiplication method

- Ex 1: 12 x 43
- Step 1: Multiply vertically on right:

12
43
6

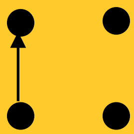


12
43
$_{1}16$



- Step 2: Multiply crosswire and add
- (3x1)+(4x2)=11
- Step 3: Multiply vertically on left and add the carry over:

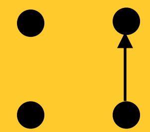
12
43
516



Vertically and crosswire multiplication method

- Ex 2: 78 x 69
- Step 1: Multiply vertically on right:

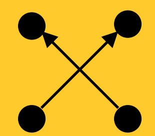
78
69
$_{7}2$



- Step 2: Multiply crosswire and add

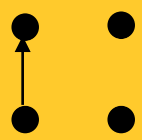
((7x9)+(6x8)=111

78
69
$_{11}82$



- Step 3: Multiply vertically on left and add the carry on:

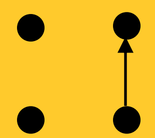
78
69
5382



Vertically and crosswie multiplication method

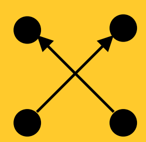
- Ex 3: Multiplication of 2 digit no. to 1 digit no.: 34x8
- Step 1: Multiply vertically on right:

34
08
$_3$2



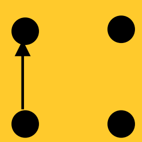
- Step 2: Multiply crosswire and add:

34
08
$_2$72



- Step 3: Multiply vertically on left and add the carry on:

34
08
272



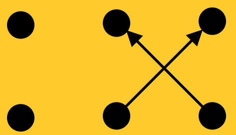
Vertically and crosswire multiplication method

- For 3 digit numbers: 123 x 456
- Step 1: Multiply vertically on right:

123
456
$_1$8

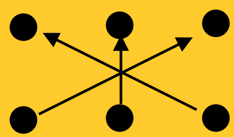
- Step 2: Multiply crosswire and add:

123
456
$_2$88



- Step 3: Multiply and add as shown in the pattern (6x1)+(4x3)+(5x2) =28:

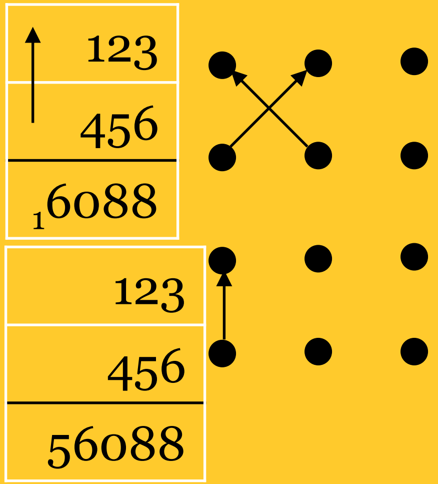
123
456
$_3$088



Vertically and crosswire multiplication method

- Step 4: Multiply left cross and add (5x1)+(4x2)=13:

- Step 5: Multiply left vertical:



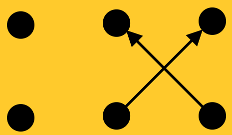
Vertically and crosswise multiplication method

- Ex 2: 231 x 745
- Step 1: Multiply vertically on right:

231
745
5

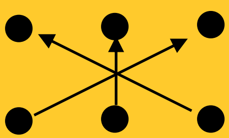
- Step 2: Multiply crosswire and add:

231
745
$_1$95



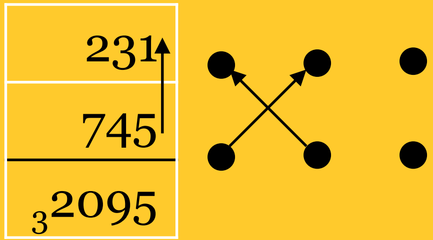
- Step 3: Multiply and add as shown in the pattern:

231
745
$_3$095

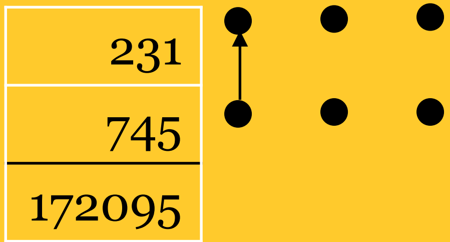


Vertically and crosswire multiplication method

- Step 4: Multiply left cross and add:



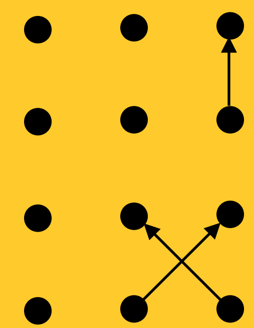
- Step 5: Multiply left vertical:



Vertically and crosswise multiplication method

- 3 digit by 2 digit multiplication: 321 x 42
- Step 1: Multiply vertically on right:

321
042
2

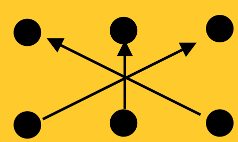


- Step 2: Multiply crosswire and add:

321
042
82

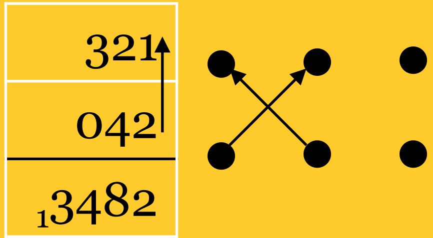
- Step 3: Multiply and add as shown in the pattern:

321
042
$_1$482

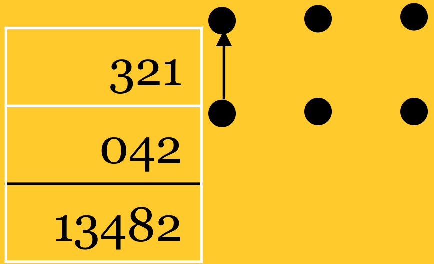


Vertically and crosswire multiplication method

- Step 4: Multiply left cross and add:



- Step 5: Multiply left vertical:



Vedic Mathematics

203
1

ADDITION

Left to right addition

- Conventionally of addition is done right to left
- In vedic mathematics, addition is done left to right
- It makes addition easier. Ex. 1: 78+45
- **STEP 1:** We add the leftmost digits of both number

$$\begin{array}{r} 78 \\ +\ 45 \\ \hline 11 \end{array}$$

Left to right addition

- **STEP 2:** We add the rightmost digits of both numbers

```
     78
+    45
  -------
   11, 13
```

- **STEP 3:** We combine or add the middle digits

```
     78
+    45
  -------
   11, 13
  -------
    123
```

Left to right addition

- Ex. 2: 87 + 69
- **STEP 1:** We add the leftmost digits of both number
- **STEP 2:** We add the rightmost digits of both numbers
- **STEP 3:** We combine or add the middle digits

$$\begin{array}{r} 87 \\ +\ 69 \\ \hline 14,\ 16 \\ \hline 156 \end{array}$$

Left to right addition

- Ex. 3: 48 + 97
- **STEP 1:** We add the leftmost digits of both number
- **STEP 2:** We add the rightmost digits of both numbers
- **STEP 3:** We combine or add the middle digits

$$\begin{array}{r} 48 \\ +\ 97 \\ \hline 13,\ 15 \\ \hline 145 \end{array}$$

Left to right addition

- Ex. 4: 582 + 759
- **STEP 1:** We add the leftmost digits of both number
- **STEP 2**: Add middle digits
- **STEP 3:** We add the rightmost digits of both numbers
- **STEP 4:** We combine or add the middle digits

```
     582
+    759
  ----------
  12, 13, 11
  ----------
     1341
```

Left to right addition

- Ex. 5: 983 + 694
- **STEP 1:** We add the leftmost digits of both number
- **STEP 2**: Add middle digits
- **STEP 3:** We add the rightmost digits of both numbers
- **STEP 4:** We combine or add the middle digits

```
    983
+   694
  ---------
  15, 17, 07
  ---------
    1677
```

Left to right addition

- Addition of multiple numbers
- Ex. 1: 5273 + 7372 + 6371 + 9782

	5273
+	7372
+	6371
+	9782
	27, 15, 29, 08
	28798

Left to right addition

- Addition of multiple numbers
- Ex. 2: 8336 + 4283 + 3428 + 9373

$$\begin{array}{rr} & 8336 \\ + & 4283 \\ + & 3428 \\ + & 9373 \\ \hline & 24,\ 12,\ 20,\ 20 \\ \hline & 25420 \end{array}$$

Vedic Mathematics

2040

SUBTRACTION

Subtraction from a base number

- This method is applicable to subtraction from a base number which is multiple of 10.
- Rule-All from 9 and last from 10
- Ex 1: 1000 – 283
- **STEP 1:** We subtract left to right all from 9 and last from 10

1 0 0 0	–	2	8	3
		↓	↓	↓
		Subtract from 9	Subtract from 9	Subtract from 10
	=	7	1	7

So our answer is 1000 – 283 = 717

Subtraction from a base number

- Ex 2: 1000 – 476

1 0 0 0	–	4	7	6
		↓	↓	↓
		Subtract from 9	Subtract from 9	Subtract from 10
	=	5	2	4

So our answer is 1000 – 476 = 524

Subtraction from a base number

- Ex 3: 10000 – 1234

1 0 0 00	–	1	2	3	4
		↓	↓	↓	↓
		Subtract from 9	Subtract from 9	Subtract from 9	Subtract from 10
	=	8	7	6	6

So our answer is 10000 – 1234 = 8766

Subtraction from a base number

- Ex 4: 10000 – 5389

1 0 0 00	–	5	3	8	9
		↓	↓	↓	↓
		Subtract from 9	Subtract from 9	Subtract from 9	Subtract from 10
	=	4	6	1	1

So our answer is 10000 – 5389 = 4611

Super Subtraction

- Step-1: Subtract natural way from left to right
- Step-2: If bottom digit is higher than top digit, then borrow 1 from left and reduce the digit at left by 1
- Step-3: Continue this process till last:

$$\begin{array}{r} 651 \\ -\ 297 \\ \hline 4\ \ \ \end{array}$$

- Ex 1: 651 – 297
- **STEP 1:** We will subtract left to right. We have 6 – 2 = 4

Super Subtraction

- **STEP 2:** In the second column, we see that 9 in the bottom is higher than 5 on top. So we will borrow 1 from 4 in the previous step to 3 and carry over 1 to the next digit which is 5. So the top number becomes 15. Now 15 – 9 = 6. That's our second digit

$$\begin{array}{r} 6\,{}^{1}5\,1 \\ -\quad 2\,9\,7 \\ \hline \end{array}$$

~~4~~

36

Super Subtraction

- **STEP 3:** In the third column, again 7 in the bottom is higher than 1 on top. So we reduce 6 in the previous step to 5 and carry over 1 to the next step. So the top number becomes 11. Now 11 – 7 = 4. That's our second digit and the result is 354.

$$
\begin{array}{r}
6^{1}5^{1}1 \\
-\quad 297 \\
\hline
\not{4} \\
3\not{6} \\
354
\end{array}
$$

Super Subtraction

- Ex 2: 425 – 168

$$
\begin{array}{r}
4^{1}2^{1}5 \\
-\quad 168 \\
\hline
\end{array}
$$

~~3~~

2~~6~~

257

Super Subtraction

- Ex 3: 7643 – 4869

$7^{1}6^{1}4^{1}3$

– 4869

~~3~~

2~~8~~

27~~8~~

2774

Super Subtraction: large

- Ex 1: 638475 – 429763

$63^{1}8^{1}475$

– 429763

2~~1~~

20~~9~~

208712

Super Subtraction: large

- Ex 2: 82 – 8

$$\begin{array}{r} 8^{1}2 \\ - \quad 08 \\ \hline \not{8} \\ 74 \end{array}$$

Super Subtraction: large

- Ex 3: 94 – 20

$$\begin{array}{r} 94 \\ -\quad 20 \\ \hline 74 \end{array}$$

Super Subtraction: large

- Ex 4: 94 − 17

$$\begin{array}{r} 9\,{}^{1}4 \\ -\quad 17 \\ \hline \not{8} \\ 77 \end{array}$$

Vedic Mathematics

DIVISION

Divisor × Quotient + Remainder

= Dividend

Division – Conventional way

Divisor->31

Dividend->848

62

228

217

110

93

Remainder->17

Quotient->27.3

Flag Method

- Say we have to divide 848 by 31
- 848 is dividend and 31 divisor. The layout will be:

3^1	8	4	8

- 3 is our flagpole and 1 is the flag
- **Rule 1:** We divide by the flagpole.
- **Rule 2:** We subtract the sum of the flag multiplied by the previous quotient digit, and subtract it from the result of rule 1.

Flag Method

- **STEP 1:** We divide 8 by 3.
- It gives us 2 as quotient and 2 as remainder.
- We put the quotient in its appropriate place.
- Then we take the remainder 2 and prefix it to 4 making it 24.

3^1	8	$_{2}4$	8
	2		

Flag Method

- **STEP 2:** We subtract (flag x the previous quotient digit) from 24
- The flag is 1 and the previous quotient digit is 2. So 24 – 1x2 = 22

3^{1}	8	$_{2}4$	8
	2		

Flag Method

- **STEP 3:** This was the first cycle. Repeat the cycle and divide again.
- We divide 22 by 3, which gives us quotient 7 and remainder 3

3^1	$8_2 4$	$_1 8$
	27	

Flag Method

- **STEP 4:** Continuing steps, we will now subtract (1 x 7 = 7) from 18, which gives us 11.
- 11 is divided by 3, which gives quotient 3 and remainder 2

3^1	$8_2 4$	${}_1 8_2$
	27	3

- The answer is 27.3

Flag Method

- Ex 2: 651 by 31

3^1	$6_0 5$	$_0 1_0$
	21	0

- Answer is 21.0

Flag Method

- Ex 3: 5576 by 25

2^5	$5_1 5_1 7$	$_1 6_1 0$
	223	05

- Answer is 223.05

Flag Method

- Ex 3: 2924 by 72

7^{2}	$29_{1}2$	$_{4}4_{2}0$ $_{1}$
	40	61

- Answer is 40.61

Division by altered remainders

- Decreasing quotient to increase remainder.
- If the difference between number formed by remainder-next digit and flag x quotient is negative, we decrease the quotient by one and increase the remainder by the flagpole.
- Example 3: 3412 divided by 24

2^4	$3_1 4_2 1$	${}_1 2_2 0$
	$1\not{5}42$	$\not{2}1$

- The answer is 142.1

Division by altered remainders

- Ex 4: 5614 divided by 21

2^1	$5_1 6_2 1$	${}_1 4_1 0$
	267	33

- The answer is 267.33

Division by altered remainders

- Ex 5: 7943 divided by 42

4^2	$7_3 9_5 4$	$_2 3_1 0$
	$1\overset{\cancel{0}}{9}89$	12

- The answer is 189.12

Division by auxiliary fractions

- Skipped because of complexity

Vedic Mathematics

DIGIT SUM

Digit Sum

- Keep adding all digits till you get a single digit number

Number	Summing digits	Digit sum
65	6+5=11; 1+1=2	2
721	7+2+1=10; 1+0=1	1
3210	3+2+1+0=6	6
67754	6+7+7+5+4=29; 2+9=11; 1+1=2	2
82571	8+2+5+7+1=23; 2+3=5	5
1890	1+8+9+0=18; 1+8=9	9
23477	2+3+4+7+7=23; 2+3=5	5

Digit Sum: Casting out nines

- In this method, we simply cast out the nines or the digits adding up to 9
- Ex. 1: 8154912320
- We cancel 8 & 1, 5 & 4, and 9: ~~81549~~12320
- We take the sum of the rest of the digits: 1+2+3+2+0 = 6
- The digit sum is 6

Digit Sum-Casting out nines

- Ex. 2: 970230612
- We cancel 9, 7 & 2, and 3 & 6: ~~97~~0~~23~~0~~6~~12
- We take the sum of the rest of the digits: 0+0+1+2 = 3
- The digit sum is 3

Using digit sum to check answers

- Addition: The digit sum of the sum of two numbers will be equal to the sum of the digit sum of the individual numbers.
- Ex. 1: 734 + 352

$$\begin{array}{rrcr} & 734 & \rightarrow & 5 \\ + & 352 & \rightarrow & 1 \\ \hline & 1086 & \rightarrow & 6 \end{array}$$

Using digit sum to check answers

- Ex. 2: 2344 + 6235

	2344	→	4	4+7 =11, 1+1=2
+	6235	→	7	1+1=2;
	8579	→	2	

Using digit sum to check answers

- The rule works the same way for subtraction.
- Except we add nine to the negative digit sum
- Ex. 3: 4321 – 1786

	4321	→	1	
–	1786	→	– 4	$-4 + 1 = -3; -3+9= 6$
	2535	→	6	

Using digit sum to check answers

- Multiplication: The digit sum of the product of two numbers is equal to the digit sum of the digit sums of the individual numbers.
- Ex .1: 62 x 83

	62	→	8
x	83	→	2
	5146	→	7

8x2=16;
1+6=7

Using digit sum to check answers

- Ex .2: 726 x 471

	726	→	6	
x	471	→	3	6x3=18; 1+8=9
	341946	→	9	

Vedic Mathematics

FRACTIONS

Addition of fractions

- Type 1 Fraction: The denominator is same

Rule-Simply add or subtract the numerator

- Ex 1: $\frac{5}{11} + \frac{3}{11} = \frac{5+3}{11} = \frac{8}{11}$
- Type 2 Fraction: When one denominator is the factor of the other.

Step-1: Find the factor.

Step-2: Multiply or divide both numerator and denominator with the factor.

Step-3: Add or subtract as before

Addition of fractions

- Ex 2: $\frac{2}{5}+\frac{9}{20}=\frac{2\times4}{5\times4}+\frac{9}{20}=\frac{8+9}{20}=\frac{17}{20}$ (Factor = 4)

- Ex 3: $\frac{11}{15}+\frac{7}{30}=\frac{11\times2}{15\times2}+\frac{7}{30}=\frac{22+7}{30}=\frac{29}{30}$ (Factor-2)

Addition of fractions

- Addition with vertically crosswire method
- **STEP 1:** For numerator: multiply crosswise and add
- **STEP 2:** For denominator: multiply denominators
- Ex. 1: $\frac{3}{5} + \frac{1}{4}$
- For numerator:

3	**1**
5	4

3x4 + 5x1 = 17

- For denominator: 5x4 = 20
- The answer is $\frac{17}{20}$

Addition of fractions

- Ex. 2: $\frac{2}{11} + \frac{7}{9}$
- For numerator: 2x9 + 11x7 = 18+77=95
- For denominator: 11x9 = 99
- The answer is $\frac{95}{99}$

Subtraction of fractions

- For type 1 and type 2 fractions, process is similar to addition
- Subtraction with vertically crosswise method
- **STEP 1:** For numerator: multiply crosswise and subtract
- **STEP 2:** For denominator: multiply denominators
- Ex. 1: $\frac{6}{7} - \frac{1}{2}$
- For numerator: 6x2 – 7x1 = 12-7=5
- For denominator: 7x2 = 14
- The answer is $\frac{5}{14}$

Subtraction of fractions

- Ex. 2: $\frac{12}{25} - \frac{3}{50}$
- For numerator: 12x50 – 25x3 = 600-125=525
- For denominator: 25x50 = 1250
- The answer is $\frac{525}{1250} = \frac{21}{50}$

DECIMALS

Principle of place value

- In decimal number system, position of the digit determines its value
- Like 4 placed before 7, makes it 47. That means 4 tens and 7 ones.
- This is place value of numbers.

Principle of place value

- Hence, the principle of place value is that the value of the place immediately to the left of any given place is ten times as great.
- In the same way a position to the right is ten times as small or 1/10th of the value of the place immediate to the left.
- We put unit's column at the middle
- The column to the left of the unit's column are for tens, hundreds and so on.
- Similarly, the columns on the right of the unit's column are for one-tenths, one-hundredths and so on.

Principle of place value

Thousand	TH
Hundreds	H
Tens	T
Units	U
.	.
Tenths	t
Hundredths	h
Thousandths	th

Th H T U . t h th

Addition of decimals

The Decimal Point: The decimal Point is used to distinguish between whole numbers and parts of a whole

- Rule 1: Keep the decimal points in a vertical line

$$4.34 + 3.42$$

$$\begin{array}{r} 4.34 \\ 3.42 \\ \hline 7.76 \end{array}$$

Addition of decimals

- Rule 2: Sum can be done from left to right or right to left
- Rule 3: Addition can be done without the decimals and putting the decimal points at the end
- Ex. 1: 78.3 + 2.031 + 2.3245 + 9.2

```
 78.3
  2.031
  2.3245
  9.2
---------
 91.8555
```

Addition of decimals

- Ex. 2: 0.0004 + 6.32 + 1.008 + 3.452

$$\begin{array}{r} 0.0004 \\ 6.32 \\ 1.008 \\ 3.452 \\ \hline 10.7804 \end{array}$$

Subtraction of decimals

- Rule-1: Put the decimal points one below another
- Ex-1: Subtract 2.09 from 45

    45.00 (Pad 45 with 00 after decimal place

    2.09

    - Rule-2: Subtract without considering decimal places:

        4500

        209

        4291

    - Rule-3: 42.91 (Place the decimal point. This will remove decimal phobia.

Multiplication by powers of 10

- Rule: Shift the decimal point right one place for each 0
- Ex. 1: 7.86 x 10 = 78.6
- Ex. 2: 7.86 x 100 = 786

Number	x 10	x 100	x 1000	x 10000
0.72	7.2	72	720	7200
0.91	9.1	91	910	9100
0.04	0.4	4	400	4000
42.03	420.3	4203	42030	420300

Multiplication for decimals

- Rule: For a 2 digit number, multiply vertically and crosswire, ignoring the decimals. Then put the decimal point into place
- Ex. 1: 7.3 x 1.4

$$\begin{array}{c} 7\cdot3 \\ 1.4 \\ \hline 10.{}_{3}2{}_{1}2 \end{array}$$

Multiplication for decimals

- Ex. 2: 6.2 x 5.4

$$\begin{array}{r} 6.2 \\ 5.4 \\ \hline 33.48 \end{array}$$

Division by powers of 10 (10, 100, 1000, etc)

- Rule: Shift the decimal point left one place for each power
- Ex. 1: 3.17/10 = 0.317
- Ex. 2: 4.52/100 = 0.0452

Division by whole numbers

- Rule 1: Divide ignoring the decimal
- Rule 2: Put the decimal in the same place as the dividend
- Ex. 1: 9.1/7 = 1.3
- Ex. 2: 5.26/2 = 2.63

Dividing a decimal by another decimal

- Rule 1: Ignore the decimal point
- Rule 2: Divide by flag method
- Ex. 1: 567.29/45.67
- Step 1: Divide 56729 by 4567 to get 12.4215
- Ex. 2: 7.625/0.923
- 7625/923 = 8.2611

Vedic Mathematics

RECURRING DECIMALS

Recurring Decimals
To convert Fractions to Decimals

- There are three types of decimals:
  1. Recurring decimals
  2. Non – recurring decimals
  3. Non – recurring and non ending decimals

1. Recurring: Never ending digits that repeat or recur

- Examples: 1/3 = 0.3333….; 1/9 = 0.1111; 1/99 = 0.1010101…..
- These decimals occurs when ever the denominator of the fraction has prime number other then 2 and 5, i.e, 3,7,11,13 etc. as factors.

Recurring Decimals

2. Non – recurring: A non recurring decimal occurs whenever the denominator has 2 or 5 as factors.

- These decimals terminate.
- ½ = 0.5; 1/5 = 0.2; 1/10 = 0.1: one significant digit
- ¼ = 0.25; 1/25 = 0.04; 1/100 = 0.01: two significant digits

Recurring Decimals

3. Non – recurring and non repeating: These numbers are irrational or transcendentals:

- Like: $\sqrt{2} = 1.41421356.....$
- $\pi = 3.141592653....$
- $e = 1.78....$

Conversion of reciprocal or fraction into a decimal

- Conventional Practice in to divide numerator by denominator
- It can be achieved in a simple way
  (1) Reciprocal of number ending with 9
- Rule-1: one more then one before
  It means that One move that the number before that a

Case-1: For 19: One before 9 is 1 and one more then one before nine is 2

Case-2: For 29 : One before 9 is 2 and one more then one before nine is 2+1=3

These goes equally for others.

Conversion of reciprocal or fraction into a decimal

Rule-2: Since Only 9 is dropped, replace the decimal of the numerator by shifting the decimal one place to the Left

Example-1: Convert the fraction 1/19 to decimal form

Step-1: One more then one before nine is 2 (1+1)

Step-2: As 9 will be dropped, replace 1 by 0.1

So that problem becomes 1/19=1/20=0.1/2

- Chapter-8

- Percentage

Vedic Mathematics

CHAPTER-9
DIVISIBILITY

Some known divisibility rules

- Divisibility by 2: The last digit of the given number must be even, like – 2, 4, 6, 8, 0
- Divisibility by 3: The given number's digit sum must be divisible by 3 for the number to be divisible by 3. Ex. 345 –> 3+4+5 = 12 –> 1+2 = 3. Hence 345 is divisible by 3.
- Divisibility by 4: The given number's last two digits must be divisible by 4 for the number to be divisible by 4. Ex. 8732 –> 32 is divisible by 4. Hence 8732 is also divisible by 4.

Some known divisibility rules

- Divisibility by 5: If the last digit of the given number is 5 or 0, the number is divisible by 5.
- Divisibility by 6: If the given number is divisible by both 2 and 3, then the number is divisible by 6. Ex. 42 = 2x21 = 3x14. Hence 42 is also divisible by 6.
- Divisibility by 8: If the last three digits of the given number are divisible by 8, then the number is divisible by 8. Ex. 337312 –> 312 = 8x39. Hence 337312 is also divisible by 8.

Some known divisibility rules

- Divisibility by 9: If the digit sum of the given number is 9, then the number is divisible by 9.

Osculation

- This is a method to ascertain whether a number is divisible by another number or not.
- This is done through a method called osculation.

1. Divisibility rule for 7, prime numbers and large numbers:

Rule – the OSCULATOR

- Osculator is the number one more than one before when a number ends in a 9 or a series of 9s.
- Example: Osculator for 9 is 1; 19 is 2; 29 is 3 and so on.

Osculation

- For 13, the osculator is 4, because to obtain 9 at the end, we require to multiply 13 by 3. We get 39, and the osculator for 39 is 4.
- Similarly, the osculator for 7 (7x7=49, 4+1=5) is 5.
- For 17, the osculator is (17x7=119, 11+1=12) 12.
- For 23, the osculator is (23x3=69, 6+1=7) 7

The osculator method for divisibility

- Ex. 1: Find if 112 is divisible by 7
- STEP 1: Osculator of 7 is 5
- STEP 2: Osculate 112 with 5.
- RULE: To osculate a number, multiply its last figure by osculator of the divisor (7) and add the result to the previous figure

–> 112 –> 11 + (2x5) –> 11 + 10 = 21

–> 21 –> 2 + 1x5 = 7

As the osculation result is the divisor itself, 112 is divisible by 7.

The oscular method for divisibility

- Ex. 2: Find if 49 is divisible by 7
- STEP 1: Osculator of 7 is 5
- STEP 2: Osculate 49 with 5.

–> 49 –> 4 + (9x5) –> 49->4+9x5 = 49

As the osculation result is the dividend itself, 49 is divisible by 7.

The osculator method for divisibility

- Ex. 3: Find if 2844 is divisible by 79
- STEP 1: Osculator of 79 is 8
- STEP 2: Osculate 2844 with 8.

–> 2844 –> 284 + (4x8) –> 284 + 32 = 316

–> 316 –> 31 + (6x8) –> 31 + 48 = 79

As the osculation result is the divisor itself, 2844 is divisible by 79.

The osculator method for divisibility

- Ex. 4: Find if 1035 is divisible by 23
- STEP 1: Osculator of 23 is 7
- STEP 2: Osculate 1035 with 7.

–> 1035 –> 103 + (5x7) = 103 + 35 = 138

–> 138 –> 13 + (8x7) = 13 + 56 = 69

69 is a multiple of 23, hence 1035 is divisible by 23.

The osculator method for divisibility

- Ex. 5: Find if 6308 is divisible by 38
- If 6308 is also divisible by 19 and 2, then 6308 is divisible by 38
- STEP 1: Osculator of 19 is 2
- STEP 2: Osculate 6308 with 2.

–> 6308 –> 630 + (8x2) = 630 + 16 = 646

–> 646 –> 64 + (6x2) = 64 + 12 = 76

–> 76 –> 7 + (6x2) = 7 + 12 = 19; hence 6308 is divisible by 19.

The osculator method for divisibility

- Ex. 6: Find if 334455 is divisible by 39
- STEP 1: Osculator of 39 is 4
- STEP 2: Osculate 334455 with 4

–> 334455 –> 33445 + (5x4) = 33445 + 20 = 33465

–> 33465 –> 3346 + (5x4) = 3346 + 20 = 3366

–> 3366 –> 336 + (6x4) = 336 + 24 = 360

–> 360 –> 36 + (0x4) = 36

Since 36 I below the divisor, 334455 is not divisible by 39.

The oscilator method for divisibility

- Ex. 7: Find if 3588 is divisible by 69
- STEP 1: Osculator of 69 is 7
- STEP 2: Osculate 3588 with 7

–> 3588 –> 358 + (8x7) = 358 + 56 = 414

Since 69 is the divisor itself, 3588 is divisible by 3588.

The negative osculator method for divisibility

- If the divisor ends with 1, or a multiple of 1, just drop 1 from the divisor.
- For example, 21 –> just drop 1 and the osculator is 2
- For 51, osculator is 5, and so on
- For 9, osculator is 9x9 = 81 –> 8

The negative osculator method for divisibility

- Ex. 1: Find if 6603 is divisible by 31
- STEP 1: Negative osculator of 31 is 3
- STEP 2: Osculate 6603 with 3

–> 6603 –> 660 – (3x3) = 660 – 9 = 651

–> 651 –> 65 – (1x3) = 65 – 3 = 62

–> 62 –> 6 – (2x3) = 6 – 6 = 0

If the result of the osculation is the divisor itself, 0 or a repetition of the previous result, then the number is divisible by the divisor.

The negative osculator method for divisibility

- Ex. 1: Find if 11234 is divisible by 41
- STEP 1: Negative osculator of 41 is 3
- STEP 2: Osculate 11234 with 4

–> 11234 –> 1123 – (4x4) = 1123 – 16 = 1107

–> 1107 –> 110 – (7x4) = 110 – 28 = 82

–> 82 –> 8 – (2x4) = 8 – 8 = 0

Hence, 11234 is divisible by 41

The negative osculator method for divisibility

- Ex. 1: Find if 2275 is divisible by 7
- STEP 1: Negative osculator of 7 is 2 (7x3 = 21)
- STEP 2: Osculate 2275 with 2

–> 2275 –> 227 – (5x2) = 227 – 10 = 217

–> 217 –> 21 – (7x2) = 21 – 14 = 7

Hence, 2275 is divisible by 7

The negative osculator method for divisibility

- Ex. 1: Find if 2275 is divisible by 7
- STEP 1: Negative osculator of 7 is 2 (7x3 = 21)
- STEP 2: Osculate 2275 with 2

–> 2275 –> 227 – (5x2) = 227 – 10 = 217

–> 217 –> 21 – (7x2) = 21 – 14 = 7

Hence, 2275 is divisible by 7

Points to be noted

- The sum of the positive and negative osculator is equal to the divisor
- For divisors ending with 1 and 7, negative osculator is smaller than the positive osculator, so better use negative osculator.
- For divisors ending with 3 and 9, positive osculator is smaller than the negative osculator. So better use the positive osculator.

Vedic Mathematics

SQUARES

Squaring by Duplex method

- Duplex simply means dual or something related to two.
- There are three types of duplexes:
  - Duplex of individual digits
  - Duplex numbers with even digits
  - Duplex of odd digited numbers

Squaring by Duplex method

- Duplex of individual digits: If the number is a, then the duplex is a^2

$$Duplex(1) = 1^2 = 1$$
$$Duplex(2) = 2^2 = 4$$
$$Duplex(3) = 3^2 = 9$$
$$Duplex(4) = 4^2 = 16$$
$$Duplex(5) = 5^2 = 25$$
$$Duplex(6) = 6^2 = 36$$
$$Duplex(7) = 7^2 = 49$$
$$Duplex(8) = 8^2 = 64$$
$$Duplex(9) = 9^2 = 81$$

Squaring by Duplex method

- Duplex of number of even digit:
- Note: Even digit numbers are not even numbers. They are the numbers with even number of digits.
- Examples: 46, 23, 2315, 231679 etc., whose number of digits are 2, 2, 4, 6 etc.
- For a 2 digit number, Duplex = 2ab, where a and b are the digits of the number.
- Ex. Duplex(81)=2x8x1=16; Duplex(73)=2x7x3=42

Squaring by Duplex method

- For a 4 digit number, the duplex is 2ad+2bc
- Example: Duplex(1234) = 2x1x4+2x2x3 = 8+12 = 20
- Duplex(8231) = 2x8x1+2x2x3 = 16+12 = 28

Squaring by Duplex method

- Duplex of odd digited numbers:
- This is a combination of the duplex of individual digits and the duplex of even digited numbers.
- Formula to be used is $m^2 + 2ab$ where m is the middle digit and a and b are the first and third digit.
- Example: Duplex(372) = 7^2+2x3x2 = 49+12 = 61
- Duplex(286) = 8^2+2x2x6 = 64+24 = 88
- Duplex(789) = 8^2+2x7x9 = 64+126 = 190

Squaring by Duplex method

- For 2 digit squares: $(ab)^2$ = Duplex(a|ab|b) = 100xDuplex(a)+10xDuplex(ab)+Duplex(b)
- Ex. 1: Find the square of 57
- 57^2 = duplex(5|57|7)
- Duplex(5) = 25; Duplex(57) = 70; Duplex(7) = 49
- Now the square of 57 is:

$$\begin{array}{r} 25 \\ 70 \\ 49 \\ \hline 3249 \end{array}$$

Squaring by Duplex method

- Ex. 2: Find the square of 74
- 74^2 = duplex(7|74|4)
- Duplex(7) = 49; Duplex(74) = 56; Duplex(4) = 16
- Now the square of 74 is:

$$\begin{array}{r} 49 \\ 56 \\ 16 \\ \hline 5476 \end{array}$$

Squaring by Duplex method

- For 3 digit squares: $(abc)^2$ = Duplex(a|ab|abc|bc|c)
- Ex. 1: Find the square of 746
- 746^2 = duplex(7|74|746|46|6)
- Duplex(7) = 49; Duplex(74) = 56; Duplex(746) = 100; Duplex(46) = 48; Duplex(6) = 36
- Now the square of 746 is:

```
49
 56
 100
   48
    36
------
556516
```

Squaring by Duplex method

- Ex. 2: Find the square of 357
- 357^2 = duplex(3|35|357|57|7)
- Duplex(3) = 9; Duplex(35) = 30; Duplex(357) = 67; Duplex(57) = 70; Duplex(7) = 49
- Now the square of 357 is:

$$\begin{array}{r} 9 \\ 30 \\ 67 \\ 70 \\ 49 \\ \hline 127449 \end{array}$$

Squaring by Duplex method

4
3 2
1 0 0
1 6 0
1 4 5
7 2
1 6
---
8 3 7 5 2 3 6

- For 4 digit squares: $(abcd)^2$ = Duplex(a|ab|abc|abcd|bcd|cd|d)
- Ex. 1: Find the square of 2894
- 2894^2 = duplex(2|28|289|2894|894|94|4)
- Duplex(2) = 4; Duplex(28) = 32; Duplex(289) = 100; Duplex(2894) = 160; Duplex(894) = 145; Duplex(94) = 72; Duplex(4) = 16
- Now the square of 2894 is:

Squaring by Duplex method

- Ex. 2: Find the square of 1234
- 1234^2 = duplex(1|12|123|1234|234|34|4)
- Duplex(1) = 1; Duplex(12) = 4; Duplex(123) = 10; Duplex(1234) = 20; Duplex(234) = 25; Duplex(34) = 24; Duplex(4) = 16
- Now the square of 2894 is:

```
1
  4
  1 0
    2 0
      2 5
        2 4
          1 6
-----------------
1 5 2 2 7 5 6
```

Squaring by Duplex method

- Ex. 3: Find the square of 73
- 73^2 = duplex(7|73|3)
- Duplex(7) = 49; Duplex(73) = 42; Duplex(3) = 9
- Now the square of 73 is:

$$\begin{array}{r} 49 \\ 42 \\ 9 \\ \hline 5329 \end{array}$$

Squaring by Duplex method

- Ex. 3: Find the square of 86
- 86^2 = duplex(8|86|6)
- Duplex(8) = 64; Duplex(86) = 96; Duplex(6) = 36
- Now the square of 86 is:

$$\begin{array}{r} 64 \\ 96 \\ 36 \\ \hline 7396 \end{array}$$

Vedic Mathematics

213
8

CUBES

Cubes

- The general algebraic formula for calculation of $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$ is extended here
- The formula is broken down in two lines:

$$(a+b)^3 = a^3 + a^2b + ab^2 + b^3$$

$$+\ 2a^2b + 2ab^2$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

- For the first row elements on the RHS, we see that:

$$a^3 + a^2b + ab^2 + b^3 = a^3\left(1 + \frac{b}{a} + \frac{b^2}{a^2} + \frac{b^3}{a^3}\right)$$

Cubes

- The second row elements on the RHS are just the double of the two middle elements in the first row
- Ex.1: Find 12^3
- Here a = 1, b = 2, b/a = 2
- $a^3 = 1$; Now we write the first and second row elements as follows:

$$\begin{array}{cccc} 1 & +\ 2 & +\ 4 & +\ 8 \\ & +\ 4 & +\ 8 & \\ \hline 1 & 7 & 2 & 8 \end{array}$$

Cubes

- Ex.2: Find 13^3
- Here a = 1, b = 3, b/a = 3
- $a^3 = 1$; Now we write the first and second row elements as follows:

1	**+**	**3**	**+**	**9**	**+**	**27**
	+	**6**	**+**	**18**		
2		**1**		**9**		**7**

Cubes

- Ex.2: Find 32^3
- Here a = 3, b = 2, b/a = 2/3
- $a^3 = 27$; Now we write the first and second row elements as follows:

27	**+**	**18**	**+**	**12**	**+**	**8**
	+	**36**	**+**	**24**		
32		**7**		**6**		**8**

Cubes

- Ex.2: Find 38^3
- Here a = 3, b = 8, b/a = 8/3
- $a^3 = 27$; Now we write the first and second row elements as follows:

27		62		51		
27	+	72	+	192	+	512
	+	144	+	384		
54		8		7		2

Vedic Mathematics

SQUARE ROOTS

Finding square roots of perfect squares

- To obtain he square root of a number, it is better to understand the pattern of squares and square roots.
- Following table gives the numbers from 1 to 9 with their respective squares, last digits and digit sums

Finding square roots of perfect squares

Number	Square	Last digit	Digit sum
1	1	1	1
2	4	4	4
3	9	9	9
4	16	6	7
5	25	5	7
6	36	6	9
7	49	9	4
8	64	4	1
9	81	1	9

Finding square roots of perfect squares

- The pattern behind the numbers:
  - The square numbers only have a digit sum 1, 4, 7, 9
  - The square numbers only end in 1, 4, 5, 6, 9, 0
- With this information, we can find out if the given number is a perfect square or not

Finding square roots of perfect squares

- Ex. 1: Find the square root of 3249
- STEP 1: put the digit in pairs 32 49
- STEP 2: Find the number of digits in the square roots the number of pairs (2 in this case)
- STEP 3: The first pair is 32. 32 is more than $5^2 = 25$ and less than $6^2 = 36$. Hence the square root will be in between 50 and 60.
- STEP 4: Focus on the last digit of the number. The last digit is 9. We get 9 by 3x3 = 9, or 7x7 = 49

Finding square roots of perfect squares

- So the second number will be 3 or 7.
- So the square root of 3249 is 53 or 57.
- STEP 5: Check the digit sum
- Digit sum of 3249 = 9
- Digit sum of 53 = 8
- So digit sum of 53^2 = Digit sum of 8^2 = digit sum of 64 = 1
- Digit sum of 57^2 = Digit sum of 3^2 = 9
- Hence the square root of 3249 is 57

Finding square roots of perfect squares

- Ex. 2: Find the square root of 2401
- 24 lies between $4^2 = 16$ and $5^2 = 25$. So the square root is between 40 and 50.
- The last digit is 1, so the last digit of the square root can be 1 or 9. SO the square root is 41 or 49
- Digit sum of 2401 = 7
- Digit sum of $41^2 = 7$; Digit sum of $49^2 = 7$
- So both can be the answer.

Finding square roots of perfect squares

- Now let's take a number between 41 and 49, say 45.
- $45^2 = 2025 < 2401$
- So square of 2401 must be greater than 45.
- Hence our final answer is 49

Finding square roots of perfect squares

- Ex. 3: Find the square root of 24964
- There will be 3 pairs: 02 49 64. So the square root will have 3 digits
- 249 lies between $15^2 = 225$ and $16^2 = 296$. So the square root is between 150 and 160.
- The last digit is 4, so the last digit of the square root can be 2 or 8. So the square root is 152 or 158
- Digit sum of 24964 = 7
- Digit sum of $152^2 = 1$; Digit sum of $158^2 = 7$
- So the answer is 158

Finding square roots of perfect squares

- Ex. 3: Find the square root of 32761
- There will be 3 pairs: 03 27 61. So the square root will have 3 digits
- 327 lies between $18^2 = 324$ and $19^2 = 369$. So the square root is between 180 and 190.
- The last digit is 1, so the last digit of the square root can be 1 or 9. So the square root is 181 or 189
- Digit sum of 32761 = 1
- Digit sum of $181^2 = 1$; Digit sum of $189^2 = 9$
- So the answer is 181

Vedic Mathematics

CUBE ROOTS

Cube roots

- The table given below shows the cubes of numbers 1 to 9:

Number	Cube	Last digit
1	1	1
2	8	8
3	27	7
4	64	4
5	125	5
6	216	6
7	343	3
8	512	2

Cube roots

- If the cube ends with 1, 4, 5, 6, 9 , the cube root also ends with 1, 4, 5, 6, 9 respectively
- If the cube ends with 8, the cube root ends with 2 and vice versa
- If the cube ends with a 7, its cube root ends with a 3 and vice versa
- With this information, finding cube root becomes easy.

Cube roots

- Ex. 1: Find the cube root of 3375
- STEP 1: Put the numbers in groups of 3, starting from right to left. In this case, the groups will be 3 375.
- STEP 2: To find the last digit of the cube root, we check the last digit of the number. In this case the last digit is 5, so the last digit of the cube root will also be 5.

Cube roots

- STEP 3: To get the first digit of the cube root, simply find a perfect cube less than or equal to 3. In this case it is 1. The cube root of 1 is also 1.
- So the answer is 15.

Cube roots

- Ex. 2: Find the cube root of 328509
- In this case, the groups will be 328 509.
- STEP 2: In this case the last digit is 9, so the last digit of the cube root will also be 9.
- STEP 3: Find a perfect cube less than or equal to 328. In this case it is 216. The cube root of 216 is 6.
- So the answer is 69.

Cube roots

- Using digit sum to check if a given number is a perfect cube

Number	Cube	Last digit	Digit Sum
1	1	1	1
2	8	8	8
3	27	7	9
4	64	4	1
5	125	5	8
6	216	6	9
7	343	3	1
8	512	2	8

Cube roots

- We see a repeating pattern of 1-8-9. SO any number which is a perfect cube must have a digit sum of 1, 8 or 9.

Cube roots

- Ex. 3: Find the cube root of 175616
- In this case, the groups will be 175 616.
- STEP 2: In this case the last digit is 6, so the last digit of the cube root will also be 6.
- STEP 3: Find a perfect cube less than or equal to 175. In this case it is 125. The cube root of 125 is 5.
- So the answer is 56.

NCERT – Class-1 Syllabus

Fermat's Last Theorem

Fermat's Last Theorem (sometimes called **Fermat's conjecture**, especially in older texts) states that no three positive integers *a*, *b*, and *c* satisfy the equation $a^n + b^n = c^n$ for any integer value of *n* greater than two. The cases $n = 1$ and $n = 2$ have been known to have infinitely many solutions since antiquity

Fermat's Little Theorem

Fermat's little theorem states that if p is a prime number, then for any integer a, the number $a^p - a$ is an integer multiple of p. In the notation of modular arithmetic, this is expressed as

For example, if $a = 2$ and $p = 7$, $2^7 = 128$, and $128 - 2 = 7 \times 18$ is an integer multiple of 7.

Goldback Conjecture : All even number is a sum of two prime numbers.

3. Axioms are the statements which are considered as true without proof.

A 1.5 : Mathematical proof

- Verification -> Trial
- Proof -> Logical Argument.

LOGO

What is Logo?
Why it is required to learn LOGO?

LOGO

What is Logo?

LOGO is pronounced as Low-go and is a high-level **programming** language known for its graphics capabilities, created by Seymour Papert in 1967.

Why it is required to learn LOGO?

With LOGO one can learn mathematics, drawing, design, logical thinking, programming, pattern identification.

LOGO

Parts of LOGO program:

1. Logo screen with a Turtle
2. Commander

How to Learn Logo?
Think of an obedient Turtle who obeys, follow and perform all the commands you provide to it through the commander.

Contents

Lession	Topics
1	Basics of Drawing
2	Looping - Repeat
3	Polygons - Variable
4	Randomness
5	Colors
6	Pen Control
7	Cartesian Coordinate System
8	Recursion and Fractals
9	Logical Reasoning
10	Pattern Recognition

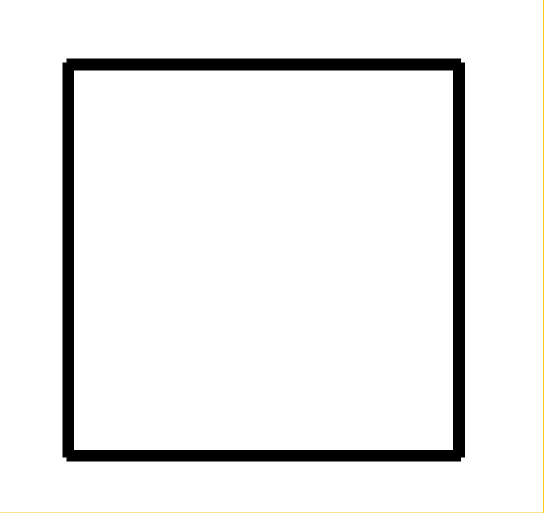
LOGO Commands

Procedure	Example	What Happens
FORWARD *number*	FORWARD 100	The turtle walks forward 100 screen dots.
BACK *number*	BACK 100	The turtle walks backward 100 screen dots.
RIGHT *number*	RIGHT 90	The turtle turns 90 degree to the right.
LEFT *number*	LEFT 90	The turtle turns to the left.
CLEARSCREEN	CLEARSCREEN	The turtle erases everything that he has drawn and goes back to where he started.

Note: Turtle always faces North or up in the screen

1. Teaching Turtle to Draw a Line

Objective-To teach forward and turn command



Fd 100
Draw Multiple Line
(Combine Command)
Fd 100 rt 90
Fd 100 rt 90 fd 100
Fd 100 rt 90 fd 100 rt 90
Fd 100 rt 90 fd 100 rt 90 fd 100
Fd 100 rt 90 fd 100 rt 90 fd 100 rt 90
Fd 100 rt 90 fd 100 rt 90 fd 100 rt 90 fd 100

LOGO

Few Commands:

Clear Screen-cs

Forward-fd 100>>>Result: Turtle Move 100 pixel

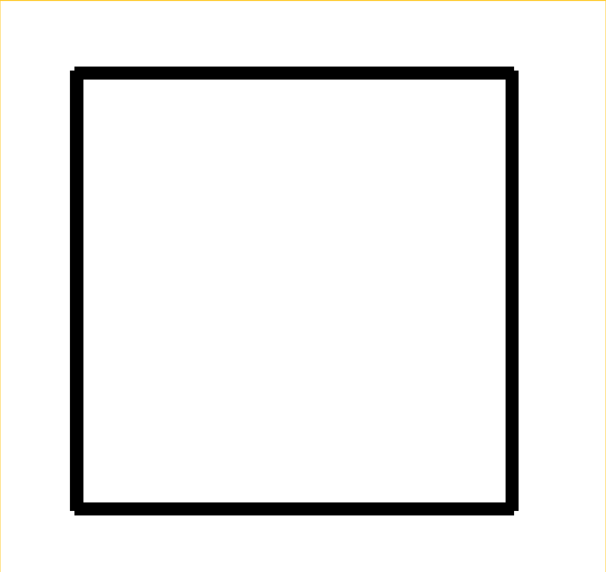
Back-bk 100>>>Result: Turtle Back 100 pixel

Right Turn-rt 90 >>Result: Turtle Turn Right 90 degree

Left Turn – lt 90>>>Result: Turtle Turn Left 90 degree

2. Teaching Turtle to Draw a Square

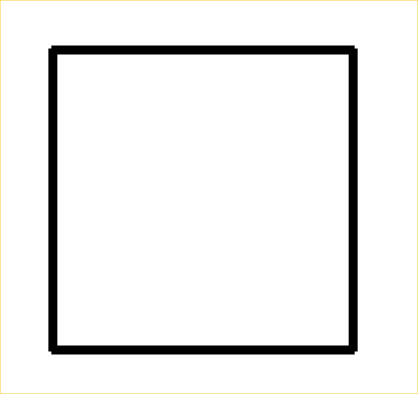
Fd 100
Fd 100 rt 90
Fd 100
rt 90
Fd 100
rt 90
Fd 100
rt 90
Fd 100
rt 90



3. Teaching Turtle to use Repeat

Objective –To follow same command repeatedly

Fd 100
rt 90
Fd 100
rt 90
Fd 100
rt 90
Fd 100
rt 90
Use of repeat command



REPEAT 4 [FD 100 RT 90]

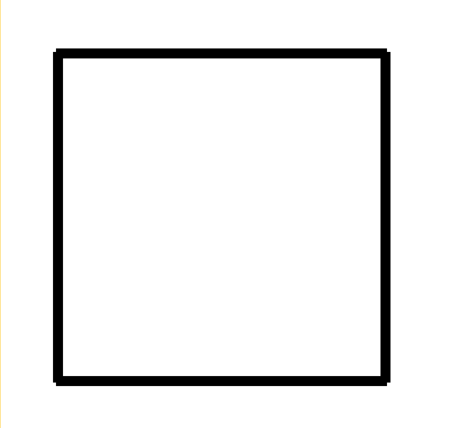
Procedure	Example	What Happens
REPEAT *number* [*instructions*]	REPEAT 4 [FORWARD 100 RIGHT 90]	The turtle does the following four times: moves forward 100, turns right 90°.

6. Teaching Turtle to use Procedure

Objective-To reproduce same object when ever required

- **Enter through edall, type command, save and exit**
- **Then whenever you type "square" at commander, turtle will draw a square**

```
To square
Repeat 4 [fd 100 rt 90]
end
```



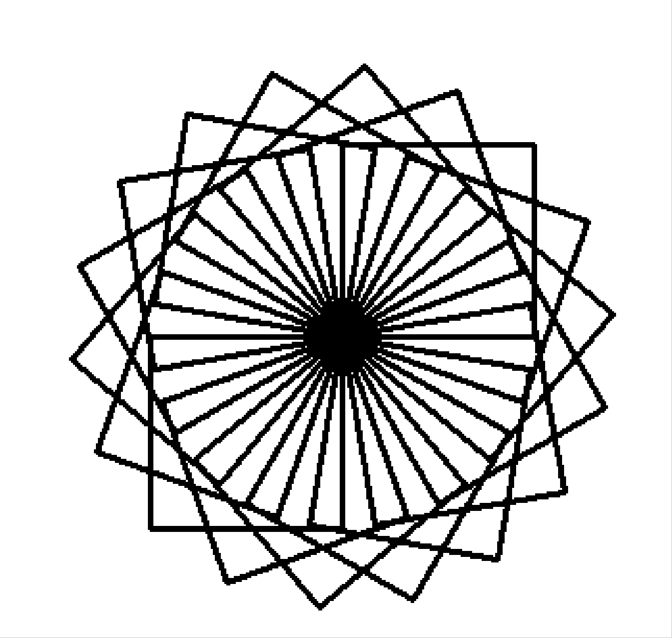
Save the file and picture for future use

7. Procedure calls Procedure

Objective-To create pattern, we have to draw same object with different orientation. This is easily can be accomplished by calling a procedure within another procedure

We will create a flower from the procedure of square created earlier

```
To square
Repeat 4 [fd 100 rt 90
End
To flower
Repeat 18 [ square rt 20]
end
```



11. Basic Loops - Repcount

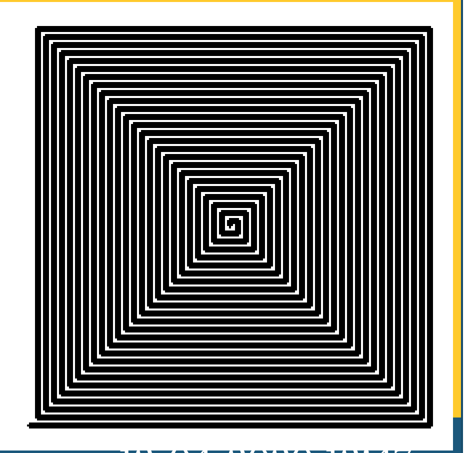
Objective: With “REPEAT” we can draw same objects of similar size many times but if we want to change size then “loops” are best options.

REPCOUNT is the loop counter, it counts the number of time one command repeats.

With repcount, we will draw a square spiral which start with 1 unit and increases 1 unit every time

REPEAT 100 [FORWARD REPCOUNT * 2 RIGHT 90]

repeat 10 [print 11 - repcount]



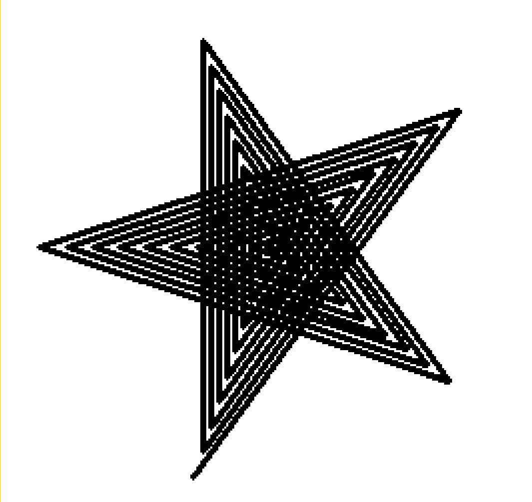
11. Basic Loops - Repcount

To type "10,9,8,7,6,5,4,3,2,1, End"

repeat 10 [print 11 - repcount]
Print "Blastoff

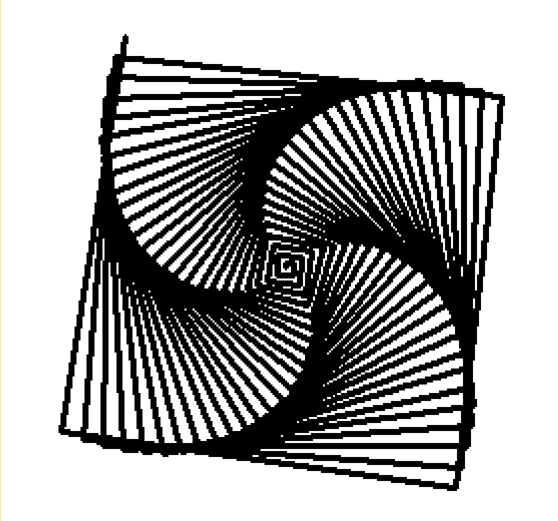
To a sequence from 10 to 20

repeat 10 [print 9 + repcount]



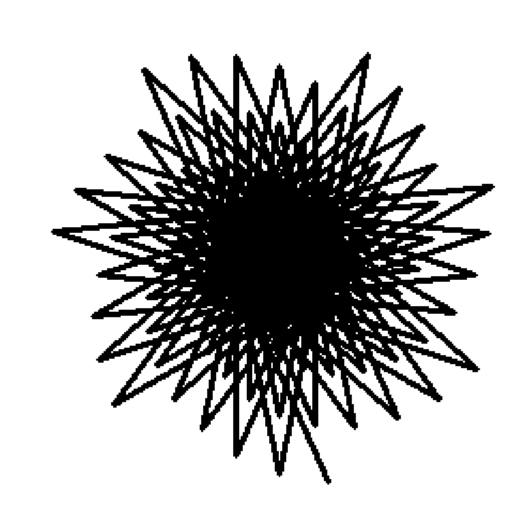
```
TO STARSPIRAL
 REPEAT 45 [ FORWARD REPCOUNT * 5 RIGHT 144 ]
END
```

11. Basic Loops - Repcount



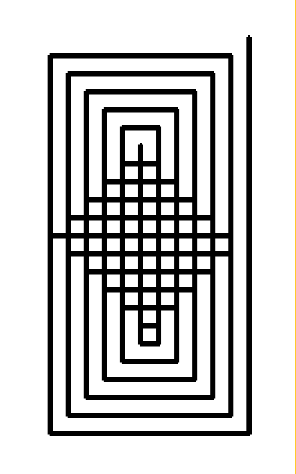
TO SQUIRAL
REPEAT 100 [FORWARD REPCOUNT * 2 RIGHT 91]
END

TO EXPLOSION
REPEAT 120 [FORWARD REPCOUNT * 2 RIGHT 204]
END

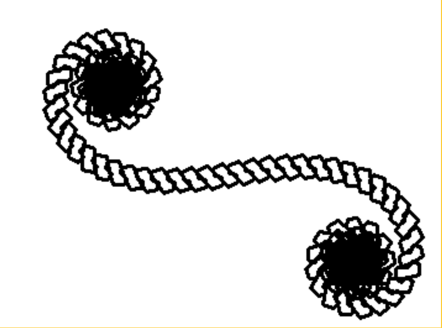


11. Basic Loops - Repcount

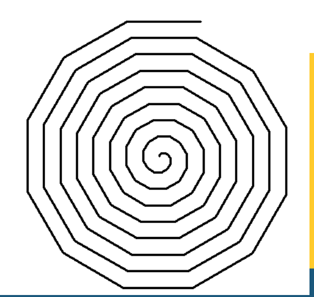
```
TO PATTERN1
REPEAT 22 [ RIGHT 90 FORWARD 110 - REPCOUNT * 10
RIGHT 90
FORWARD REPCOUNT * 10 ]

END
```



```
TO SSHAPE
REPEAT 750 [ RIGHT 90
REPEAT 4 [ FORWARD REPCOUNT * 3 RIGHT 72 ]

RIGHT REPCOUNT ]
END
```

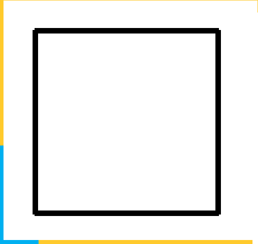


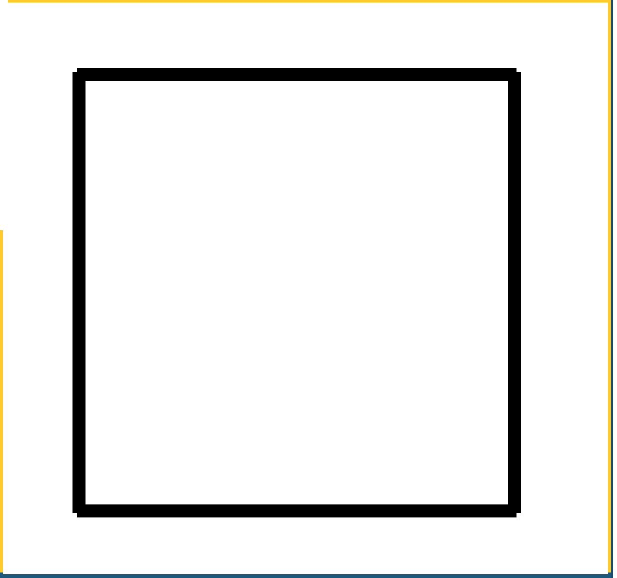
```
REPEAT 100 [FD REPCOUNT RT 30]
```



17 Teaching Turtle to add Variable

Objective: With procedure we can draw different shape when ever required but the dimensions of the objects remain same. We . With "parameter" we can create objects of different sizes.





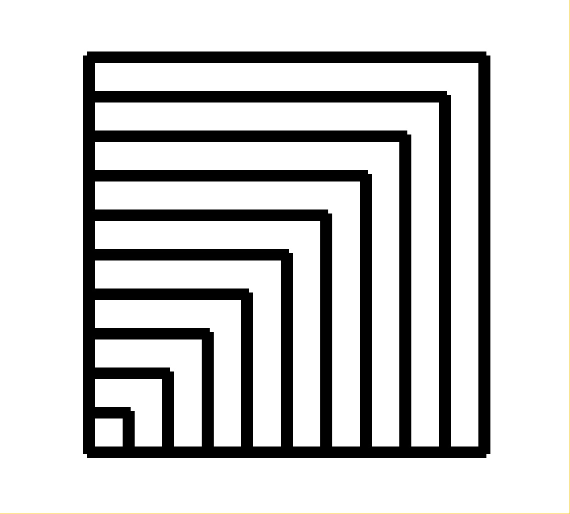
```
TO SQUARE1
 REPEAT 4 [ FORWARD 100 RIGHT 90 ]
END
```

```
TO SQUARE2  :length
 REPEAT 4 [ FORWARD :length RIGHT 90 ]
END
```

17 Teaching Turtle to add Variable

Objective: With procedure we can draw different shape when ever required but the dimensions of the objects remain same. We . With “parameter” we can create objects of different sizes.

```
to  square :Length
repeat 4 [ fd :Length rt 90]
End
to msquare
repeat 10 [square repcount * 10]
end
```

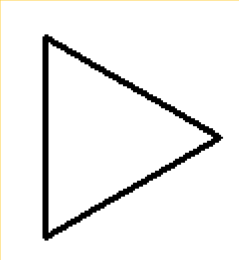
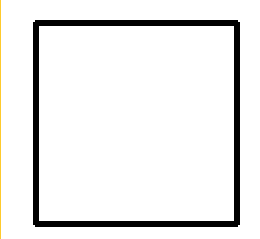
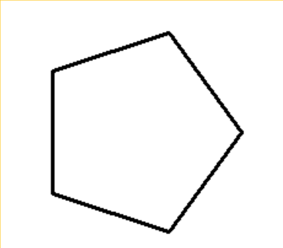
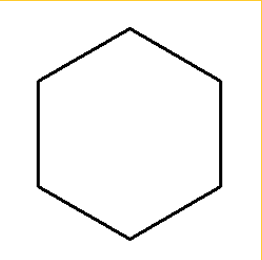
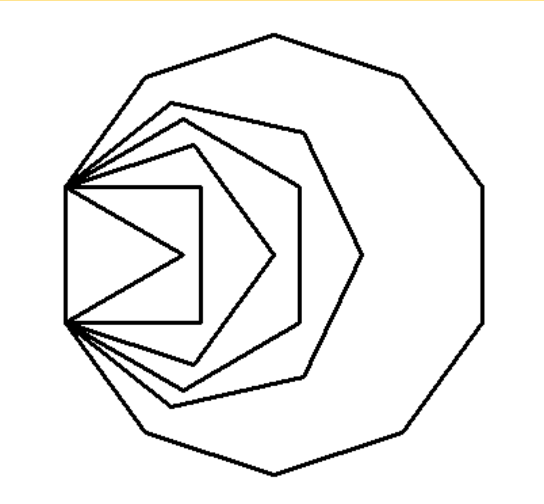


17 Teaching Turtle to draw Polygon

To Draw a triangle: repeat 3 [fd 100 rt 120]

To Draw a square: repeat 4 [fd 100 rt 90]

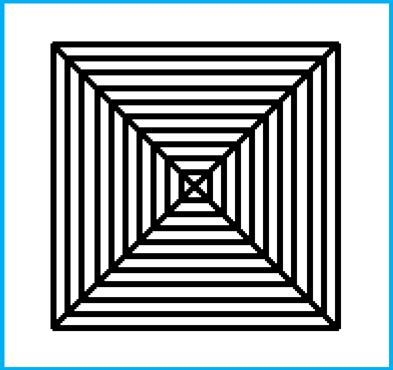
To Draw a Pentagon: repeat 5 [fd 100 rt 72]

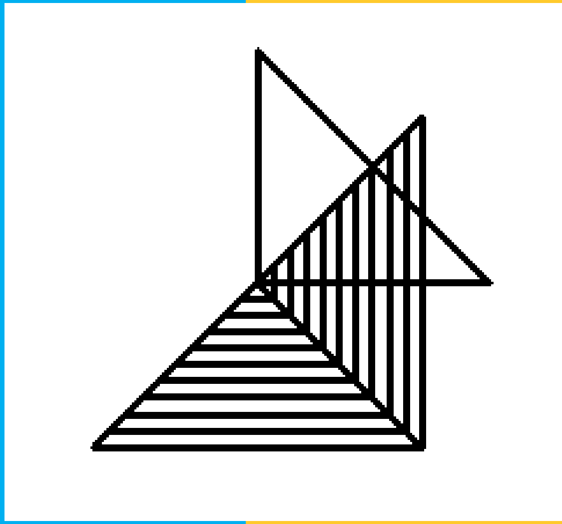






To Polygon : side :length
Repeat :side [fd :length rt 360/:side]
end

17 Teaching Turtle to draw Pattern

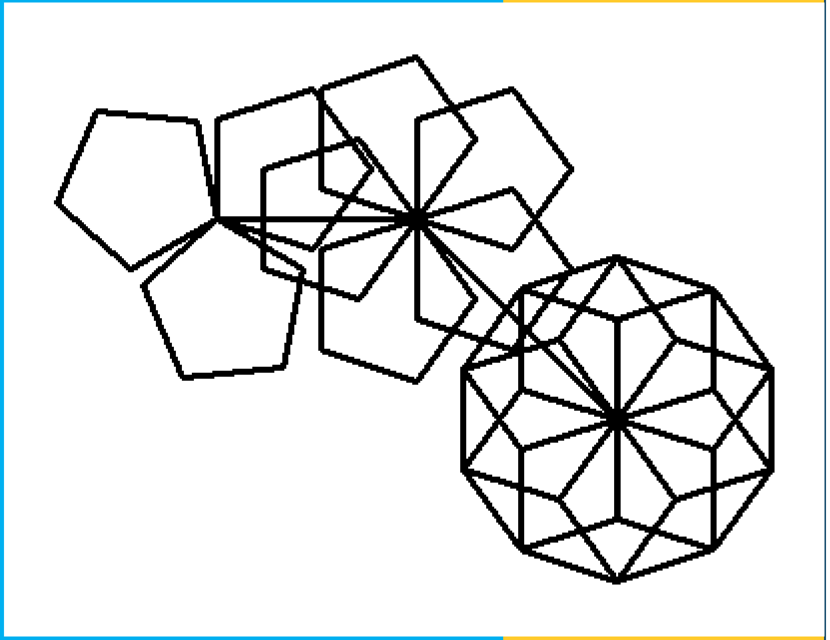
```
TO RIGHTTRIANGLE :LENGTH
FORWARD :LENGTH
RIGHT 135
FORWARD :LENGTH * SQRT 2
 RIGHT 135
 FORWARD :LENGTH
RIGHT 90
END


TO PYRAMID
RIGHT 45
REPEAT 4 [ REPEAT 10 [ RIGHTTRIANGLE REPCOUNT * 10 ]
RIGHT 90 ]
LEFT 45
END
```





17 Teaching Turtle to draw Pattern

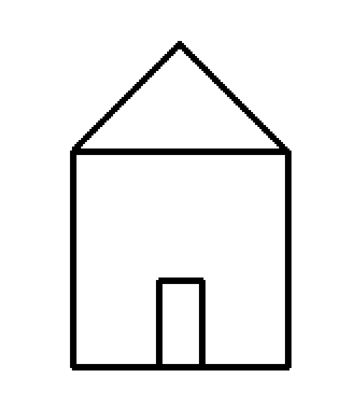
```
TO POLYGON1 :SIDES :LENGTH
  REPEAT :SIDES [
    FORWARD :LENGTH
    RIGHT 360 / :SIDES
  ]
END


TO FLOWER :PETALS
  REPEAT :PETALS [
    POLYGON1 5 50
    RIGHT 360 / :PETALS
  ]
END
```



17 Teaching Turtle to draw House

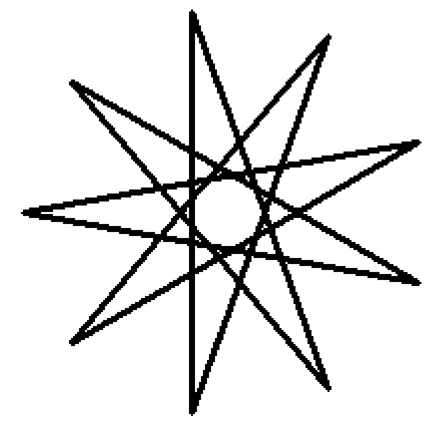
```
TO RECTANGLE :HEIGHT :WIDTH
 REPEAT 2 [
  FORWARD :HEIGHT
  RIGHT  90
  FORWARD :WIDTH
  RIGHT  90
 ]
END

TO TRIANGLE :LENGTH
 RIGHT 45
 FORWARD :LENGTH * (SQRT 2) / 2
 RIGHT 90
 FORWARD :LENGTH * (SQRT 2) / 2
 RIGHT 135
 FORWARD :LENGTH
 RIGHT 90
END
```

```
TO HOUSE
   ; draw the house
   RECTANGLE 100 100
    ; draw the roof
   FORWARD   100
   TRIANGLE 100
   BACK      100
   ; draw the door
   RIGHT      90
   FORWARD    60
   LEFT      180
   RECTANGLE 20 40
   FORWARD    60
   RIGHT      90
END
```

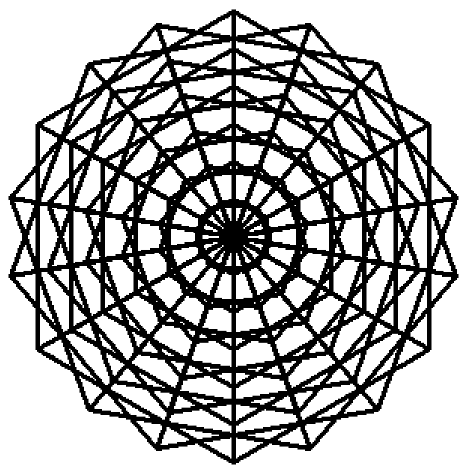


17 Teaching Turtle to draw Star

```
TO STAR :LENGTH :POINTS
  REPEAT :POINTS [     FORWARD :LENGTH
    RIGHT   180 - (180 / :POINTS)  ]
END
```



```
TO TRIANGLE :LENGTH
  REPEAT 3 [FORWARD :LENGTH RIGHT 120 ]
END
TO TRIANGLEFLOWER :LENGTH :COUNT
  REPEAT :COUNT [
    TRIANGLE  :LENGTH
    RIGHT     360 / :COUNT
  ]
END
TO WEB
  REPEAT 6 [ TRIANGLEFLOWER REPCOUNT * 25 18 ]
END
```



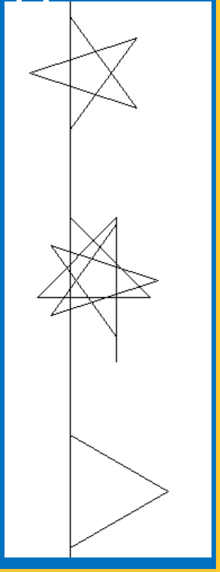
26 Moving from regular to random

REPEAT 1000 [FORWARD 10 RIGHT RANDOM 360]

27 Teaching Turtle to pick

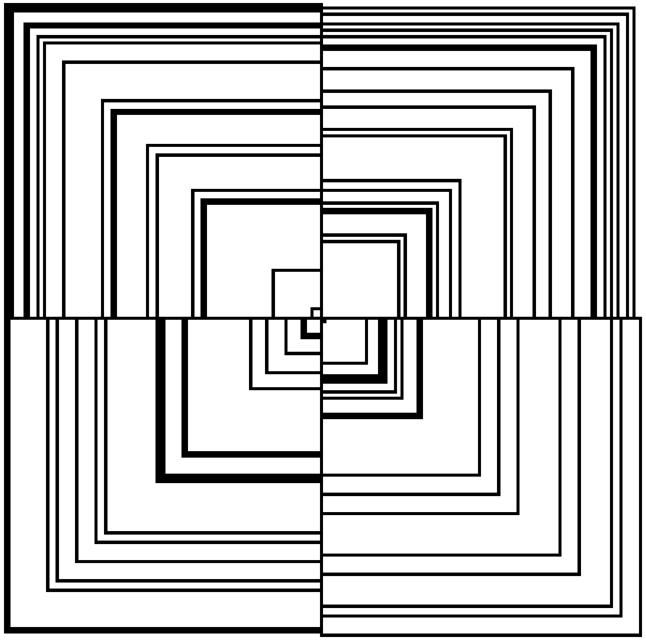
run pick [[forward 100] [rt 90][back 100] [lt 90]]

```
to star :points
repeat :points [forward 100   rt 180 - (180/:points)]
end
to randomstar
star pick [ 3 4 5]
end
to manystar
repeat 5 [randomstar fd random(500)]
end
```



27 Teaching Turtle to pick

```
REPEAT 4 [ FORWARD :SIZE
RIGHT 90 ]
END

TO RANDOMBOXES
  REPEAT 10 [ SQUARE RANDOM
100 ]
END

TO BOXPICTURE
  REPEAT 4 [ RANDOMBOXES
RIGHT 90 ]
END
```



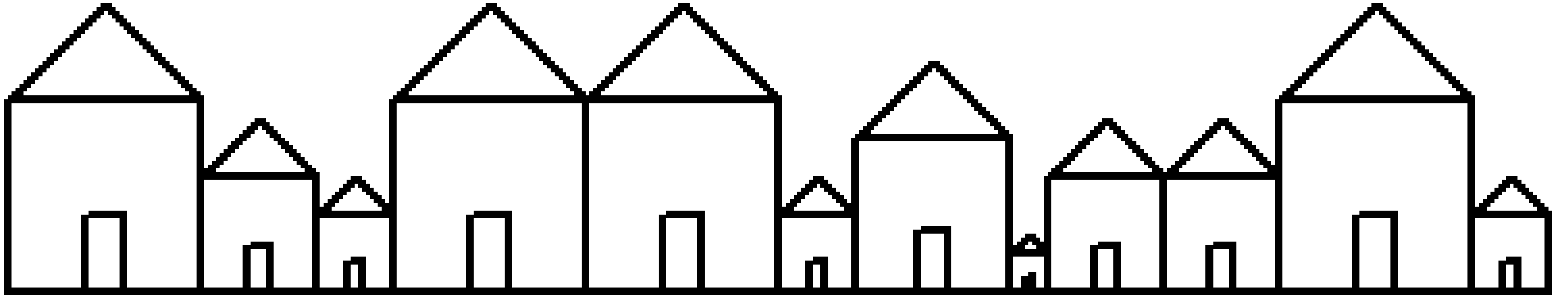
27 Teaching Turtle to use Random

```
TO HOUSE :SIZE
   ; draw the house
RECTANGLE (10 * :SIZE)(10*
:SIZE)

   ; draw the roof
   FORWARD  10 * :SIZE
   TRIANGLE 10 * :SIZE
   BACK     10 * :SIZE

   ; draw the door
   RIGHT     90
   FORWARD   6 * :SIZE
   LEFT      180
   RECTANGLE (2 * :SIZE) (4 *
:SIZE)
   BACK      4 * :SIZE
   RIGHT     90
END
```

```
TO HOUSEROW
   REPEAT 15 [ HOUSE RANDOM 6]
END
TO RECTANGLE :HEIGHT :WIDTH
   REPEAT 2 [
   FORWARD :HEIGHT RIGHT   90
   FORWARD :WIDTH RIGHT  90]
END
TO TRIANGLE :LENGTH
 RIGHT 45
 FORWARD :LENGTH * (SQRT 2)/2
 RIGHT 90
 FORWARD :LENGTH * (SQRT 2)/2
 RIGHT 135
 FORWARD :LENGTH
  RIGHT 90
END
```

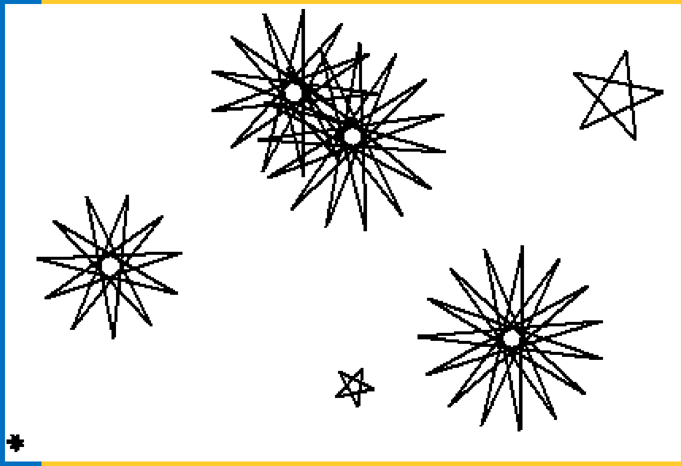
27 Teaching Turtle to use Random

HOUSEROW



27 Teaching Turtle to use Random

```
TO STAR :LENGTH :POINTS
  REPEAT :POINTS [
    FORWARD :LENGTH
    RIGHT   180 - (180 / :POINTS)   ]
END

TO STARRYNIGHT
  REPEAT 30 [
    TELEPORT
    STAR (RANDOM 100 + 20) ((RANDOM 6) * 2 + 5)   ]
END

TO TELEPORT
  PENUP
  RIGHT   RANDOM 360
  FORWARD RANDOM 1000
  PENDOWN
END
```



27 Teaching Turtle to Use Color

Commands:

1. Setpencolors amount
2. Setscreencolors amount

There are two options to set color –

1. 16 commonly used colors with specific code
2. Create color by mixing – Red, Green, Blue (RGB)

COLOR INDEX	COLOR NAME	R G B

Color index varies from 0 to 15 which means that there are 16 commonly used colors. These 16 colors are show below:

27 Teaching Turtle to Use Color

COLOR INDEX	COLOR NAME	R G B

Color Index	Color Name	[R G B]	Color
0	black	[0 0 0]	
1	blue	[0 0 255]	
2	green	[0 255 0]	
3	cyan (light blue)	[0 255 255]	
4	red	[255 0 0]	
5	magenta (reddish purple)	[255 0 255]	

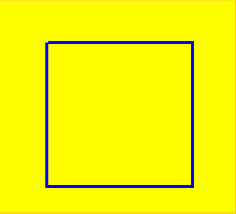
27 Teaching Turtle to Use Color

COLOR INDEX	COLOR NAME	R G B
6	yellow	[255 255 0]
7	white	[255 255 255]
8	brown	[155 96 59]
9	light brown	[197 136 18]
10	dark green	[100 162 64]
11	darkish blue	[120 187 187]
12	tan	[255 149 119]
13	plum (purplish)	[144 113 208]
14	orange	[255 163 0]
15	gray	[183 183 183]

Mixing of Base Colors

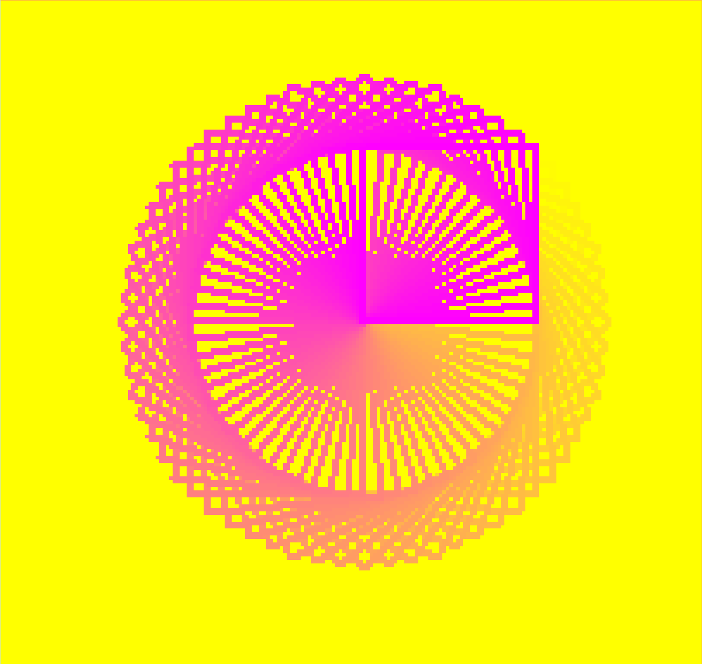
RGB means Red Green and Blue. By mixing these three colors at different proportions, 16 million colors can be created (255 x 255 x 255 =16581375). These colors are related to light. It is related to the receptor of human eye.
Red Yellow and Blue are primary pigment colors related to paints. Actualy, the pigments colors are Magneta (Redish Purple) Yellow Cyan (light blue).

```
Setscreencolor 6
Setpencolor 1
Repeat 4 [fd 100 rt 90]
```



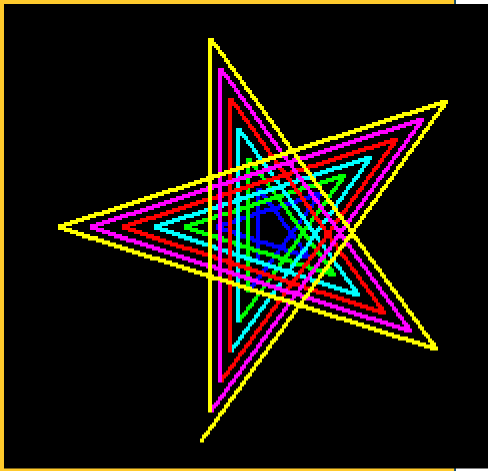
Teach Turtle to use color


```
TO SQUARE
  REPEAT 4 [ FORWARD 50 RIGHT 90 ]
END

TO SETPEN :BRIGHTNESS
  SETPENCOLOR ( LIST
     255                ; red
     255 - :BRIGHTNESS  ; green
     :BRIGHTNESS        ; blue  )
END

TO SQUAREFLOWER
  REPEAT 64 [
    SETPEN REPCOUNT * 4 - 1
    RIGHT 360 / 64
    SQUARE  ]
END
```



Teach Turtle to use color


```
TO COLORSTAR
  SETSCREENCOLOR 0
  REPEAT 35 [
    SETPENCOLOR INT (REPCOUNT - 1) / 5
    FORWARD    REPCOUNT * 6
    RIGHT      144  ]
END
```



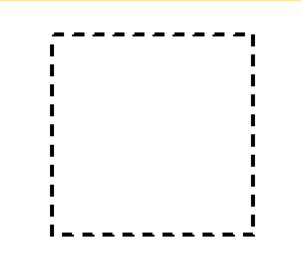
To teach Turtle to use Pen

Till now, we have used continuous solid line. Now we will create dashed line by controlling pen.

Commands:

1. PENUP
2. PENDOWN

Repeat 10 [fd 5 penup fd 5 pendown]



Repeat 4 [Repeat 10 [fd 5 penup fd 5 pendown] rt 90]

Coloring objects

Till now, we have created objects with lines. Now we will fill the objects with color.
Commands:

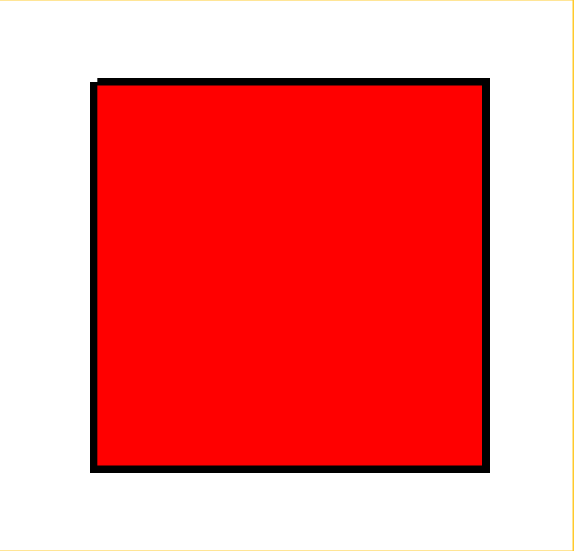
1. Setfloodcolor amount
2. Fill

Repeat 10 [fd 5 penup fd 5 pendown]

Repeat 4 [Repeat 10 [fd 5 penup fd 5 pendown] rt 90]

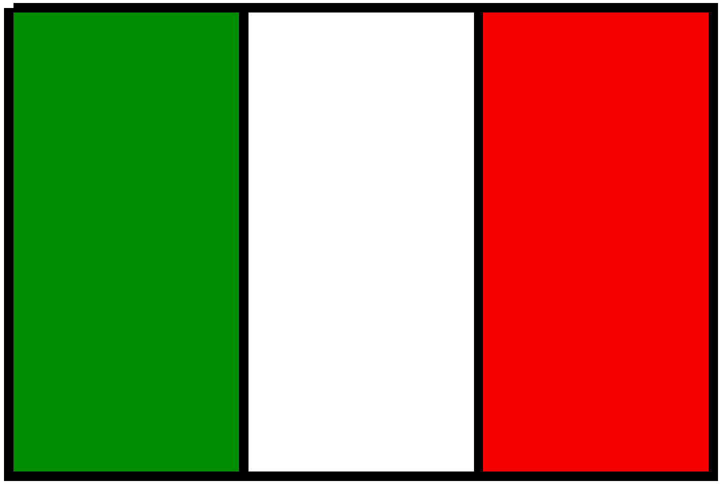
Creating a red suare


```
TO REDSQUARE
  ; draw the outline
  REPEAT 4 [FORWARD 100 RIGHT 90]

  ; move into the square
  PENUP
  RIGHT 45
  FORWARD 4

  ; fill the square with red
  SETFLOODCOLOR 4
  FILL

  ; move back
  BACK 4
  LEFT 45
  PENDOWN
END
```



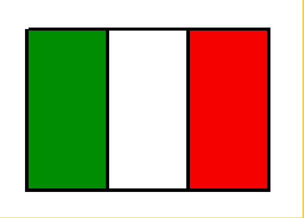
Coloring disconnected object


```
TO RECTANGLE :WIDTH :HEIGHT
  REPEAT 2 [ FORWARD :HEIGHT
RIGHT   90  FORWARD :WIDTH
RIGHT   90  ]
END
TO TRICOLOR_FLAG :COLOR1 :COLOR2
:COLOR3
  ; Draw each of the three stripes
  REPEAT 3 [ RECTANGLE REPCOUNT *
50 100 ]
  ; Get set to fill in each rectangle
  PENUP
  FORWARD 10  RIGHT   90
  ; fill the first stripe
  FORWARD 25
  SETFLOODCOLOR :COLOR1
  FILL
```



Coloring disconnected object


```
  ; fill the second stripe
  FORWARD 50
  SETFLOODCOLOR :COLOR2
  FILL
  ; fill the third stripe
  FORWARD 50
  SETFLOODCOLOR :COLOR3
  FILL
 ; go back where we started from
  BACK 50 * 2 + 25
  LEFT 90   BACK 10    PENDOWN
END

TO ITALY_FLAG
  ; Italy's flag is green|white|red
  TRICOLOR_FLAG [0 140 0] [255 255
255] [247 0 0]
END
```



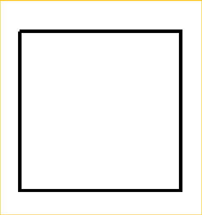
Introducing Cartesian Coordinate System

Commands:

1. Home
2. Setxy x y

To create a square with coordinate system:

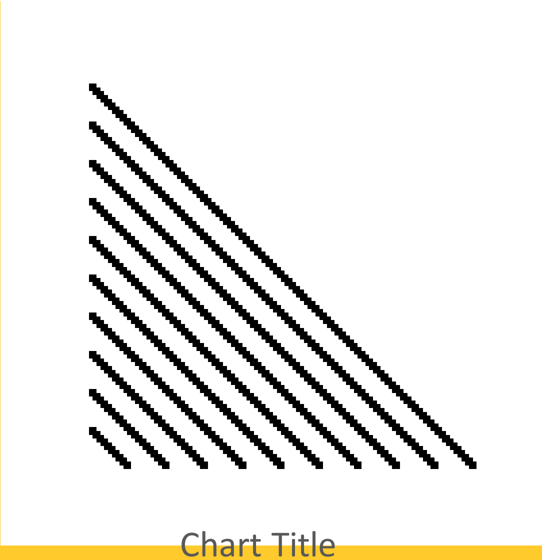
Setxy 0 100
Setxy 100 100
Setxy 100 0
home

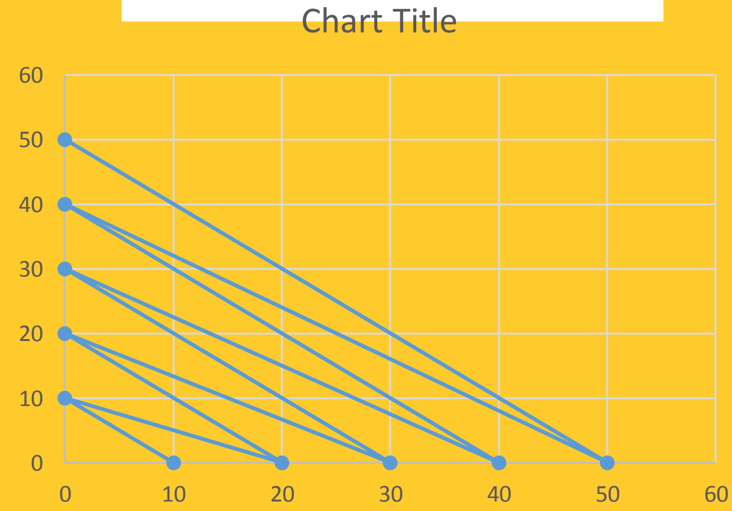


String Art with Cartesian Coordinate System


```
TO STRINGART
 REPEAT 10 [
  PENUP
  SETXY (REPCOUNT * 10) 0
  PENDOWN
  SETXY 0 (REPCOUNT * 10)
 ]
END
```

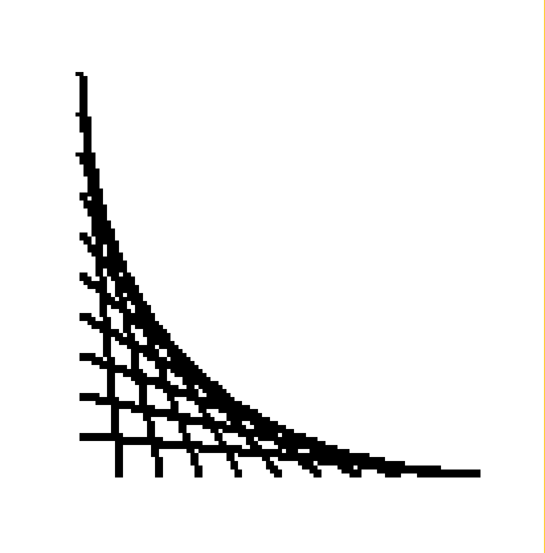
Repcount	Initial Points		End Points	
	x	y	x	y
1	10	0	0	10
2	20	0	0	20
3	30	0	0	30
4	40	0	0	40
5	50	0	0	50
6	60	0	0	60
7	70	0	0	70
8	80	0	0	80
9	90	0	0	90
10	100	0	0	100





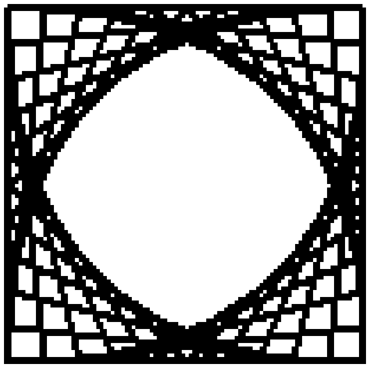
String Art with Cartesian Coordinate System


```
TO STRINGART
 REPEAT 10 [
   PENUP
   SETXY (REPCOUNT * 10) 0
   PENDOWN
   SETXY 0 (110 - REPCOUNT * 10)   ]
END
```



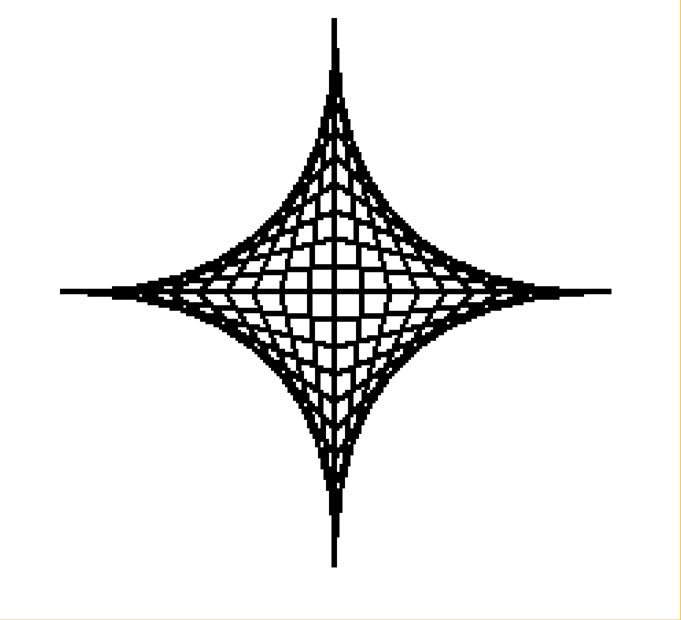
String Art with Cartesian Coordinate System


```
TO CYCLE :INDEX :LAST
  SETXY :INDEX          0
  SETXY :LAST           :INDEX
  SETXY (:LAST - :INDEX)  :LAST
  SETXY 0               (:LAST - :INDEX)
  SETXY :INDEX          0
END

TO SQUAREART
  REPEAT 10 [ CYCLE REPCOUNT * 10 100 ]
END
```



String Art with Cartesian Coordinate System

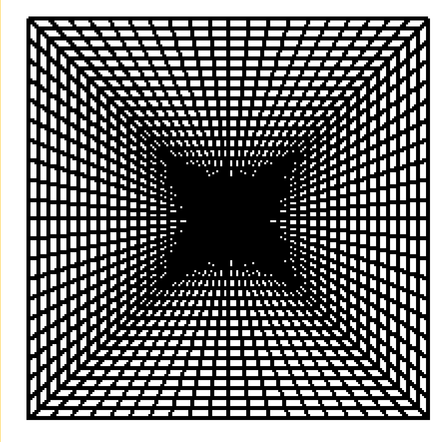

```
TO CYCLE :INDEX :LAST
  PENUP
  SETXY :INDEX 0
  PENDOWN
  SETXY 0        (:LAST - :INDEX)
  SETXY -:INDEX  0
  SETXY 0        (:INDEX - :LAST)
  SETXY :INDEX   0
END

TO PLUSART
  REPEAT 11 [ CYCLE ((REPCOUNT - 1) * 
10) 100 ]
END
```



String Art with Cartesian Coordinate System


```
TO SPIKEDSQUARE
  ; left-right lines
  REPEAT 20 [
    HOME
    SETXY (100)  (REPCOUNT * 10 - 110)
    SETXY (-100) (110 - REPCOUNT * 10)
    HOME
  ]

  ; up-down lines
  REPEAT 20 [
    HOME
    SETXY (REPCOUNT * 10 - 100) (100)
    SETXY (100 - REPCOUNT * 10) (-100)
    HOME
  ]
END
```

```
TO SQUARE :HALF_LENGTH
  SETXY  :HALF_LENGTH   :HALF_LENGTH
  SETXY -:HALF_LENGTH   :HALF_LENGTH
  SETXY -:HALF_LENGTH  -:HALF_LENGTH
  SETXY  :HALF_LENGTH  -:HALF_LENGTH
  SETXY  :HALF_LENGTH   :HALF_LENGTH
  HOME
END

TO PIT
  REPEAT 20 [SQUARE REPCOUNT * 5]
  SPIKEDSQUARE
END
```



Moving into the world of Fractals

Fractals are the objects whose a small part is look like the overall object.

In this session we will explore Recursion tool:

1. Recursion is the process where the command calls itself.
2. Recursive commands all follow same pattern.
3. They do little work and then calls themselves with simpler inputs.
4. This in turn do a little more work and call itself even more simpler inputs.
5. When the input is so simple that there's essentially nothing to be done. The command just stops without doing anything.

Moving into the world of Fractals

Repeat command vs Recursion:

1. Any thing we do with repeat command can be done with recursion
2. In "REPEAT" command, we fix the number of times the command is to be repeated.
3. In "RECURSION", we create a loop which will be repeated till certain condition and criteria is met.

There are two parts of recursion:

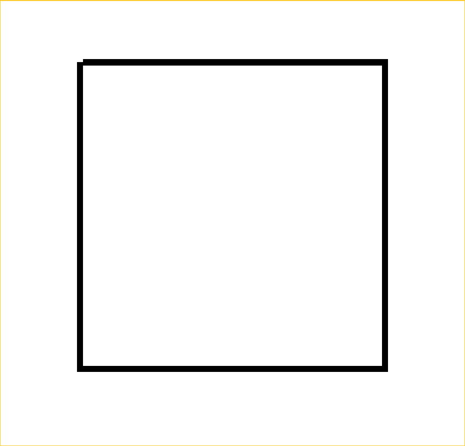
1. Base Case
2. Recursive case

The base case is used to set the criteria when the command will stop calling itself. Without base case, the command will be repeated for ever.

Moving into the world of Fractals

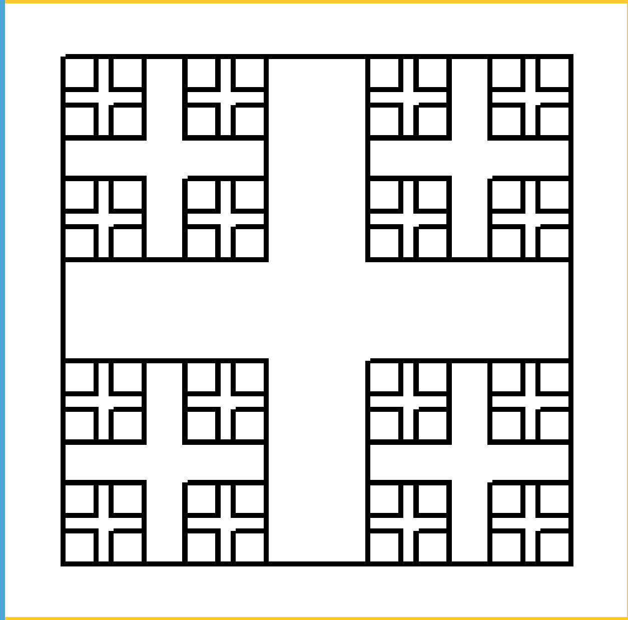

```
TO SQUARE
  REPEAT 4 [ FORWARD 100 RIGHT 90 ]
END

SQUARE
```

```
TO SQUARE.RECURSIVE :SIDES.TO.GO
  ; base case: do nothing
  IF :SIDES.TO.GO = 0 [ STOP ]

  ; recursive case: draw a side and call
recursively
  FORWARD 100
  RIGHT   90
  SQUARE.RECURSIVE :SIDES.TO.GO - 1
END
```



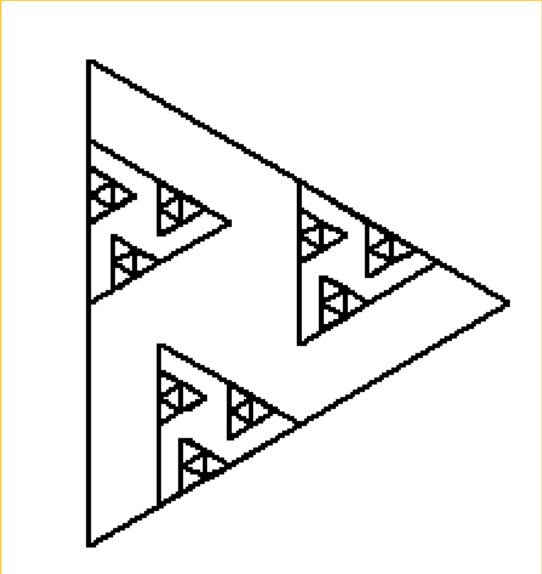
Moving into the world of Fractals


```
TO SQUARE.FRACTAL :LENGTH :DEPTH

  ; base case: no more squares
  IF :DEPTH = 0 [ STOP ]

  ; recursive case: draw a square such that each corner
  ; of the square has SQUARE.FRACTAL in it.
  REPEAT 4 [
    FORWARD :LENGTH
    RIGHT   90
    SQUARE.FRACTAL :LENGTH * 0.4 :DEPTH - 1   ]

END
```



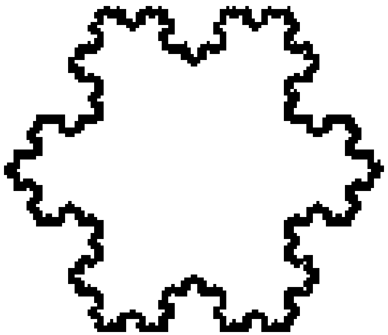
Moving into the world of Fractals


```
TO TRIANGLE.FRACTAL :LENGTH :DEPTH
  ; base case:
  ; just move forward, no more squares
  IF :DEPTH = 0 [
    FORWARD :LENGTH
    STOP  ]

  ; recursive case:
  ; draw a triangle such that each side of
  ; the triangle has TRIANGLE.FRACTAL in it.
  REPEAT 3 [
      FORWARD :LENGTH / 3
      TRIANGLE.FRACTAL :LENGTH / 3 :DEPTH - 1
      FORWARD :LENGTH / 3

      RIGHT 120  ]
END
```



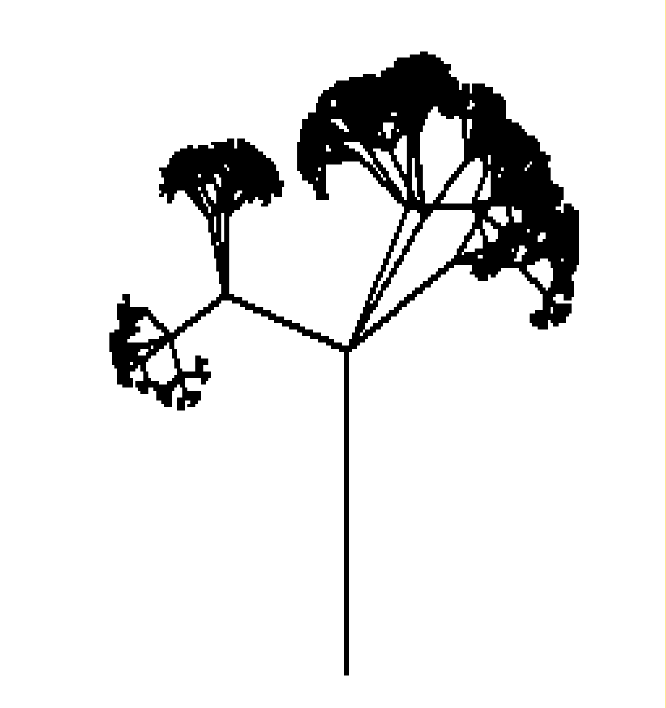
Moving into the world of Fractals


```
TO SNOWFLAKE.SIDE :LENGTH :DEPTH

  IF :DEPTH = 0 [
   FORWARD :LENGTH    STOP   ]
  SNOWFLAKE.SIDE :LENGTH  / 3 :DEPTH - 1
  LEFT    60
  SNOWFLAKE.SIDE :LENGTH  / 3 :DEPTH - 1
  RIGHT   120
  SNOWFLAKE.SIDE :LENGTH  / 3 :DEPTH - 1
  LEFT    60
  SNOWFLAKE.SIDE :LENGTH  / 3 :DEPTH - 1
END

TO SNOWFLAKE :LENGTH :DEPTH
  REPEAT 3 [    SNOWFLAKE.SIDE :LENGTH :DEPTH
    RIGHT 120  ]
END
```



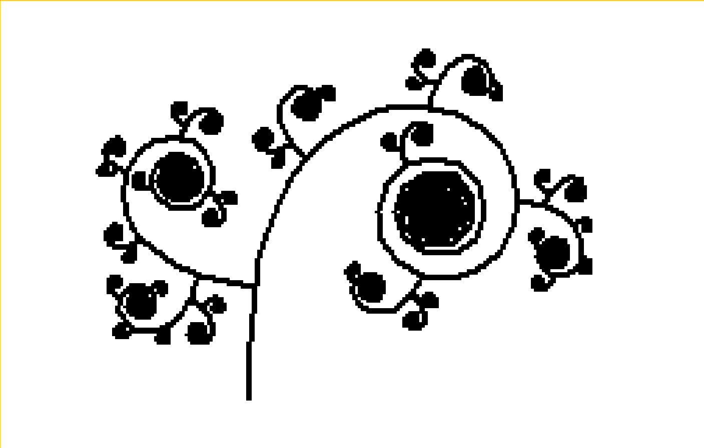
Moving into the world of Fractals


```
TO PLANT :SIZE :ANGLE
  IF :SIZE < 1 [ STOP ]

  RIGHT :ANGLE
  FORWARD :SIZE
  REPEAT 4 [
    PLANT :SIZE / 2 DIFFERENCE
RANDOM 160 80
  ]
  BACK :SIZE
  LEFT :ANGLE
END
```



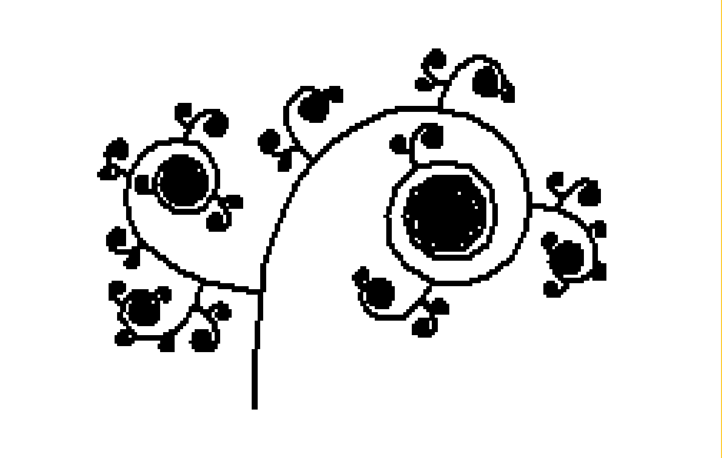
Moving into the world of Fractals


```
TO CURLY.FRACTAL :SIZE
  IF :SIZE < 0.5 [ STOP ]
  REPEAT 360 [
      IF REPCOUNT = 5 [
         LEFT 90
         CURLY.FRACTAL :SIZE / 2
         RIGHT 90      ]

      IF REPCOUNT = 10 [
         LEFT 90
         CURLY.FRACTAL :SIZE / 5
         RIGHT 90      ]

      IF REPCOUNT = 15 [
         LEFT 90
         CURLY.FRACTAL :SIZE / 5
         RIGHT 90      ]
```



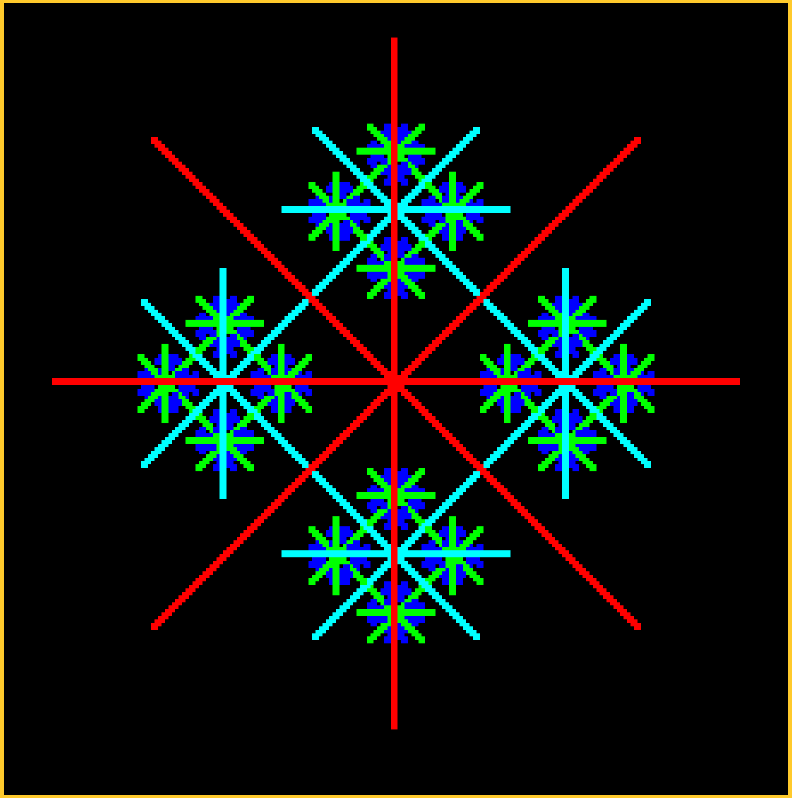
Moving into the world of Fractals


```
IF REPCOUNT = 20 [
      LEFT 90
      CURLY.FRACTAL :SIZE / 4
      RIGHT 90     ]
    IF REPCOUNT = 25 [
      LEFT 90
      CURLY.FRACTAL :SIZE / 5
      RIGHT 90     ]
    IF REPCOUNT = 30 [
      LEFT 90
      CURLY.FRACTAL :SIZE / 8
      RIGHT 90     ]
    FORWARD :SIZE
    RIGHT REPCOUNT   ]

  RIGHT 180
END
```



Moving into the world of Fractals


```
TO CRISSCROSS :SIZE :DEPTH
 IF :DEPTH = 0 [ STOP ]
 SETPENCOLOR :DEPTH
 REPEAT 4 [
   FORWARD :SIZE / 2
   CRISSCROSS :SIZE / 3 :DEPTH - 1
   FORWARD :SIZE / 2     BACK :SIZE

   RIGHT 45     FORWARD  :SIZE
   BACK :SIZE     RIGHT 45  ]
 SETPENCOLOR :DEPTH + 1
END

TO CRISSCROSSPICTURE
 SETSCREENCOLOR 0
 CRISSCROSS 100 4
END
```



Teaching Turtle to Talk

- Teaching Grammar

Working with words

- Parts of Speech

- A "Part of Speech" is a way of grouping words that have similar uses. For example "run", "catch", "throw", and "kick" are all action words. Some parts of speech are given in the table below.

Part of Speech	Description	Examples
noun	a person, place, or thing	Spiderman, Bellevue, table
verb	an action word	run, catch, throw, kick
adjective	words that describe nouns	fast, tall, slow, strong
adverb	words that describe verbs	quickly, slowly, very

Working with words

Command	Example	What Happens
LOCALMAKE *name value*	LOCALMAKE "length 100	Creates a variable named :length and assigns it the value 100.
PRINT *list*	PRINT [Hello World!]	Displays "Hello World!" in the Commander window.
LIST *value1 va lue2 ...*	(LIST "Hello "World!)	Creates a list from whatever follows it. Each element is evaluated before being put into the list.

Teaching Turtle English


```
TO MADLIBS
  LOCALMAKE "adjective1 "slow
  LOCALMAKE "opposite1  "fast
  LOCALMAKE "adjective2 "steady
  LOCALMAKE "adjective3 "short
  LOCALMAKE "bodypart   "feet
  LOCALMAKE "contest    "race
  LOCALMAKE "loser      "Hare
  LOCALMAKE "winner     "Tortoise
  SHOWSTORY
END
TO SHOWSTORY
  PRINT (LIST
```

Teaching Turtle English

```
TO SHOWSTORY
  PRINT (LIST
"\
"One "day, "a :loser "made "fun "of "the "\
:adjective3 :bodypart "and :adjective1 "pace "of "\
"a :winner ".\
"The :winner "replied: ""Even "though "I\
"have :adjective3 :bodypart ", "I "will "beat\
"you "in "a :contest "."\
"The :loser "thought "that "this "was "impossible.\
"So "he "challenged "the :winner "to "a :contest ".\
"On "the "day "of "the :contest "the "two\
"started "together.\
```

Teaching Turtle English


```
"The :winner "never "stopped "for "a "moment,\
"but "went "on "with "a :adjective1 "but :adjective2 "pace.\
"The :loser "laid "down "and "fell "asleep.\
"When "the :loser "woke "up,\
"he "went "as :opposite1 "as "he "could, "but "the\
:winner "had "already "won "the :contest "\
"and "was "comfortably "dozing.\
"\
"The "moral "of "the "story "is\
:adjective1 "and :adjective2 "wins "the :contest ".\)
END
```

madlibs

- One day, a Hare made fun of the short feet and slow pace of a Tortoise .
- The Tortoise replied: "Even though I have short feet , I will beat you in a race ."
- The Hare thought that this was impossible.
- So he challenged the Tortoise to a race .
- On the day of the race the two started together.
- The Tortoise never stopped for a moment, but went on with a slow but steady pace.
- The Hare laid down and fell asleep.
- When the Hare woke up, he went as fast as he could, but the Tortoise had already won the race and was comfortably dozing.
- The moral of the story is slow and steady wins the race .

Teaching Turtle To Talk

- TO PICKWORD
- OUTPUT PICK [
- A THE
- MAN DOG BANANA BOOK
- ATE BIT TOOK KICKED
-]
- END

- TO TALK
- PRINT (LIST PICKWORD PICKWORD PICKWORD PICKWORD PICKWORD)
- END

REPEAT 10 [TALK]

- BIT TOOK BIT MAN THE
- REPEAT 10 [TALK]
- BOOK THE MAN DOG ATE
- BANANA TOOK THE ATE A
- BIT BOOK BOOK BIT A
- BANANA BIT MAN A DOG
- BIT THE THE DOG THE
- MAN MAN THE BANANA BIT
- BOOK THE A BOOK DOG
- TOOK KICKED BANANA MAN DOG
- BOOK DOG MAN KICKED THE
- DOG MAN ATE BIT DOG

Parts of Speech

Part of Speech	Examples
Article	A THE
Noun	MAN DOG BOOK BANANA
Verb	ATE BIT TOOK KICKED

- THE BANANA KICKED THE BANANA
- THE BANANA TOOK THE MAN
- THE MAN KICKED A MAN
- THE BANANA BIT THE BOOK
- A DOG TOOK THE MAN
- A BOOK TOOK THE DOG
- THE DOG ATE A DOG
- THE MAN KICKED A MAN
- THE MAN BIT A DOG
- A BOOK BIT A BANANA

```
TO ARTICLE
  OUTPUT PICK [ A THE ]
END

TO NOUN
  OUTPUT PICK [ MAN DOG BANANA BOOK
DAVID ]
END

TO VERB
  OUTPUT PICK [ ATE BIT TOOK KICKED]
END

TO TALK
  PRINT (LIST ARTICLE NOUN VERB
ARTICLE NOUN)
END
```

```
THE DOG TOOK A BOOK
THE BANANA KICKED A BOOK
A MAN BIT A BOOK
THE MAN BIT THE BANANA
A BANANA ATE THE DOG
THE DOG TOOK A BANANA
A MAN ATE THE DOG
THE BOOK KICKED A BANANA
THE DOG BIT A BOOK
```

Writing Rule

TALK -> NOUN_PHRASE VERB NOUN_PHRASE
NOUN_PHRASE -> ARTICLE NOUN
NOUN_PHRASE -> PROPER_NOUN
VERB -> ate
VERB -> bit
VERB -> took
VERB -> kicked
NOUN -> man
NOUN -> dog
NOUN -> bananna
NOUN -> book
PROPER_NOUN -> david
ARTICLE -> a
ARTICLE -> the

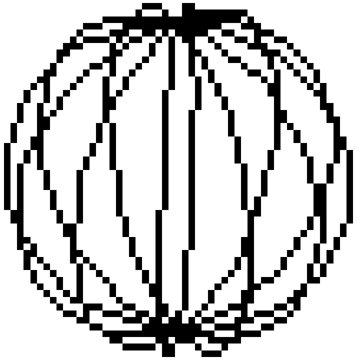
Learning Language


```
TO ARTICLE
 OUTPUT PICK [ A THE ]
END
TO NOUN
  OUTPUT PICK [ MAN DOG BANANNA BOOK ]
END
TO NOUN_PHRASE
  OUTPUT RUN PICK [
    [ (LIST ARTICLE NOUN) ]
    [ (LIST PROPER_NOUN)  ]  ]
END
TO PROPER_NOUN
  OUTPUT PICK [ DAVID JIM ]
END
TO VERB
  OUTPUT PICK [ ATE BIT TOOK KICKED ]
END
TO TALK
  PRINT (LIST NOUN_PHRASE VERB
NOUN_PHRASE)
END
```

- [THE DOG] BIT [A MAN]
- [THE MAN] KICKED [THE BANANNA]
- [JIM] KICKED [A BANANNA]
- [JIM] KICKED [THE MAN]
- [THE DOG] TOOK [JIM]
- [DAVID] ATE [THE BOOK]
- [DAVID] ATE [THE BOOK]
- [A MAN] KICKED [THE MAN]
- [JIM] ATE [DAVID]
- [JIM] TOOK [THE BANANNA]

Moving to 3d World


```
TO SPHERE
PERSPECTIVE
REPEAT 12 [circle 25 rr 30]
END
```



To rotate the sphere:

```
FOR [p 0 3600000 1][CS RR :P SPHERE]
```

Numerical Integration - Requirement

In two cases Numerical Integration is required:

1. When analytical solution is a problem or not possible
2. When data is generated experimentally

Numerical Integration – What for

Main objective of the numerical integration:

1. Finding area below the curve
2. Solving Differential Equations

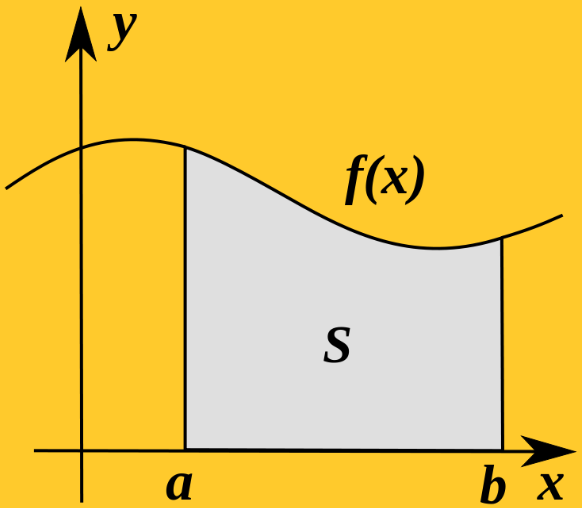
Numerical Integration – Methods

There are different method available for numerical integration:

1. Left end approximation
2. Right end approximation
3. Midpoint Rule
4. Trapezoidal Rule
5. Simpson's 1/3 Rule

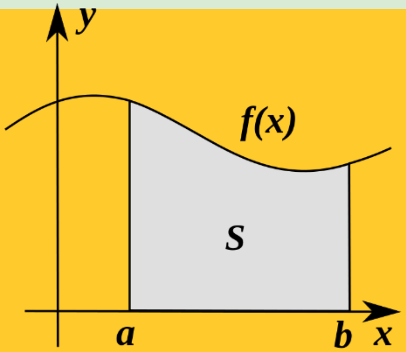
Numerical Integration – General Approach

- In numerical integration, we are interested to find the area below the curve enclosed by the interval a<=x<=b and x axis.
- In a regular rectangle, we can calculate the area as Area=Height x Breadth



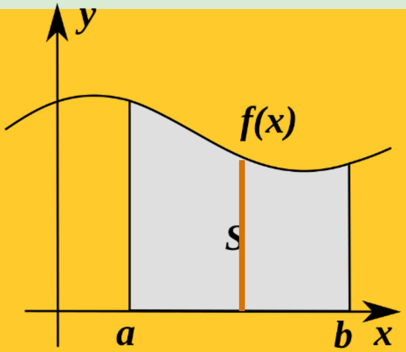
Numerical Integration – General Approach

- In a regular rectangle, we can calculate the area as Area=Height x Breadth
- As the given figure is not a rectangle, we can not use the above formula directly.
- We will play a trick. We will divide the area vertically and get large number of small areas.
- Then calculate these small areas considering them as rectangle.
- In this case, we may not get exact area but we can get approximate area very close to the exact area.



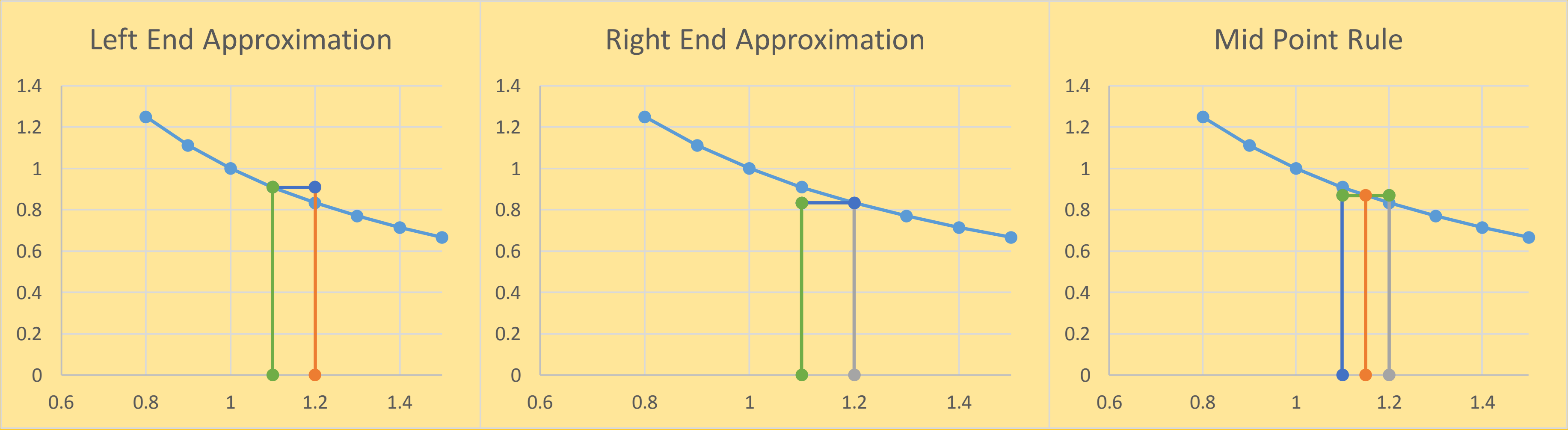
Numerical Integration
Issues in General Approach

- In a regular rectangle, we can calculate the area as Area=Height x Breadth
- Suppose, we divide the area in 'n' intervals.
- Then we have to calculate 'n' small areas but when we divide the main area into 'n' small areas, we get 'n+1' heights or 'n+1' data points.
- Now we have three choices for Height – Left, Right and Middle



Numerical Integration
Issues in General Approach

- Now we have three choices for Height – Left, Right and Middle
- Depending upon the value of left height, right height and middle height, calculate areas will differ



Numerical Integration
Left End Approximation

- Find the Integral of the function y= 1/x from interval a=1 to b=2.

Step-1: Suppose we will divide the area into n parts where n=7
Step-2: Calculate the breadth of each part, h=(b-a)/n.
Hence, h=(2-1)/7=0.1428

Step-3: Calculate Left x, Height =Left y, Breadth=h, Area=y*x, Cumulative Area = cum sum of Area as shown in next slide

Numerical Integration
Left End Approximation

- Find the Integral of the function y= 1/x from interval a=1 to b=2.

Sl No	x	Left End (y)	h	y*h	SUM(y*h)
1	1	1	0.1429	0.1429	0.14286
2	1.1429	0.875	0.1429	0.125	0.26786
3	1.2857	0.777778	0.1429	0.1111	0.37897
4	1.4286	0.7	0.1429	0.1	0.47897
5	1.5714	0.636364	0.1429	0.0909	0.56988
6	1.7143	0.583333	0.1429	0.0833	0.65321
7	1.8571	0.538462	0.1429	0.0769	0.73013
8	2	0.5	0.1429	0.0714	

Calculated Area= 0.73013, Actual Area (Calculated Analytically) = 0.6931471805599

Numerical Integration
Left End Approximation

- Formula for Left Hand Approximation:
  - h=(b-a)/n
- I=$\sum_{i=1}^{n} f(x(i-1)$ *h

Numerical Integration
Right End Approximation

- Find the Integral of the function y= 1/x from interval a=1 to b=2.

Sl	x	Right y	h	y*h	cum(y*h)
1	1	1	0	0	0
2	1.1429	0.875	0.1429	0.125	0.125
3	1.2857	0.7778	0.1429	0.1111	0.236111
4	1.4286	0.7	0.1429	0.1	0.336111
5	1.5714	0.6364	0.1429	0.0909	0.42702
6	1.7143	0.5833	0.1429	0.0833	0.510354
7	1.8571	0.5385	0.1429	0.0769	0.587277
8	2	0.5	0.1429	0.0714	0.658705

Calculated Area= 0.658705, Actual Area (Calculated Analytically) = 0.6931471805599

Numerical Integration
Right End Approximation

- Formula for Right End Approximation:
- h=(b-a)/n
- $I=\sum_{i=1}^{n} f(xi) * h$

Numerical Integration
Mid Point Rule

- Find the Integral of the function y= 1/x from interval a=1 to b=2.

SL	x	mid x	y	h	y*h	cum(y*h)
1	1	0	0	0	0	0
2	1.1429	1.0714	0.9333	0.1429	0.1333	0.133333333
3	1.2857	1.2143	0.8235	0.1429	0.1176	0.250980392
4	1.4286	1.3571	0.7368	0.1429	0.1053	0.35624355
5	1.5714	1.5	0.6667	0.1429	0.0952	0.451481645
6	1.7143	1.6429	0.6087	0.1429	0.087	0.538438167
7	1.8571	1.7857	0.56	0.1429	0.08	0.618438167
8	2	1.9286	0.5185	0.1429	0.0741	0.692512241

Calculated Area= 0.692512, Actual Area (Calculated Analytically) = 0.6931471805599

Numerical Integration
Mid Point Approximation

- Formula for Mid Point Approximation:

$$h=(b-a)/n\ , \qquad \overline{xi} = \frac{(xi-1+x1)}{2},\ i=1 \text{ to } n$$

- $$I=\frac{h}{2}[f(\overline{x1})+f(\overline{x2})+..+f(\overline{x(n-2)})+f(\overline{x(n-1)})+f(\overline{xn})]$$

Numerical Integration
Trapezoidal Rule

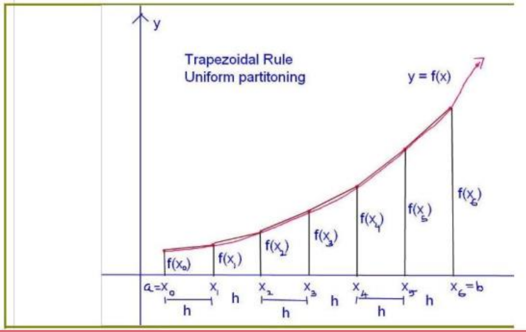
- Find the Integral of the function y= 1/x from interval a=1 to b=2.

Sl No	x	y	y_bar= (yi+yi+1)/2	h	y_bar*h	cum(y_bar*h)
1	1	1	0	0.1429	0	0
2	1.1429	0.875	0.9375	0.1429	0.13393	0.133928571
3	1.2857	0.777778	0.826389	0.1429	0.11806	0.251984127
4	1.4286	0.7	0.738889	0.1429	0.10556	0.357539683
5	1.5714	0.636364	0.668182	0.1429	0.09545	0.452994228
6	1.7143	0.583333	0.609848	0.1429	0.08712	0.54011544
7	1.8571	0.538462	0.560897	0.1429	0.08013	0.620243645
8	2	0.5	0.519231	0.1429	0.07418	0.694419469

Calculated Area= 0.692512, Actual Area (Calculated Analytically) = 0.6931471805599

Numerical Integration
Trapezoidal Rule

Basis: Area of a Trapezium = (a+b)/2*h, a and b are the length of parallel sides and h is the distance between them



- Formula for Trapezoidal Rule:

- $$I=\frac{h}{2}\left[\sum_{i=1}^{n} f(xi+1)+\sum_{i=1}^{n} f(xi)\right]$$

$$I=\frac{h}{2}\left[f(x0)+2*f(x1)+2*f(x2)+..+2*f(xn-1)+f(xn)\right]$$

Numerical Integration
Simpson's Rule

- Find the Integral of the function y= 1/x from interval a=1 to b=2.

Sl	x	y	coefficie	y=coeff/	h/3	h/3*y	cum(h/3*y)
1	1	1	1	1	0.0333	0.033333333	0.033333333
2	1.1	0.9091	4	3.6364	0.0333	0.121212121	0.154545455
3	1.2	0.8333	2	1.6667	0.0333	0.055555556	0.21010101
4	1.3	0.7692	4	3.0769	0.0333	0.102564103	0.312665113
5	1.4	0.7143	2	1.4286	0.0333	0.047619048	0.36028416
6	1.5	0.6667	4	2.6667	0.0333	0.088888889	0.449173049
7	1.6	0.625	2	1.25	0.0333	0.041666667	0.490839716
8	1.7	0.5882	4	2.3529	0.0333	0.078431373	0.569271088
9	1.8	0.5556	2	1.1111	0.0333	0.037037037	0.606308125
10	1.9	0.5263	4	2.1053	0.0333	0.070175439	0.676483564
11	2	0.5	1	0.5	0.0333	0.016666667	0.693150231

Calculated Area= 0.69315, Actual Area (Calculated Analytically) = 0.6931471805599

Numerical Integration
Simpson's Rule

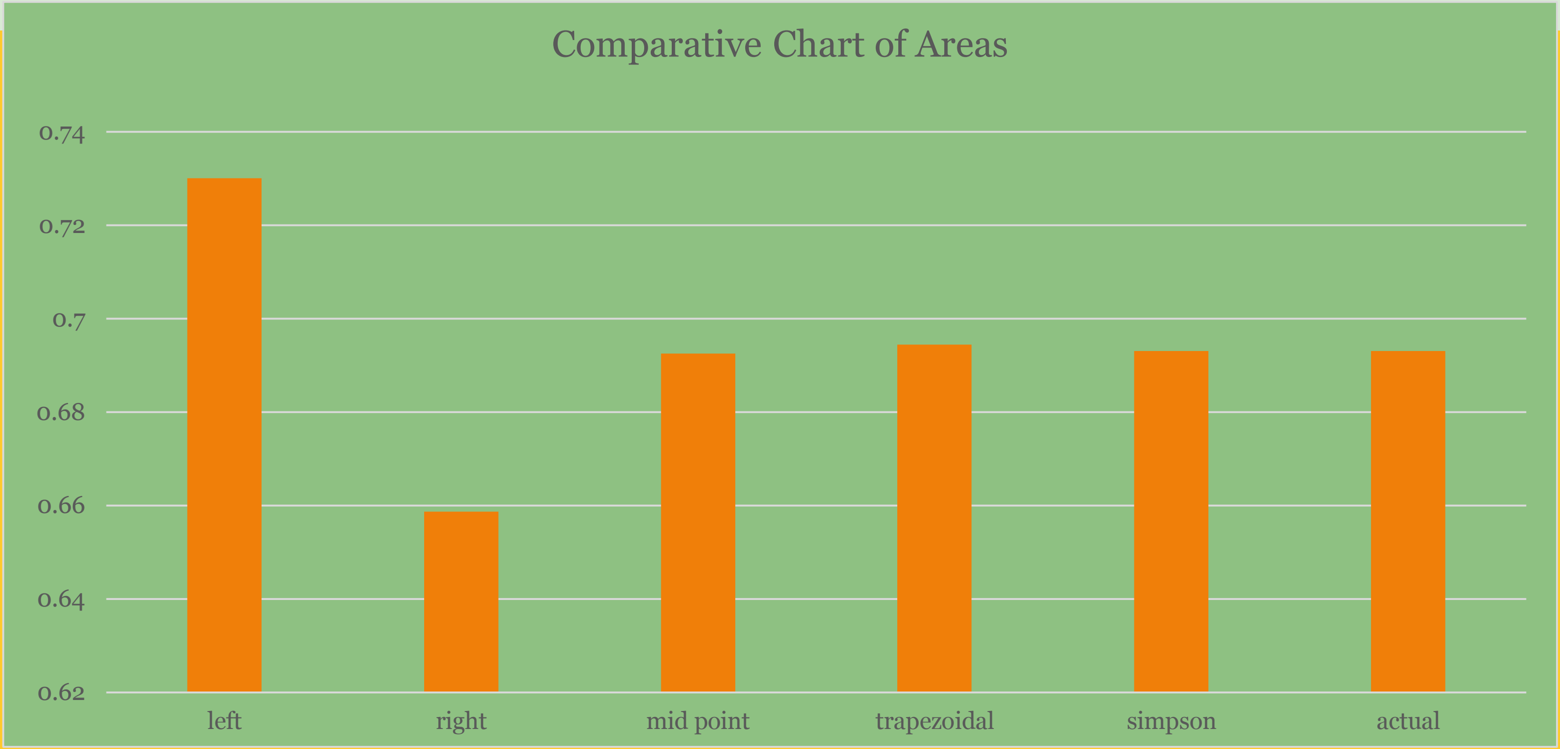
- Formula for Trapezoidal Rule:

$$I = \frac{h}{3}\left[f(x0) + 4 * f(x1) + 2 * f(x2) + .. + 4 * f(xn - 1) + f(xn)\right]$$

n=Even integer

Numerical Integration
Comparison of Areas

- Find the Integral of the function y= 1/x from interval a=1 to b=2.



Actual Area (Calculated Analytically) = 0.69314718055 99

Numerical Integration
Formula Trapezoidal Rule

- Find the Integral of the function y= 1/x from interval a=1 to b=2.
- Formula for calculating the Area. If n = 7, then points = 8

1	x0	a	y0	0	y0	h/2*y0
2	x1	a+h	y1	(y0+y1)/2	2 y1	h/2*2 y1
3	x2	a+2h	y2	(y1+y2)/2	2 y2	h/2*2 y2
4	x3	a+3h	y3	(y2+y3)/2	2 y3	h/2*2 y3
5	x4	a+4h	y4	(y3+y4)/2	2 y4	h/2*2 y4
6	x5	a+5h	y5	(y4+y5)/2	2 y5	h/2*2 y5
7	x6	a+6h	y6	(y5+y6)/2	2 y6	h/2*2 y6
8	x7	b	yn	(y6+y7)/2	y7	h/2*2 y7

Area = h/2(y0+2 y1 +2 y2 + 2 y3 +2 y4 + 2 y5 +2 y5 +2 y6 + y7)

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule:

- In rectangular form or trapezoidal form, we have used straight lines to approximate the curve.
- In Simpson's Rule, instead of straight line, we use parabolas to approximate the curve.
- The standard formula of a parabola is given by :

$$y = a\,x^2 + b\,x + c$$

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

- When the parabola passes through three points, where x0=-h, x1=0, x2=h, Then the Area can be calculated as:

$$Area = \int_{-h}^{h} (ax^2 + bx + c)\,dx$$

$$Area = \left[a\frac{x^3}{3} + b\frac{x^2}{2} + c\,x\right]_{-h}^{h}$$

$$Area = \left[a\frac{h^3}{3} + b\frac{h^2}{2} + c\,h\right] - \left[a\frac{-h^3}{3} + b\frac{-h^2}{2} + c\ -h\right]$$

$$Area = \left[2\,a\frac{h^3}{3} + +2c\,h\right]$$

$$Area = \frac{h}{3}[2\,ah\text{^}2 + +6c\ ]$$

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

$$Area = \frac{h}{3}[2\,ah\text{^}2 + 6c\,]$$

- It can be seen that the coefficient b has no role in calculating area.
- Since the parabola passes through three point of the curve, these points are (-h, yo), (o, y1) and (h, y2)
- We are interested to calculate the area under the parabola passing through these points.

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

- Since the parabola passes through (-h, y0), (0, y1) and (h, y2), we can put the equation of parabola passing through these points.
- We are interested to calculate the area under the parabola passing through these points.
- For this, first we have to calculate the values of the coefficients a, b and c for the desired parabola.

$$Area = \frac{h}{3}[2\ ah\text{^}2 + 6c\ ]$$

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

$$Area = \frac{h}{3}[2\;ah\hat{}2 + 6c\;]$$

- For this, first we have to calculate the values of a and c in terms of the given point:

- Equation-1: y0=a (-h)^2+b (-h) +c
- Equation-2: y1=a (0)^2+b 0 +c
- Equation-3: y2=a (h)^2+b h +c

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

- Equation-1: y0=a (-h)^2+b (-h) +c
- Equation-2: y1=a (0)^2+b 0 +c
- Equation-3: y2=a (h)^2+b h +c
- Equation-1a: y0=a h^2-b h +c
- Equation-2a: y1=c
- Equation-3a: y2=a (h)^2+b h +c

Now, y0+4 y1+y2 =a h^2-b h +c +4 c + a h^2+b h +c

Hence, y0+4 y1+y2 =2 a h^2+6 c

$$Now, Area = \frac{h}{3}[2\;ah\text{^}2 + 6c\;] = \frac{h}{3}[y0 + 4\;y1 + y2\;]$$

Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

$$Now, Area = \frac{h}{3}[2\,ah\hat{}2 + 6c\,] = \frac{h}{3}[y0 + 4\,y1 + y2\,]$$

The above formula has been calculated for the area passing through the point points (x0,y0),(x1,y1), (x2,y2).

Similarly we can calculate the area for the points [(x2,y2),(x3,y3), (x4,y4)], [(x4,y4),(x5,y5), (x6,y6)], [(x6,y6).(x7,y7)],[(x8,y8)],[(x8,y8),(x9,y9),(x10,y10)

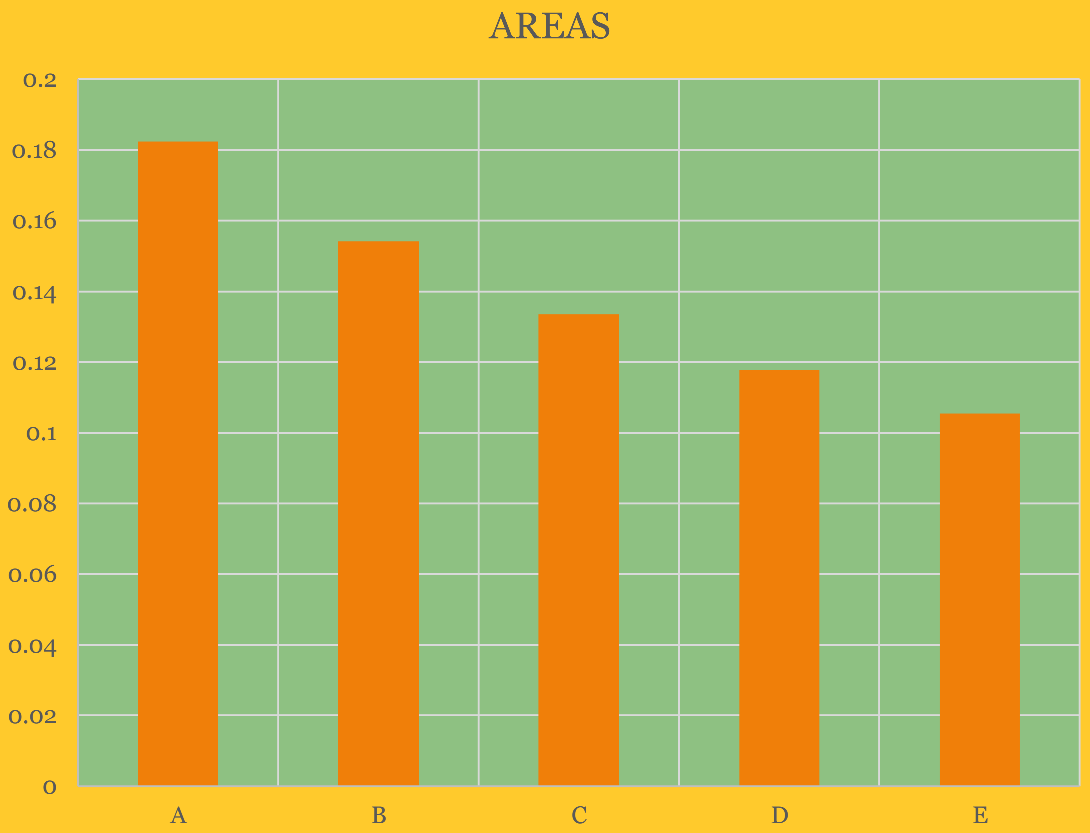
So, Area = $\frac{h}{3}[y0 + 4\,y1 + y2\,] + \frac{h}{3}[y2 + 4\,y3 + y4\,] + \frac{h}{3}[y4 + 4\,y5 + y6\,] + \frac{h}{3}[y6 +$

Numerical Integration
Simpson's Rule

x	y	coeff	y/3	area	area
1.1	0.9091	1	0.0333	0.0303	0.0303
1.2	0.8333	4	0.0333	0.1111	0.1414
1.3	0.7692	1	0.0333	0.0256	0.1671
				0.0513	

Numerical Integration
Simpson's Rule

A	0.1823
B	0.1542
C	0.1335
D	0.1178
E	0.1054
TOTAL	0.6932



Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

- *Now, we have got the formula of the area in terms of the selected point, as*

$$Area = \frac{h}{3}[y0 + 4\,y1 + 2\,y2 + 4\,y3 + 2\,y4 + 4\,y5 + 2\,y6 + 4\,y7 + 2\,y8 + y9 + y10]$$

- *Now How can we get the equation of the parabola that passes through these three points?*
- *We have to solve three linear equations with three unknowns:*

x	a	b	c	y
1.1	1.21	1.1	1	0.91
1.2	1.44	1.2	1	0.83
1.3	1.69	1.3	1	0.77

a	0.5828
b	-2.098
c	2.5117

Equation of the Parabola: y= 0.5828 x^2-2.098 x + 2.5117

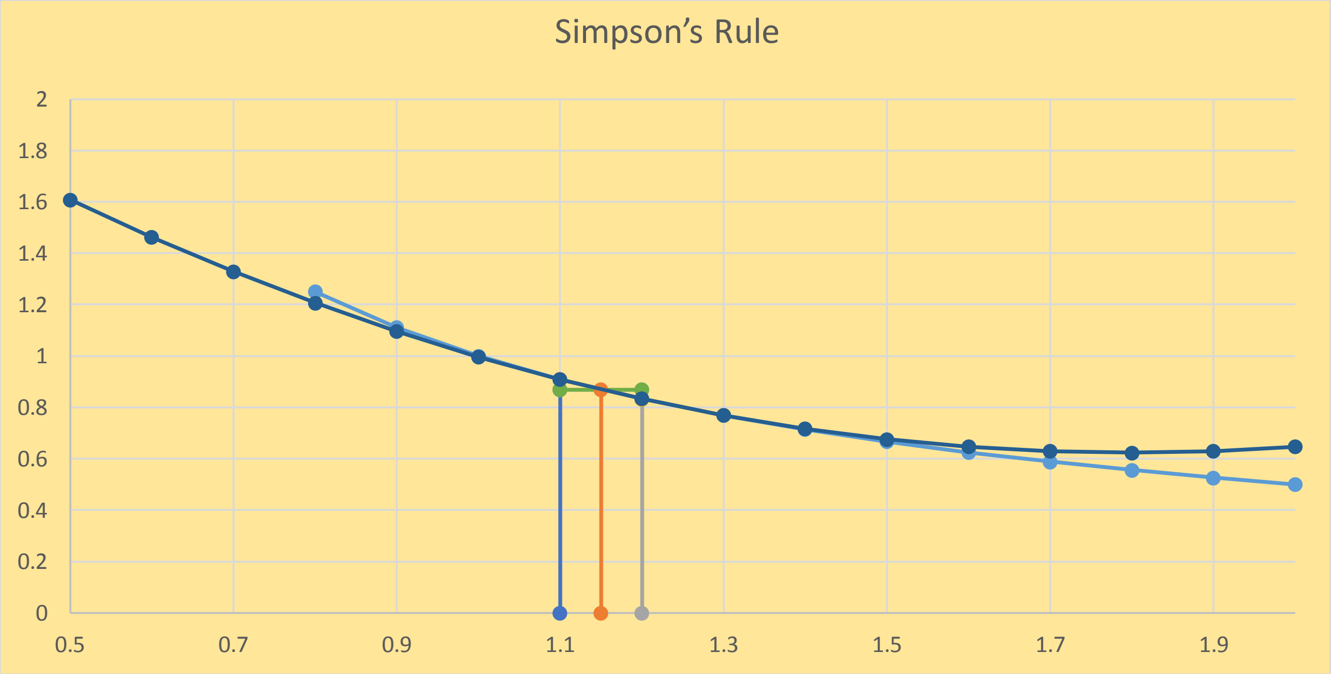
Numerical Integration : Simpson's Rule

Origin of Simpson's Rule: $y = a\,x^2 + b\,x + c$

x	a	b	c	y
1.1	1.21	1.1	1	0.91
1.2	1.44	1.2	1	0.83
1.3	1.69	1.3	1	0.77

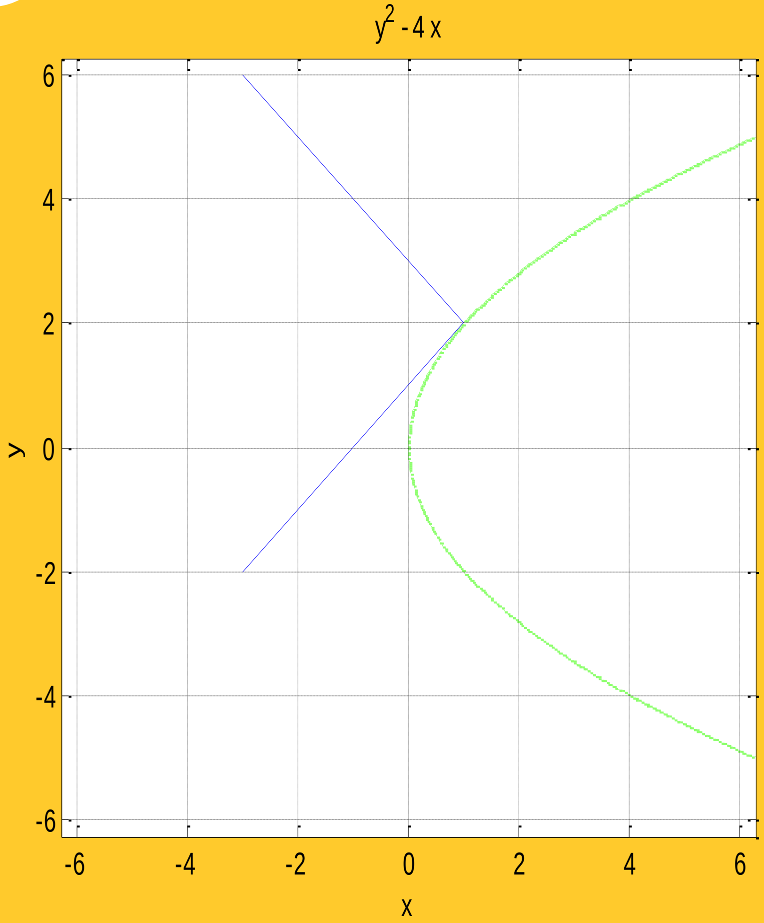
a	0.5828
b	-2.098
c	2.5117

Equation of the Parabola:
y= 0.5828 x^2-2.098 x + 2.5117
Given Function, y=1/x



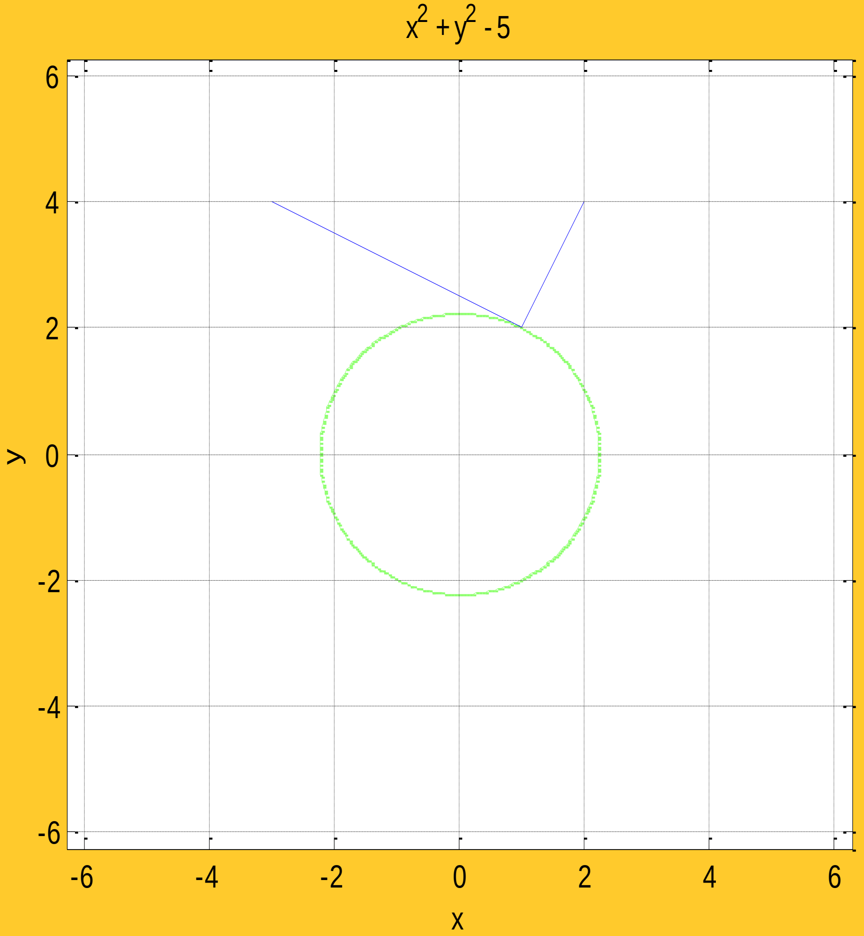
Direction Ratios

- syms f x y
- f=y^2-4*x
- dfdx=diff(f,x)
- dfdy=diff(f,y)
-
- ezplot(y^2-4*x)
- grid
-
- hold on
- plot([1,-3],[2,6])
- plot([1,-3],[2,-2])
- hold off



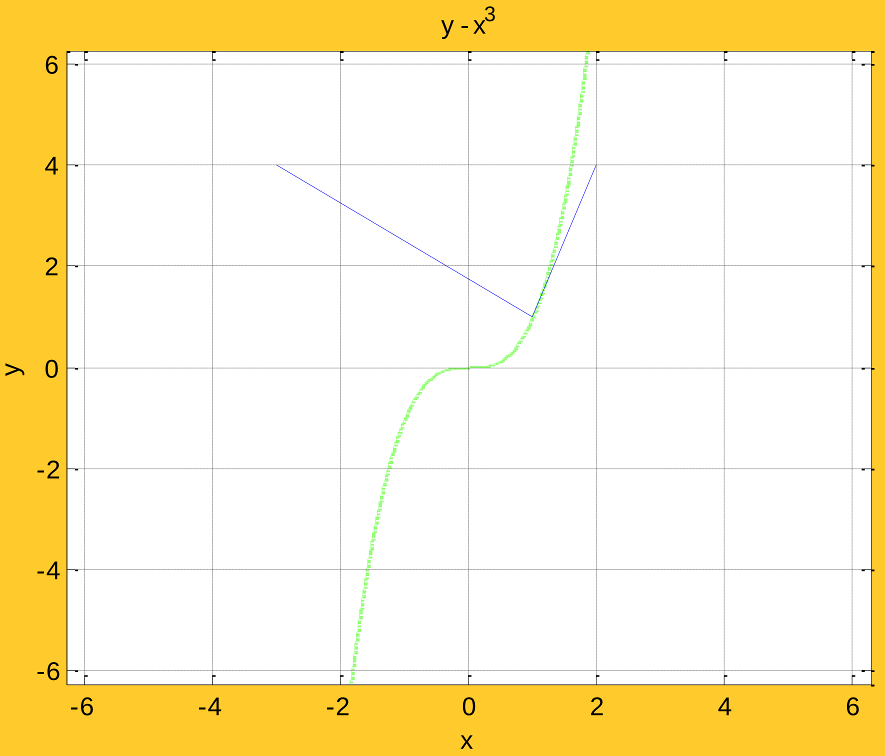
Direction Ratios

- f=x^2+y^2-5
- dfdx=diff(f,x)
- dfdy=diff(f,y)
- figure
- ezplot(x^2+y^2-5)
- grid
-
- hold on
- plot([1,2],[2,4])
- plot([1,-3],[2,4])



Direction Ratios

- f=y-x^3
- dfdx=diff(f,x)
- dfdy=diff(f,y)
- figure
- ezplot(y-x^3)
- grid
-
- hold on
- plot([1,2],[1,4])
- plot([1,-3],[1,4])



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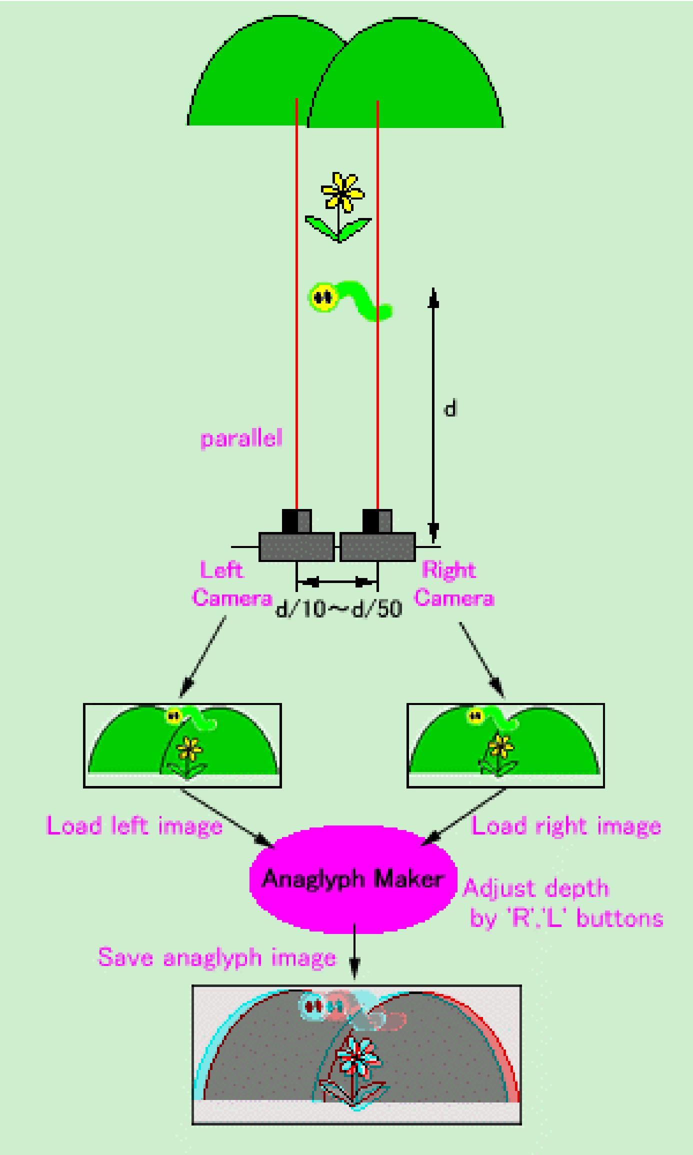
Anaglyphs



parallel
d
Left Camera
Right Camera
d/10~d/50
Load left image
Load right image
Anaglyph Maker
Adjust depth by 'R','L' buttons
Save anaglyph image

Anaglyph

- 3D photography or stereoscopic photography is the art of capturing and displaying two slightly offset photographs to create three dimensional images.

  The 3D effect works because of a principle called stereopsis. Each eye is in a different location, and as a result, it sees a slightly different image. The difference between these images is what lets us perceive depth. This effect can be replicated with photography by taking two pictures of the subject that are offset by the same distance as your pupils (about 2.5 inches or 63 mm). The two images are then viewed so that each eye sees only the corresponding picture. Your brain puts the two images together just as it does for normal vision and you perceive a single three dimensional image.

  This project will give you a brief introduction to the methods for taking and viewing 3D photographs.

Step 1: How to Take Stereoscopic 3D Pictures

- Taking stereoscopic pictures is simple. All you need is a camera and a tripod. Set up your camera and tripod on a level surface. Compose your shot with the main subject in the center and take a picture. Then slide the tripod 2.5 inches (about 63 mm) to either the right or the left. If necessary adjust the direction of your camera so that the subject is again in center of the shot. This should only be necessary for close up shots. Then take a second picture from the new position.

  This method works great for subjects that are still. But if you want to capture 3D images of moving objects, then you will need some additional hardware. If you have two cameras, then you can construct a simple two camera rig that mounts onto your tripod. In this kind of setup, the cameras are mounted 2.5 inches apart from center to center. To see a good example, check out this rig by user ciscu92. Then when taking the picture, you need to activate both cameras at the same time.

  If you don't have two cameras, you can construct a mirror splitter like this one by user courtervideo. This rig uses mirrors to split the image and space each part at the appropriate distance. This lets you capture both views with a single camera.

Step 2: Methods for Display and View 3D Images

- There are many different ways to display and view a stereoscopic 3D image. Here are some of the most common forms.

  <u>3D viewing systems with glasses</u>: These systems superimpose the right and left views on the screen. The observer wears glasses that filter the image so that each eye sees only the appropriate view.

  Color filtering glasses: The picture is displayed in two colors (one for each view). These glasses use a colored gels to selectively filter out the opposite color image. The most common colors used are Red/Cyan, Green/Magenta, and Blue/Yellow

  Polarized glasses: Polarized systems use two sets of polarized light filters. The picture is projected through one pair of polarized filters. The right and left view have opposite polarity. The viewer wears glasses with another pair of polarized filters. Each filter lets the image with matching polarity pass through but blocks the opposite polarity. This system has an advantage over colored filter systems in that it is able to display full color pictures. The disadvantage of this system is that it either requires two projectors (like you see in movie theaters) or your resolution is limited (such as in interleaved television displays).

  Active shutter 3D glasses: These systems switch the display between the right and left views every other frame. The glasses are wirelessly synced to the display and use LCD's in each lens to black out the appropriate eye at the appropriate time. This requires the displays to run at 48 frames per second instead of 24. These systems give a superior picture quality but cost substantially more than other systems.

Step 2: Methods for Display and View 3D Images

- There are many different ways to display and view a stereoscopic 3D image. Here are some of the most common forms.

<u>3D viewing systems without glasses</u>

Wiggle 3D: The picture is rapidly switched between the left and right views about every 0.10 seconds. This approximates a 3D effect without glasses. However, many people find it disorienting to view these images and the rate of frame switching makes it impractical for viewing moving images.

Mirror Split: This system uses one or two mirrors to virtually overlap the images. One of the views is often mirrored horizontally.

Parallel: The two views are displayed side by side. The easiest way to view these pictures is with a tool called a stereoscope. I will discuss this in more detail in later steps.

Cross-eyed: The two views are place side by side like with the parallel viewing system. However, in this system the right view is placed on the left side and the left view is placed on the right side. They are viewed by the observer crossing their eyes to look at the appropriate image. I will discuss this in more detail in later steps.

Step 3: How to View Cross-eyed 3D Images

- The simplest method of displaying and viewing 3D images is the cross-eyed method. This is the only method that doesn't require any additional viewing tools. To display these images, the two pictures are positioned side by side with the right view on the left side and the left view on the right side. Occasionally, a small dot is added above each picture to mark the center point.

  To view these images, place the pictures centered in front of you. Then gradually cross your eyes so that the pictures appear to overlap. Eventually you will see three images. Try to bring the center image into focus. When in focus, this center image will appear to be in 3D. This is techniques is also used to view many Magic Eye puzzles.

  Unfortunately many people find the cross-eyed viewing method uncomfortable to maintain for more than a few seconds. If you experience this problem, you may wish to use the parallel viewing method detailed in the next step.

Step 4: How to View Parallel 3D Images With a Stereoscope

- Parallel 3D images are typically viewed using a tool called a stereoscope. This device uses lenses to help the observer to focus one eye on each picture. There are many different styles of stereoscopes. You are probably most familiar with the View-Master that is produced by Fisher-Price. Older styles such as the Brewster stereoscope and the Holmes stereoscopes can still be found in many antique stores. The viewing cards (called stereographs) can also be found at some antique stores or you can make your own. Just print off a pair of stereoscopic pictures so that each image is about 2.5-3 inches in width (depending on the style of stereoscope).

  These viewers are quite simple to operate. You just place the picture card in the picture holder and look through the viewing lenses. Some models let you adjust the position of the picture to be more adaptable to different users.

Step 5: How to Make Your Own Simple Stereoscope

- To make a simple stereoscope, all you need is a pair of reading glasses and a small machine screw (at least 1/2 inch long). When choosing a pair of reading glasses, there are two traits that you want to look for. It needs to have a high magnifying power (preferably 3.0 to 3.5), and it needs to have temples (the bar on the side of your face) that are wide enough to fit a machine screw through them.

  Start by cutting the glasses in half at the middle of the bridge. Then use a file or grinder to round off the cut edges. Next, cut each temple about 1/2" past the hinge. Again round off the cut ends. Drill a hole in the centers of the remaining temple pieces that is just large enough to tightly fit the machine screw. Position the two eye pieces so that the temples are about 1/2 inch apart. Then screw the machine screw through one temple and into the second temple. Now you have a simple pocket sized stereoscope.

  To use your new stereoscope, hold it up to your face with the temples and bolt sticking out on the side that is nearest to your face. Position it so that the lenses are about two inches away from your eyes. Then hold the stereograph card about 12 inches away from your face. You will probably need to make adjustments to make it is easier to view based on your eyes and the lenses that you are working with. Play around with the spacing between your eyes, the lenses and the card. You can also adjust the spacing between the two lenses. I have found that the temples can be spaced anywhere from 1/4" apart to 1" apart and it still works. The spacing that you use will depend how what you find more comfortable.

Abstract Algebra

- Abstract algebra deals with structural features of numbers.
- Algebraic structures include groups, rings, fields, modules, vector spaces, lattices, and algebras.

- It started with symmetry: Symmetry is the transformations or operations that preserve the structure
- A square when rotated by 90 degree or reflected, it preserves its structure. This is true for many algebraic or arithmetic operations. In abstract algebra these type of objects or numbers which preserve their structure or values are called groups.
- The study of such objects or numbers lead to the development of group theory.

Group Theory

Gradient Divergence Curl

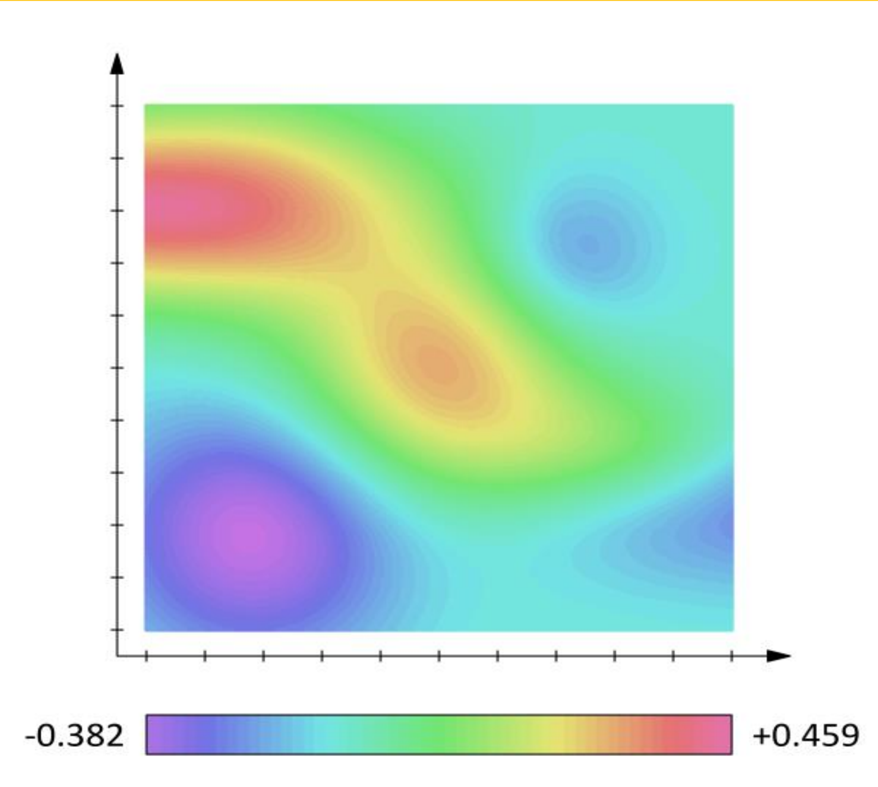
- Field
- Scalar Field
- Vector Field

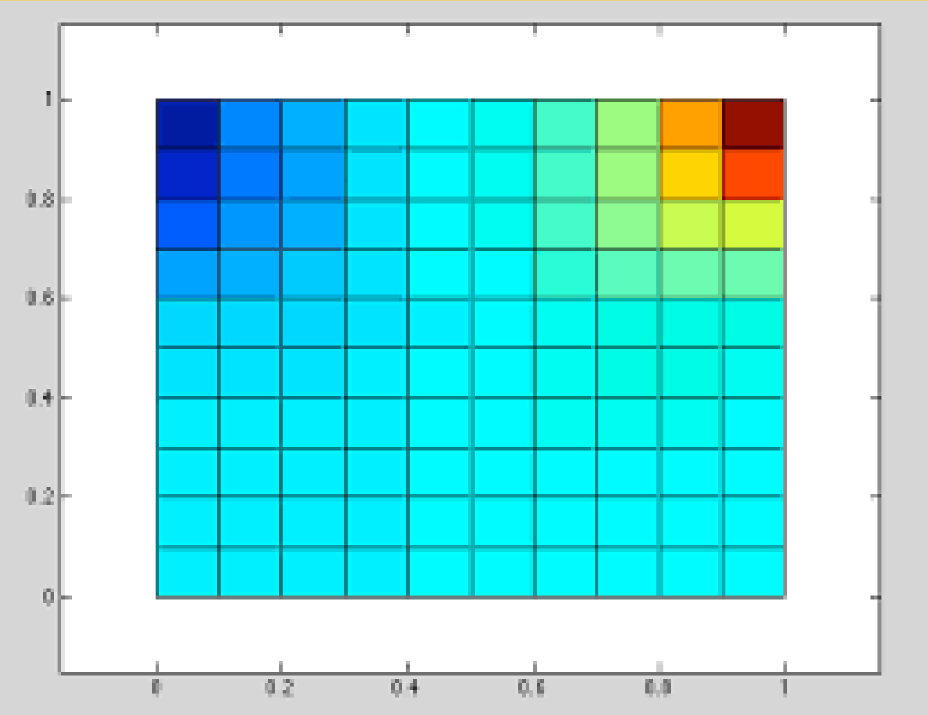
Field

Field

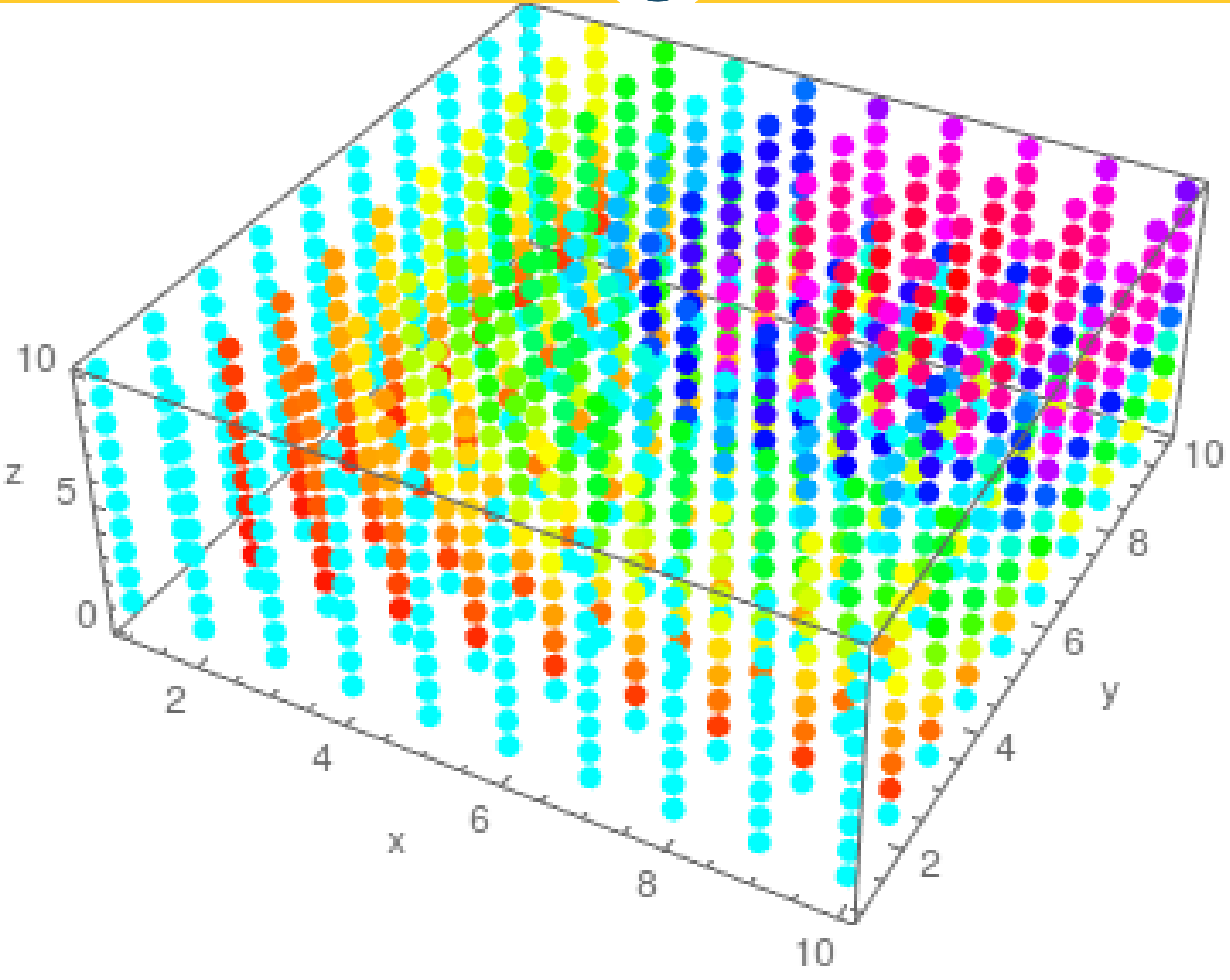
Field

Scalar Field 2d

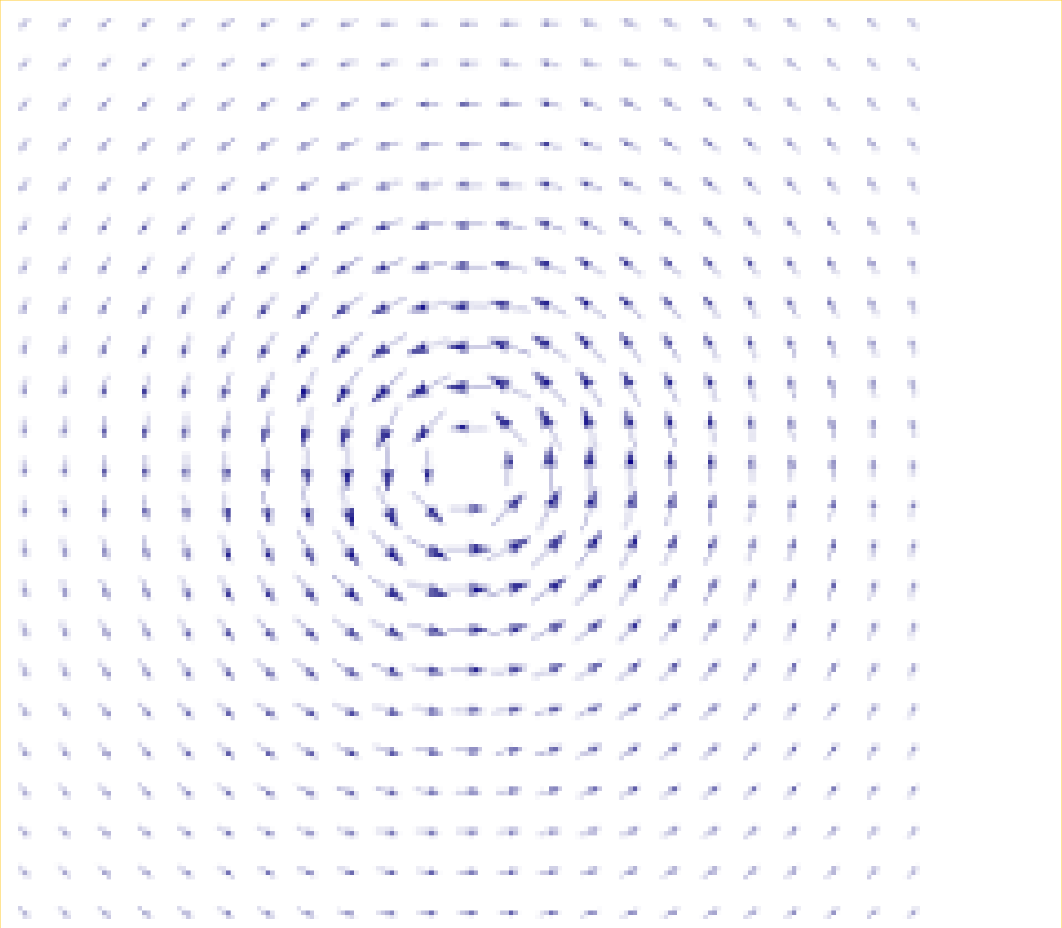


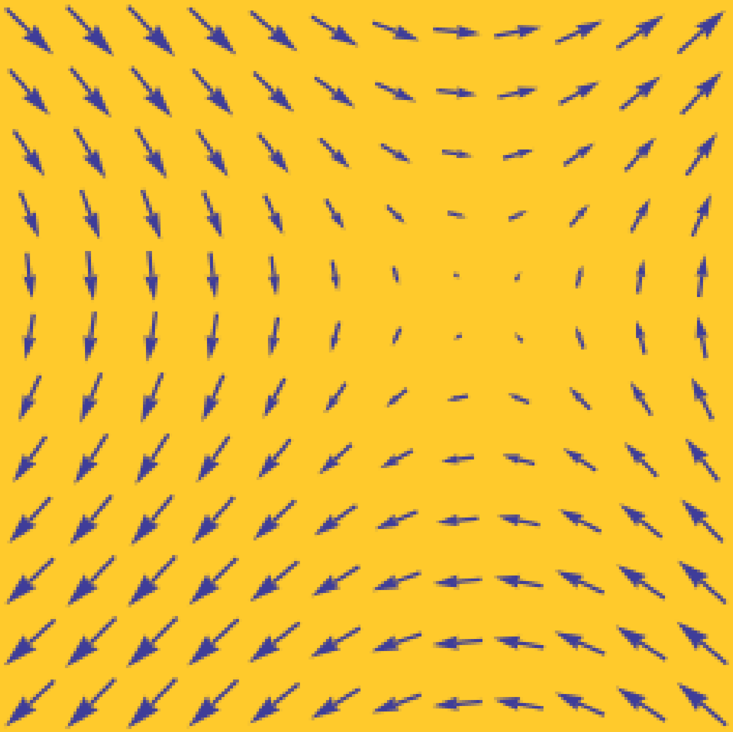


Scalar Field 3d



Vector Field 2d





Vector Field 3d

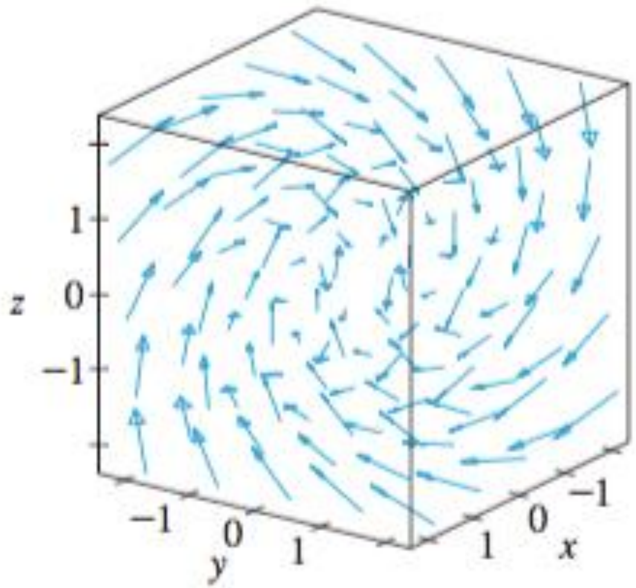
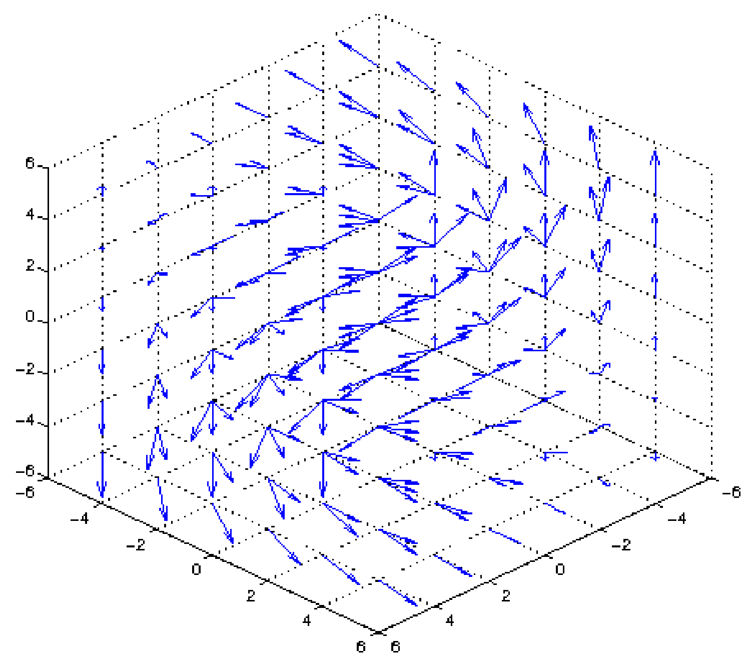
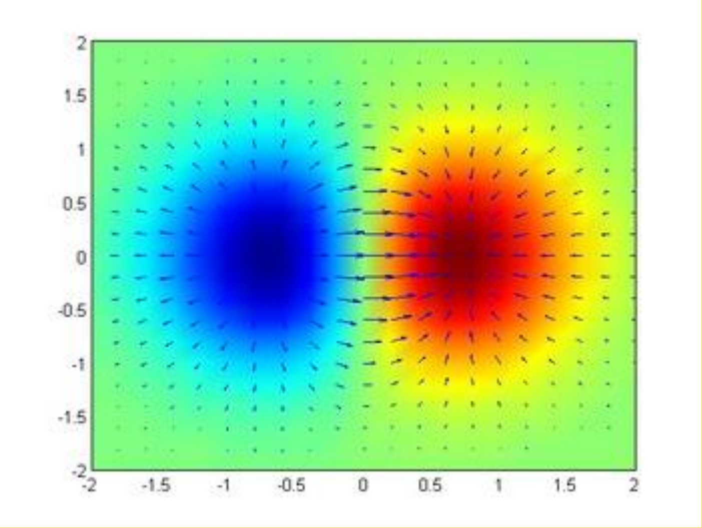


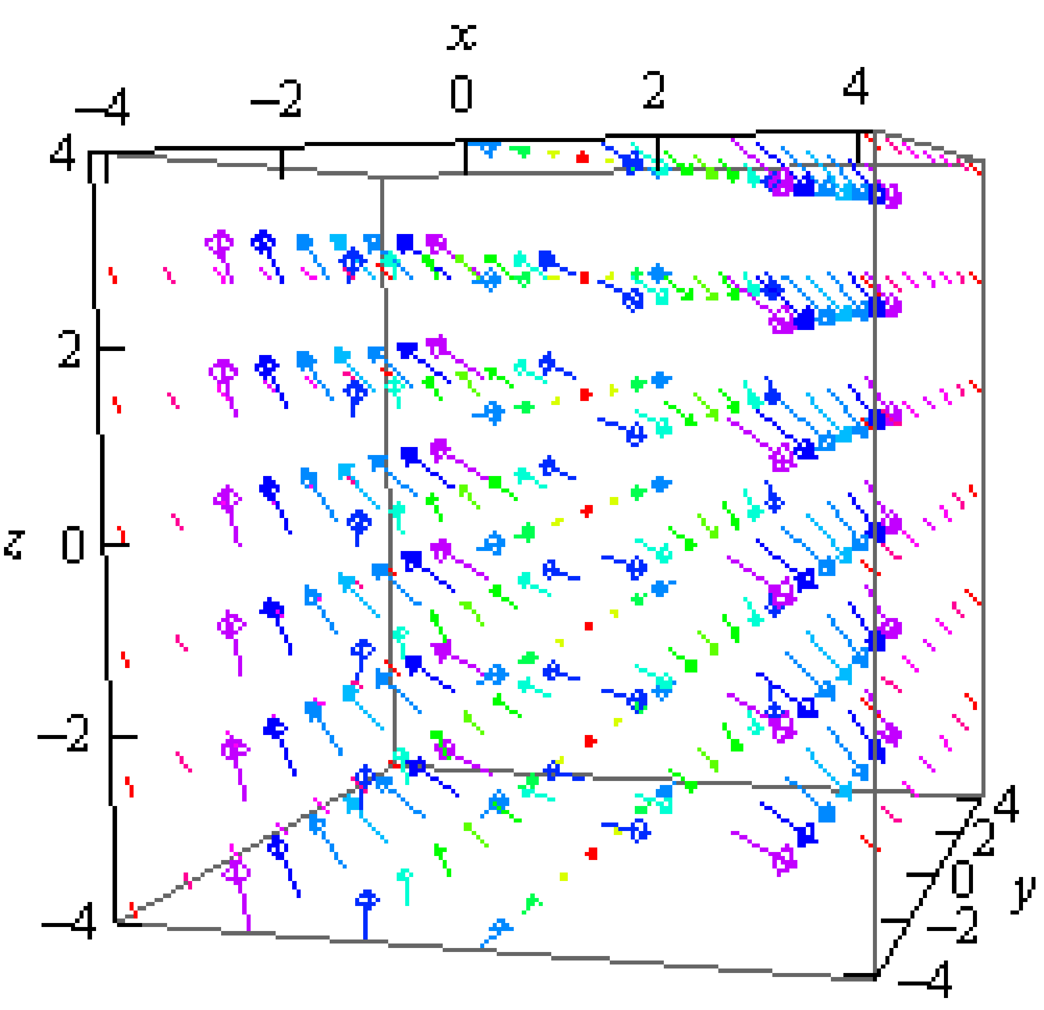
FIGURE 10
$\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} + x\,\mathbf{k}$



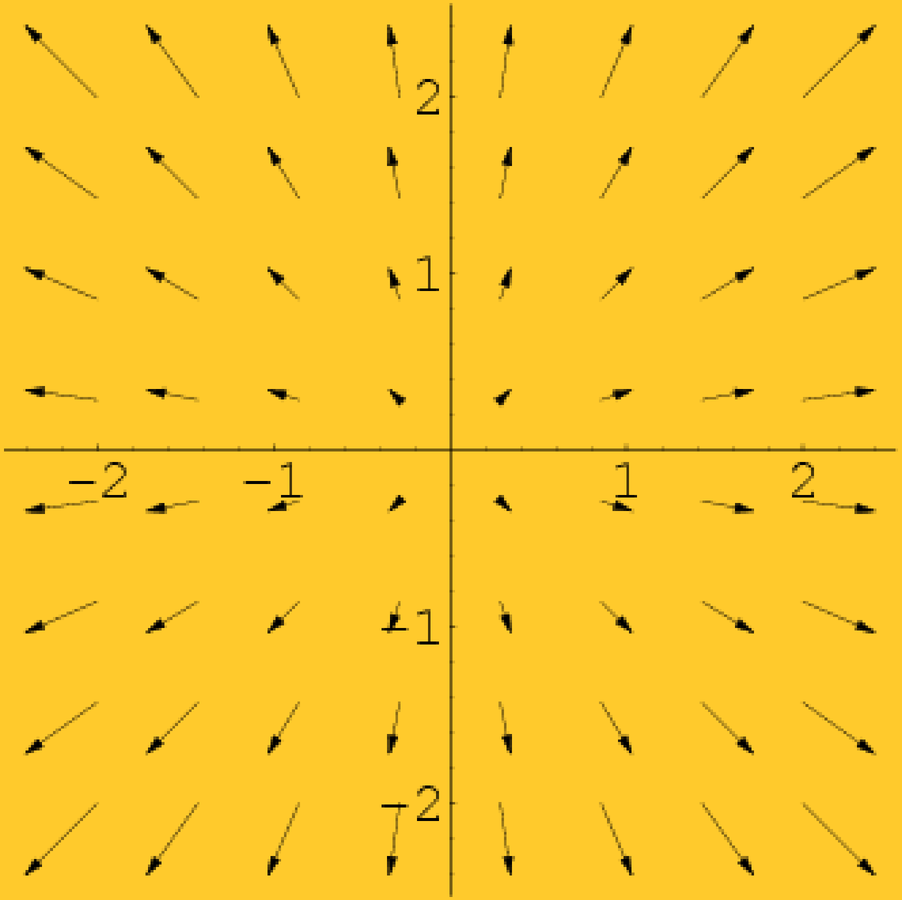
Gradient Field 2d



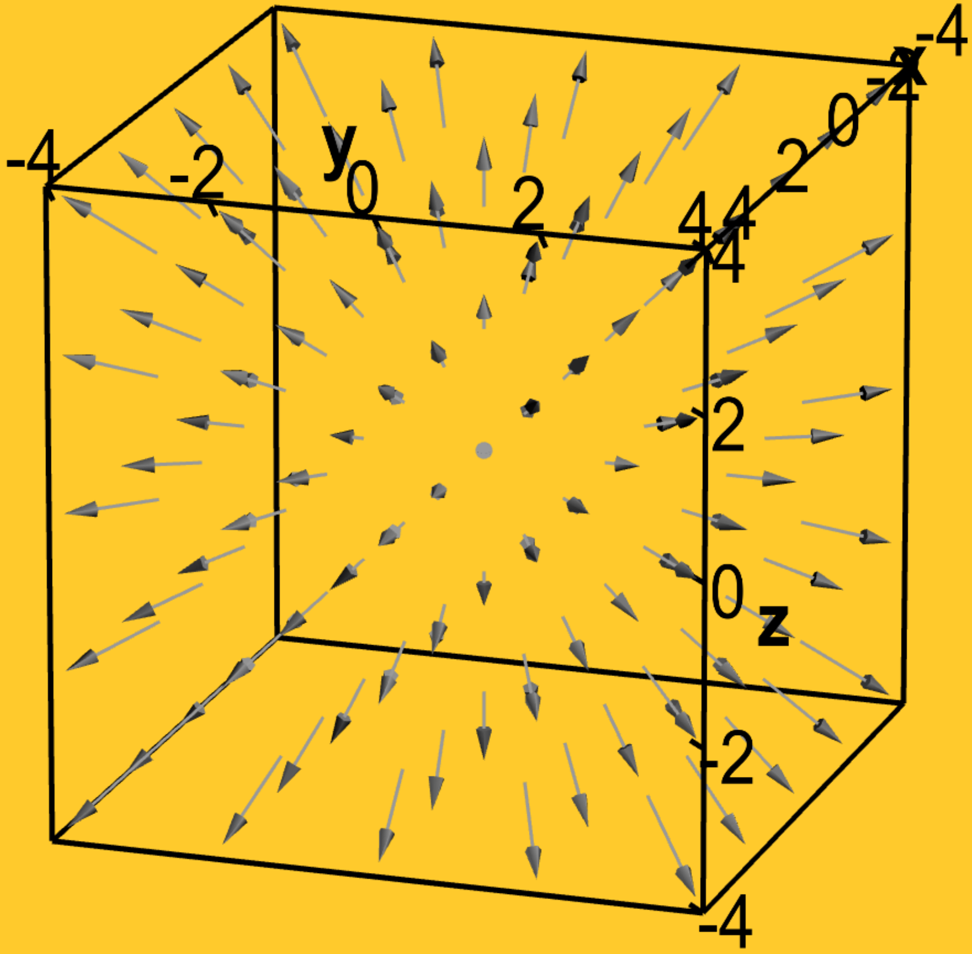
Gradient Field 3d



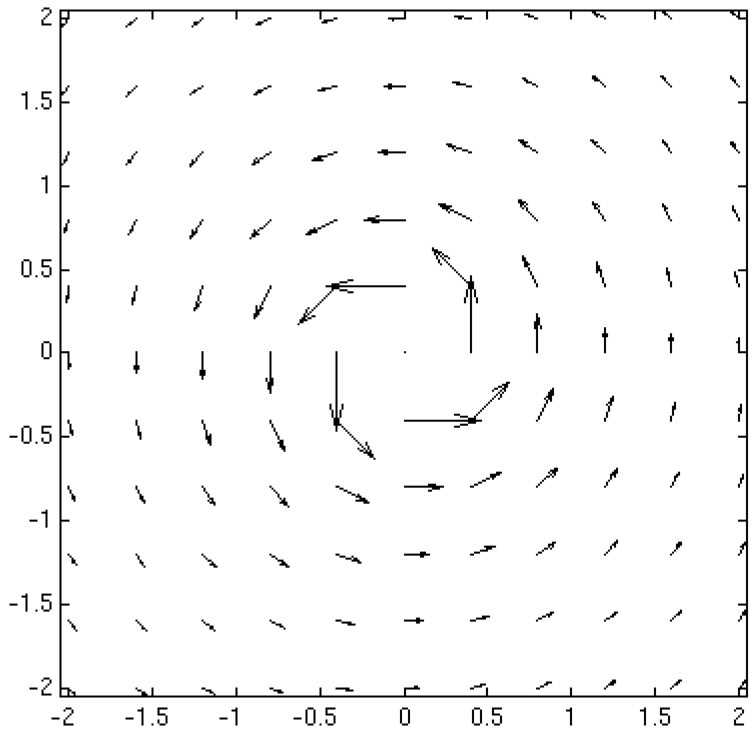
Divergence Field 2d



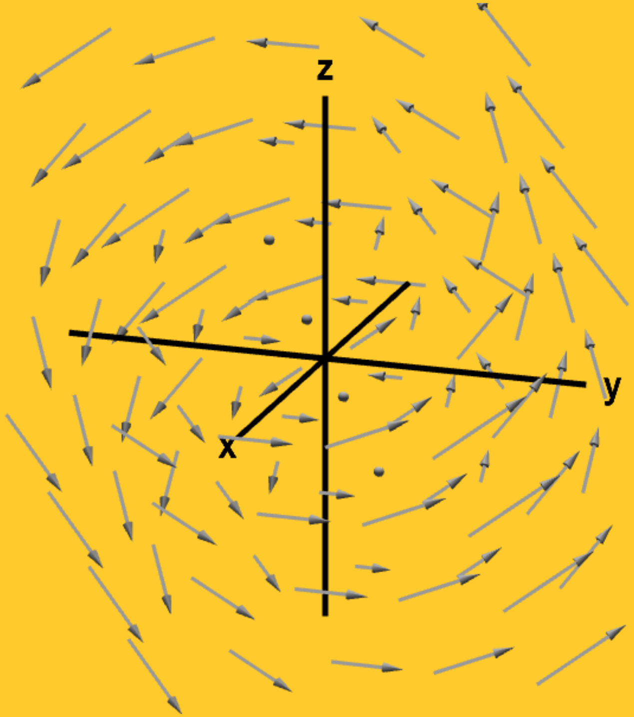
Divergence Field 3d



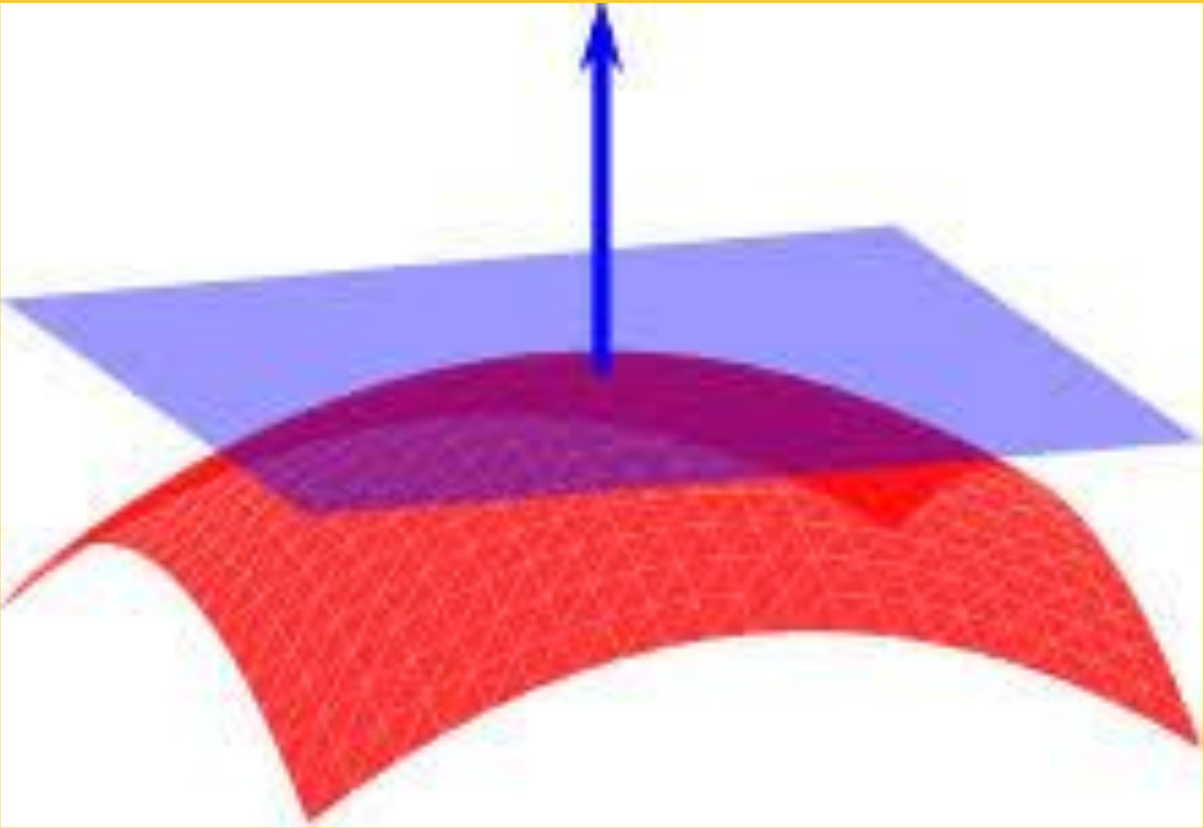
Curl Field 2d



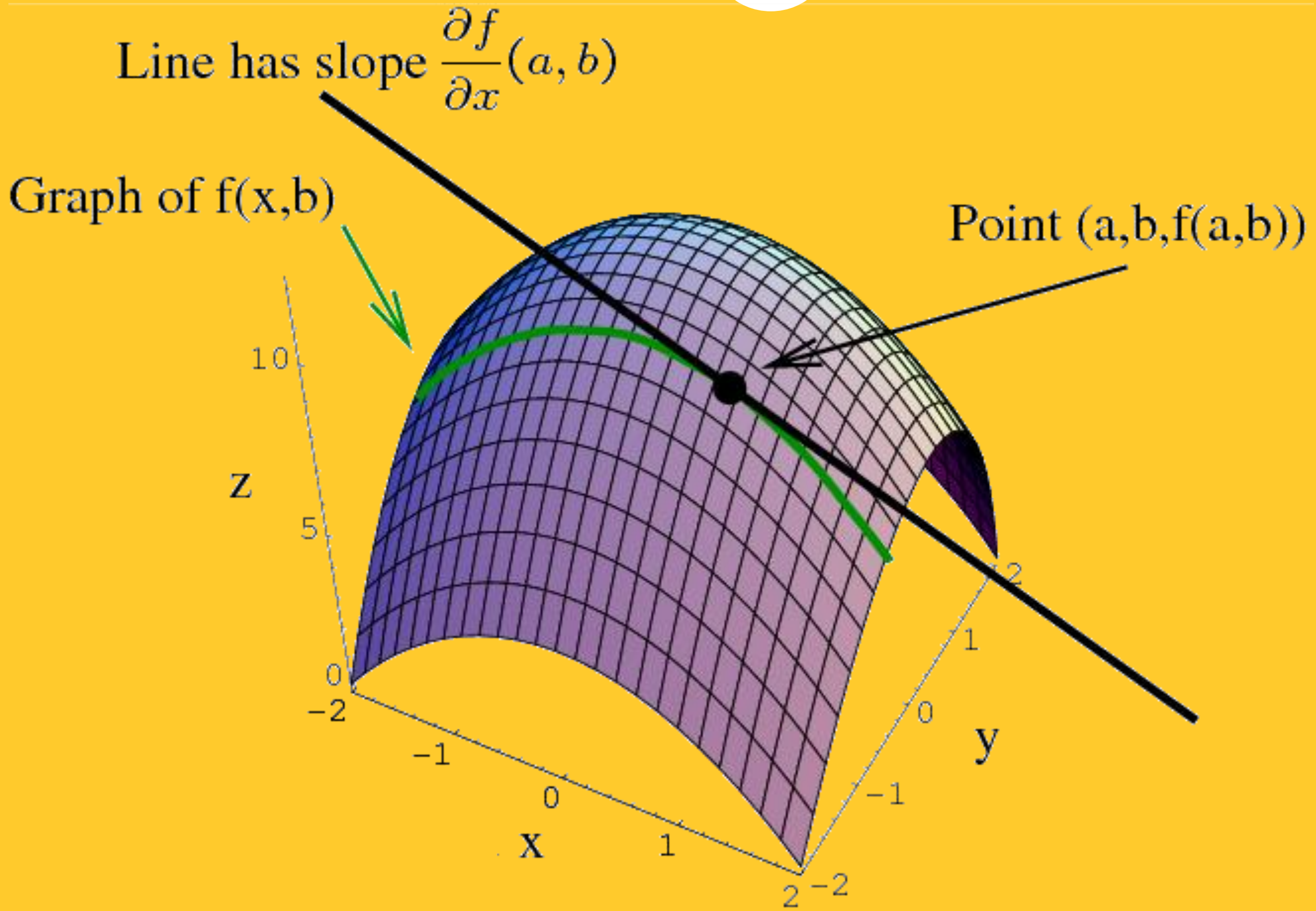
Curl Field 3d



Normal and Tangent Plane



Partial Derivative



Direction Derivative